MAKING MATHEMATICAL LITERACY A REALITY IN CLASSROOMS

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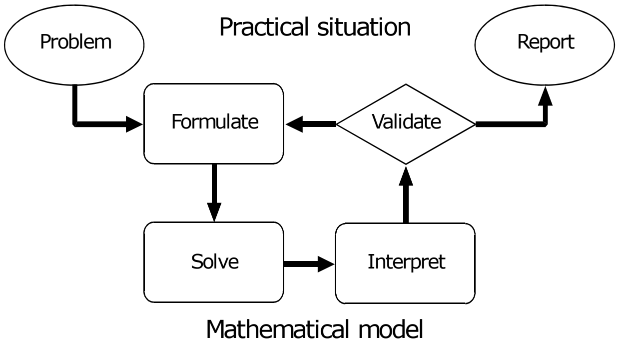
Abstract

Modelling of new problems is at the heart of mathematical literacy, because many situations that arise in adult life and work cannot be predicted, let alone taught at school. There are now plenty of examples of the successful teaching of modelling at all levels – yet it is to be found in few classrooms. How can every mathematics teacher be brought to teaching modelling reasonably effectively? This paper discusses how progress may be made, illustrating it with examples of „thinking with mathematics" about everyday life problems of concern to most citizens. It discusses the role that curriculum materials, professional development and various kinds of assessment may play, together with the challenges at system level. There are some reasons to be optimistic.

1. Background: the story so far

In a recent paper (Burkhardt, with Pollak 2006), we reviewed the history of the teaching of modelling in school mathematics curricula, focusing on developments in the UK and the US. The early explorations in the 1960s were followed by twenty years of more systematic development, so that by about 1990 there were proof-of-concept courses and course components of various kinds across the age range 10-21. These demonstrated that typical teachers can teach modelling skills if they have well-engineered teaching materials and some, relatively modest, professional development support. Students in these courses demonstrate a power over practical problems, from real-life or'fantasy' worlds, in which their mathematical toolkits play an important role in the analysis and reporting. They handle, for non-routine problems of appropriate complexity, the various phases of modelling shown in the diagram, and not only the *solve* phase on which school mathematics is normally focused.



Because these are *switch-on effects*, where students are showing kinds of performance that are new to school mathematics, evidence of progress does not require tightly structured research studies. Further, the social value of the skills involved is obvious, and rarely questioned[1]. The change in student motivation when working on real-life problems is equally dramatic.

The importance of these clear qualitative gains have kept the focus of work so far on development rather than insight-focussed research in depth, on the *engineering* rather than the *science* of the teaching and learning of modelling. There have been a few studies in greater depth with some interesting results, such as Vern Treilibs detailed study of formulation processes[2] (Treilibs et al. 1980). Among other things, it documented the „few year gap" between the mathematics students can do in imitative exercises and those that they choose and use when modelling (some recent studies suggest that this gap is narrowed by teaching modelling). These examples underline the need and opportunities for research to provide further insights into the processes of modelling, how students learn the skills involved, and how teachers can help them. I hope that some studies will focus on design research that can help the field move practice forward, rather than simply academic studies (see Burkhardt 2006).

In summary, we know how to teach modelling, have shown how to develop the support necessary to enable typical teachers to handle it, and it is happenng in many classrooms around the world. The bad news? „Many" is compared with one; the *proportion* of classrooms where modelling happens is close to zero.

Why is this, and how can the situation be transformed so that modelling is a feature of the mathematics curriculum for every student – the prerequisite for mathematical literacy? I shall first look at what we mean by mathematical literacy, its importance as a life skill, and its role in making mathematics itself meaningful and useful to most people. Then I shall list *barriers* that obstruct the large-scale implementation of modelling and, indeed, other curriculum improvements, linking these to various *levers* that promise progress.

From a societal perspective, the school mathematics curriculum is worse than regrettable; it is scandalous. Currently *most people in their adult lives use none of the mathematics they are first taught after age 11*. Further, study after study has shown that school mathematics gives them none of the aesthetic satisfactions that people get from, say, music or literature. Modelling is the missing ingredient.

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[1] Though some pure mathematicians argue that modelling should be deferred until more mathematics has been learnt – indeed, to a stage that most students never reach.

[2] 120 students age 17 of high ability in mathematics, but untutored in modelling, were tested on real world problems. Despite 5 years successful experience in algebra, none used it for modelling; though it seemed the obvious tool, they chose to rely on more elementary methods: numbers, tables and, sometimes, graphs.

2. What is mathematical literacy?

Many different terms are used in various places and circumstances: *mathematical literacy* (ML) is the most widespread, *quantitative literacy* is favoured in the US, *functional mathematics* is now fashionable in the UK, while *numeracy* was originally defined as „the mathematical equivalent of literacy“. Distinctions between these terms are not widely agreed; for our purposes, they are unimportant.

PISA (Programme for International Student Assessment, OECD 2003) defines ML:

> Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen.

However, such verbal descriptions on their own are ambiguous, particularly across countries and cultures – they are easy to re-interpret in terms of one’s own experience. It is useful to complement them with examples – in education, of the kinds of task that represent learning goals. The following illustrate what I (and many others) mean by mathematical literacy. I begin with a PISA task.

ROCK CONCERT *M552Q01*

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A. 2 000

B. 5 000

C. 20 000

D. 50 000

E. 100 000

While the length and multiple choice format are limiting, this kind of ‚back of the envelope’ estimation is central to ML. So are the following types of task.

MAKING A CASE

The spreadsheet contains 2 sets of reaction times – 100 each for Joe and Maria.

Using this data, construct and justify two arguments:

A: that Joe is quicker than Maria, **and**

B: that Maria is quicker than Joe

SUDDEN INFANT DEATHS

In the general population, about 1 baby in 8,000 dies in an unexplained "cot death". The cause or causes are at present unknown. Three babies in one family have died. The mother is on trial for murder.

An expert witness says:

"One cot death is a family tragedy; two is deeply suspicious; three is murder. The odds of even two deaths in one family are 64 million to 1"

Discuss the reasoning behind the expert witness' statement, noting any errors, and write an improved version to present to the jury.

PRIMARY TEACHERS

In a country with 60 million people, about how many primary school teachers will be needed? Try to estimate a sensible answer using your own everyday knowledge about the world. Write an explanation of your answer, stating any assumptions you make.

HOW RISKY IS LIFE?

"My parents won't let me go out on my own. They think I'll be mugged, or run over."

"My sixty year old granny is terrified by the stories she reads in the newspapers. One day she is afraid of being assaulted, the next she is frightened of terrorists."

What do you think? Collect and use data on different causes of death to estimate the chances of people becoming a victim of these and other events. Compare the likelihoods of these events with each other, with other risks, and with the 'base' risk – the probability that people of different ages will die in the next year.

It is clear from the above that mathematical literacy involves complex reasoning, linking models of the situation to data. Lynn Steen (2002) describes it as „The sophisticated use of elementary mathematics“, in contrast to school mathematics.

3. Barriers to large-scale improvement

Here I shall list some of the key implementation challenges we face. These are discussed in more detail in (Burkhardt with Pollak 2006) and Burkhardt (2006)

- **System inertia**: The limited large-scale implementation of modelling is not unique; it has proved difficult in many countries to establish *any* profound innovation in the mainstream mathematics curriculum. This should not surprise us. Teaching modelling requires changes in the well-grooved practices of teachers, their teaching skills, and their beliefs about the nature of mathematics – and those of parents and politicians. To become part of the mainstream curriculum, it is not enough to be "good" and "important".
- **The real world** is an unwelcome complication in many mathematics classrooms. The "purity" of the subject is something that attracts people to teach mathematics; for them, using mathematics to tackle real world problems is not their job. (First language teachers welcome the motivation it provides)

- **Limited professional development** In many countries teachers are expected to deliver a curriculum on the basis of the skills they acquired in their pre-service education, consolidated in early years in the classroom. In a changing world, continuing professional development is essential but in most countries is not yet an integral part of most teachers' week-by-week work.
- **The role and nature of research and development** in education, as compared with other applied fields, is not well organised for turning research into practice. Burkhardt and Schoenfeld (2003) looked at how this process can be improved, learning from research-based improvement in medicine, engineering and other fields. The growing role of ‚design research' in education is a move in this direction but more is needed if policy makers with problems are to turn to the research community to solve them.

The research and development agenda that these barriers imply is huge and work on it is at an early stage. Here I can only sketch some of the key ingredients that are likely to be important in establishing modelling. They are all worth working on.

4. The importance of communication

The story of modelling in school mathematics is one of mutual incomprehension between leaders in mathematical education and those they seek to serve. The public and most politicians see mathematics as "What I learned at school". The mathematical limitations of many students, which they regularly deplore, are seen as a failure to make every child mathematically ‚like them'. The changes in the mathematical skills that society needs are acknowledged, but their implications are not understood. This needs greatly improved communication[3].

Contributions to the media are the first area that needs attention.

- These need to explain and illustrate the changes. The mathematics curriculum is still focused on developing reliable technical skills in well-practised procedures; everywhere except in schools, these are now performed by technology. In this more technical world, where computers do the routine things that clerks used to do, people need a broader range of higher level skills so as to be flexible problem solvers who can handle change.
- This is not an easy communication challenge – people don't want to read about mathematics, so media are reluctant to publish such pieces. Skilled writers of ‚popular science' can provide help.
- Assessment tasks can be useful tools – they communicate new goals in a vivid and compact form, bringing to life verbal explanations; otherwise these are interpreted within each reader's experience.

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[3] It is a salutary exercise to try to trace the paths by which your (excellent) research might influence practice in typical classrooms, who is responsible for each step, and how likely it is to get through. What changes in research might make its influence more direct? (see Burkhardt and Schoenfeld 2003)

Meetings with policy makers, both politicians and their senior civil servants, are crucial to improving the communication process. In addition to the above kinds of input, they will respond well to:

- Suggestions that are aligned with their existing policies – look hard for elements of declared policy to which you can attach the initiative you want, and adapt your proposals to maximise the alignment;
- Evidence on the learning outcomes from curriculum components of a similar kind that have been tried elsewhere, from evaluations and/or independent research studies;
- More comprehensive and detailed descriptions of the proposed changes, preferably with examples of assessment tasks, lesson materials and the professional development needs and methods;
- Estimates of the likely costs of development, and of implementation – it pays to offer alternative models, with varying scales and pace of change, including some that start inexpensively;
- Evidence of some public support for the changes proposed – policy makers are pressured to provide support for many things; they are more likely to respond to ideas that have public support.

Mathematicians are a key group that may need particular attention. In the US in particular, a small well-organised group („Mathematically Correct") with conservative political support have led a highly effective opposition to reform. Most research mathematicians have little understanding of the complex dynamics of learning and teaching mathematics. Ignorant of the associative nature of learning, they tend to assume that the logical structure of mathematics provides the best learning sequence[4]. Further, pure mathematicians work in a field in which logical consistency is the sole criterion, so are often naive about empirical evidence, downplaying its decisive role. Perhaps most important of all, their unspoken priority is the education of students of high-ability like themselves. They emphasise particularly fluency and accuracy in manipulating algebra, the key language of specialist mathematics that only a few will ever use in their adult lives.

Thus the mathematical literacy of the many is sacrificed to the very-real specialist needs of the few who will work in engineering, science or economics. This important group can be catered for by additional options in specialist mathematics; the priority of the core mathematics curriculum should be high quality mathematical literacy.

The following can help to get support:

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[4] Except in the UK, this plausible assumption was the basis of many mathematician-driven „new math" movements in the 1960s. In one project that followed the Bourbaki reconception of pure mathematics, itself motivated by mathematics education issues, the curriculum began with a set-theory course for 5-year olds. The empirical evidence on these experiments was largely negative, but this is often forgotten.

- Local support from sensible university mathematicians who: are willing to spend some time learning about how school students learn; recognise that their limitations in this area; accept that mathematics education is an empirical field in which evidence is a better guide than pure reasoning.
- Formal and informal involvement of the representative societies of research mathematicians and scientists, negotiating with their leadership to ensure that those they nominate satisfy these same criteria. Formal approval by these societies of the processes by which reforms are developed and evaluated is an important asset.
- Professional associations of those involved in the teaching of mathematics, science and social studies form a key constituency. I mention them last because those who drive mathematics reform are often in close contact with some of these. However, for mathematical literacy the mathematics-focussed associations are not enough; those in science and social science can be powerful allies, or foes. Science teaching associations will be concerned that the mathematics needed for physics may suffer. Many social studies teachers will downplay the need for mathematics, often reflecting their own insecurity with it. Good ongoing communication, with reassuring evidence, is important.

5. The roles of assessment

In trying to reform curriculum, assessment is often an afterthought – important for evaluating progress and, perhaps, for holding schools to account but not a core part of planning and development. This attitude leads to a tragically missed opportunity. Why? There are two key reasons, one already noted:

- Assessment tasks provide a clear and vivid statement of the learning and performance goals of the change. Teachers, students, politicians and the public can understand them. In contrast, lesson materials are too bulky to be easy to comprehend – or for policy makers to read – while „standards“ alone, focussing on separate ingredients of mathematics, do not specify performance.
- In systems with strong ‚accountability' pressures on schools, most teachers „teach to the tests“. (WYTIWYG) Many people deplore this but the tests, whatever their limitations, are the main target that society sets for successful learning. *Thus the tests effectively define the implemented curriculum.* ‚Authority' is often reluctant to accept this, perhaps because it implies a responsibility for designing high-stakes assessment that reflects all the performance goals of the curriculum in a balanced way – this costs more.

However, viewed positively, this influence offers unmatched leverage for effecting changes in the implemented curriculum. Because everyone likes simple tests, this leverage usually impoverishes the curriculum, narrowing the range of classroom learning activities. Multiple-choice tests, dominant in the US, assess very short chains of reasoning, and favour elimination tactics focused on the wrong answers – performances that are only indirectly connected to curriculum goals. It is argued that these correlate with better measures but that is rarely proved by research and, crucially, ignores WYTIWYG. England mostly uses short items with constructed responses – better, but again with short chains of reasoning, totally different from those needed for modelling or most other *thinking with mathematics*.

What of systems that do not have high-stakes tests? They depend on teachers' professionalism, and are protected against the negative effects of WYTIWYG. However, it would be unfortunate if they lose the benefits that high-quality assessment has to offer. Professionals, generally speaking, are good at sustaining established practice; the introduction of improvements is more problematic, especially when these require new teaching skills. The modified professional practice that is needed to encourage greater student autonomy in non-imitative tasks needs explicit *support*. Assessment tasks are a key part of that support. Change also requires *pressure*. The anglophone countries tend to rely on pressure alone, with negative consequences; it remains to be seen if support alone can establish modelling or other curriculum reforms.

6. Tools for teachers

Here I shall be brief since this area is relatively familiar. People in all fields are much more effective when they have well-engineered tools. What are they here?

- **Classroom teaching materials** are part of the professional practice of most teachers, even in familiar areas. For new curriculum elements that need extended teaching styles, classroom materials are even more important – as is the design and development challenge they present.

  To develop such materials requires the ‚engineering research' approach that is used to develop effective tools in other fields, from consumer electronics to new medicines (Burkhardt 2006). What does this approach involve? Input from prior research and from other designs with similar goals. The design skills to turn these into draft materials that match the goals. Rich and detailed feedback from a sequence of trials that informs each revision, until the outcomes with users, representative of the target populations, match the goals. Gathering this feedback needs the methods of insight-focussed research. Finally, ongoing feedback ‚from the field' informs subsequent improvements.

  This methodology is more elaborate than the ‚author' model, more usual in education; however, it pays off – no-one would fly in an airplane or take a medicine that had been developed by the craft methods still used in education.

- **Professional development** (PD) support has an important role to play. Methodology is important here. Most PD is delivered ‚live' or on-line and designed by those who give it. It is usually evaluated by questionnaire, asking participants whether they found the experience valuable; feedback is powerful so the response is usually positive. However, *there is no feedback on whether the PD leads to any changes in teacher's classroom behaviour* – surely the main objective. The few studies that have used classroom observation before, during and after PD found no significant changes in teaching style.

It is not always like this. When observational feedback is part of the development, it leads to a different style of the PD – less concerned with teaching general principles and more with specific experiences, in the course sessions and in the classroom between them. Sharing these experiences among participants leads to general discussion on mathematics, learning and teaching, from which the principles emerge. It is constructivist learning for teachers. Well-engineered materials are important here, too.

The issue of *transfer* needs more research. How much of this kind of PD experience do teachers at various levels of sophistication need before they adopt the same broader teaching style in other teaching – of concepts and skills, for example. While these *can* be taught by ‚direct instruction', this is ineffective for resolving mistakes and misconceptions. The investigative, discussion-based methods that are effective (see, e.g. Swan 2005) have much in common with those needed for teaching modelling.

7. Models for systemic change

These components of successful change will only be effective if integrated. Piecemeal changes of the right kind have often been tried: new textbooks, but with the same tests; more professional development, but on an occasional basis; changes in policy involving new ‚standards'; and so on. Such attempts have proved inadequate, so that mathematics classrooms today are much like those our grandparents were taught in. What are key characteristics of a model that *is* likely to prove effective? Experience in other domains suggests:

- **Coherence** Policy, curriculum specification, classroom materials, assessment and professional development support all need to be closely aligned, developed together, and clearly communicated.
- **Sensible pace of change** Politicians, and many in education, like ‚Big Bang' solutions that will ‚fix the problem' once and for all. However, there is much to be said for gradual change. It gives the many groups, particularly teachers, who have to absorb profound changes time to absorb them. It also offers year-by-year gains that reconcile the few-year timescale of elections that drives politicians with the decade timescale of significant improvement in education.

  This model has proved effective. The Shell Centre (1984-86) worked with a leading English examination board to introduce specific profound changes to the mathematics examination at age 16, providing assessment, curriculum and professional development materials. These units were popular with teachers.
- **Realistic costing** In government initiatives the challenges are usually underestimated and the money provided for development is grossly inadequate. This guarantees failure. It is better to scale down or spread out the goals so that realistic costing can be reconciled with spending limits.

Success is never, of course, guaranteed but this kind of *sensible planning avoids guaranteed failure*. The need for further research and development is clear; the above analysis is a contribution to specifying such a program.

8. Scenarios for the future: optimistic and otherwise

History should make us cautious. The most likely scenario is little or no change. Most of those involved will be happy to avoid extra challenges in their already busy lives. However, there are some things that allow us to be more optimistic.

PISA is now the prime international comparison between countries' performance in mathematics, and it is designed to assess mathematical literacy. Politicians care about the results. Some countries are making policy moves to bring modelling into mathematics. Following the high-level Tomlinson (2004) and Smith (2004) enquiries, the British Government has made „Functional Mathematics" a central goal for English schools. Time will tell whether the government will make the moves needed to make functional mathematics a reality (Shell Centre 2005).

The problem of establishing modelling as a regular part of school mathematics remains work in progress – but progress there is.

References

Burkhardt, H., & Schoenfeld, A.H. (2003). Improving Educational Research: toward a more useful, more influential and better funded enterprise. *Educational Researcher*, 32, 3-14

Burkhardt, H. (2006). *From design research to large-scale impact: Engineering research in education*. In J. Van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research*. London: Routledge.

Burkhardt, H. with contributions from Pollak, H. O. (2006). Modelling in Mathematics Classrooms: reflections on past developments and the future. *Zentralblatt für Didaktik der Mathematik,* 39 (2), 178-195.

NTPS, Swan, M., Binns, B., Gillespie, J., and Burkhardt, H, with the Shell Centre team (1987-89). *Numeracy Through Problem Solving*. Five curriculum and assessment modules. Harlow: Longman, reissued 2000. Nottingham: Shell Centre Publications

OECD (2003). *The PISA 2003 Assessment Framework: Mathematics, Reading, Science and Problem Solving Knowledge and Skills*. Paris: OECD.

Shell Centre, Burkhardt, H., Bell, A., Pead, D., & Swan, M. (2005). *Making functional mathematics happen. C*omments to the qualifications and Curriculum Authority Nottingham. Nottingham: Shell Centre Publications

Shell Centre (1984-86). *Problems with Patterns and Numbers* and *The Language of Functions and Graphs.* Manchester: Joint Matriculation Board, reissued 2000 Nottingham: Shell Centre Publications.

Smith Report. (2004). *Making Mathematics Count.* London: Department For Education and Skills, HMSO. Retrieved Nov. 13, 2005, http://www.mathsinquiry.org.uk/report /index.html

Steen, L. A. (Ed). (2002). *Mathematics and Democracy: the case for quantitative literacy*, the National Council on Education and the Disciplines (NCED), USA.

Tomlinson Report (2004). *14-19 Curriculum and Qualifications Reform*. London: Department For Education and Skills, HMSO. Retrieved Nov. 13, 2005, http://www.14-19reform .gov.uk/

Treilibs, V., Burkhardt, H., & Low, B. (1980). *Formulation processes in mathematical modelling.* Nottingham: Shell Centre Publications.

Swan, M. (2005) *Improving Learning in Mathematics.* Sheffield: Department for Education and Skills. (DfES ref: 1599-2005DOC-EN)