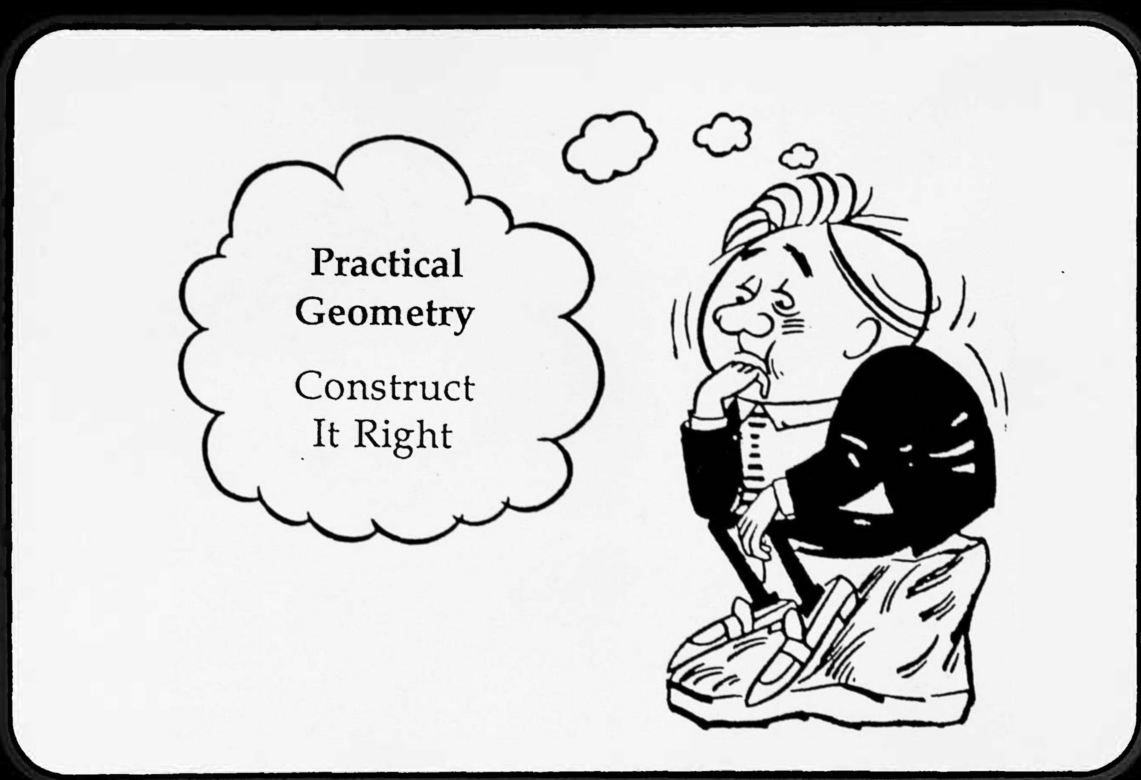
EXTENDED TASKS FOR GCSE MATHEMATICS

A series of modules to support school-based assessment



MIDLAND EXAMINING GROUP

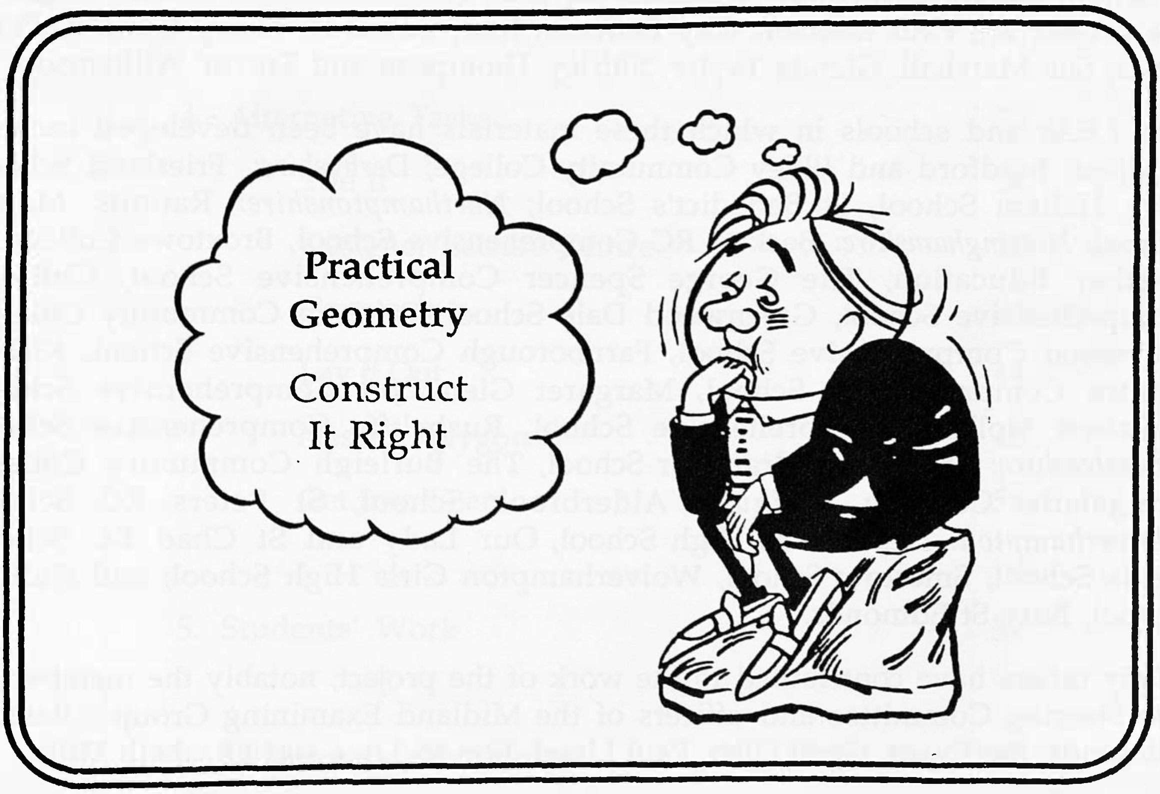
SHELL CENTRE FOR MATHEMATICAL EDUCATION

EXTENDED TASKS

FOR GCSE

MATHEMATICS

A series of modules to support school-based assessment



MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

Authors

This book is one of a series forming a support package for GCSE coursework in mathematics. It has been developed as part of a joint project by the Shell Centre for Mathematical Education and the Midland Examining Group.

The books were written by

Steve Maddern and Rita Crust

working with the Shell Centre team, including Alan Bell, Barbara Binns, Hugh Burkhardt, Rosemary Fraser, John Gillespie, Richard Phillips, Malcolm Swan and Diana Wharmby.

The project was directed by Hugh Burkhardt.

A large number of teachers and their students have contributed to this work through a continuing process of trialling and observation in their classrooms. We are grateful to them all for their help and for their comments. Among the teachers to whom we are particularly indebted for their contributions at various stages of the project are Paul Davison, Ray Downes, John Edwards, Harry Gordon, Peter Jones, Sue Marshall, Glenda Taylor, Shirley Thompson and Trevor Williamson.

The LEAs and schools in which these materials have been developed include *Bradford:* Bradford and Ilkley Community College; *Derbyshire:* Friesland School, Kirk Hallam School, St Benedict's School; *Northamptonshire:* Raunds Manor School; *Nottinghamshire:* Becket RC Comprehensive School, Broxtowe College of Further Education, The George Spencer Comprehensive School, Chilwell Comprehensive School, Greenwood Dale School, Fairham Community College, Haywood Comprehensive School, Farnborough Comprehensive School, Kirkby Centre Comprehensive School, Margaret Glen Bott Comprehensive School, Matthew Holland Comprehensive School, Rushcliffe Comprehensive School; *Leicestershire:* The Ashby Grammar School, The Burleigh Community College, Longslade College; *Solihull:* Alderbrook School, St Peters RC School; *Wolverhampton:* Heath Park High School, Our Lady and St Chad RC School, Regis School, Smestow School, Wolverhampton Girls High School; and Culford School, Bury St Edmonds.

Many others have contributed to the work of the project, notably the members of the Steering Committee and officers of the Midland Examining Group - Barbara Edmonds, Ian Evans, Geoff Gibb, Paul Lloyd, Ron McLone and Elizabeth Mills.

Jenny Payne has typed the manuscript in its development stages with help from Judith Rowlands and Mark Stocks. The final version has been prepared by Susan Hatfield.

Contents

1 *Introduction*

CONSTRUCT IT RIGHT is one of eight such 'cluster books', each offering a lead task which is fully supported by detailed teacher's notes, a student's introduction to the problem, a case study, examples of students' work which demonstrate achievement at a variety of levels, together with six alternative tasks of a similar nature. The alternative tasks simply comprise the student's introduction to the problem and some brief teacher's notes. It is intended that these alternative tasks should be used in a similar manner to the lead task and hence only the lead task has been fully supported with more detailed teacher's notes and examples of students' work.

The eight cluster books fall into four pairs, one for each of the general categories: Pure Investigations, Statistics and Probability, Practical Geometry and Applications. This series of cluster books is further supported by an overall teacher's guide and a departmental development programme, IMPACT, to enable teacher, student and departmental experience to be gained with this type of work.

The material is available in two parts

Part One		The Teacher's Guide
		IMPACT
	Pure Investigations	I1 - Looking Deeper
		I2 - Making The Most Of It
	Statistics and Probability	S1 - Take a Chance
		S2 - Finding Out
Part Two	Practical Geometry	G1 - Pack It In
		G2 - Construct It Right
	Applications	A1 - Plan It
		A2 - Where There's Life, There's Maths

This particular 'cluster book', CONSTRUCT IT RIGHT, offers a range of support material to students as they pursue practical geometry tasks within any GCSE mathematics scheme. The material has been designed and tested, as extended tasks, in a range of classrooms. A total of about twelve to fifteen hours study time, usually over a period of two to three weeks, was spent on each task. Many of the ideas have been used to stimulate work for a longer period of time than this, but any period which is significantly shorter has proved to be rather unsatisfactory. The practical geometry tasks are intended to stimulate students' interest in, and understanding of, the spatial world in which they live. Many geometrical discoveries are made experimentally. However, this experimental approach can be followed up and reinforced, using reasoning and proof. Geometry also provides excellent opportunities for making and testing hypotheses.

It is important that students should experience a variety of different types of extended task work in mathematics if they are to fully understand the depth, breadth and value of the subject. The tasks within this cluster begin within real life situations, and they are intended to be tackled practically. However, it is important that this practical approach should be followed up using reasoning, calculation and proof, according to the individual need and ability of each student. The common element amongst all the items within this cluster is the idea that they are designed to develop spatial awareness and geometrical drawing skills.

Clearly, there are many styles of classroom operation for GCSE extended task work and it is intended that this pack will support most, if not all, approaches. All the tasks outlined within the cluster books may be used with students of all abilities within the GCSE range. The lead task *Get Drawing* may be used with a whole class of students, each naturally developing their own lines of enquiry. It is intended that all the tasks within the cluster may be used in this manner. However, an alternative classroom approach may be to use a selection, or even all, of the ideas within the cluster at one time, thus allowing students to choose their preferred context for their practical geometry task. There is, however, a further more general classroom approach which may be adopted. This is one that does not even restrict the task to that of a practical geometry nature. In this case some, or all, of the items within this cluster may be used in conjunction with those from one or more of the other cluster books, or indeed any other resource. The idea is that this support material should allow individual teacher and class style to determine the mode of operation, and should not be restrictive in any way.

Teachers who are new to this type of activity are strongly advised to use the lead tasks.

These introductory notes should be read in conjunction with the general teacher's guide for the whole pack of support material. Many of the issues implied or hinted at within the cluster books are discussed in greater detail in The Teacher's Guide.

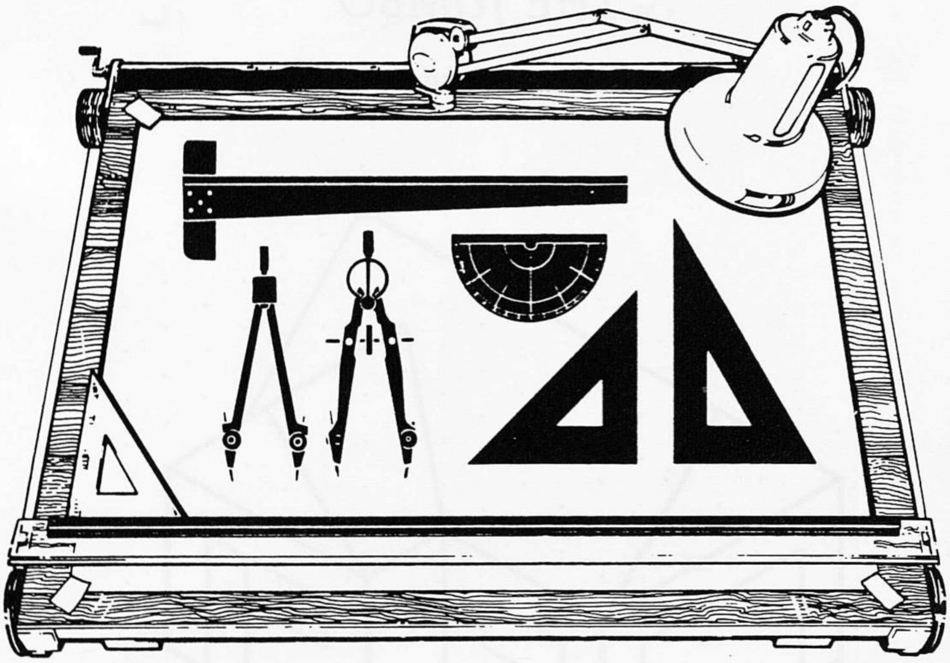
2 *Get Drawing*

The lead task in this book is called *Get Drawing*. It is based on a real life situation and provides a rich and tractable environment for practical geometry coursework tasks at GCSE level.

The task is set out on pages 7-14 in a form that is suitable for photocopying for students.

The Teacher's Notes begin on page 15. These pages contain space for comments based on the school's own experiences.

GET DRAWING



During this task, you are going to be involved in producing a scale drawing of a building. This could be your home, classroom or any other building you choose. This means that you have to choose a problem you would like to tackle, and then go ahead with it in any way which you like. You will need to measure various things, make rough sketches and draw accurate plans.

During the time spent on this task, you will have a chance to talk to your friends, and explain your ideas to each other. You can extend the problem in lots of different ways if you wish, and if you have time. Your ideas about the task will build up as you go along.

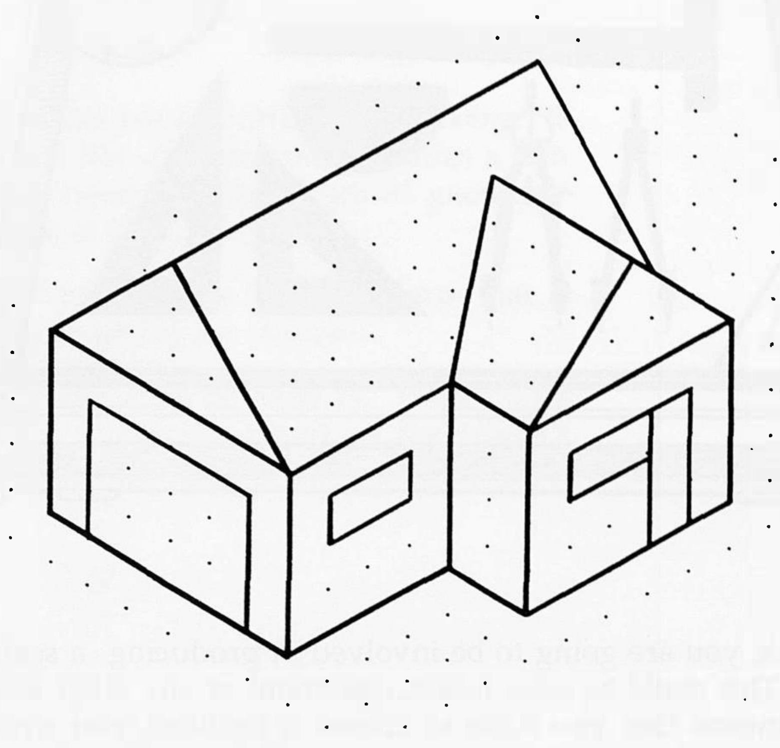
Your report at the end of this task should explain to someone, exactly what you have done and why you did it. This is an important part of your assessment, so include everything that helps you to explain your work.

During the first few lessons, try to set yourself a definite problem to tackle. Write down your problem on paper, and then make sure that you try to solve this problem over the two to three weeks which you have for this task. Naturally, you can bring in other ideas as you go along, but setting yourself a main problem will help you to keep moving forward.

ISOMETRIC DRAWING

THE JUMBO

Single Garage and Workshop



Dimensions :	Garage :	Length 18'
		Width 12'
	Workshop area :	Length 10'
		Width 4'
	Height :	Walls 8'
		Main apex 14'
	Door size :	8' x 6'
Specification :	Brick built	Side door
	Tiled roof	Opening window (2)

PERSPECTIVE DRAWING I

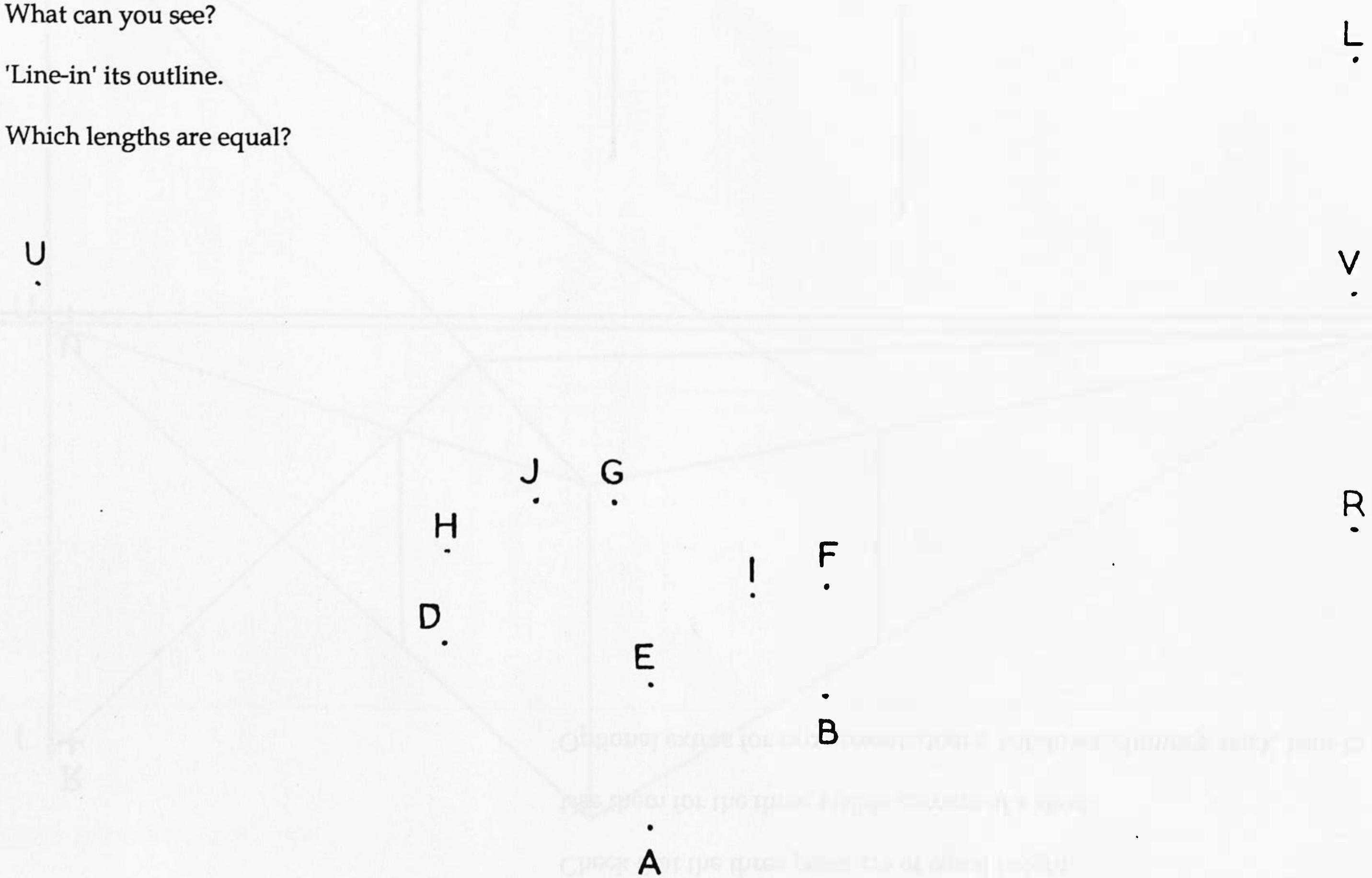
With faint straight lines, join the following:

* UV
* sets of points which lie '3 in a straight line.'
* the pairs of points HD, EA and FB.

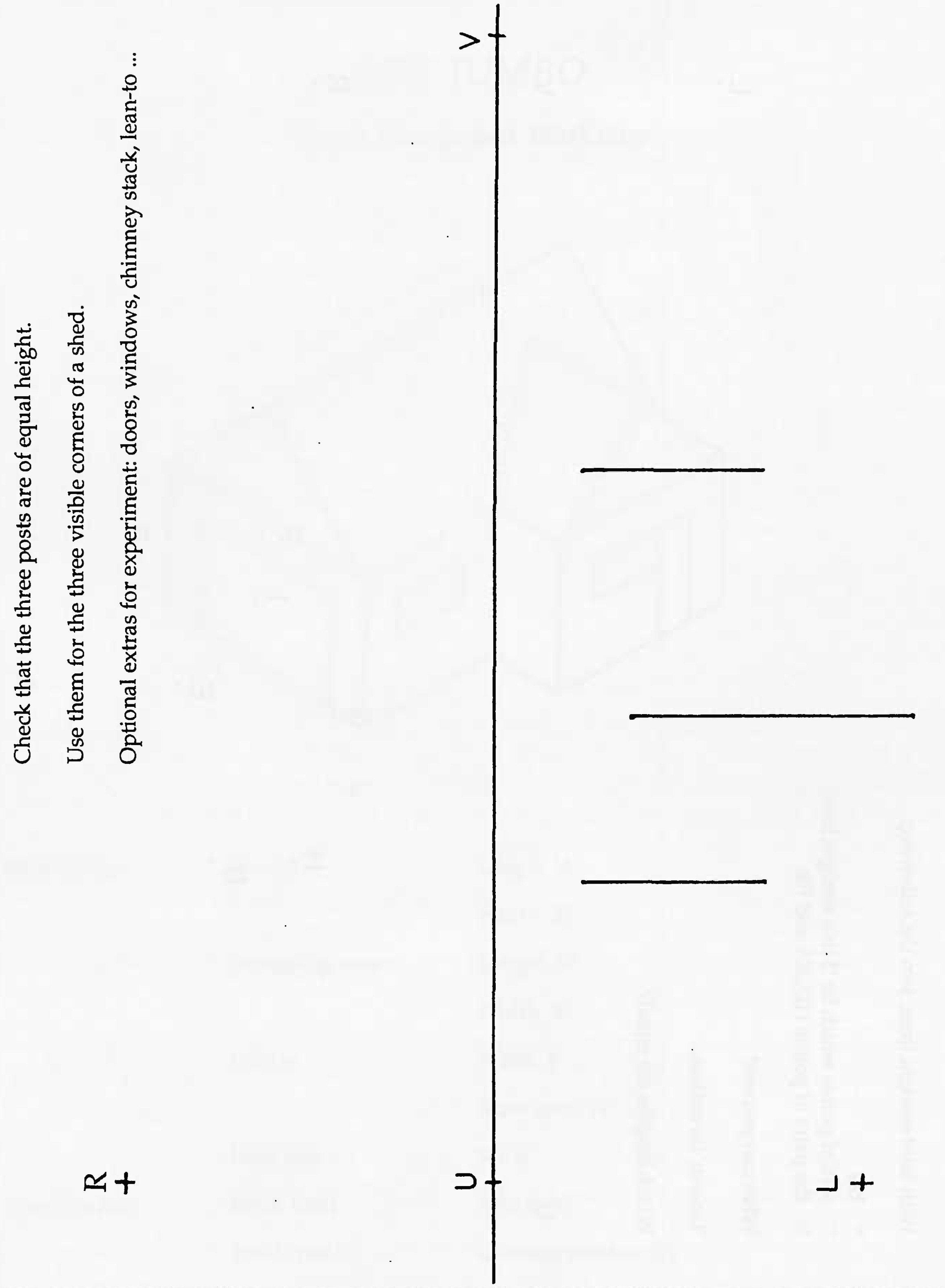
What can you see?

'Line-in' its outline.

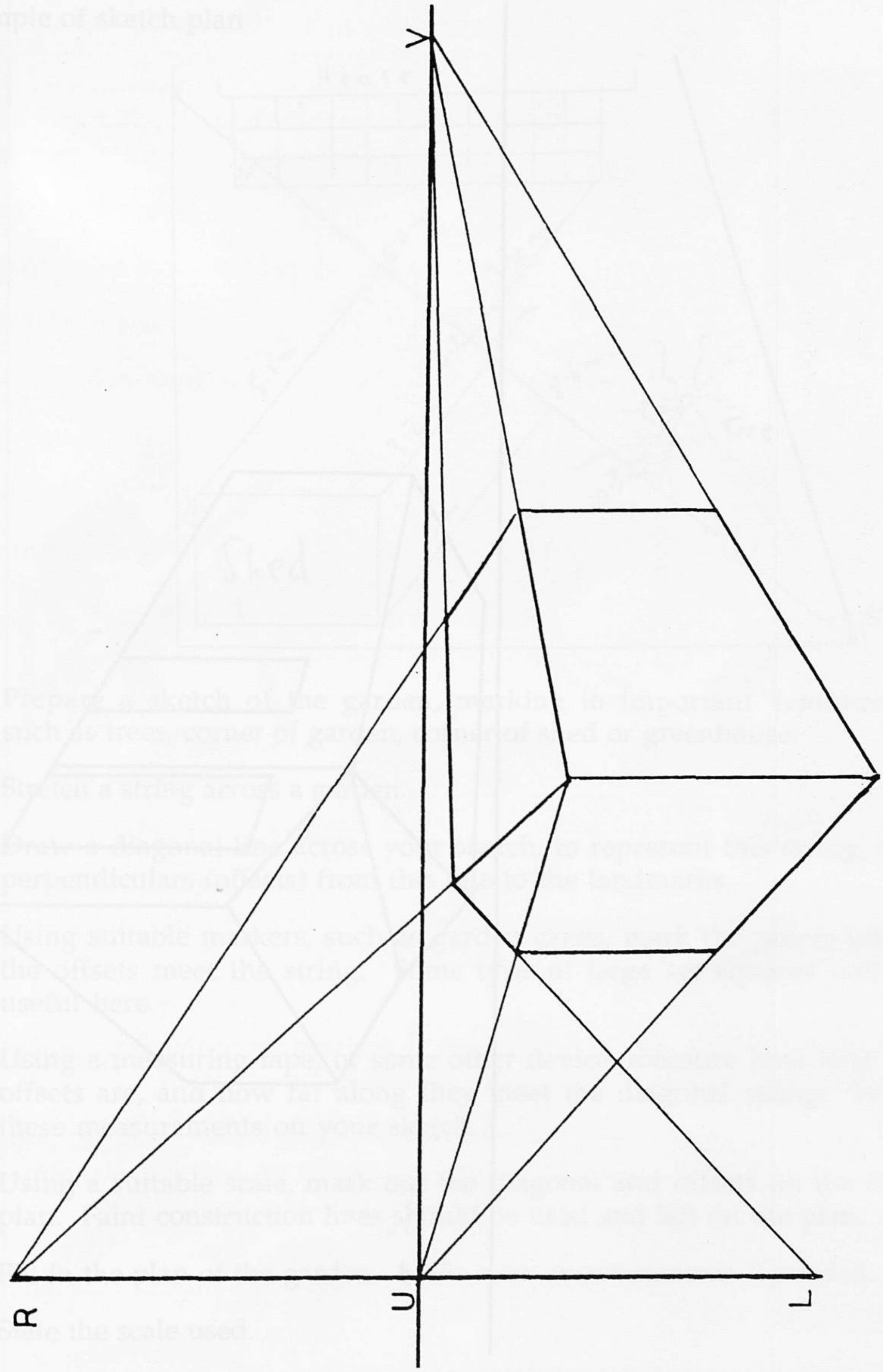
Which lengths are equal?



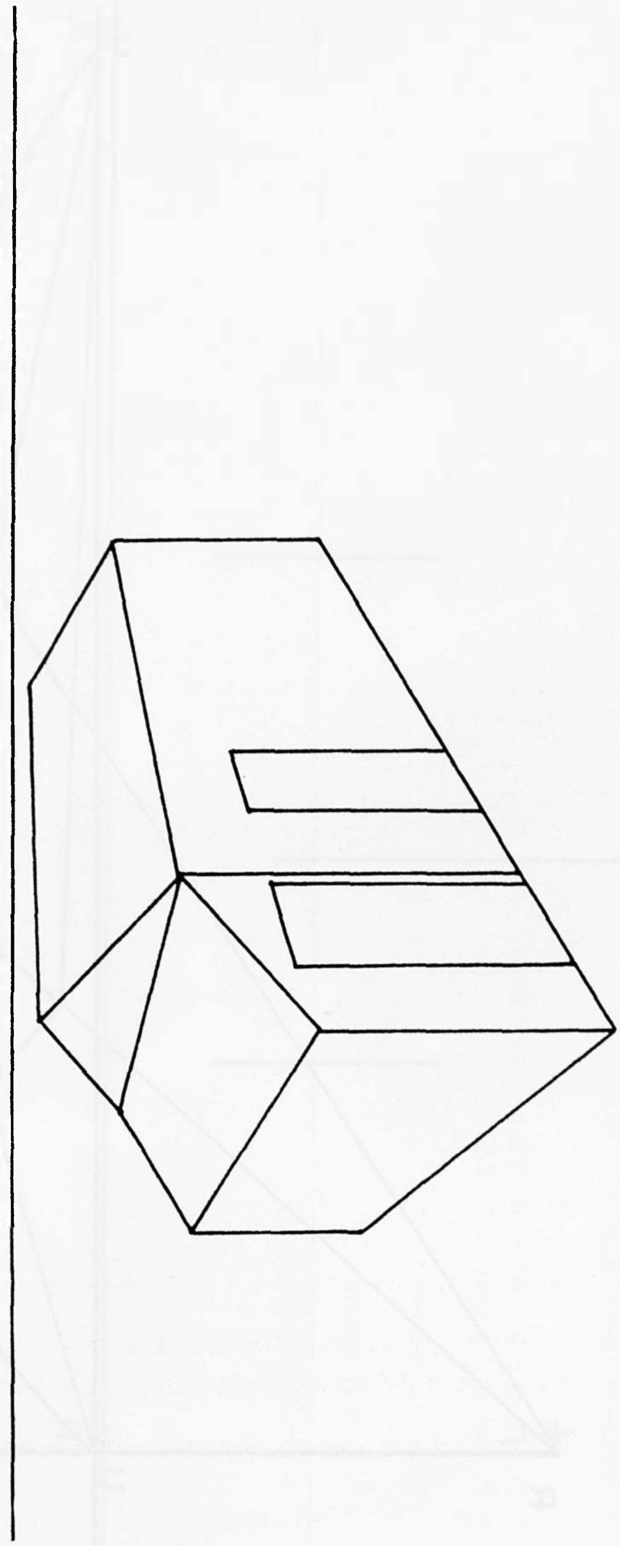
PERSPECTIVE DRAWING II



PERSPECTIVE DRAWING III



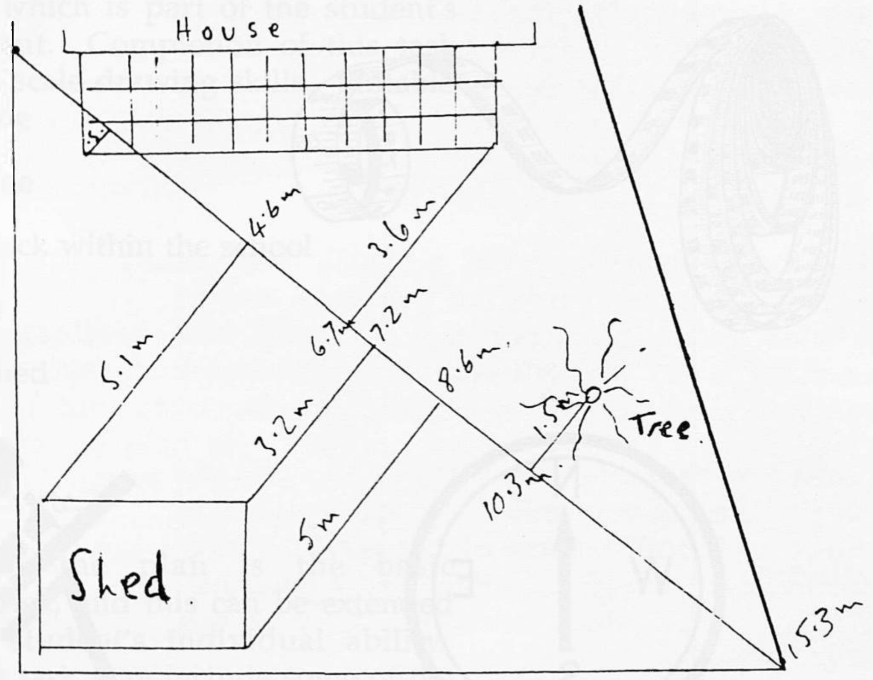
PERSPECTIVE DRAWING IV



Put in the construction lines.

OFFSET SURVEYS

Example of sketch plan



* Prepare a sketch of the garden, marking in important 'landmarks', such as trees, corner of garden, corner of shed or greenhouse.

* Stretch a string across a garden.

* Draw a diagonal line across your sketch, to represent this string, and perpendiculars (offsets) from this line to the landmarks.

* Using suitable markers, such as garden canes, mark the points where the offsets meet the string. Some type of large set squares will be useful here.

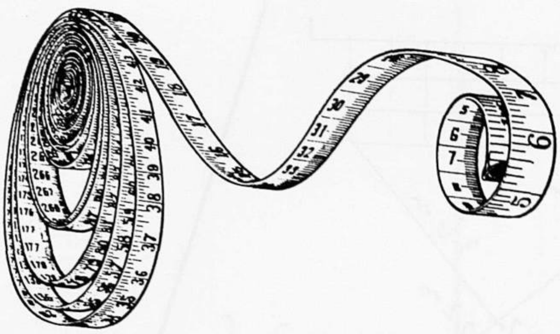
* Using a measuring tape, or some other device, measure how long the offsets are, and how far along they meet the diagonal string. Write these measurements on your sketch.

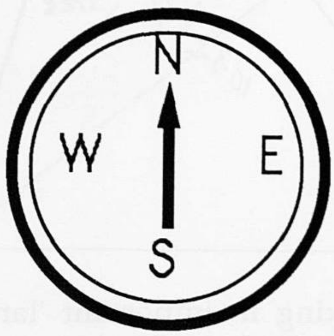
* Using a suitable scale, mark out the diagonal and offsets on the final plan. Faint construction lines should be used and left on the plan.

* Fill in the plan of the garden. Make extra measurements if needed.

* State the scale used.

TAPE AND COMPASS SURVEYS





There are many ways to produce a plan of something like a field using a variety of instruments. Here is one idea, using a 10 m measuring tape and a compass.

* Mark out two points somewhere in the middle of the field but choose them so that they are a known distance apart, say 10 m.
* Use your compass to take readings for each corner point of the field from one point. Record each of your results on a sketch diagram.
* Now do the same from the other point.
* You should now have enough information to produce an accurate scale drawing.

Get Drawing - Teacher's Notes

Get Drawing is a practical geometry task. The task focusses on the production of a plan for some particular building which is part of the student's everyday environment. Completion of this task requires, essentially, scale drawing skills. Suitable buildings may include

* their own home
* a particular block within the school
* the school site
* their garden shed
* the sports hall
* the local church etc.

The production of the plan is the basic requirement of the task, and this can be extended according to each student's individual ability. Further work on this task may include some of the following

* Estimation and calculation of lengths which cannot be measured.
* Various methods of measurement
* Isometric drawing
* Perspective drawing
* Making and using a clinometer
* Offset surveys
* Route planning using, say, a bearing and distance method
* Plan drawing by bearings from two different points
* Building costings
* Model construction
* Redesigning a building they have used etc.

Naturally, certain techniques need to be available to the students before they can tackle the main task, and the timing of the teaching of this part of the syllabus is left to the individual teacher. The task may be used as a revision of some of these ideas. Alternatively, the task may be used immediately after students have acquired the necessary skills, so that they can apply what they have learnt to a new situation. A further alternative is to integrate the whole teaching aspect into the task, over a longer period of time.

Most of the available time will probably be spent on individual student work, but there will also be times when group interaction and discussion will be beneficial to all. The style and depth of the work pursued will depend upon the individual group.

The series of resource sheets supplied with this task is designed to offer initial support for some of the more technical aspects of the work. These should be used only if a student has a particular need. They are not designed to be duplicated and handed out as a set of exercises to be completed as part of each student's assignment.

It is often helpful if students know that there will be an exhibition of their final drawings.

Understanding and Exploring the Problem

The task requires some form of introduction, and this is often best accomplished through discussion. This activity would provide a very limited experience for the group, and indeed for each student, if they all tackled exactly the same problem. It is often best to try to avoid this problem before it arises.

If the task could be introduced to the students through discussion, they could be asked to suggest why a plan of a building or a room may need to be produced. Initially, the task could be restricted to producing a ground floor plan of their own home. Some possible reasons for this may include

* Building an extension
* Thinking about buying carpets or some other form of floor covering

* Fire regulations
* A new form of rating system
* Including a plan on an estate agents details when selling the house
* Council records
* Planning where to put the furniture etc
* Installing a burglar alarm.

Devising and Planning Individual Studies

After they have completed their plan of the ground floor of their own home, students could then be encouraged to think about other buildings they may wish to draw a plan of. Working in small groups, they will find it helpful to discuss what they could do and what they might include in their work. This is probably best completed at the end of a couple of sessions, after the students have had an opportunity to tackle the introductory task. By this time each student will have had the opportunity to consider the basic problem, and to decide exactly what they feel it is appropriate to tackle during their major item of work on this task. At this stage they will also have more to offer to, and gain from, discussion. It is important that each student should write down the exact problem they intend to tackle, and that these should present a realistic challenge for them. This lays a firm foundation for work which has a definite focus. Such problems may be of the type

* To draw a plan of my home, garden and street to use as sales information
* To produce drawings, plans and a model of the science block
* Our group of four intends to make a scale model of the school
* To produce an accurate plan of the school for use by visitors or new students
* To estimate the cost of rebuilding our garage.
* To reorganise the layout of the youth club hall

Implementing Plans and Pursuing Ideas

During this stage, students will often work individually, getting on with their own problems. They will be involved in making rough sketches, estimating dimensions and taking measurements as they attempt to solve their problems. It is important that students should measure carefully, and record accurately. However, accurate measurements cannot be obtained individually. It will be necessary for each student to find someone to help her as she carries out this stage of her task. A willingness to co-operate with others and to work as a member of a team, is essential if this task is to be completed satisfactorily. After these measurements have been obtained, students will be in a position to produce accurate scale drawings.

When sufficient time and care has been given to the initial stage of thinking about the task and possible approaches, this stage of the work can be an extremely fruitful and productive period of activity. Naturally, according to the task which students have set themselves, the breakdown of activities in this stage will vary. Some may simply be producing their plan, while others may be taking further measurements and producing extra sketches and plans. Some, of course, may be producing 3D models or drawings.

Towards the end of this stage, it may be a good idea for the teacher to discuss with each small group the variety of activities observed during the period already spent on the task. This may be supported by asking some of the students to explain their individual work. During these discussions, students may realise that there are one or two extra ideas which they can apply to their own work. Naturally, at this stage students will be thinking about their final reports and completing their task. What form this report should take, and what should be included, are interesting points for discussion. Such discussions may be organised on a small group, or whole class basis. They may vary in length from just a few minutes, to in-depth and longer activities.

Reviewing and Communicating Findings

At this stage students will be looking forward to completing their final drawings, and carrying out any necessary additional tasks which have arisen. They will also be thinking about, and completing, their reports. If students have worked together on this project they will now need to write up separately, in a way which demonstrates and explains their individual contribution to the overall task.

It is probably best if A3 plain paper, or larger, is available for this task, since this is much more realistic than using A4. The final report should outline what each student did and why she did it. It should also include any rough sketches, intermediate scale drawings, as well as the final polished version of their drawings and any other developments and/or mathematics which she has produced. This report, together with the teacher's ongoing observations of each student over the three week period, will form the basis of the assessment.

At the end of such a project, it is often valuable to organise an exhibition of students' work.

Many teachers believe that it is necessary to buy expensive equipment, if their students are to produce good coursework which involves surveying activities. However, this is not the case. Good coursework can be produced using, say, the plastic theodolites, clinometers and compasses which are available from educational suppliers. For minimal costings, students can measure fields etc. using ropes or washing lines which have been marked at appropriate points. What is essential, is that students should work carefully and as accurately as their equipment permits. The PE department are often helpful with the loan of long measuring tapes and sometimes compasses.

3 A Case Study

Further Education

Intermediate Level GCSE Group

Before tackling the project we spent an afternoon looking at different ways of calculating the height of the classroom above the ground, giving students a chance to become familiar with the theodolite, tapes and poles. Explanatory sheets on 'triangulation' and 'chain and compasses' were discussed in class, and we gave out the project guidelines. What could possibly go wrong?

It was February; a wet and cold February. Never mind, wrap up well and off we go, if somewhat reluctantly. The first problem was trying to get the students to work out angles using compass bearings. The main difficulty emerged when North fell between the two bearings; for example, if one was 340 and the other 050, most students found it difficult to work out that the angle was 70: even when calculated, transferring the information to a scale drawing was beyond their capabilities. With 20 students working in 6 groups, getting around to see them all was a problem and the allocated class time of about 10 hours soon proved to be inadequate.

The area they were surveying was more difficult than it appeared, and the maps they drew looked nothing at all like the real thing. The errors in calculating the bearings were huge, but then they weren't helped by painters arriving and erecting scaffolding around the college rendering the compasses useless in their vicinity. We tried to make it easier for our students by getting them to do a 'chain and compass' survey, calculating angles with the theodolite or the blackboard protractor and some washing line. Some gave up in disgust, not enamoured of weaving tapes in and out of scaffolding in the rain, with a compass that gave four different directions for North.

We had seriously underestimated the task, our guidelines weren't good enough and many didn't get as far as the extension. Marking was a nightmare as many write-ups didn't include all the results tabulated in a readable and checkable form. Curiously enough many excellent projects were received, and there were as many C's on this project as on the others. When we carried out a survey of our students opinions on coursework, this rated as the favourite with nearly 25%.

We rewrote it completely for 1988/9. Our intention was to give it more structure by breaking the tasks down into manageable chunks, so that they could learn the different techniques before applying them to some extension of their own choice.

Well it was certainly an improvement, but there were still problems. We found that we needed more equipment, having only one theodolite, four tapes and six compasses. For a group of twenty you need about twice as much. There were still problems with bearings despite spending more time practising using the equipment in the classroom. Several students did not get as far as extending the projects, taking too much time to sort out the initial problems. Less able students got confused with the variety of methods they used, and were probably given too many options - we need to simplify the task further for them. Write-ups were often well done, with poor tabulation of results and a lack of explanation of methods used. Appreciation of errors was of a variable standard, although better students showed insights.

From the assessment point of view, most students produced similar work if they worked together. We found that we needed to include marks for 'contribution to the group' in our assessment, which we were quite happy to do. We did, however, get quite a large spread of marks for four people who worked together, often due to lack of effort in the write-up. We are, of course, going to change it again for 1989/90. Our main intentions are to simplify the initial piece and give more time and credit to the extension which now lines up with the general approach of the MEG/Shell Centre project.

The general tone of this account of our activities has been 'doom and gloom', but we feel that the project has been immensely valuable in helping to teach the Geometry and Trigonometry syllabus. Much excellent work has been done, particularly by mature students, and it has motivated some of the least able highly. To finish with I'd like to quote the conclusion to her project written by a middle-aged student of about grade D/E standard.

"After completing this project I feel a great sense of achievement. When I was told we would be using compasses, theodolites and clinometers, I was totally mystified. It would have been very easy to have dropped out. By listening, watching and joining in with the other students I have gained a tremendous amount.

With many of the scale drawings I made numerous attempts before succeeding. I have felt anger and frustration, then joy when one more obstacle had been overcome.

Regardless of my result I know I am truly satisfied with the knowledge I have gained during this project."

4 *Alternative Tasks*

Tile It

Designer Leisure Centre

Constraints

Lay It Out

Nested Polygons

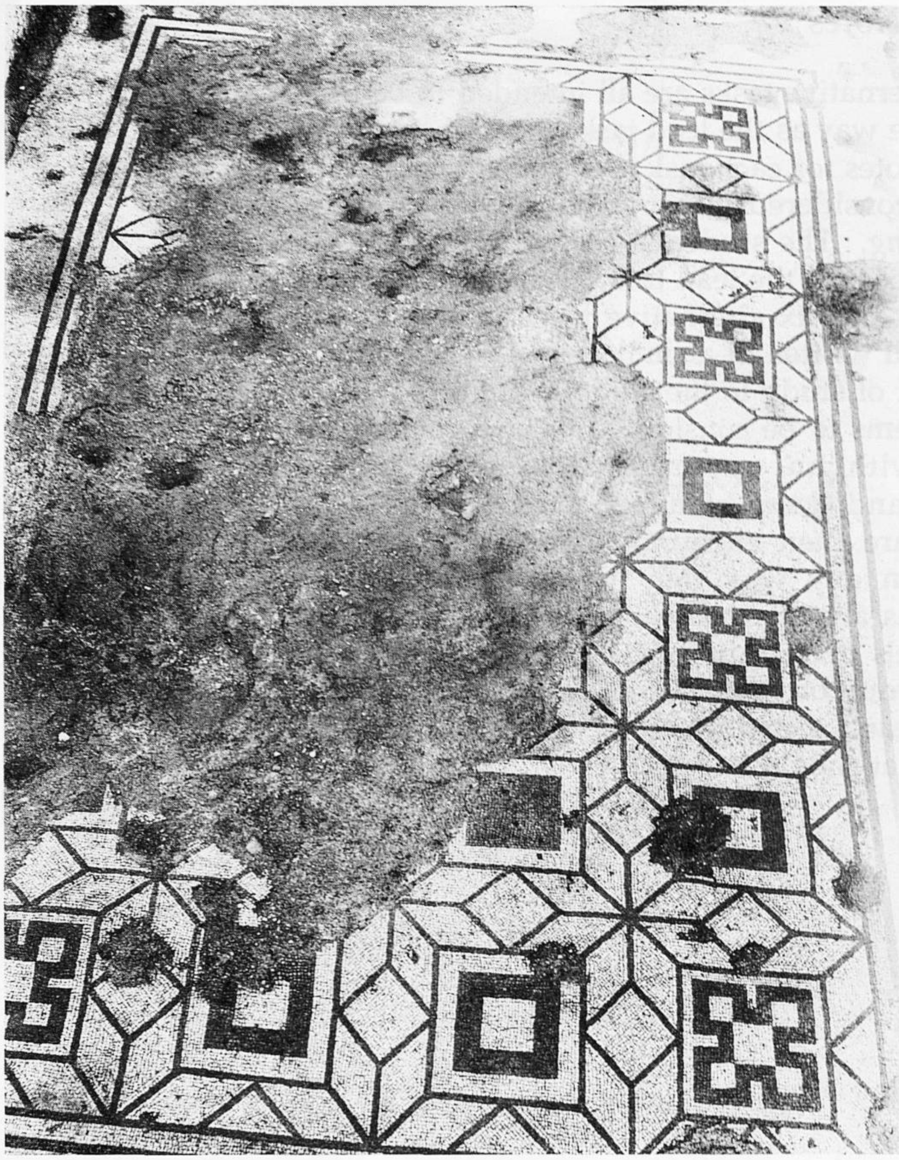
Get Into Gear

Alternative Tasks

General Notes

The six alternative tasks are all intended to be used in the same way as the lead task, *Get Drawing*. The teacher's notes for each task are brief and should be read and considered in conjunction with those for *Get Drawing*. The student's notes are in the same form as those for the lead task. The student's notes offered for the six alternative tasks in this cluster book are all written in a similar style. They outline the context of study to the student and offer one or two problems to be considered. This provides the student with an opportunity to consider the problem and gain some understanding of it. Students are then encouraged to investigate the problem in any way they wish. Some further suggestions are offered which may be used if the teacher feels this is appropriate for any individual student, group or class. These suggestions provide further ideas for investigation without prescribing exactly what should happen.

TILE IT



Mosaics have been used to decorate floors and walls for about five thousand years. The picture, shown above, is a photograph of a black and white floor mosaic in the Roman Palace at Fishbourne in West Sussex. This floor is believed to date from the end of the first century AD.

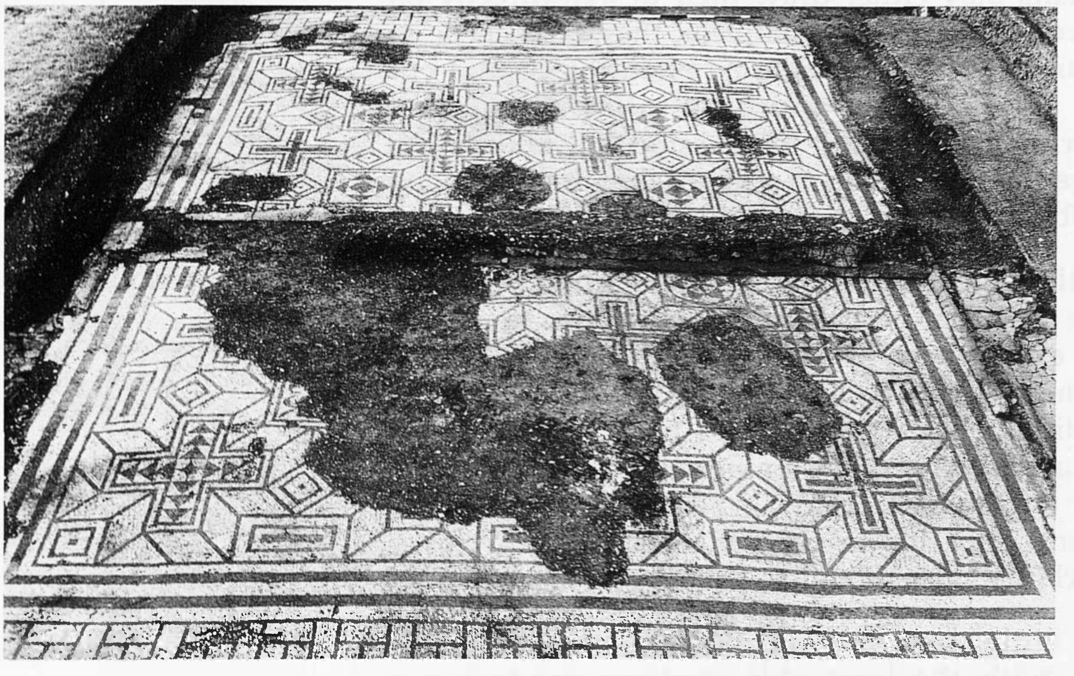
* Look carefully at the picture and try to draw what you think the whole floor looked like before it became partly worn out.

* Try to draw a variety of possible patterns.

* Use your patterns and ideas to investigate ways of tiling floors and walls.

TILE IT : continued

You may find the following photograph of another mosaic floor from the Roman Palace at Fishbourne helpful as you pursue this task.



Tile It - Teacher's Notes

The mosaic floor pattern at the Roman Palace at Fishbourne offers a very simple but extremely rich starting point for a GCSE extended task of a practical geometry nature. This task was first used in our trial classrooms following an actual visit to the Palace. There may be similar situations near other schools throughout the country.

Since it makes low entry demands, the initial task allows all students to get into the problem quickly. Students may either draw their design directly on squared paper or, alternatively, they may chose to produce a sheet of tiles to be photocopied and cut up. This latter approach allows practical experimentation and discussion to take place before the production of a set of drawings showing possible patterns.

At this stage, discussion between students is likely to be based around ideas of symmetry and their own experiences of such patterns. The teacher's role here is very much one of listening and asking questions about why certain patterns have been produced. This allows students to communicate their mathematical ideas relating to geometric pattern, and this in turn enables them to gather and clarify their thoughts.

This initial task is likely to take a considerable time, with the least able students producing a range of possible patterns, and the more able starting to link ideas of transformation geometry and construction work with their investigation.

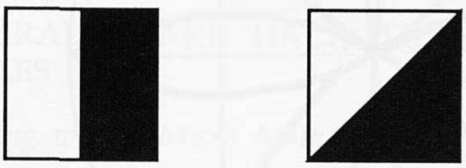
The extension ideas must naturally come from the individual students, but it is worth listing some of the range of activities which have been generated in our trial classrooms.

* Complete range of patterns using just the three visible tiles

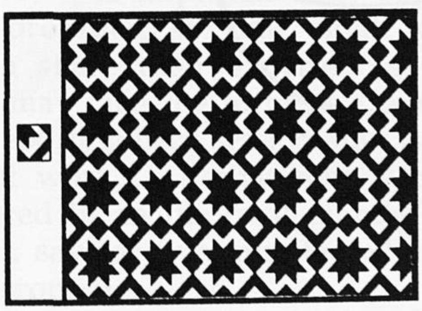
* The introduction of different tiles

* A catalogue of tiling designs which are possible using just a single tile

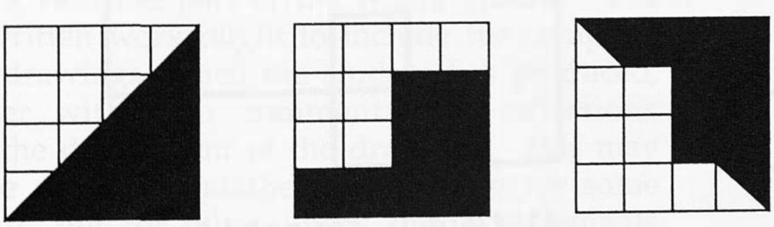
  An interesting example of a student's work on a similar activity appears in the *Teacher's Evaluating and Assessing Mathematics* material from the Mathematics Education Centre at West Sussex Institute of Higher Education. In this work, a student produces and discusses a catalogue based on simple tiles which are half black and half white.



* The use of transformations of all types in the production of possible patterns.



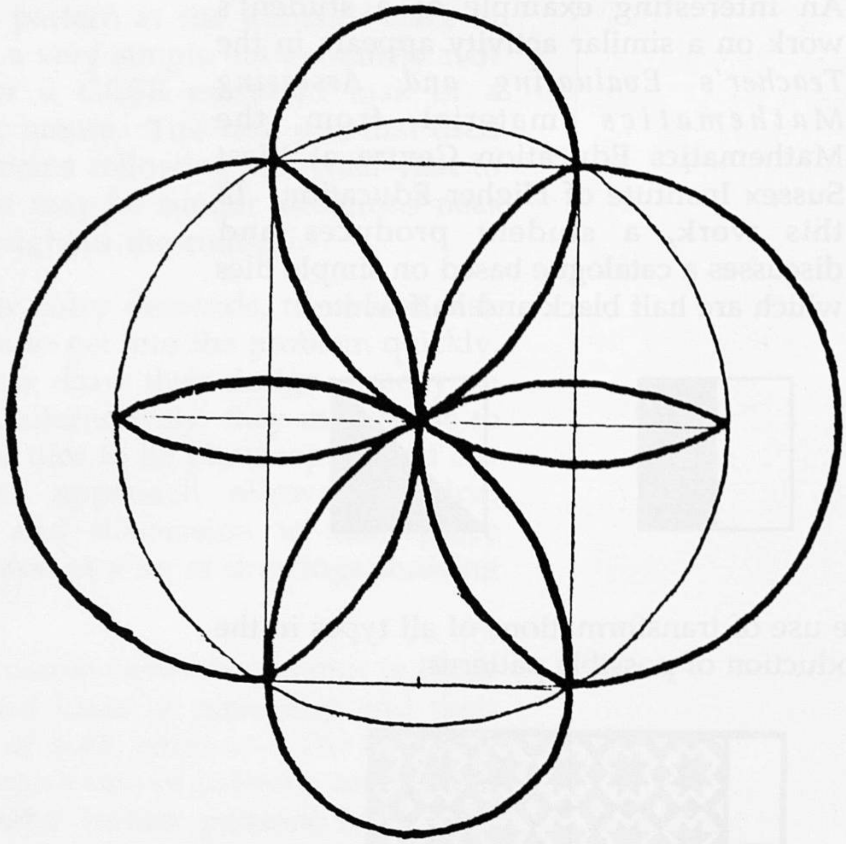
* The 'shading a half' type problem. How can it be done?



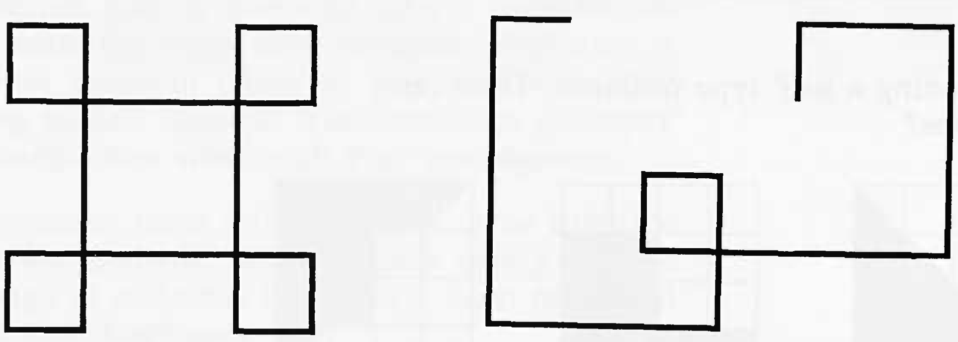
* The use of matrices to describe transformations

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad a,b,c,d \in \{-1,0,+1\}$$

* Construction work using pencil and compass techniques. The discovery and use of the available constructions.

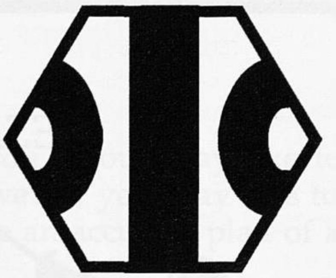


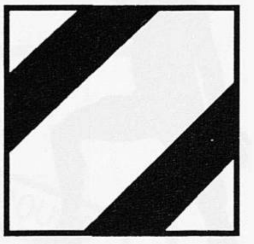
* Spirals using straight lines



(1,2,3,4,1, repeat)

* General tessellations using any shape or shapes. Pentomino and hexomino sets are interesting shapes to work with.





Apart from the material already mentioned, a whole range of resources is available for ideas relating to geometric shape and pattern work. The ATM tile generator post cards and materials are excellent resources for work in this area and fit with many of the ideas from the above list. These showing a variety of patterns which were produced using their square and hexagonal MATS by children younger than the GCSE age range. Various pieces of software are extremely valuable for work of this nature and include:

* *Microsmile : The first 30, The next 17*
    - SPIRALS, TAKE HALF, TILES, NEW TILES
* *Teaching with a Micro Maths 3*
    - SPLOT

Whilst accepting that individual students should be developing their own ideas throughout an extended task, it is important that opportunities should be provided so that small groups of students can share their experiences with each other. This may take the form of a well planned and formal reporting back session after, say, a week of individual work. Alternatively, such support may be offered in an informal manner and on a regular basis, say for five minutes when you feel this to be appropriate.

During a task like this, the majority of student time may well be spent on experimental drawing or construction work. This is to be expected, and it forms a valuable part of the whole process. The final written work ought to include the complete set of drawings which the student has produced, together with their comments and reflections about the develoment of the drawings. This may well be a detailed mathematical report for some students, but for others, very simple statements may link their drawings. This latter approach will be perfectly acceptable for many students. However, a final submission should not be just a set of drawings. In most cases the student would fail to communicate with others about their work if the final report of their submission consisted only of a set of drawings.

DESIGNER LEISURE CENTRE

Hundreds of new houses have been built on the outskirts of Blackston. The local council has agreed to build a sports and leisure centre to provide facilities for people of all ages.

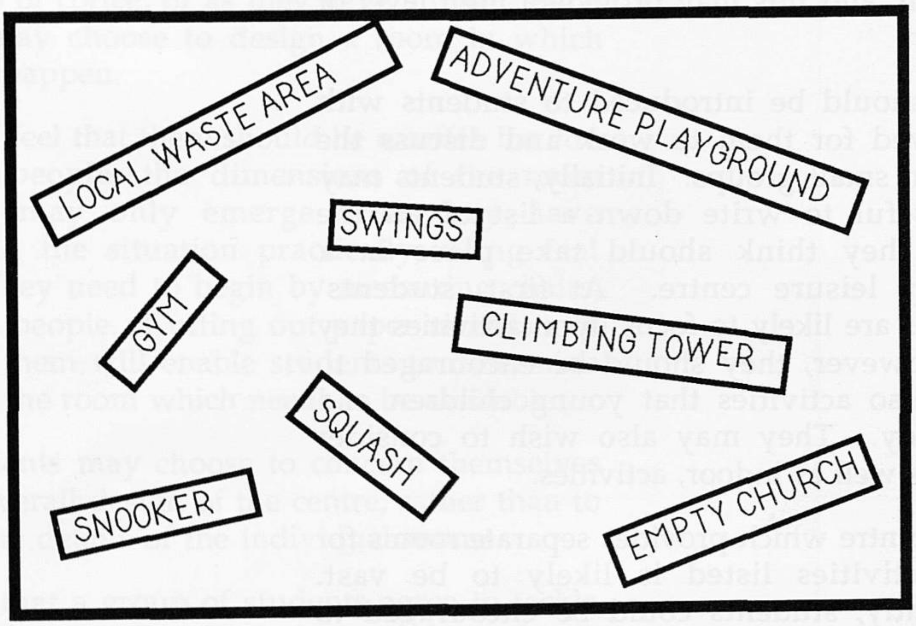
A competition has been organised to produce plans for the new centre. There is a prize for the best overall design for the centre. There are also prizes for designing the details of the rooms in which different sports and social functions will take place.

Everybody has been asked to put their ideas on paper, so that the centre can provide what the local residents really want.

What do you suggest?

DESIGNER LEISURE CENTRE : Continued

You can develop this task in any way that interests you. You may like to continue to work in more detail on the first idea. However, you may like to explore your own ideas. The main theme is to produce an accurate plan of a leisure area of some kind.



You will have to plan your work very carefully, and you may need to collect some measurements as you go along.

Designer Leisure Centre - Teacher's Notes

This task is intended to provide a context within which students develop their spatial awareness as they visualise and plan their ideal leisure centre. The task provides an opportunity for students to acquire, practise and demonstrate many of the same skills as does the lead task. However, there is much greater opportunity for students to use their imagination, and this may provide a high level of motivation.

The task should be introduced to students with time allowed for them to work and discuss the situation in small groups. Initially, students may find it useful to write down a list of all the activities they think should take place in a community leisure centre. At first, students' suggestions are likely to focus upon activities they enjoy. However, they should be encouraged to consider also activities that young children and adults enjoy. They may also wish to consider outdoor, as well as indoor, activities.

A leisure centre which provides separate rooms for all the activities listed is likely to be vast. Consequently, students could be encouraged to consider the possibility of multi-purpose rooms. Gradually, the centre may assume reasonable proportions. Some leisure centres share facilities with schools, so perhaps this possibility could be discussed.

Students who are particularly interested in, say, gymnastics may choose to concentrate upon designing a gym which contains all the equipment they would like to use. They will need to find out the measurements of all these pieces of equipment and determine how much space needs to be available if people are to be able to use it safely and effectively. The amount of space needed to play, say, table-tennis or snooker often surprises students.

Students who are interested in swimming may choose to design a swimming pool which has all the facilities and equipment they would like. They may also wish to design attractive tilings or mosaics for floors and walls.

Other students may perceive a need for a young children's play area: this could be indoor or outdoor. Designing an indoor playroom, or an outdoor adventure playground, which contains swings, slides, climbing frames, sand pits .... etc, could prove absorbing for some students.

There may be students who feel that an essential feature of a leisure centre, is that people of all ages should be able to sit down and chat in comfort, with a cup of coffee, or as they have a meal. Such students may choose to design a room in which this could happen.

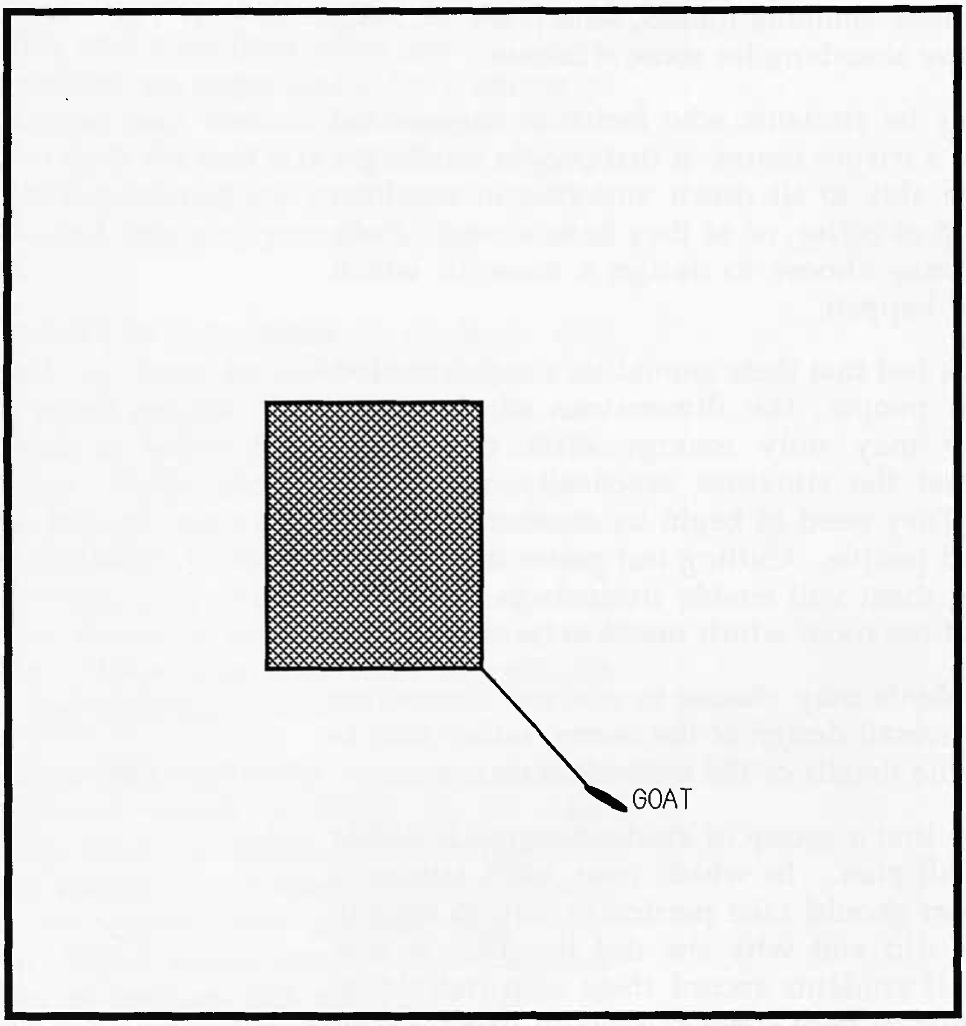
If students feel that there should be a coffee bar for, say, fifty people, the dimensions of the room necessary may only emerge after they have approached the situation practically, using real objects. They need to begin by measuring chairs, tables and people. Cutting out paper models and arranging them will enable students to determine the size of the room which needs to be available.

Some students may choose to concern themselves with the overall design of the centre, rather than to consider the details of the individual rooms.

It may be that a group of students agree to tackle one overall-plan. In which case, each student's final report should take particular care to explain what she did and why she did it. This is best achieved if students record their own individual contribution at each stage as they go along, rather than trying to do the whole report at the end. Alternatively, students may work as individuals within a whole class or small group project. For example, a small group may decide to work together to consider the development of local waste land into a leisure area. Individual jobs within this may include the overall site plan, the detailed outdoor play area, the layout of the court areas in the main indoor hall, costings etc. The latter being more appropriate to an Applications submission, rather than one of Practical Geometry.

As students work on this task the teacher's role will mainly be that of counsellor and resource. However, it is likely that students will find that other students are also useful sources of information.

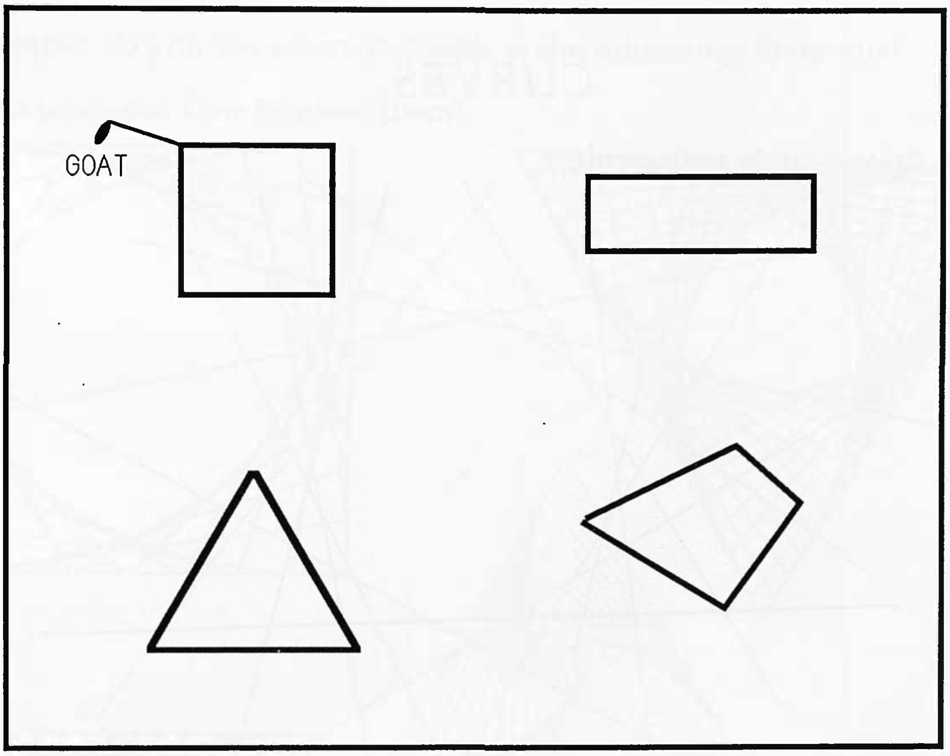
CONSTRAINTS

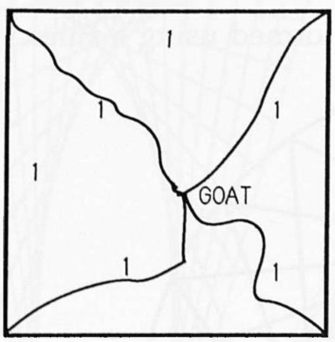


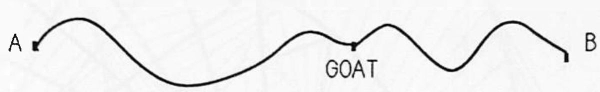
This is the plan of a rectangular shed in a large field. The shed is 5 metres long and 4 metres wide. A goat is tied to one corner of the shed by a rope 3 metres long.

* What area of grass can the goat reach?
* What happens if the length of the rope is changed?
* What happens if the shape and size of the shed are changed?
* What happens if the rope is tied to a different point of the shed?

CONSTRAINTS : continued







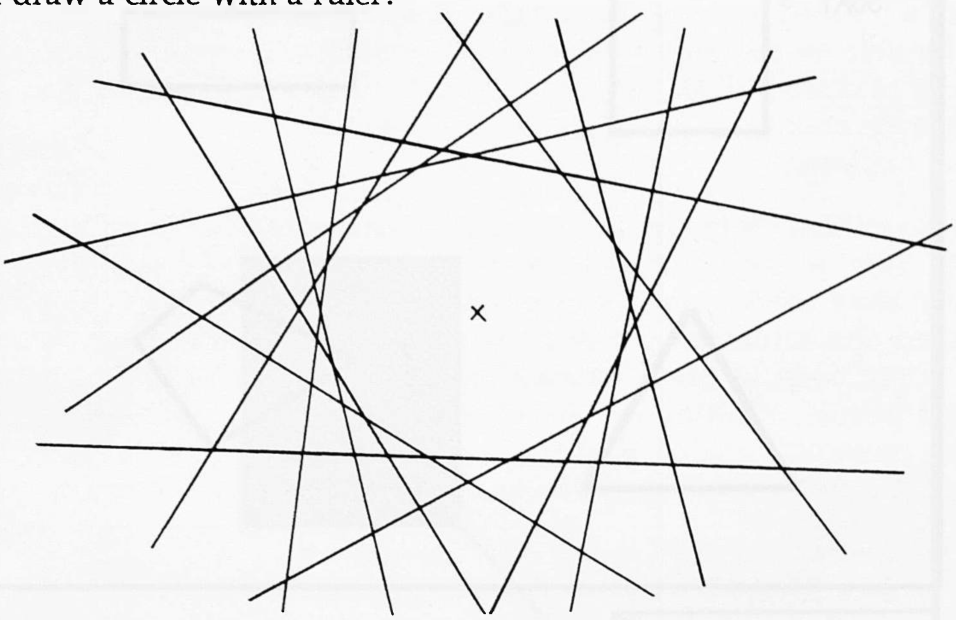
There are lots of things you could consider which are related to this problem. Here are some suggestions, but you may have other interesting ideas

* What area of grass can the goat reach if the rope can slide along a horizontal pole which forms the perimeter of a square?
* What happens if the same length pole forms the perimeter of a different shape?
* What happens if the goat is tied to more than one point?

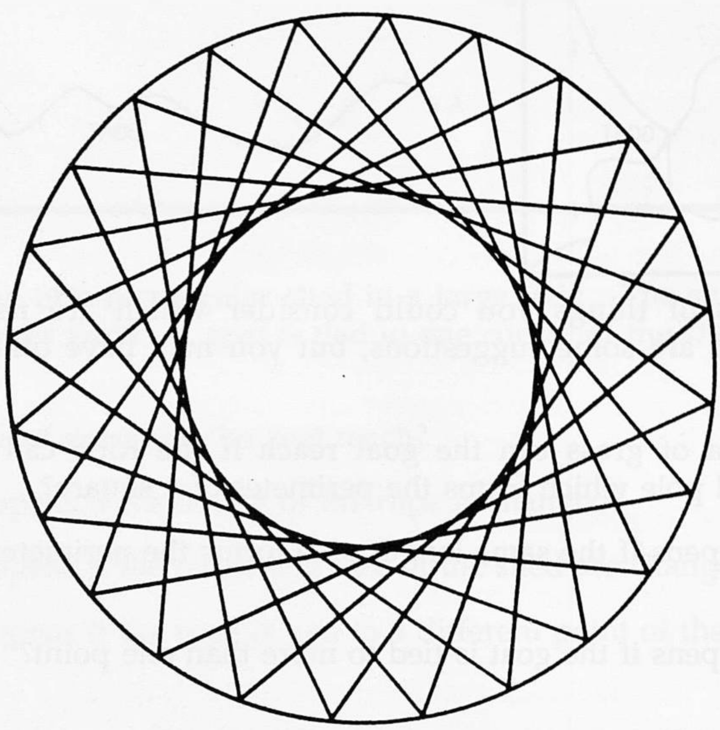
CONTRAINTS : continued

CURVES

Can you draw a circle with a ruler?



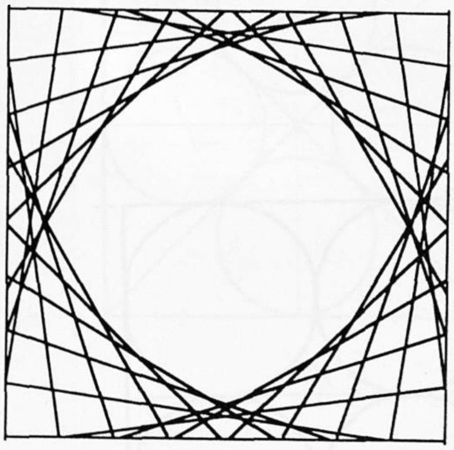
In the diagram below the smaller circle has been formed using a ruler.

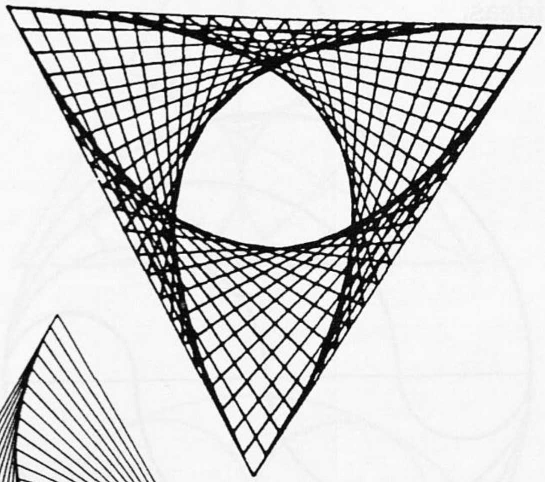


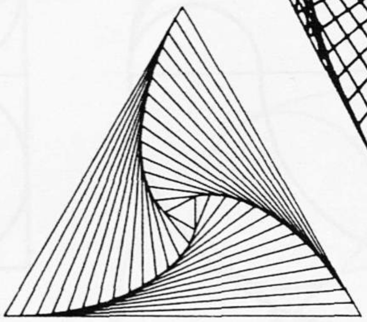
CURVES : continued

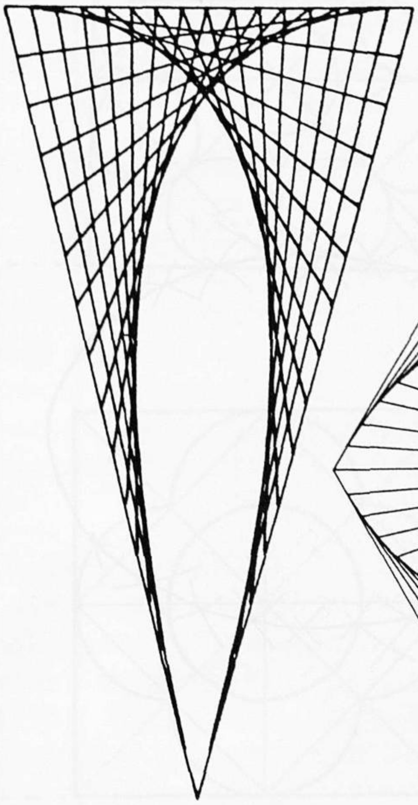
What shapes do you see when you look at the folllowing diagrams?

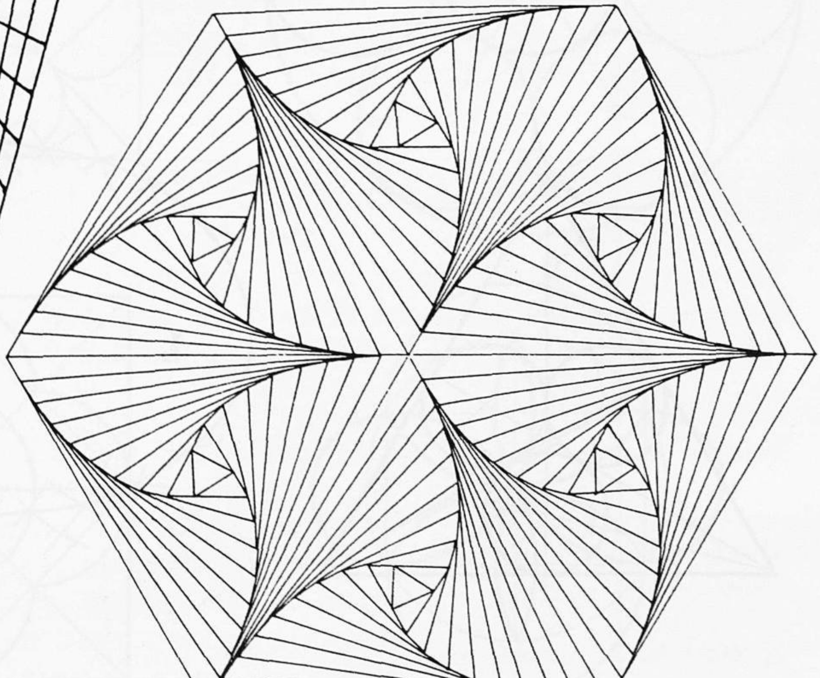
Can you work out how to draw them?







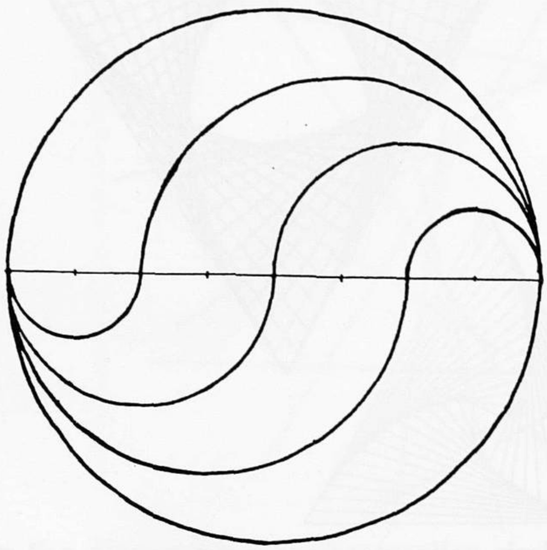


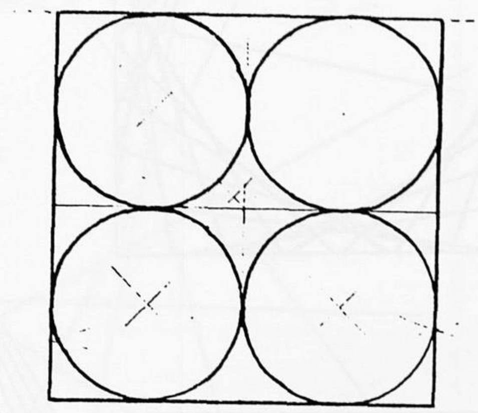


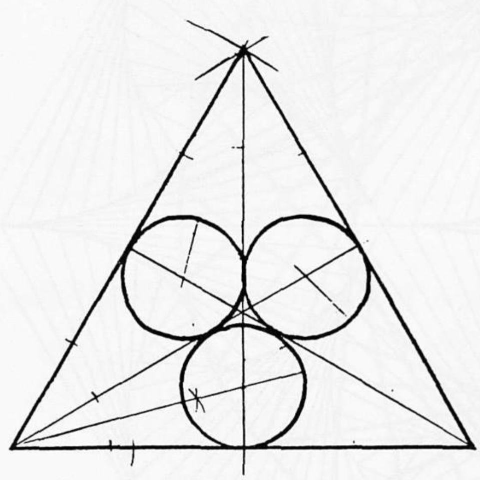
CONSTRAINTS : continued

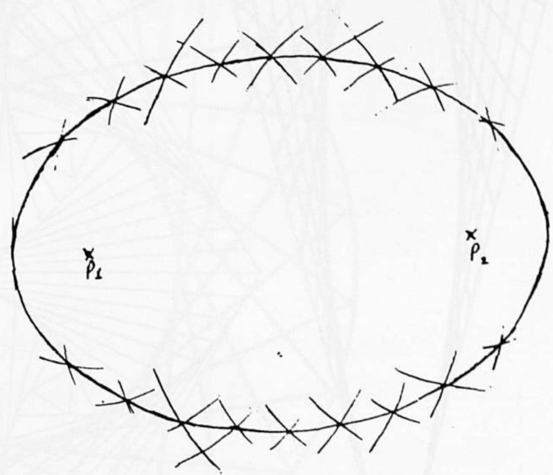
Here are some more ideas which you may like to construct.

After you have drawn these you may wish to construct some of your own ideas.

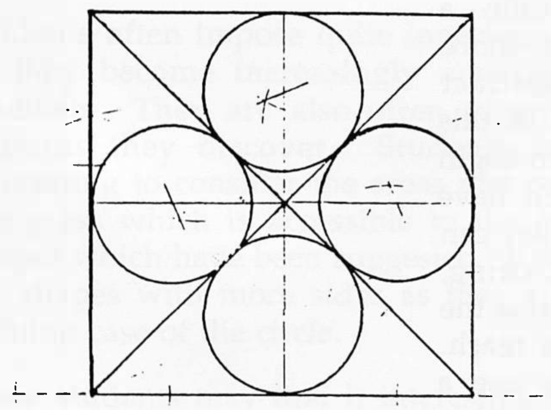


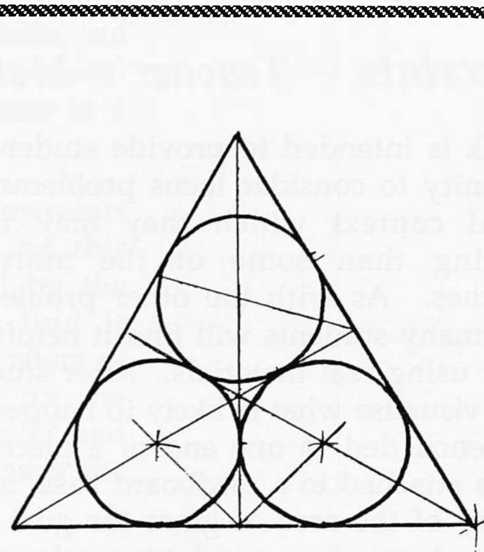


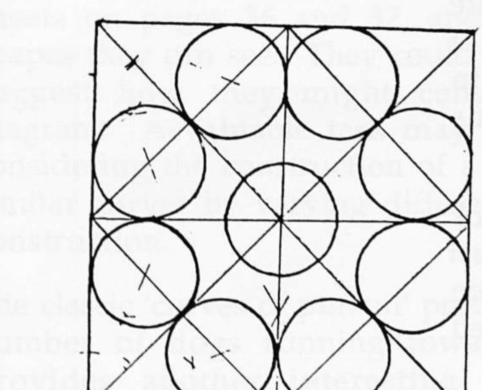


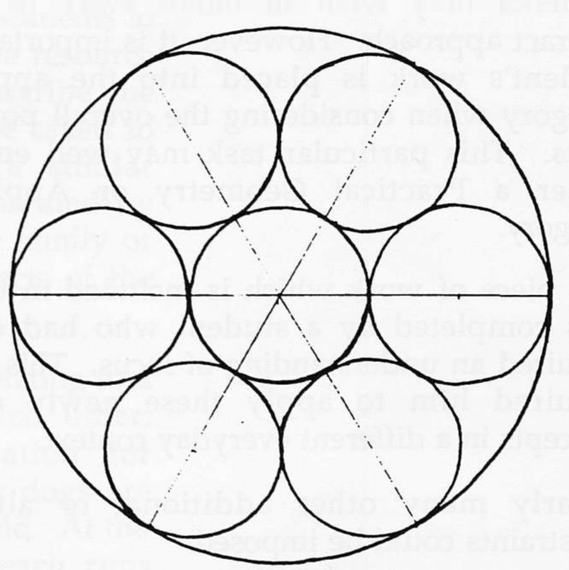


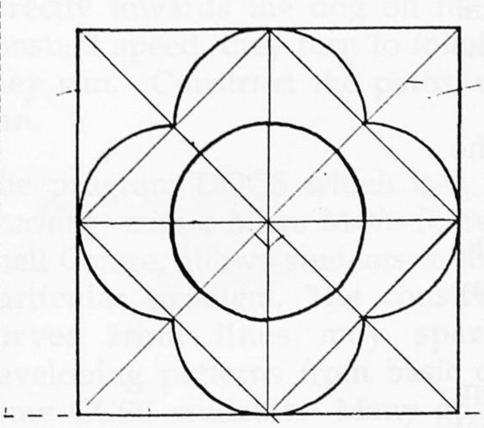
CONSTRAINTS : continued

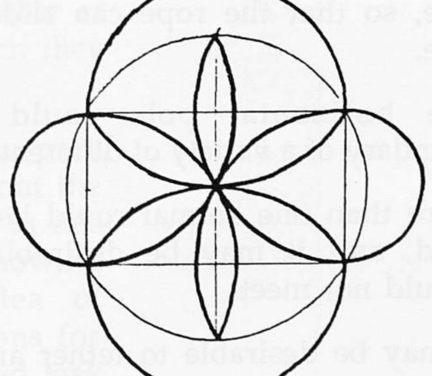












Constraints - Teacher's Notes

This task is intended to provide students with an opportunity to consider locus problems within a practical context which they may find more interesting than some of the more abstract approaches. As with the other problems in this cluster, many students will find it helpful to begin the task using real materials. After students have tried to visualise what is likely to happen, they can use a pencil tied to one end of a piece of string, which is attached to a cardboard base, to draw the boundary of the area of grass the goat can reach. Some students may need to continue to use a 'concrete' approach throughout their study. Other students may wish to move away to a more abstract approach. However, it is important that a student's work is placed into the appropriate category when considering the overall portfolio of tasks. This particular task may well end up in either a Practical Geometry or Applications category.

The piece of work which is included in this book was completed by a student who had only just acquired an understanding of locus. This problem required him to apply these newly acquired concepts in a different everyday context.

Clearly many other additional or alternative constraints could be imposed.

* The goat could be tethered to a horizontal pole, so that the rope can slide along the pole.

* The horizontal pole could form the boundary of a variety of different shapes.

* More than one animal could be tied to the shed, and it may be desirable that they should not meet.

* It may be desirable to tether animals from different points on different days, so that food is available.

* What if the goat is tethered to more than one point?

* The goat could become a guard-dog. It may be desirable to secure the dog to a point, but it may also be necessary for it to prevent access through some particular entrance to a building.

Students often impose quite ingenious constraints as they become increasingly motivated by this problem. They are also often surprised by the patterns they discover. Students may find it interesting to consider the areas and perimeters of the grass which is accessible to the goat for the shapes which have been suggested on page 35, and for shapes with more sides as they approach the limiting case of the circle.

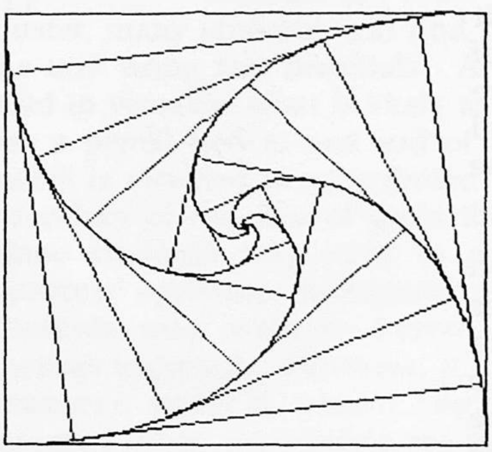
Some students may find it interesting to move on to consider curves from straight lines and curves of pursuit. One possible starter is to ask students to look at diagrams such as the ones on the resource sheets on pages 36 and 37, and to describe the shapes they can see. They could then be asked to suggest how they might construct a similar diagram. A valuable task may be generated by considering the construction of a whole family of similar curves by varying different aspects of the construction.

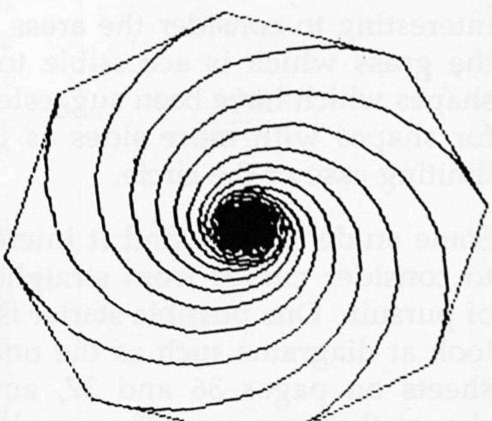
The classic 'curves of pursuit' problem relating to a number of dogs running towards each other, provides another interesting foundation for construction work. For example; four dogs are tied to posts at each corner of a square field. At the same moment they are released and each runs directly towards the dog on its left. Moving at constant speed, they turn to follow one another as they run. Construct the paths along which they run.

The program DOGS which is a part of the pack *Teaching with a Micro Maths 3*, available from the Shell Centre, allows students to look closely at this particular problem. The constructions showing curves from lines may spark the idea of developing patterns from basic constructions for some GCSE students. Many good extended task submissions have been produced simply through exploring patterns by construction, the examples on pages 38 and 39 are offered as further starters along this line.

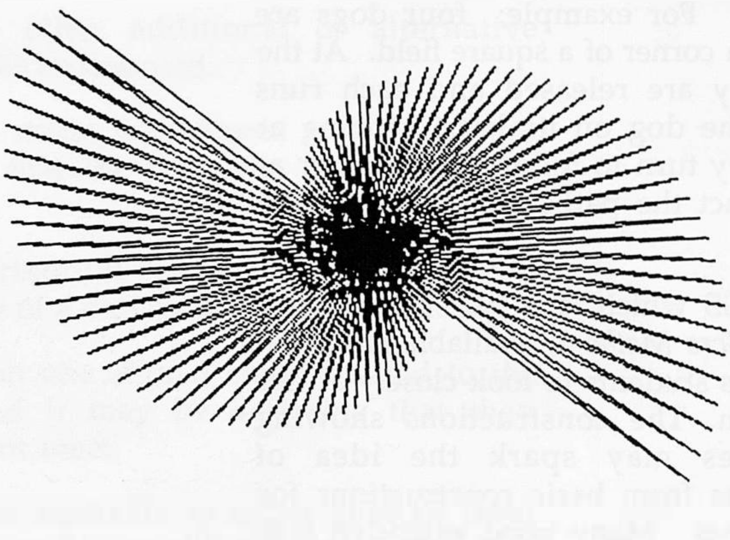
Examples of screen displays from the programs DOGS and SPLOT, both from *Teaching with a Micro Maths 3,* are shown on this and the following page.

DOGS

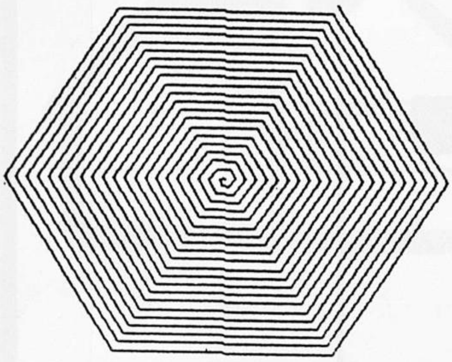


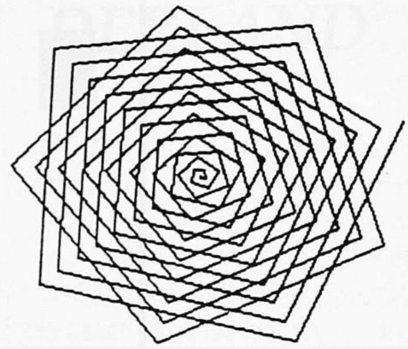


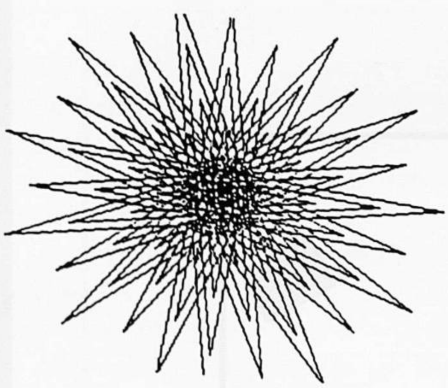
SPLOT

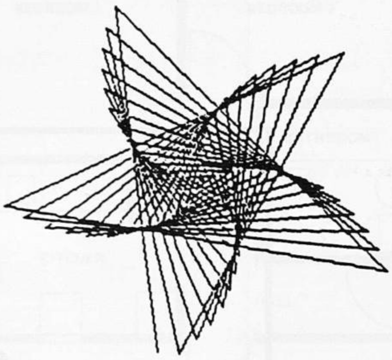


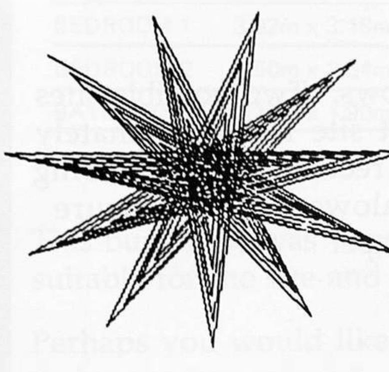
SPLOT

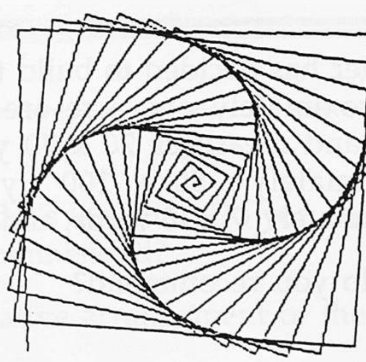




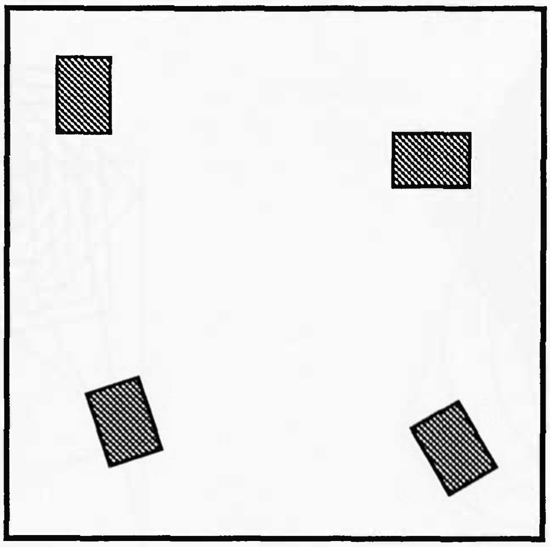


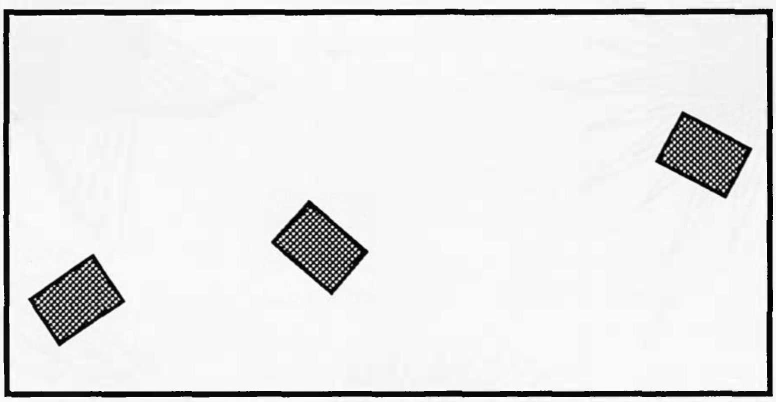






LAY IT OUT

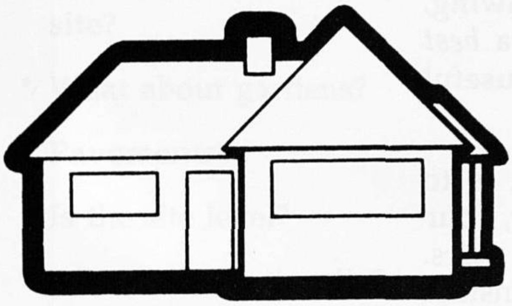




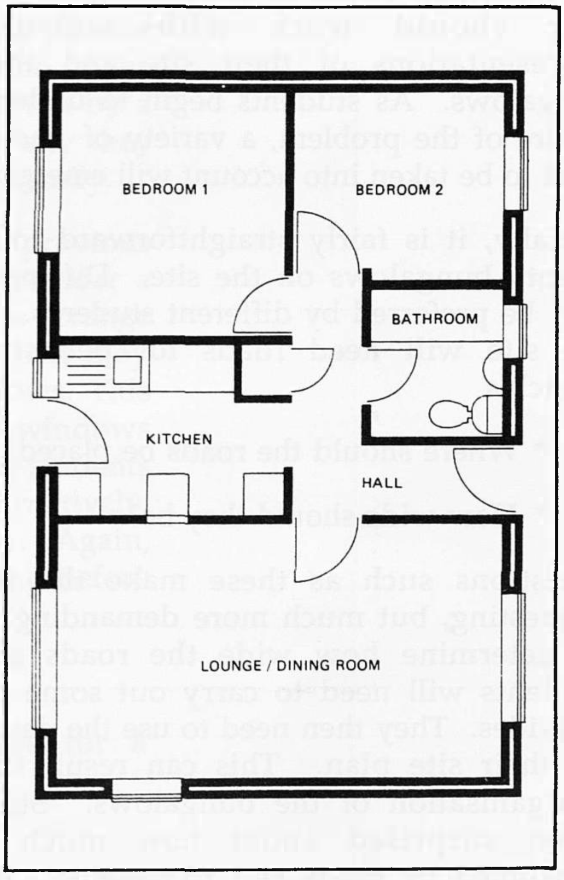
A builder has decided to build twenty small bungalows. Two possible sites of approximately one acre are available. The first site is approximately square and measures 70 x 70 yards. The other is rectangular, measuring approximately 50 x 100 yards. The bungalows will measure approximately 7 x 10 yards, and are rectangular in shape.

What do you recommend?

LAY IT OUT : continued



THE DERLAND



THE DERLAND

LOUNGE / DINING ROOM	5.74m x 3.24m	18'8" x 10'6"
KITCHEN	3.02m x 2.20m	9'9" x 7'2"
BEDROOM 1	3.02m x 3.18m	9'9" x 10'4"
BEDROOM 2	2.50m x 2.64m	8'6" x 8'2"
BATHROOM	1.71m x 1.90m	5'6" x 6'2"

This bungalow was very popular on the builder's last site. Do you think it is suitable for the site and layout you have recommended?

Perhaps you would like to suggest an alternative arrangement of the rooms inside the bungalows?

Lay It Out - Teacher's Notes

This task is designed to be tackled practically. If students approach this task using scale drawing, they will rapidly lose interest in obtaining a *best* solution, although scale drawing plays a useful role in recording the final layout.

A stimulating introduction to the problem is to organise students into small groups of, say, four students and to encourage them to work in pairs. Within each group of four, one pair could consider arranging the bungalows on the square field, while the other pair considers the rectangular field. Each pair should work with scaled cut-out representations of their site and the twenty bungalows. As students begin to understand the nature of the problem, a variety of factors which need to be taken into account will emerge.

Initially, it is fairly straightforward to place the twenty bungalows on the site. Different layouts may be preferred by different students. However, the site will need roads for pedestrians and vehicles

* Where should the roads be placed?

* How wide should they be?

Questions such as these make the task more interesting, but much more demanding. In order to determine how wide the roads should be, students will need to carry out some measuring activities. They then need to use the data gathered on their site plan. This can result in a lot of reorganisation of the bungalows. Students are often surprised about how much space is consumed by roads and pavements. Bungalows which were detached may have to become semi-detached, or even terraced. All twenty bungalows may not fit on the site. After each pair of students has reached agreement about what is possible on their site, each small group of four students should reach consensus about which site they prefer. A class discussion could try to then take place. At this stage it is useful if the bungalows have been glued on to the site, so that a rough site plan can be displayed to support the students reasoned arguments in favour of particular arrangements. Other important factors may emerge during this discussion, such as

* How does traffic flow, or gain access to the site?
* What about gardens?
* Pavements?
* Is the site level?
* What borders the site?
* What about car parking?

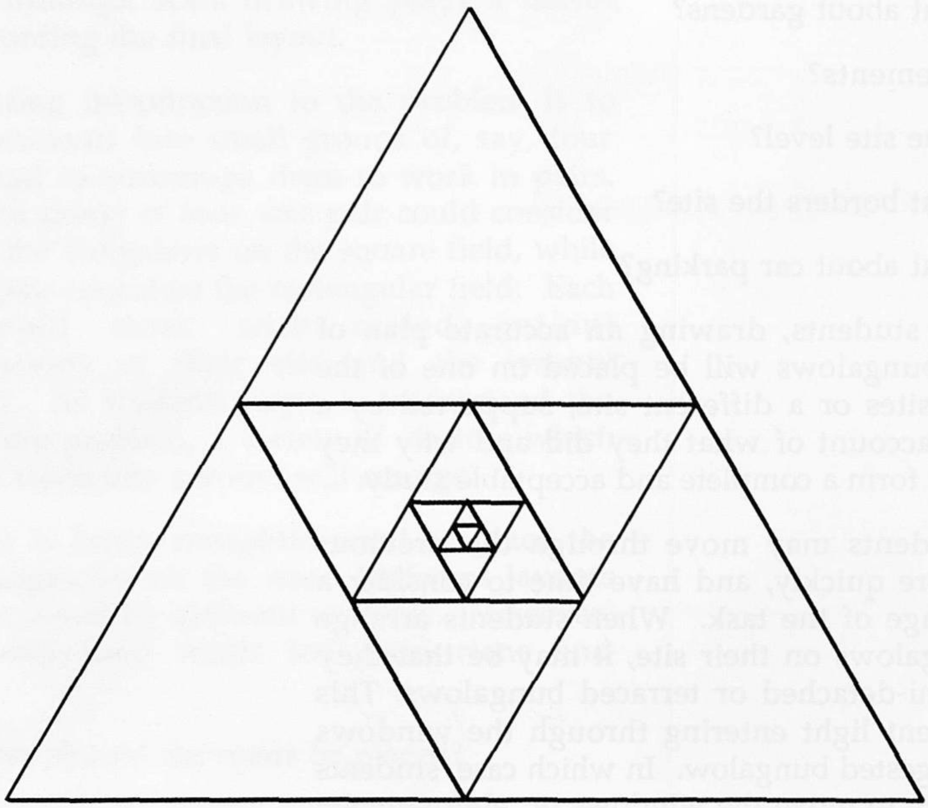
For some students, drawing an accurate plan of how the bungalows will be placed on one of the available sites or a different site, supported by a reasoned account of what they did and why they did it, will form a complete and acceptable study.

Other students may move through the previous stages more quickly, and have time to consider a second stage of the task. When students arrange their bungalows on their site, it may be that they create semi-detached or terraced bungalows. This may prevent light entering through the windows of the suggested bungalow. In which case, students may decide to move the windows or, alternatively, to reorganise the inside of the bungalow. Again, students may need to do some measuring before they can answer questions such as

* How small can a bathroom be?
* What is the most efficient shape for a kitchen?
* Drainage positions
* Economical building and overall costs

Thinking about these questions introduces a variety of interesting problems, each of which requires students to do some accurate measuring, planning and drawing activities. Some students may like to encompass all their ideas in attempting to lay out such a site on their local play area or a part of the school site, although let us all hope that such situations do not become reality.

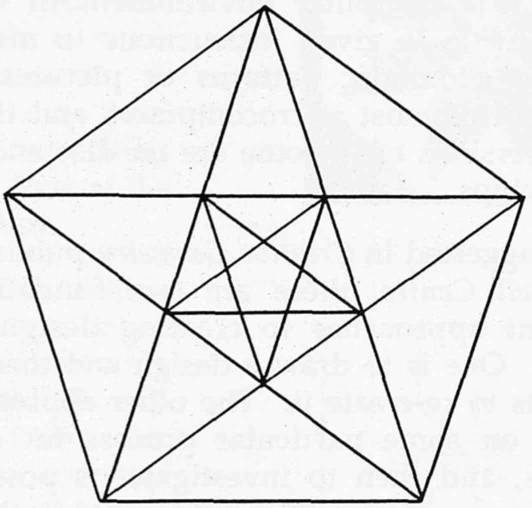
NESTED POLYGONS

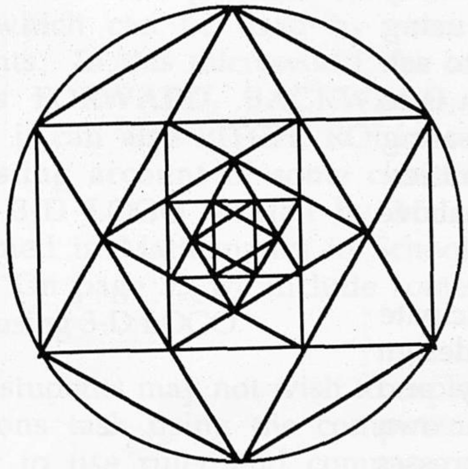


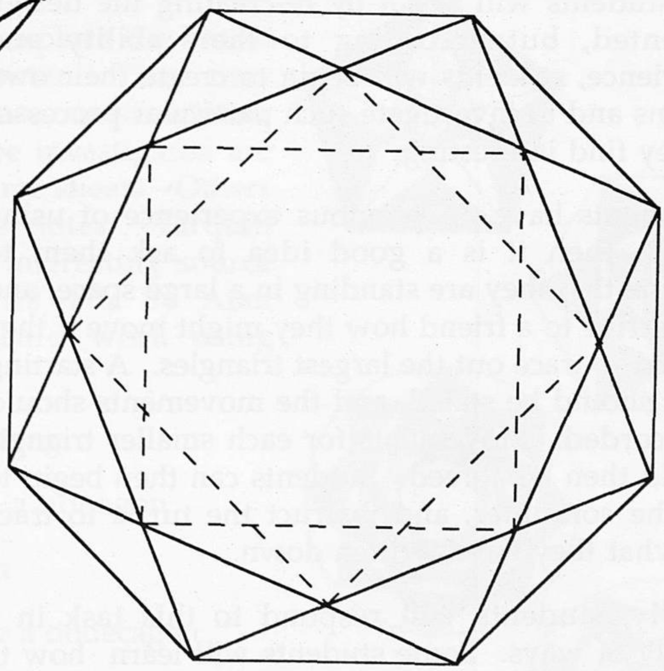
These are nested equilateral triangles

* Can you produce this diagram using Logo?
* Investigate further
* Extend the problem
* Generalise

NESTED POLYGONS : continued







You may prefer to investigate nested polygons using ruler and compass constructions.

You may wish to use some of your designs to produce interesting posters.

Nested Polygons - Teacher's Notes

LOGO is a computer environment in which a screen turtle is given instructions to move and produce geometric patterns or pictures. It is available for most microcomputers and there are many versions of it, some are on disc and others are on chips.

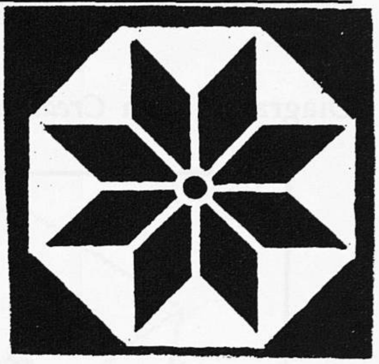
As is suggested in *Creative Geometry* published by *The Shell Centre*, there are two fundamentally different approaches to creating designs using LOGO. One is to draw a design and then to ask students to re-create it. The other approach is to decide on some particular process for creating designs, and then to investigate its possibilties. The designs on page 52 are discussed in the above mentioned book.

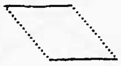
Through using the introductory task, we anticipate that students will begin by re-creating the design presented, but according to their ability and experience, students will begin to create their own designs and to investigate such particular processes as they find interesting.

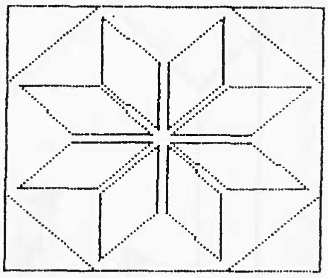
If students have no previous experience of using LOGO, then it is a good idea to ask them to imagine that they are standing in a large space, and to describe to a friend how they might move if they wanted to trace out the largest triangles. A starting point should be stated, and the movements should be recorded. Movements for each smaller triangle should then be agreed. Students can then begin to use the computer, and instruct the turtle to trace out what they have written down.

Clearly, students will respond to this task in a variety of ways. Some students will learn how to create procedures and to use variables, although other students may already have acquired such skills. The piece of student's work which is included in this book was completed by a student who had very little previous knowledge of LOGO. For her, the completion of the task was a genuine learning experience.

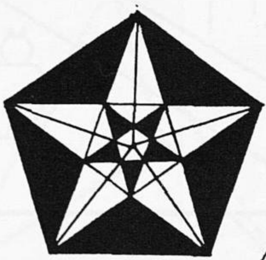
Students who are already familiar with LOGO procedures, and the use of variables, may find it stimulating to begin by using our starter and then to look around their own environment for some design or pattern in, say, their local church or perhaps a piece of Greek pottery or a floor tile. Such objects can provide them with a more challenging task of some personal interest. A delightful starter for tasks using the local environment is the *Source Book of Ideas Using LOGO* which has been produced by Reg Eyre at the College of St Paul and St Mary, Cheltenham.

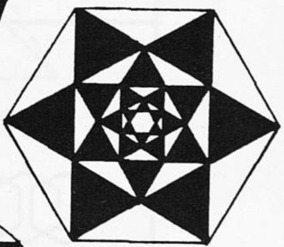




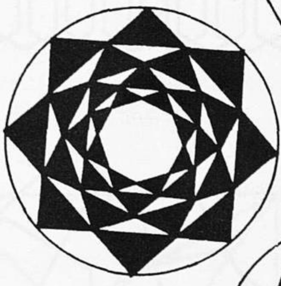


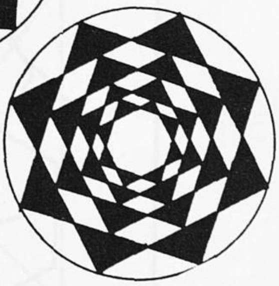
LOGOTRON now produces a 3-D LOGO extension disc which can be used by more experienced students. In this microworld, the turtle not only moves FORWARD, BACKWARD, RIGHT and LEFT, it can also PITCH, ROLL and YAW. An interesting account of some classroom activities using 3-D LOGO written by Malcolm Petts, is contained in Mathematics in Schools, November 1988. On page 53 we include some examples of work using 3-D LOGO.





Some students may not wish to explore the nested polygons task using the computer. They may prefer to use ruler and compass constructions. Some examples of what could be investigated are presented on the student's resource sheets. Others are included in these teacher's notes. Tarquin Publications have produced an interesting source of such ideas in *The Geometrics File* by Alan Wiltshire. Examples of students' work using colour are included here.



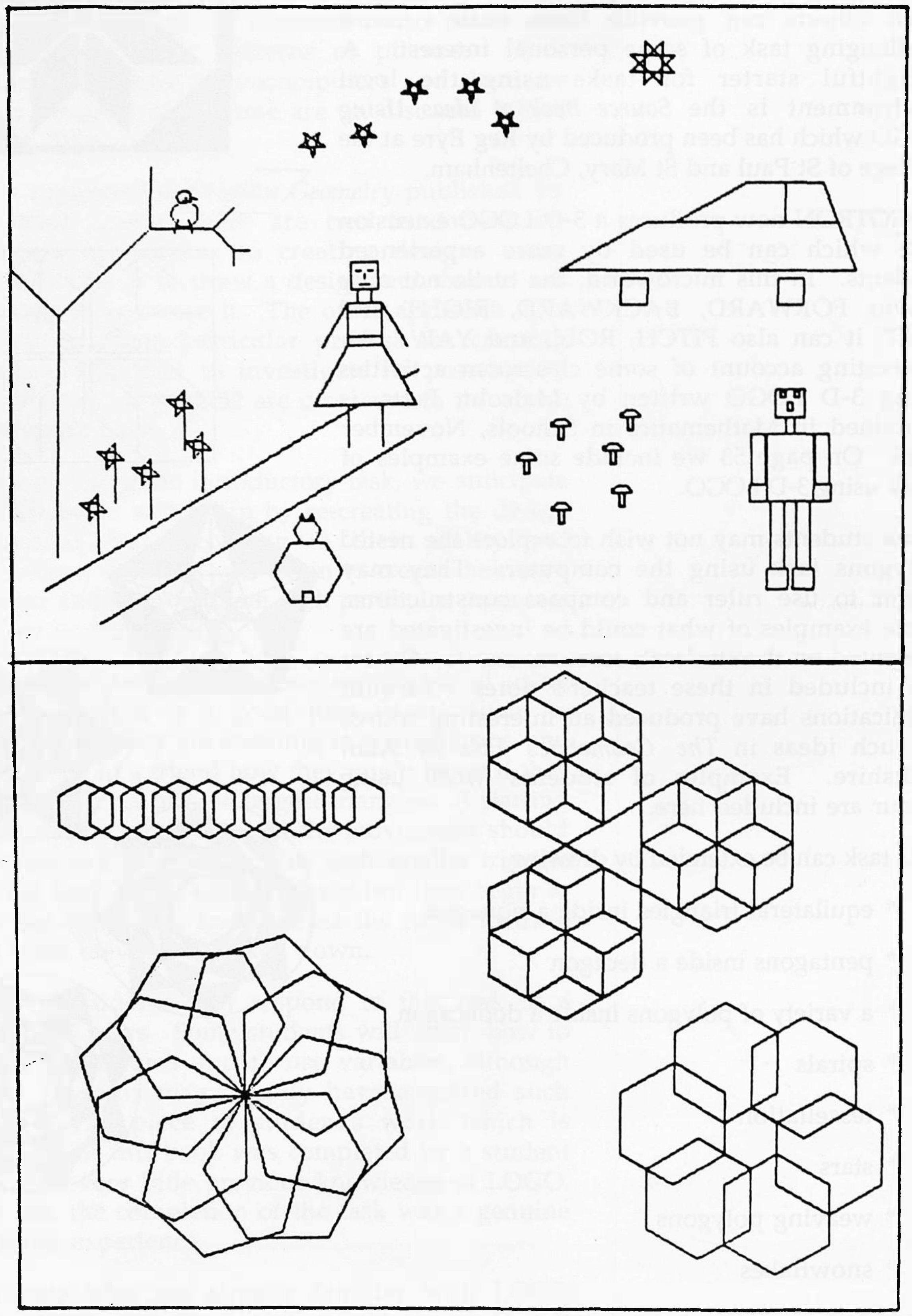


This task can be extended by drawing

- equilateral triangles inside a nonogon
- pentagons inside a decagon
- a variety of polygons inside a dodecagon
- spirals
- tessellation
- stars
- weaving polygons
- snowflakes

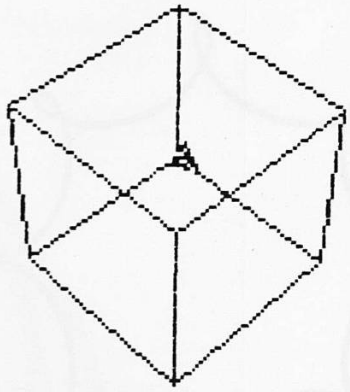
3D LOGO is available from LOGOTRON LTD, Cambridge

Diagrams from *Creative Geometry*

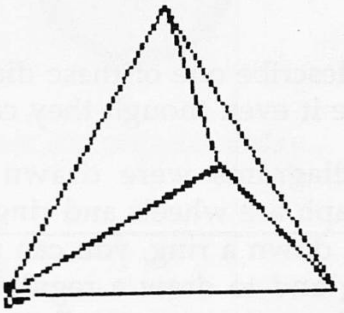


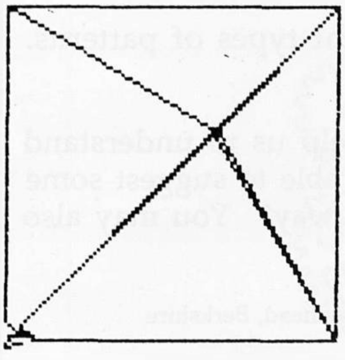
3-D LOGO

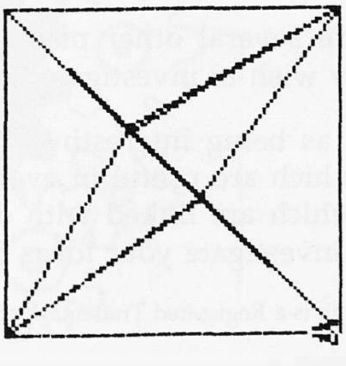
```
TO SQUARE
REPEAT 4 [FD 200 LT 90]
END
TO TRIANGLE
REPEAT 3 [FD 350 LT 120]
END
```



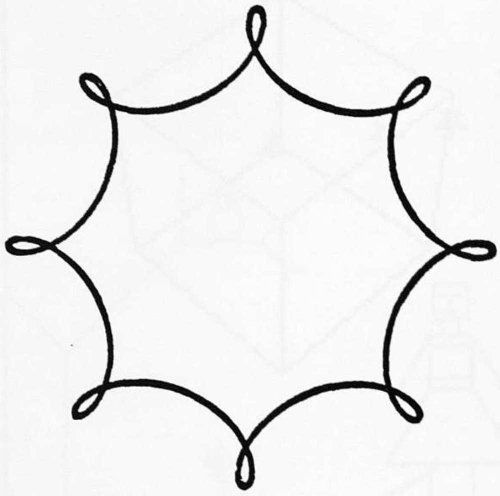
```
TO TETRAHEDRON
REPEAT 3 [RO 70.46 TRIANGLE FD 350 RO
        -70.46 LT 120]
END
TO SQUAREPYRAMID
REPEAT 4 [RO 54.5 TRIANGLE FD 350 RO
        -54.5 LT 90]
END
TO OCTAHEDRON
SQUAREPYRAMID
RO 180
LT 270
SQUAREPYRAMID
END
```

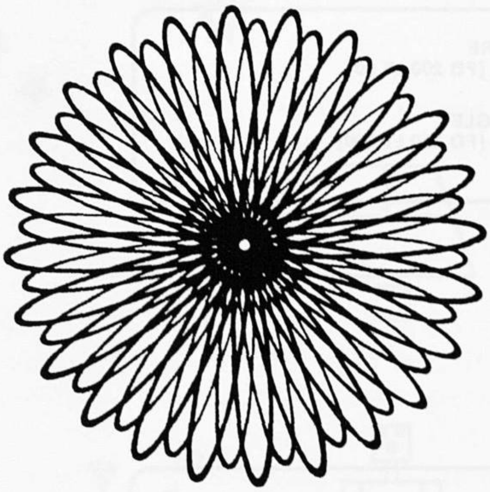






GET INTO GEAR





Try to describe one of these diagrams to a group of friends, so that they can visualise it even though they cannot see it.

These diagrams were drawn using Spirograph™. The basic pieces of Spirograph are wheels and rings. Both of these pieces have cogs or teeth. If you pin down a ring, you can use a pen to roll a wheel around the inside of the ring and to draw a regular pattern. The pattern produced depends on which ring you use, as well as which wheel you choose.

After you have drawn some designs using the inside of the ring, why not roll your wheel around the outside of the ring? What do you notice?

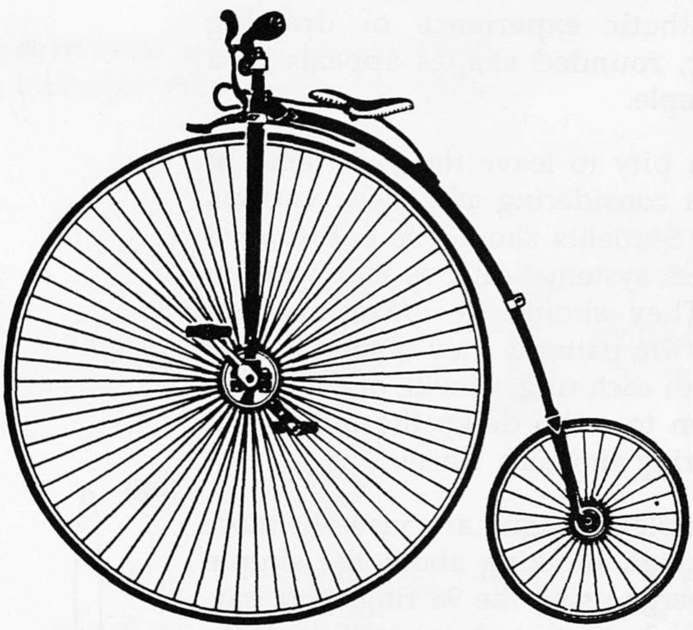
Spirograph also contains two racks; these look like rulers with curved ends. Why not experiment using these? What happens? Can you explain why it happens?

There are several other pieces which produce different types of patterns. You may wish to investigate these.

As well as being interesting, pastimes like this can help us to understand things which are useful in everyday life. You may be able to suggest some things which are linked with drawing patterns in this way? You may also wish to investigate your ideas further.

™ Spirograph is a Registered Trademark of Kenner Parker Toys Inc., Maidenhead, Berkshire

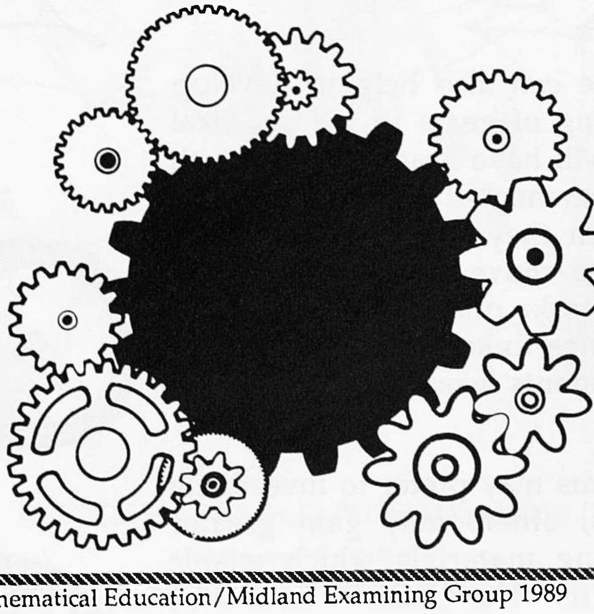
GET INTO GEAR : Continued



Early bicycles were very different from those that are popular today.

* In what ways have they changed?
* Can you explain why they have changed?

Perhaps you would like to investigate how other things which use gears work.

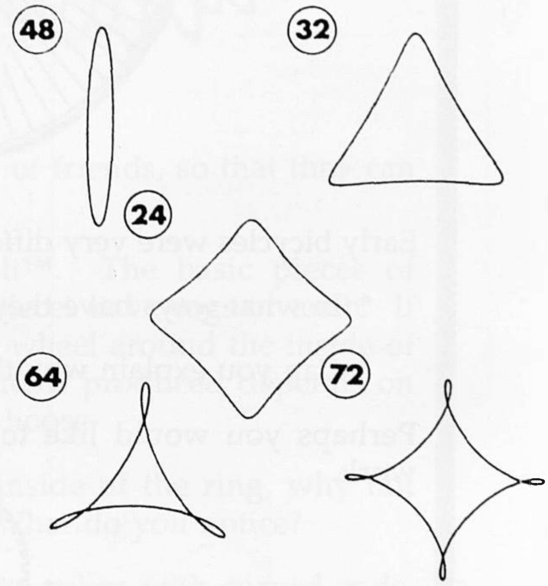


Get Into Gear - Teacher's Notes

Spirograph is a pastime which has delighted children of all ages for a considerable number of years. The aesthetic experience of drawing attractive, regular, rounded shapes appeals to a wide variety of people.

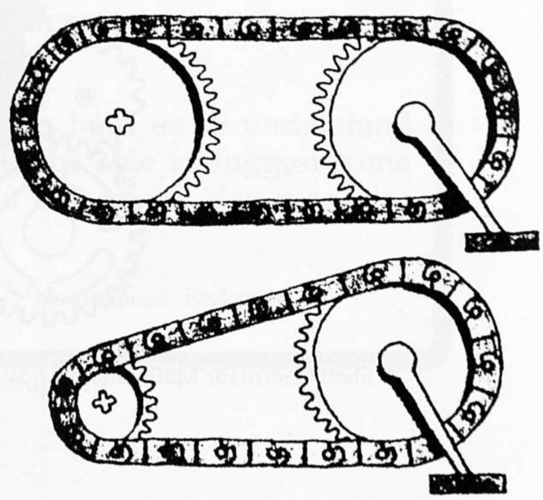
But it would be a pity to leave the experience at this level, without considering *why* these various patterns emerge. Students should be encouraged to approach the task systematically as they work in small groups. They should be encouraged to describe in words the patterns they obtain as they use each wheel with each ring. It will, of course, be necessary for them to write down the number of cogs around each ring and each wheel.

Although the complex designs are visually more exciting, it is through thinking about the simple designs which emerge using the 96 ring with, say, the 48, 32, 24 wheels that students will come to terms with *why* particular designs appear. This then makes it possible for students to try to predict what sort of pattern is likely to emerge when they use particular combinations of wheels and rings.



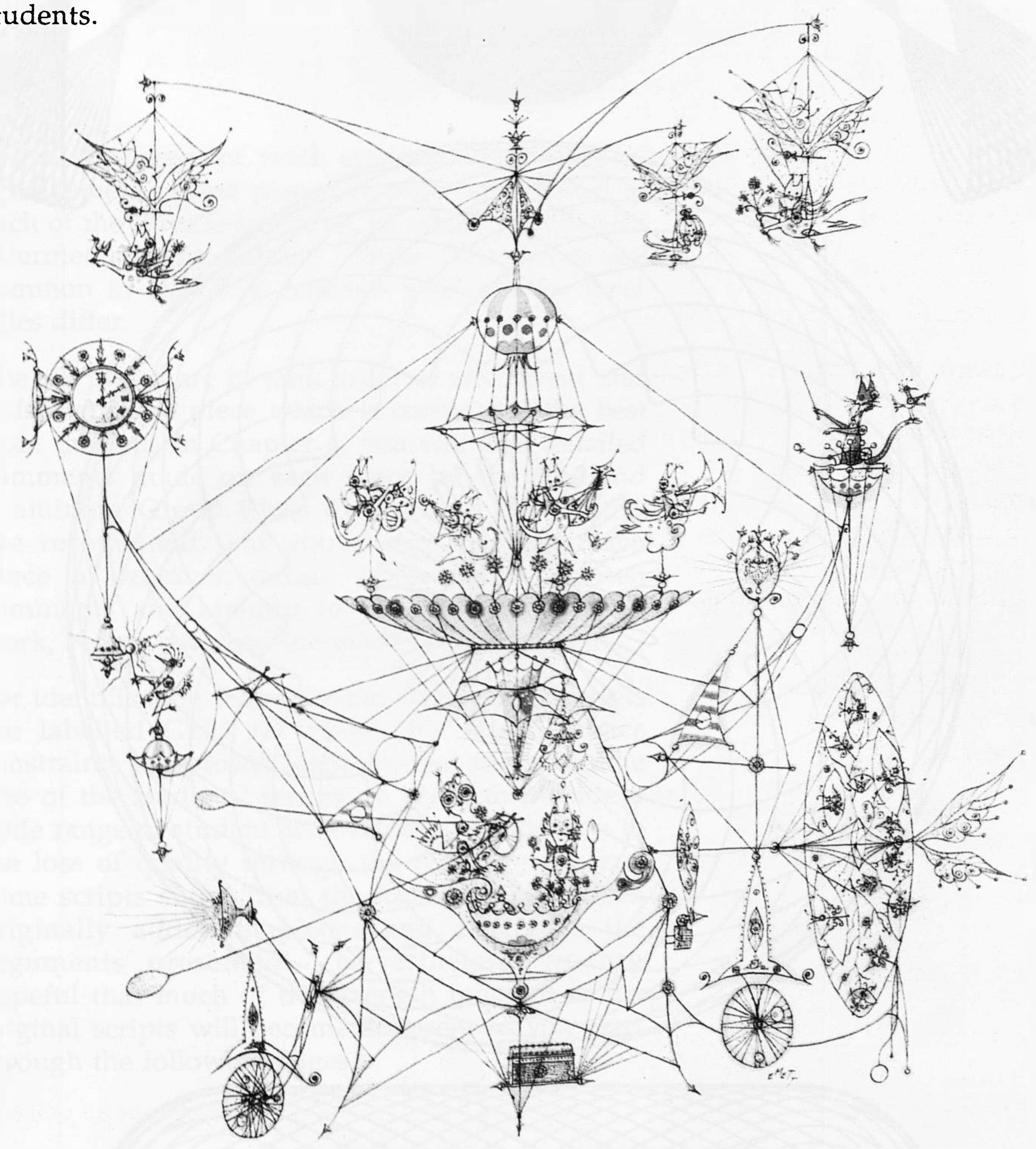
A less well known drawing aid is CYCLOGRAPH™, which is also produced by Kenner Parker. This also seems to us to offer some interesting possibilities.

This visual experience can also help to develop students' understanding of gears in the practical world. All students will have heard of bikes with gears, but not all of them will understand what actually happens when they change gear. Some students may wish to move quickly on to the second part of this task: others may prefer to concentrate on drawings linked with the starting task. Examples of students' work are included on the opposite page.



Although some students may prefer to investigate actual real-life gears, others may gain greater insights through using materials which enable them to model real-life. The LEGO™ TECHNIC Kits which are now widely available are extremely useful aids as students develop their understanding of a wide variety of situations using pulleys, gears, chain transmission, gear transmission, and worm transmission. Designing a walking robot is an interesting challenge for students.

A working model submitted with a mathematical report may be a suitable submission for some students.

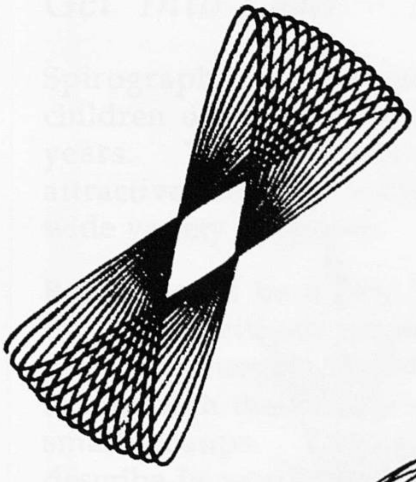


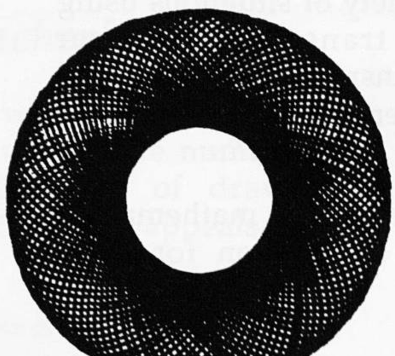
"C/C (CAT'S CRADLE) PUSSIEWILLOW III"

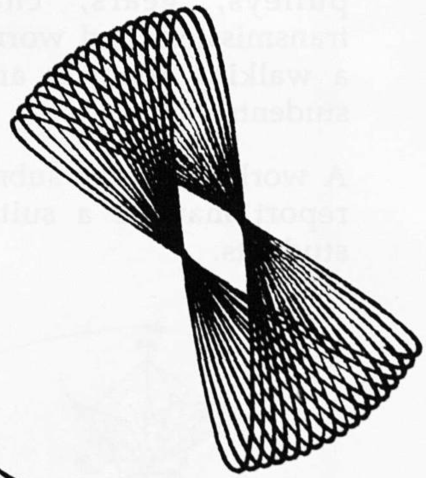
by Rowland Emett O.B.E.

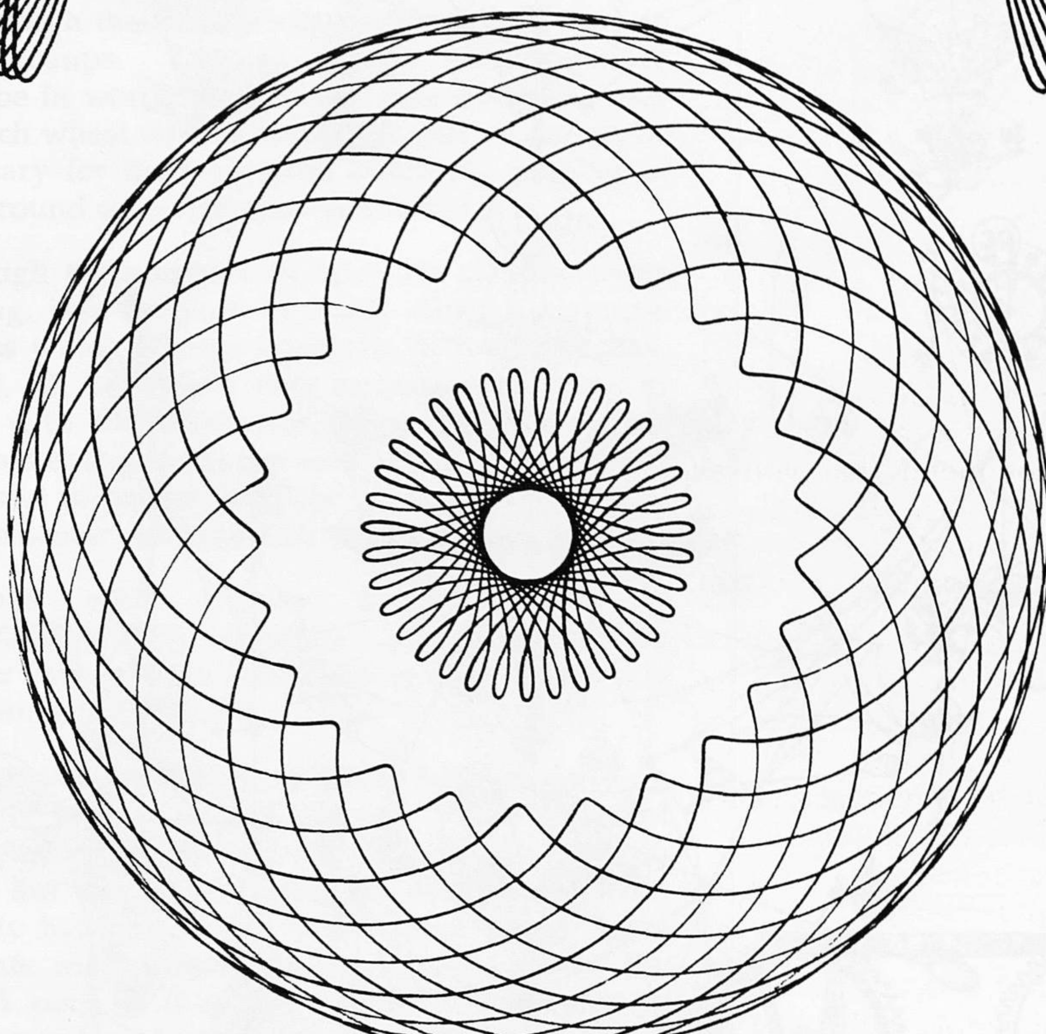
THE EASTGATE SHOPPING MALL, BASILDON, ESSEX.

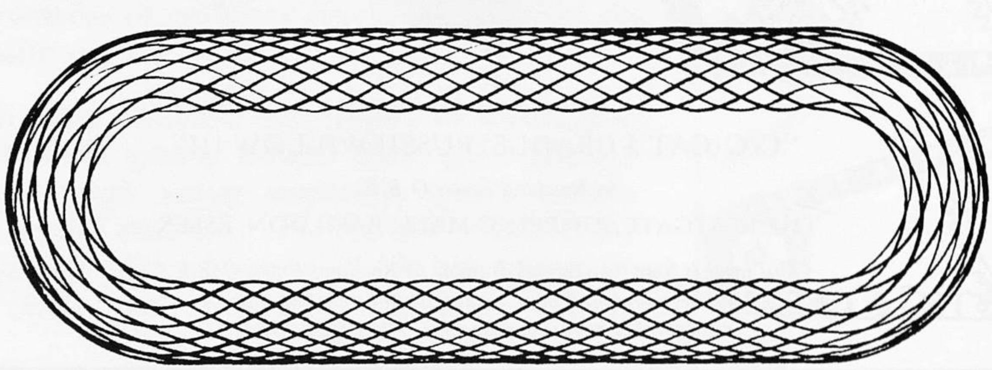
This print is from the original drawing by Rowland Emett O.B.E. for his twenty feet high, moving sculpture which was inaugurated by Michael Bentine, 7th August 1981.











5 Students' Work

These six pieces of work cover a wide range of achievement. Two pieces of work are offered at each of the three levels of GCSE study; Foundation, Intermediate and Higher. These three levels are common to all GCSE schemes although the level titles differ.

The six pieces are in rank order of attainment and finish with the piece which is considered the best from the set. In Chapter 6, you will find detailed comments made on each piece by the Midland Examining Group Chief Coursework Moderator. We recommend that you should consider each piece of work in detail, make a few written comments and attempt to grade each student's work, before you read the moderator's comments.

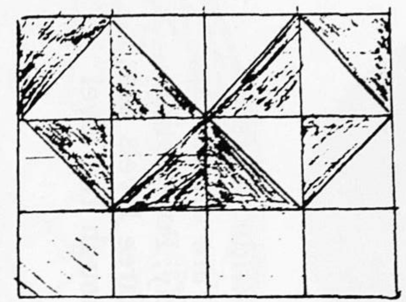
For identification purposes, the six student's scripts are labelled G2/1 to G2/6. Because of space constraints the project team decided to reduce the size of the students' scripts, in order to include a wide range of student achievement. In addition to the loss of quality through the reduction in size, some scripts suffer from the loss of colour which originally added emphasis and clarity to the arguments presented. Nevertheless, we are hopeful that much of the strength inherent in the original scripts will become apparent as you read through the following pages.

G2/1

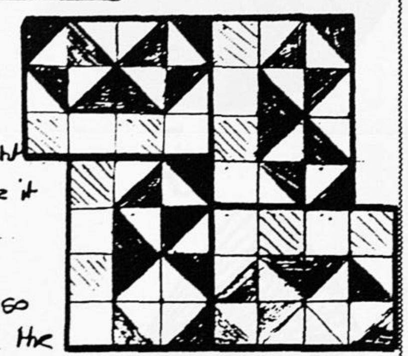
POSTER 88

Poster 88 G.C.S.E.

Our problem was to design a poster using the techniques that we have learnt e.g. translation, tessellation, reflection and rotation. In my design I had rotation and tessellation I did it in black and white as I think this lookes effetif effectif.

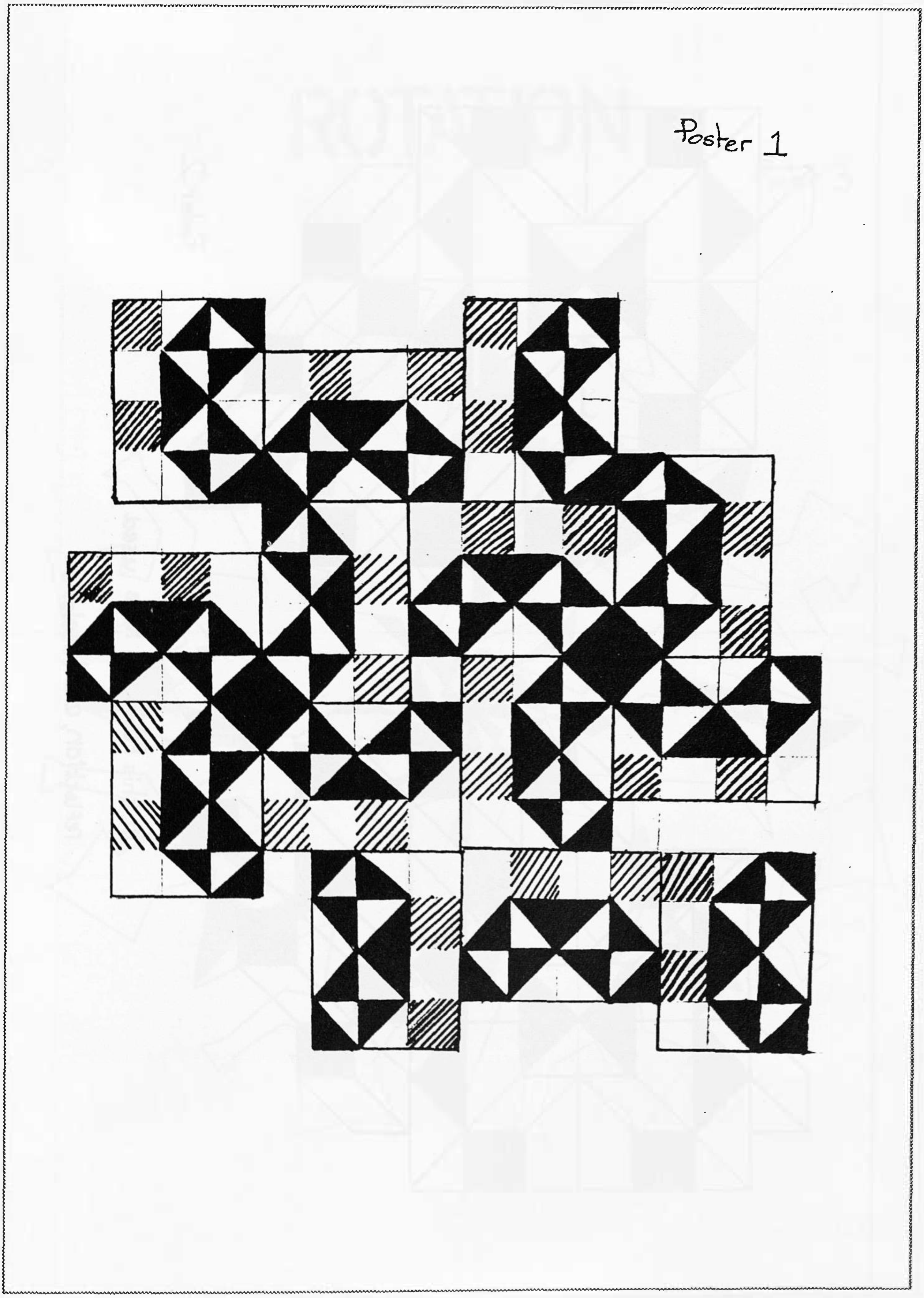


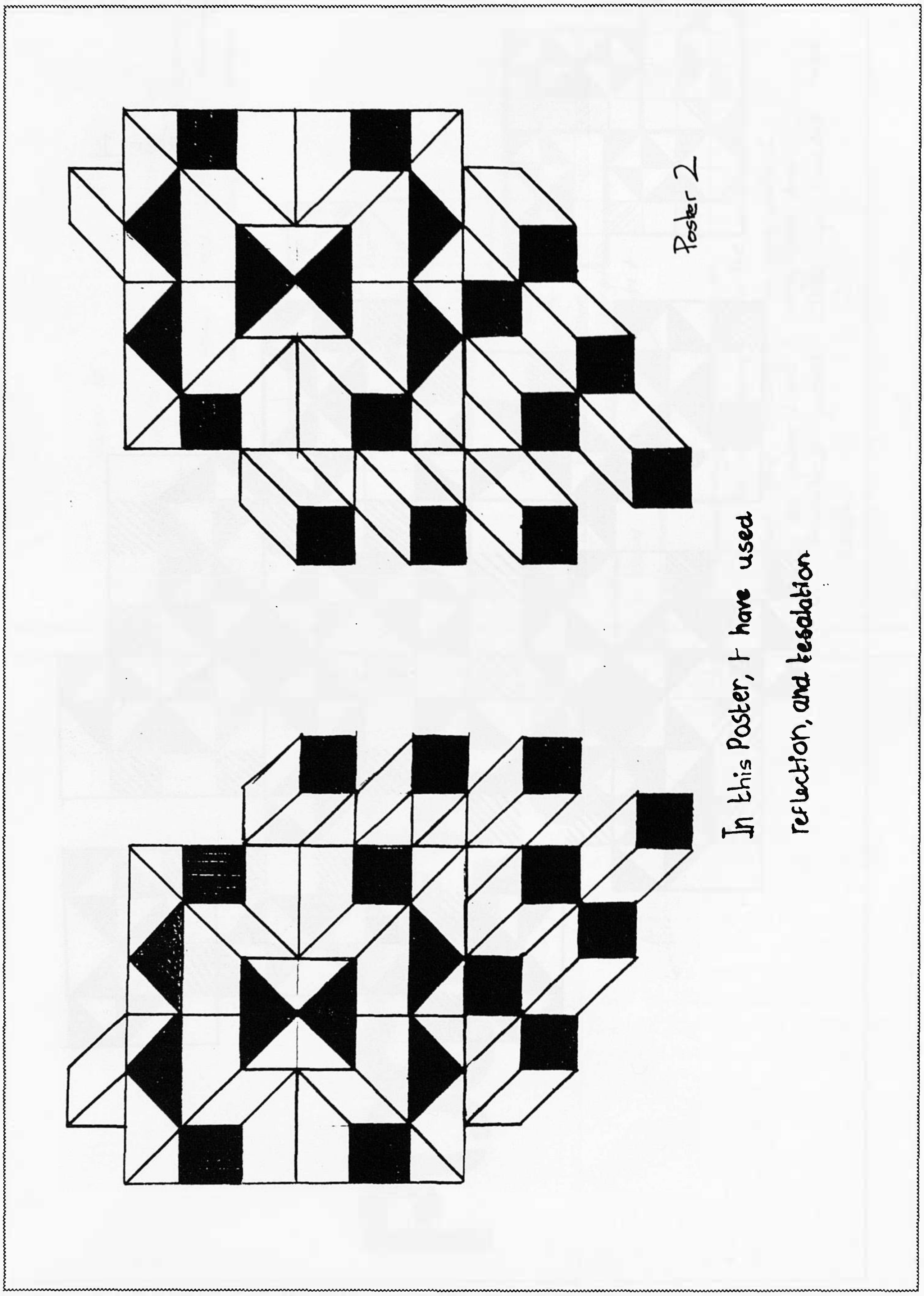
When I desiged this I was just playing around with the lines when I got this shape and I decided I liked it. I thorght I would add the lines to make it look more.

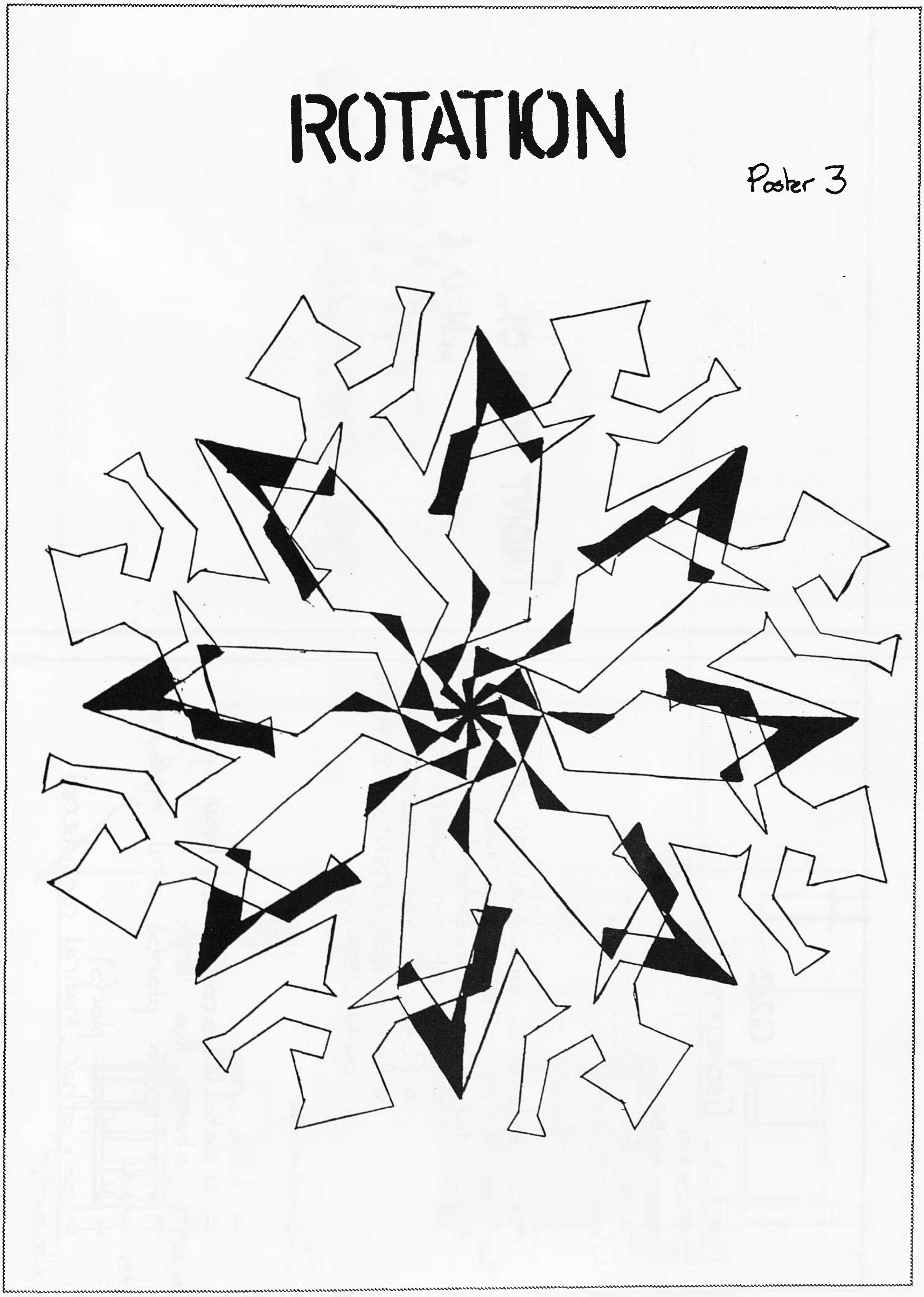


I rotated my by 90 degrees so I got the same shape all the way but keep rotating it. This was poster 1. Then I did posters 2 and 3 using other ideas.

Excellent posters. Write up needed more detail.


Poster 1


Poster 2
In this Poster, I have used
reflection, and tesalation


ROTATION
Poster 3

G2/2

PRACTICAL GEOMETRY
SURVEYING
PLANOF HOUSE
HOUSE X
OBJECT

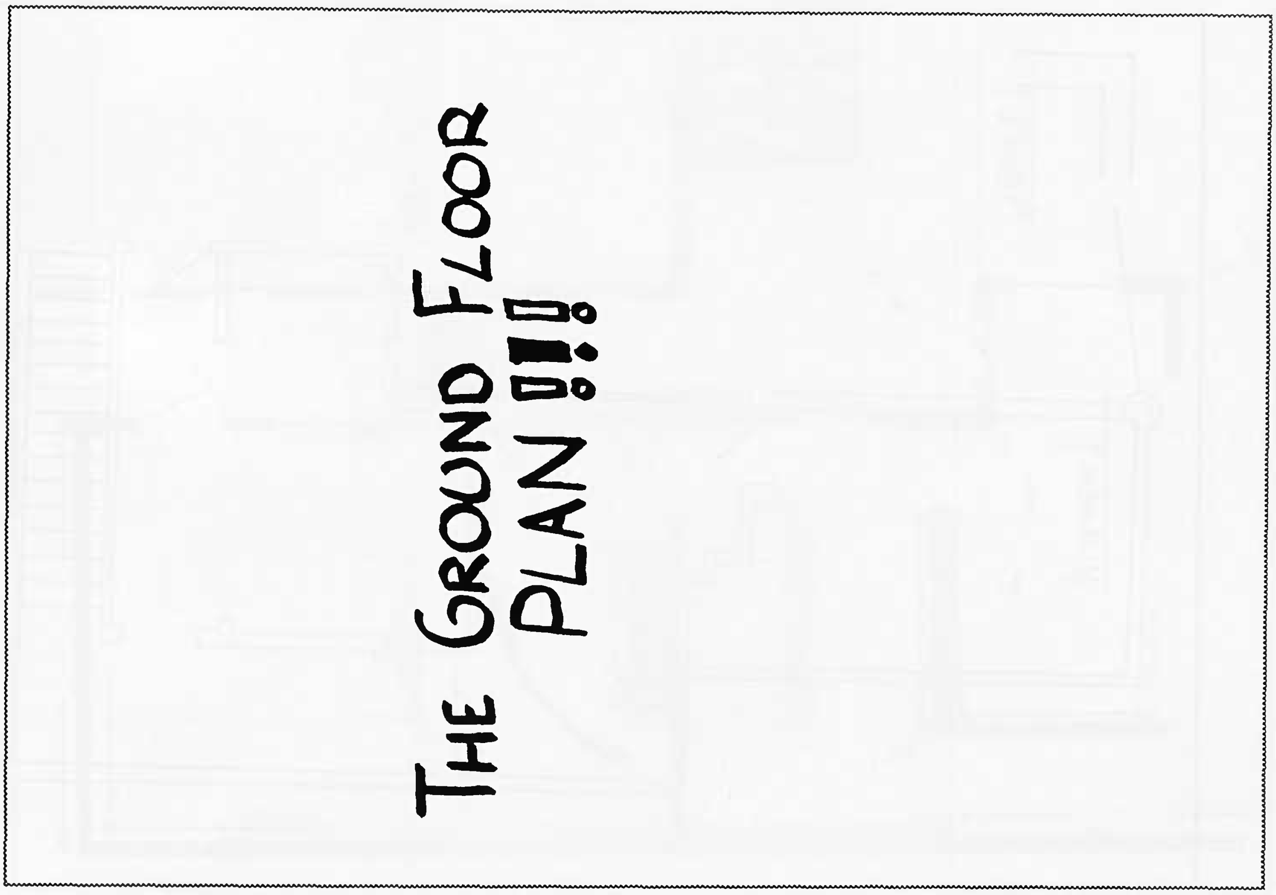
- to peduce a groundfloor plan of the house in which you live.
- presintation may be extended by adding
  - o PLAN OF THE GARDEN.
  - Δ PLAN OF THE STREET.
  - ◇ ANY OTHER EXTENTION OF YOUR OWN CHOICE.

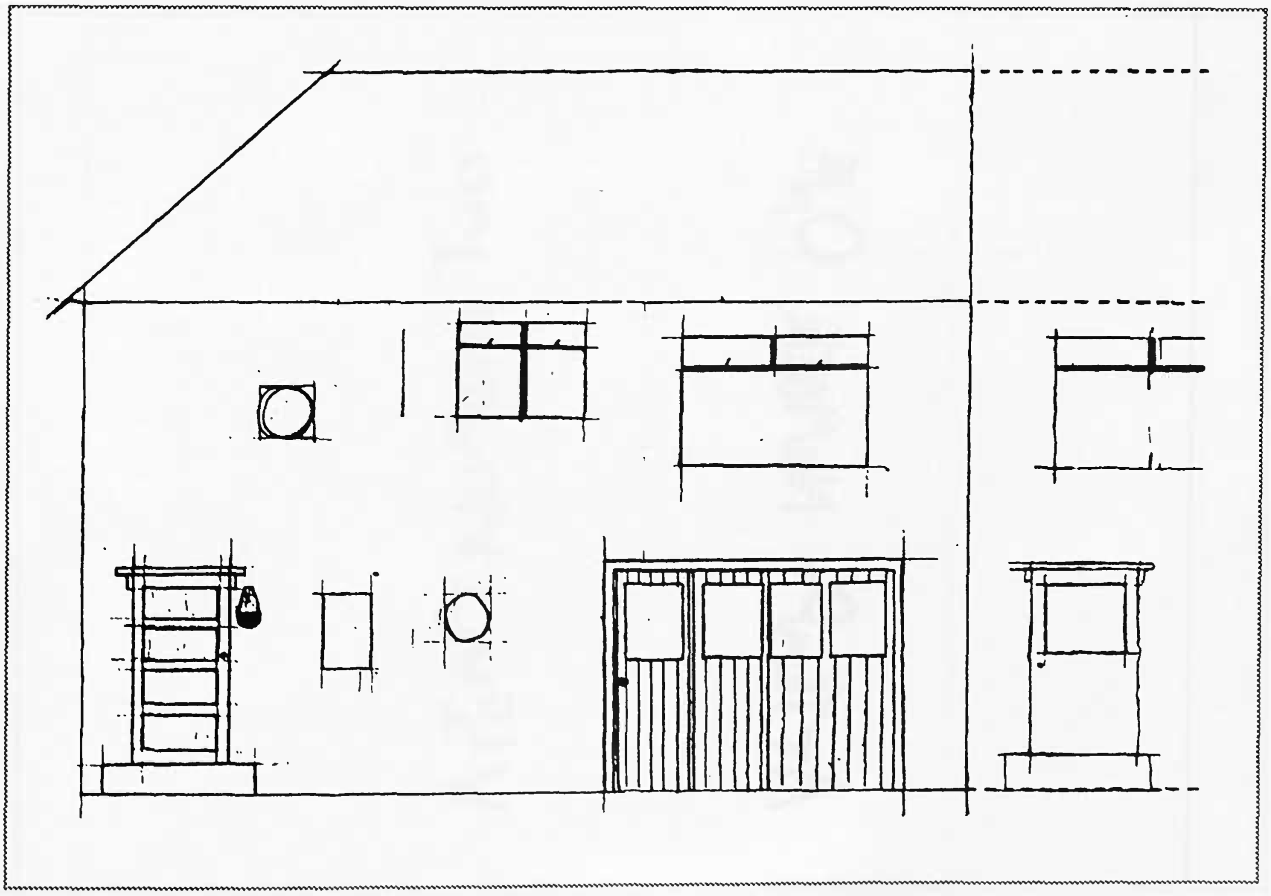
PRESINTATION

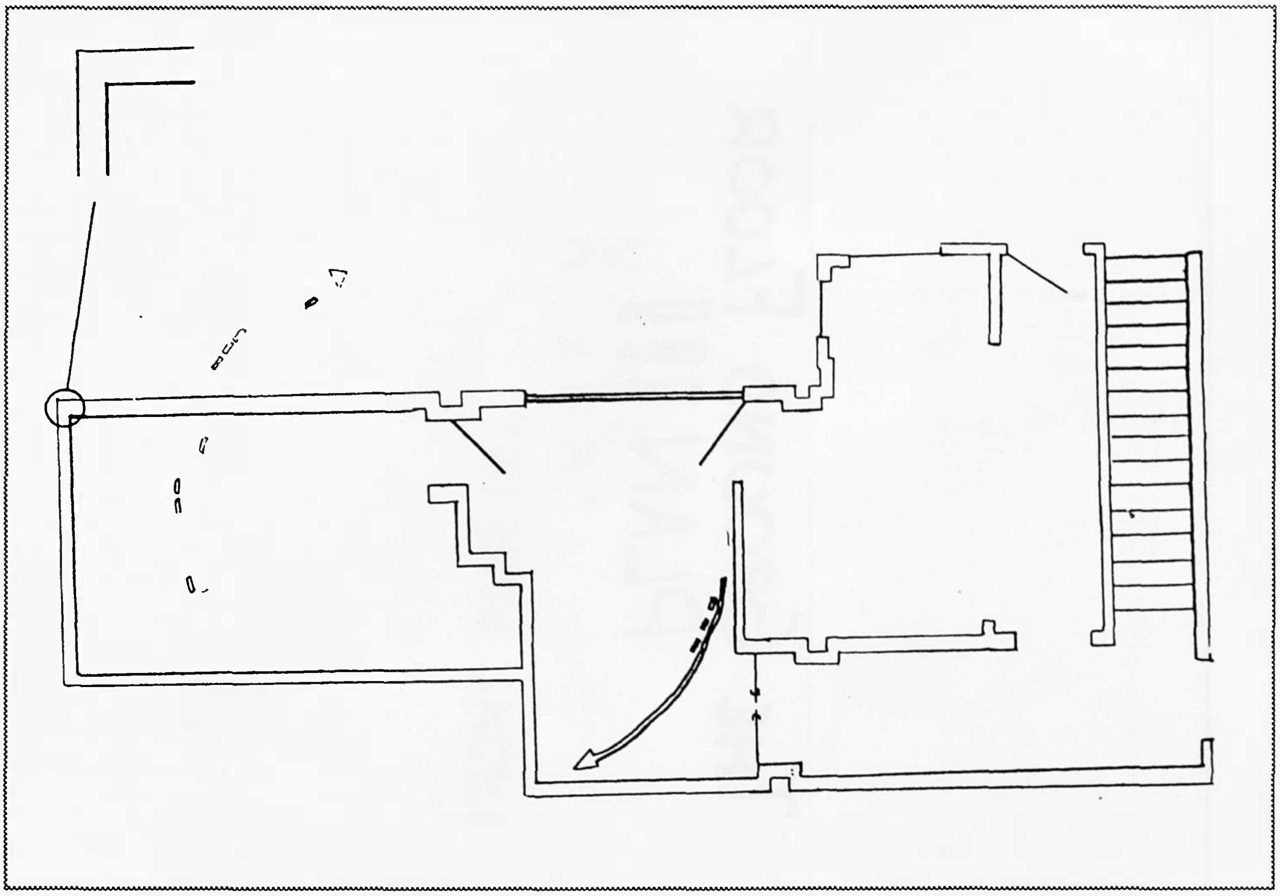
- title page
- a detailled acount of how you went abont the task
- the rough plan(s) and measurements made
- the final plan(s)
- any other matiral considered relevant.

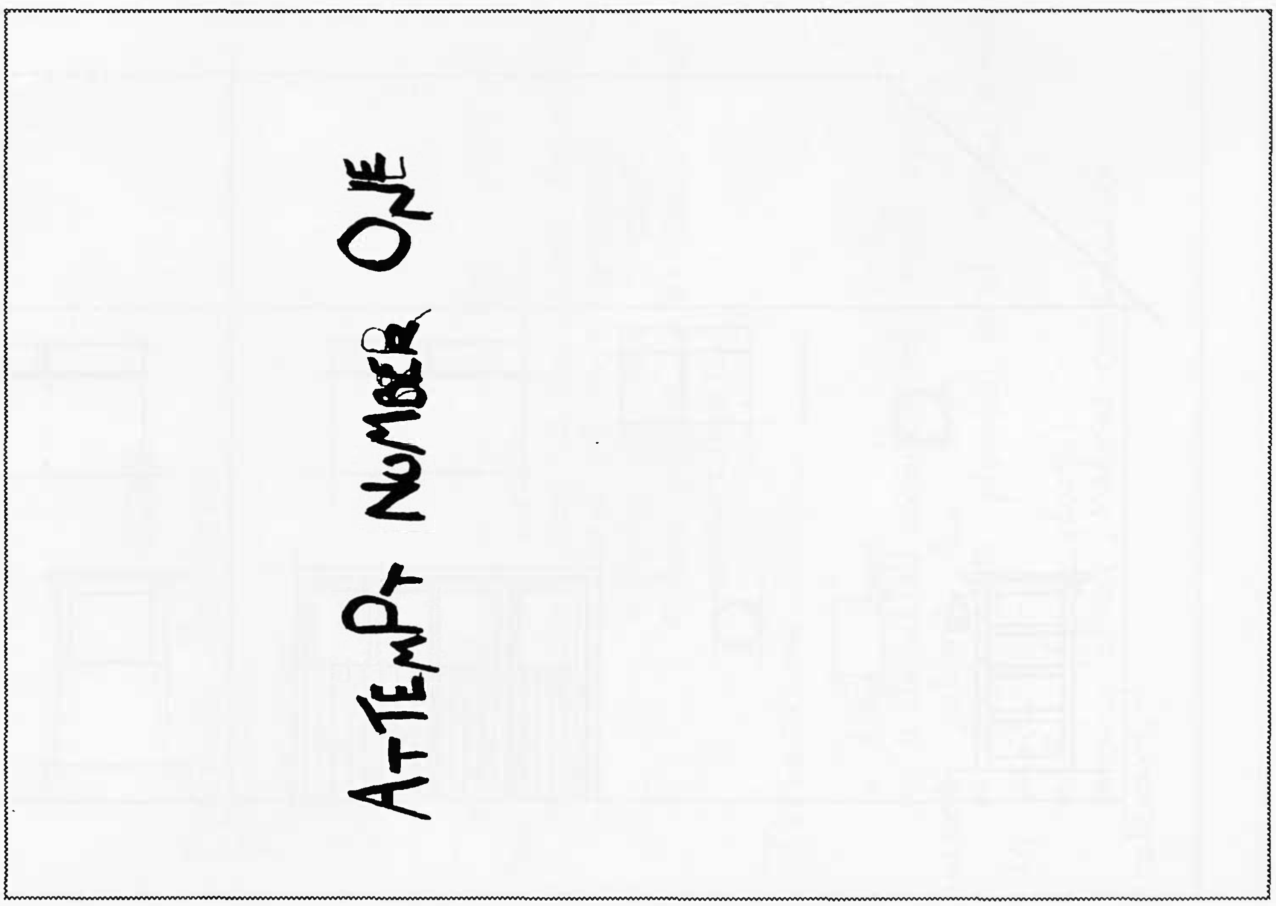
FRONT PLAN OF..

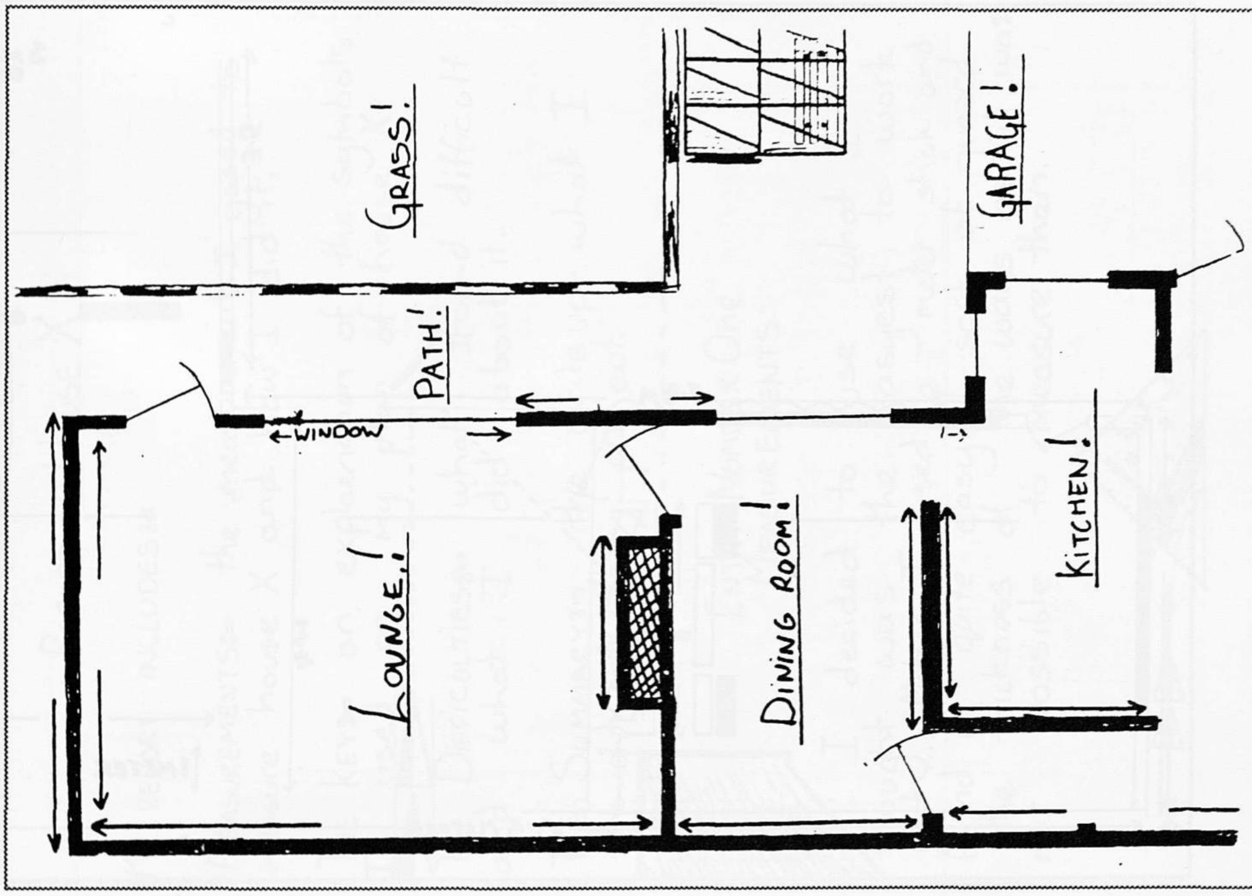
..HOUSE X


THE GROUND FLOOR
PLAN!!!

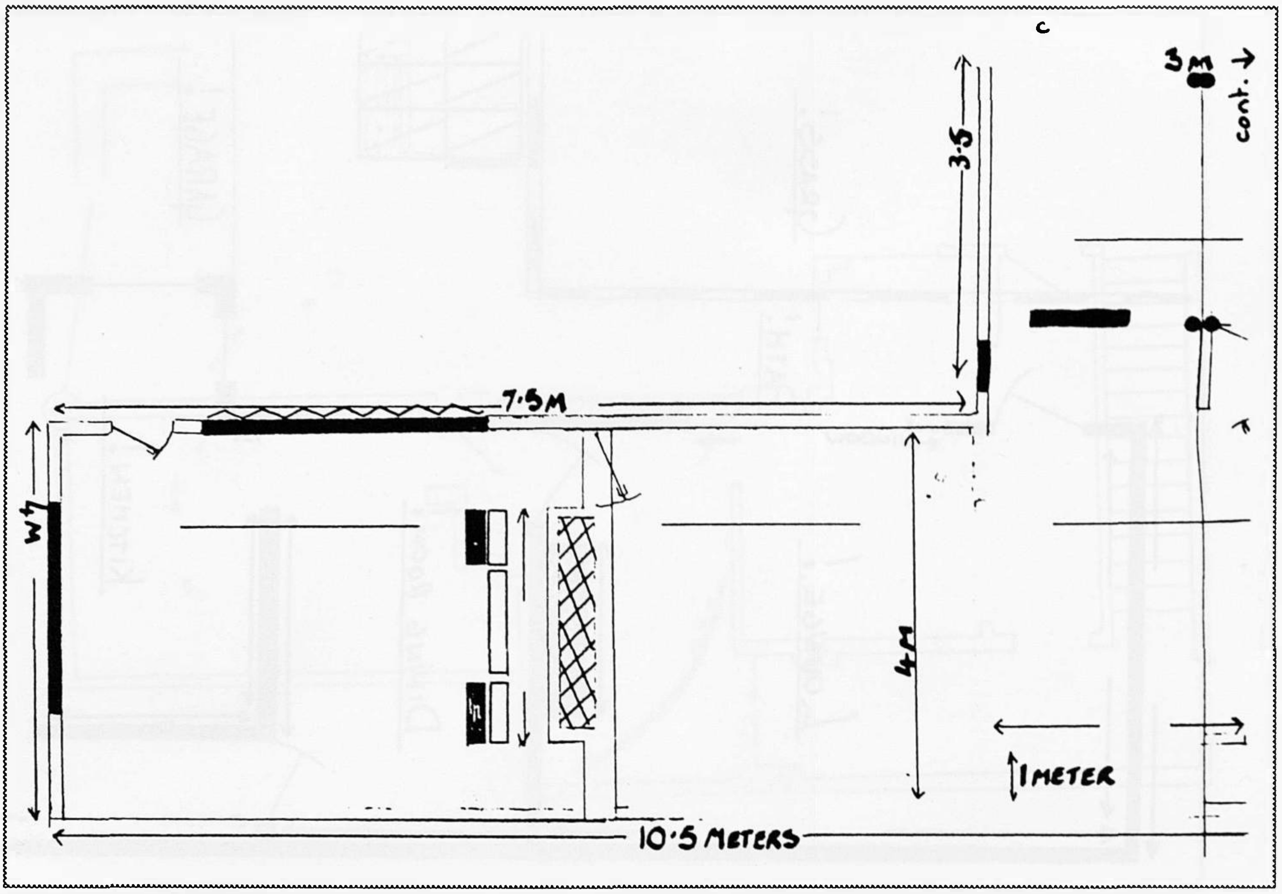


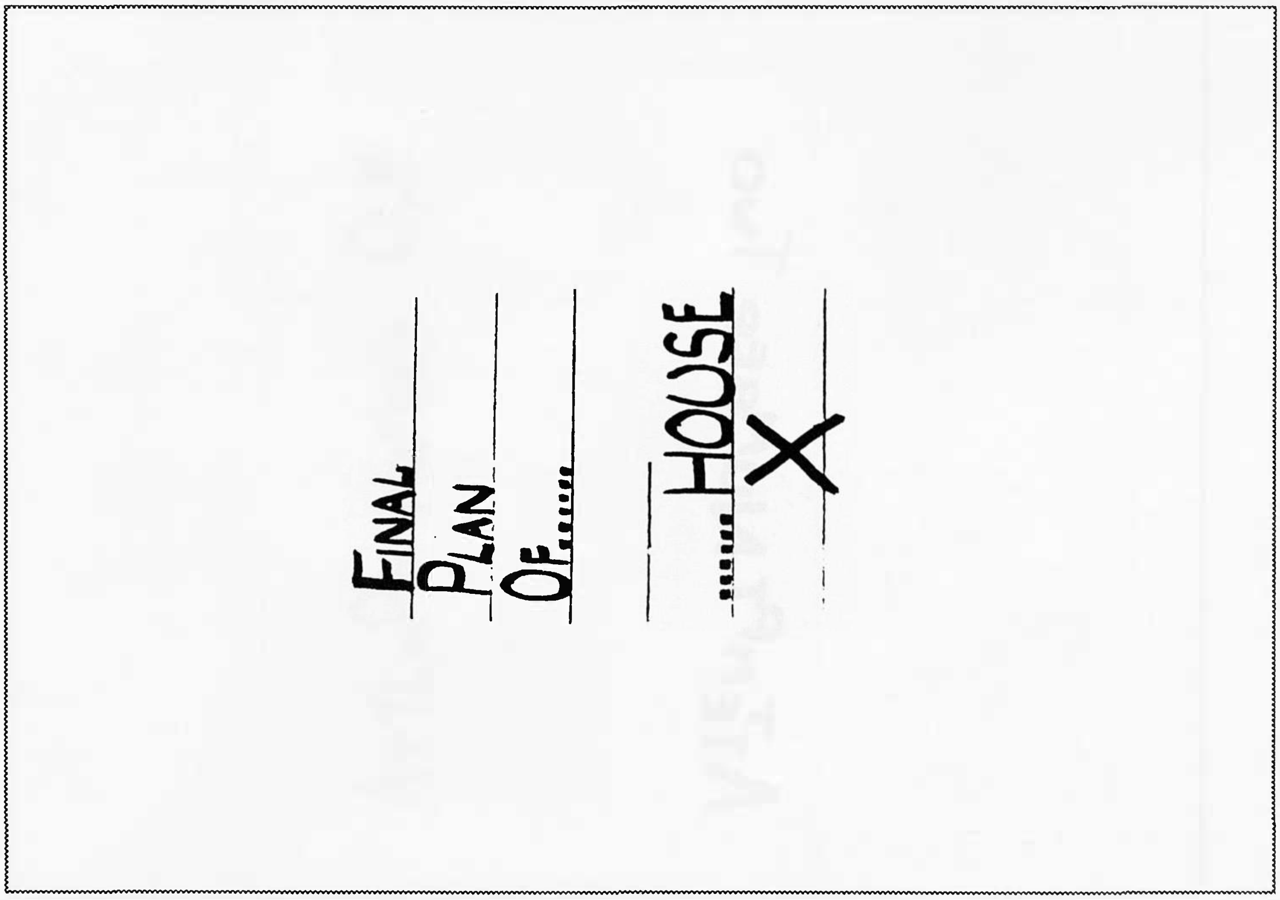


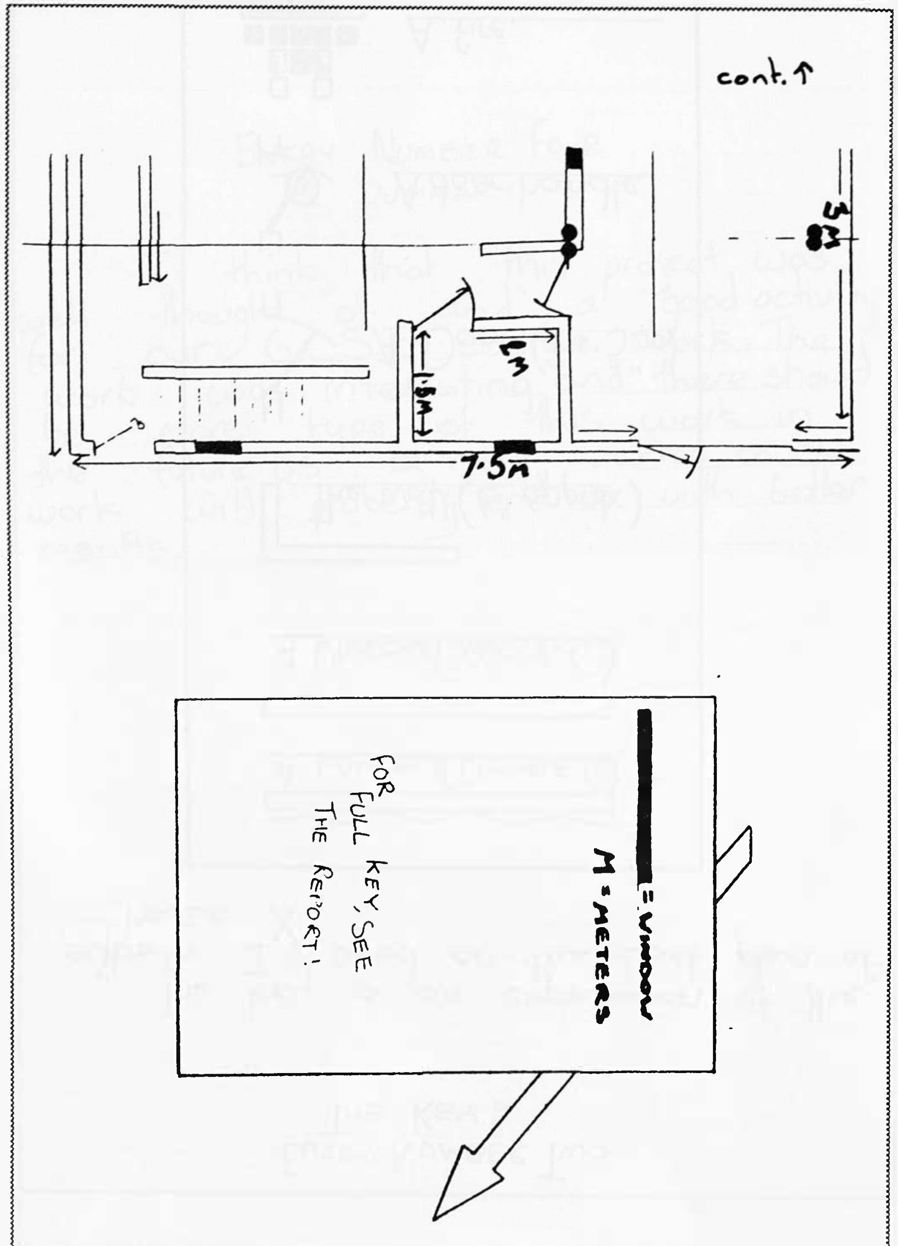

ATTEMPT NUMBER ONE



ATTEMPT NUMBER TWO


7·5M
3·5
3M
cont.
4M
1 METER
10·5 METERS


FINAL
PLAN
OF
HOUSE X



REPORT ON HOUSE X

MY REPORT INCLUDES:-

MEASUREMENTS:- the measurements I used to measure house X and how I did it.

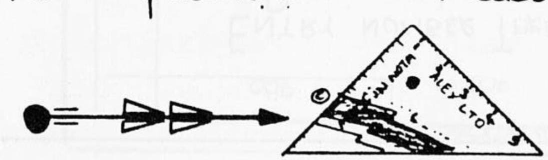
THE KEY:- an explaination of the symbols I used on my plan of house X.

THE DIFFICULTIES:- what I found difficult and what I did about it.

THE SUMMARY:- the write up - what I thought of the project.

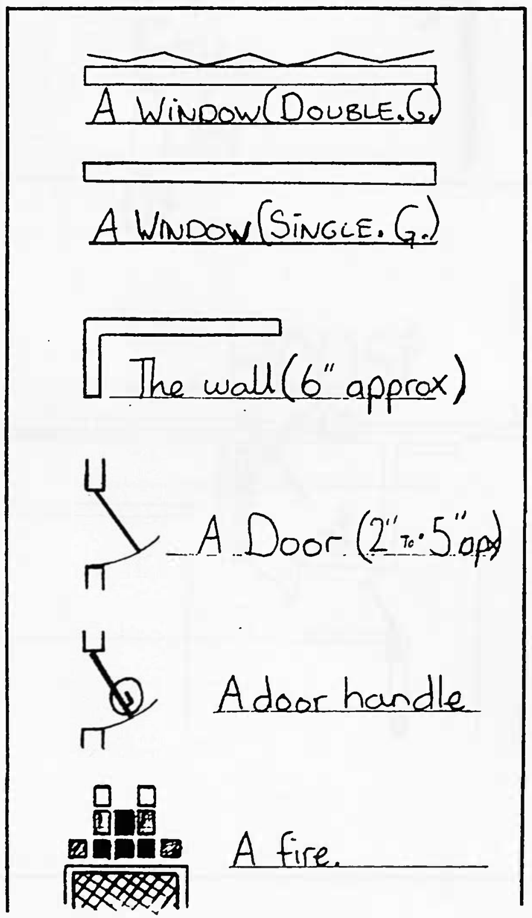
ENTRY NUMBER ONE
MEASUREMENTS

I decided to use what I thought was the easyest to work out.... Meters. I used a meter stick and found it quite easy. I sort of guesed the thickness of the walls as it was not possible to measure them.



ENTRY NUMBER TWO
THE KEY!

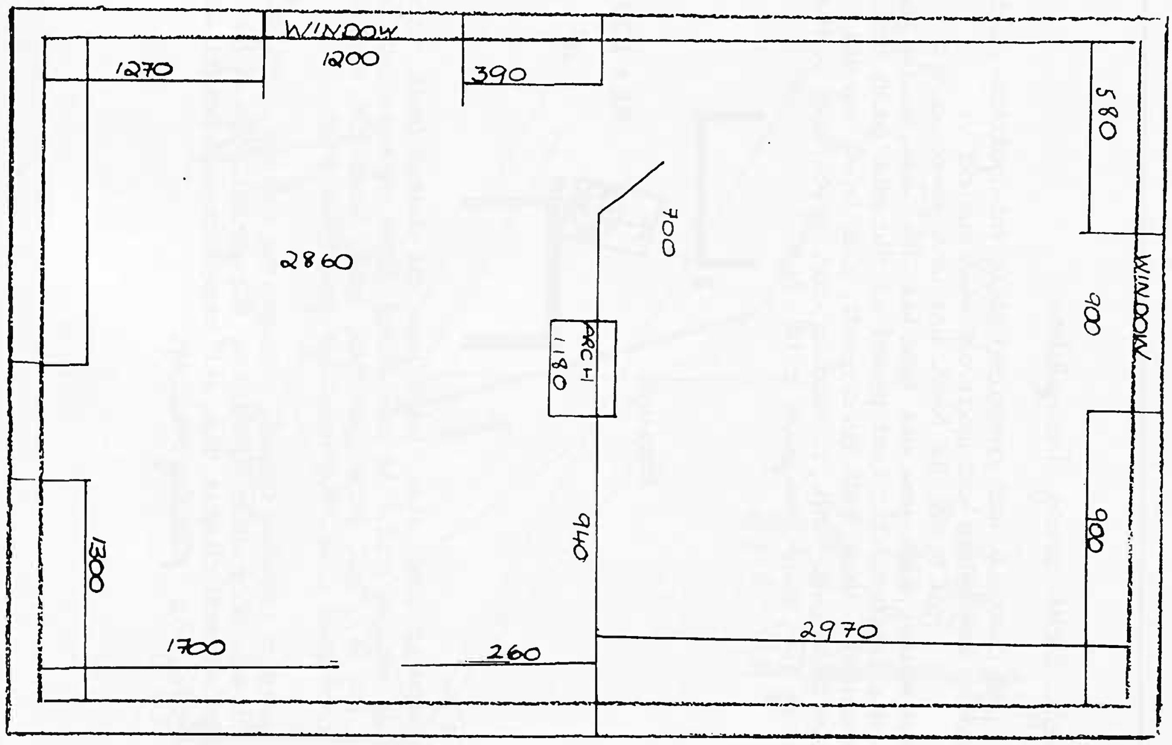
The key is an explanation of the sybols I used on the Last plan of house X.



ENTRY NUMBER THREE
DIFFICULTIES

I found quite a few difficulties in the process of designing the plan of House X. The difficulties I found were.

- △ How thick the walls were.
- ○ Measuring the fire.
- ◇ Measuring behind heavey furniture.
- ☆ Should I measure the window pains of the brakets.
- Should I measure to the skirting boards or to the wall behind them.



ENTRY NUMBER FOUR

THE SUMMARY

I think that this project was well thought of and a good activity for our G.C.S.E course work. The work was interesting and there should be more type of this work in the future as a more varied course work will probably end up with better results.

G2/3

Introduction

The project involves drawing a selection of relatively accurate maps of the college.

The following methods were used:

"Chain and compass"
"Bearings"
"Triangulation".

Contents.

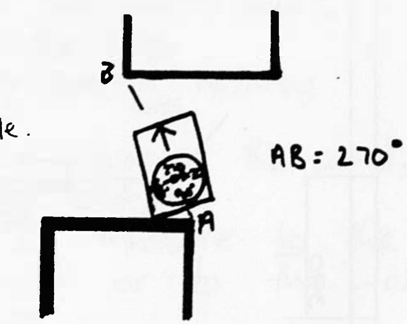
1) Rough map using triangulation.
2) Main map.
3) Diagram showing heights of college buildings using Clinometer/ Theodolite.
4) Conclusion.

Rough Sketch using Triangulation.

The first diagram was constructed using triangulation, and accurately demonstrates how inaccurate this method is.

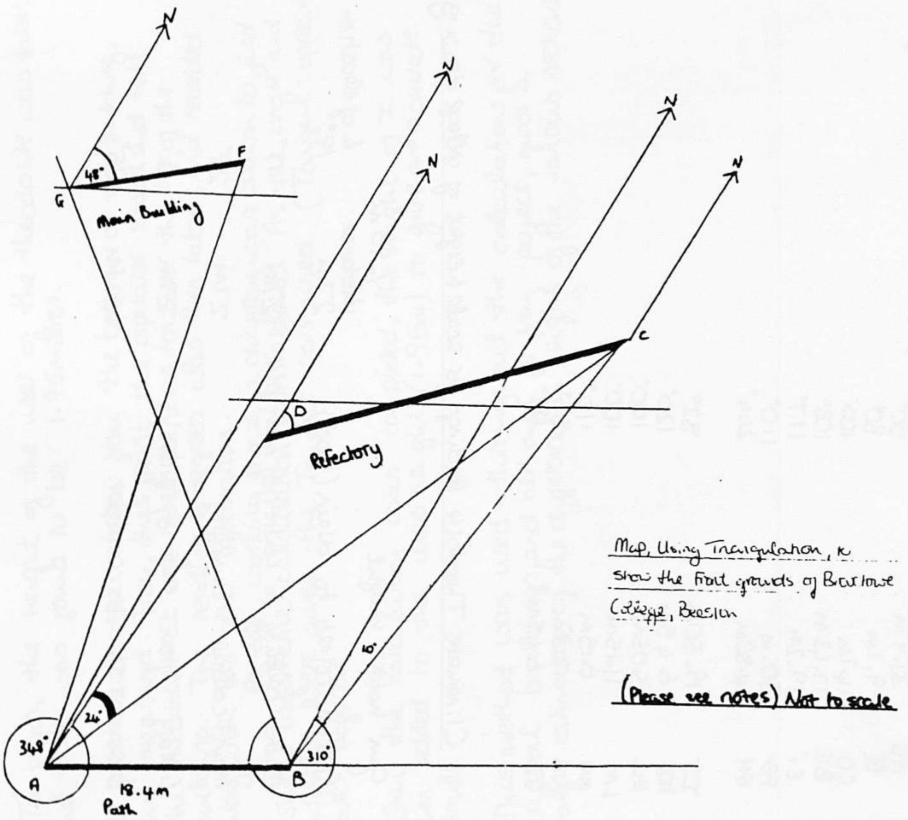
First of all, the North line was found, using a compass reading, taken from the base line, AB. Then, having found this, the compass arrow was pointed at the other points (G, F, D, C) consecutively. Using north as a guide, and lining up the compass point with North, a reading was given, and in the case of AF, this was found to be 348°.

Example.



Measurements were then taken from the various points, so that a scale drawing could be calculated. Cross references were also taken (eg GF), but these were often totally inaccurate, and did not correspond with the readings from other points.

The finished drawing vaguely resembles the college, with the two thick black lines representing the front walls of the college. However, despite this, the experience has proved helpful in creating the following drawings.



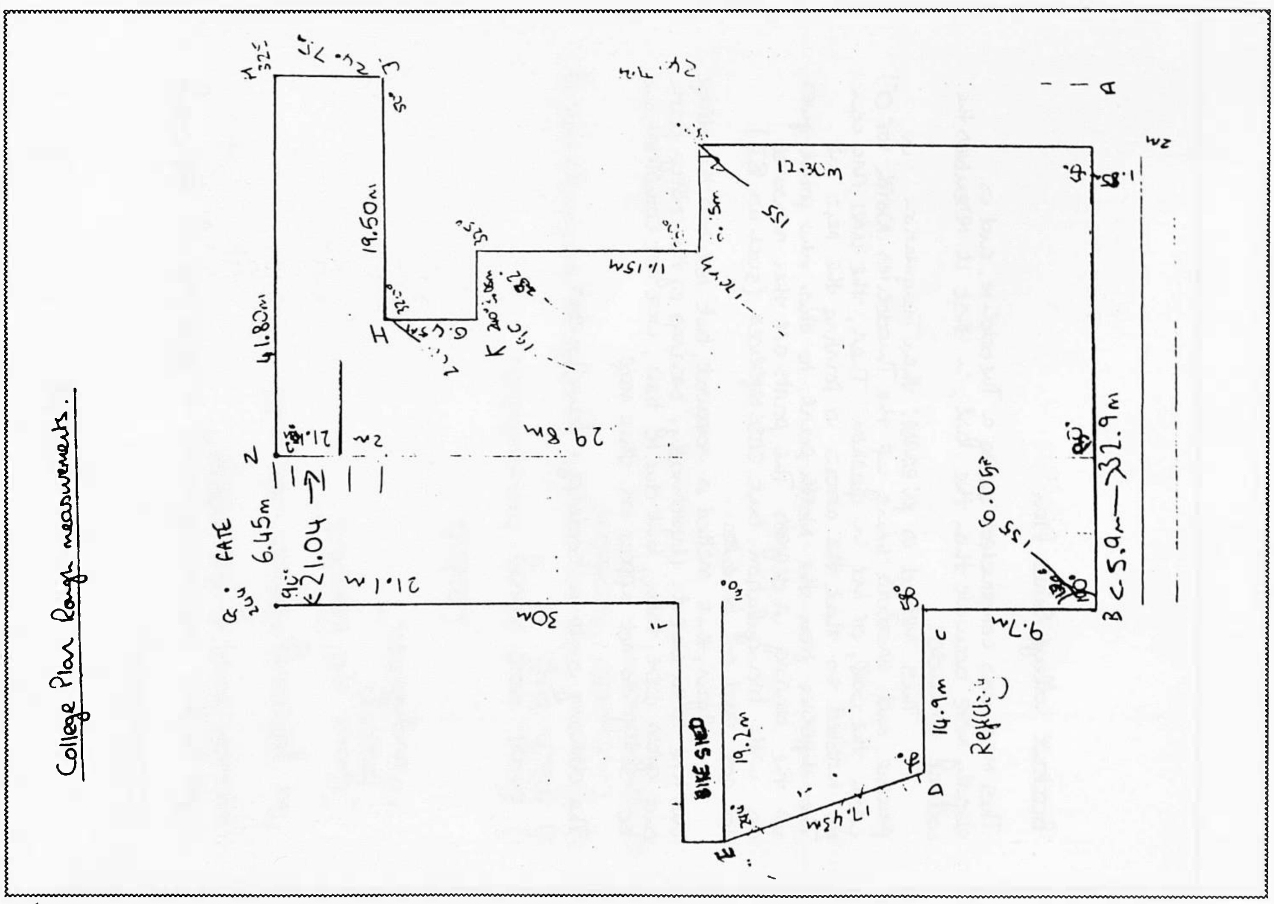
Bramcote College Scale Plan.

This map was constructed using a Theodolite, and is slightly more accurate than the last, in that it resembles the college grounds!

This method is preciser than Triangulation in practice, and involves lining up the Theodolites North (or 0°) with the wall, or line in question. Then, the inner ring can be twisted so that the arrow is pointing the new line. The difference from the North point, to this new point, gives us the bearing in degrees. The points are then measured as with triangulation, but cross references (such as BI) do not need to be taken.

Again, this method is inaccurate, but resembles the college out line to an extent. Unfortunatly, bearings of the college path and grass were taken, but due to time, were not completed, and so sadly do not appear on this map

The drawing is to a scale of: 1cm = 2metres



Rough Chain and Compass readings for College Scale drawing

AB	32.9 m	50°
BC	9.7m	80°
CD	14.7m	100°
DE	17.43 m	108°
EF	19.2m	117°
FG	30.m	110°
GH	41.80m	244°
IJ	19.50m	82°
KI	6.45m	120°
KL	3.05m	100°
LM	11.15m	100°
MN	6.15m	110°
NA	12.30m	83°
GZ	6.45m	99°

Clinometer Theodolite Readings for scale height of college drawing

Gun holders height 1.95 m

	distance	∠ of elevation
Refectory corner to drain (road)	27m	19°
Maths building to centre of main path	27m	17°
Main college building	27m	28°
Library from edge of grass	27m	5°
business studies corridor	16.1m	25°

Heights.

Drawing to show heights of college buildings using clinometer Theodolite

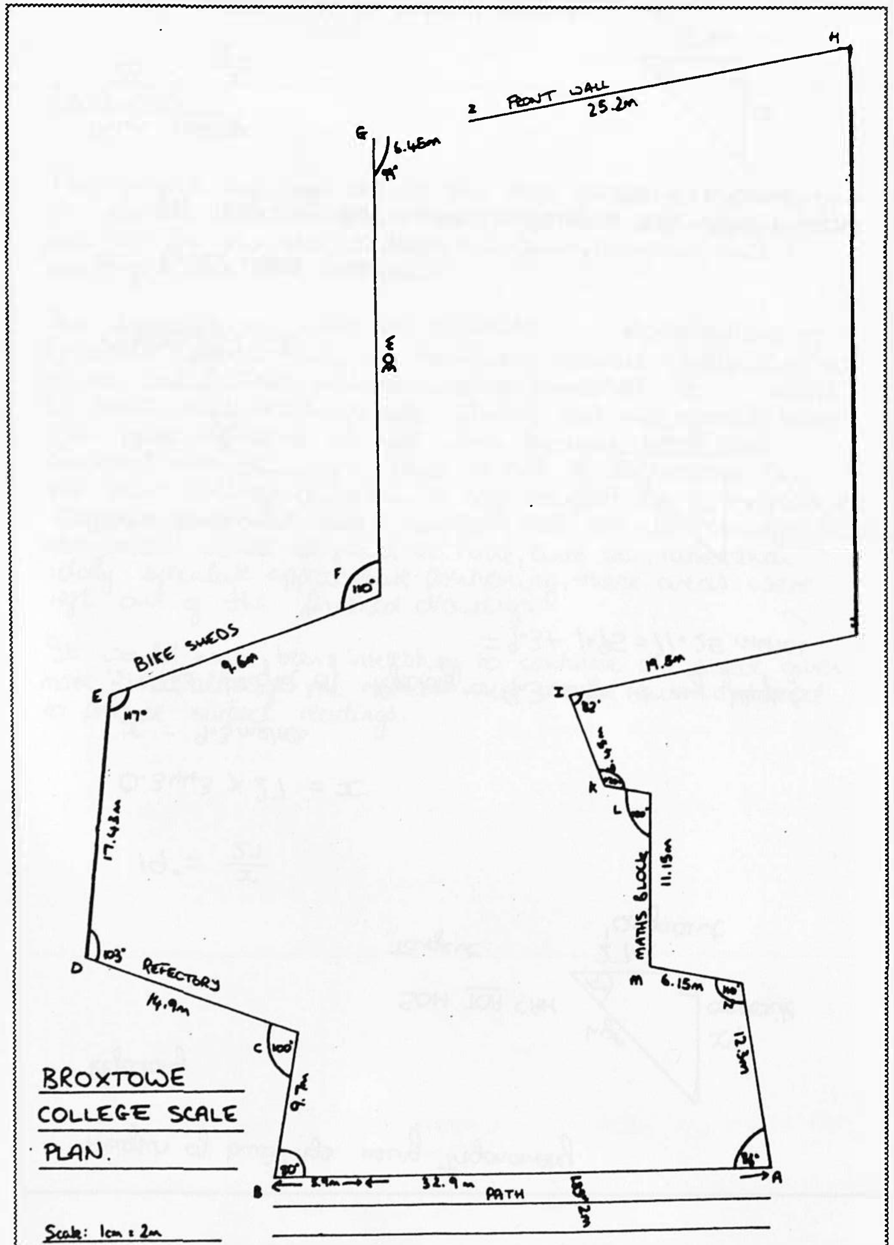
To begin, the height of the user of the theodolite was taken and this was found to be 1.95metres.

A point was then taken from the bottom of the building, and measured. From this point, the operator stood and held the theodolite at eye level, pointing toward the top of the building. The reading in degrees was then taken, and recorded along with the other information.

As the working shows, a diagram was drawn to find which Trigonometric calculation was needed. As the angle and adjacent were given, the Tangent was used (Tangent: opposite over adjacent.

When the calculation was completed, the height of x was then added to the users height (1.95m) to give an accurate result.

This method was used throughout the calculations for the different buildings, and although far from perfect, gives a rough estimation of the differences in height of the various sections

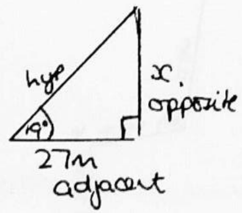


Heights of buildings using Trigonometry.

Refectory

SOH TOA CAH

Tangent



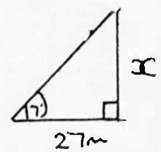
$19° = \frac{x}{27}$

$0.3443 \times 27 = x$

$x = 9.3$ metres

∴ The height of refectory = 9.3 metres + my height
= 9.3 + 1.95 = 11.25 metres ✓

Maths building



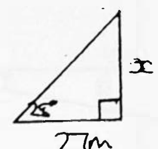
$17° = \frac{x}{27}$

$0.306 \times 27 = x$

$x = 8.262$ metres

∴ The height of the maths building = 8.3 metres + height
= 8.3 + 1.95 = 10.25m

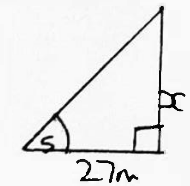
Main college



$28° = \frac{x}{27}$

$x = 0.532 \times 27 = 14.36$ metres + height
14.36 + 1.95 = 16.31 metres.

Library

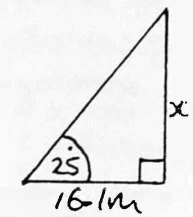


$\text{Tan } 5° = \frac{x}{27}$

$x = 0.09 \times 27$

$x = 2.43$m + height
2.43 + 1.95 = 4.38 metres ✓

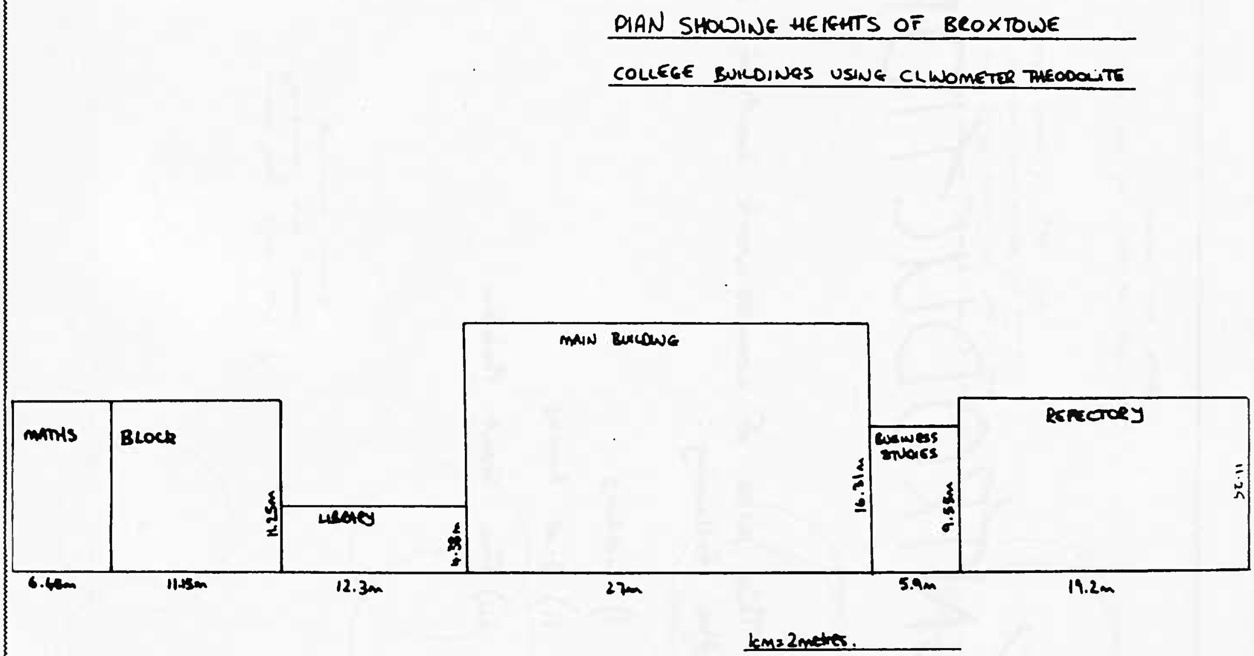
Business studies corridor



$\text{Tan } 25 = \frac{x}{16.1}$

$0.47 \times 16.1 = 7.6$ metres + height
7.6 + 1.95 = 9.65 metres

Should use 4 sf for $\tan\theta$



Conclusion

This project has been one of the most deceptive to date. On the one hand, it involves working outside, and enjoying the fresh air. On the other, it leads to confusing diagrams and bearings which don't correspond!

The diagrams within are not accurate representations of Broxtowe College. They are however, accurate (hopefully) as far as calculations and drawings are concerned. It must be said that readings taken whilst out and about have often made no sense at all when brought back and converted into drawings. This is one of the reasons why the main college diagram is not complete (car park, grass, etc. Measurements were taken, however, but not all readings which were needed could be found in time, and so, rather than idly speculate approximate positioning, these areas were left out of the finished drawings.

It would have been interesting to continue the project, given more time, although the methods used would have continued to produce suspect readings.

G2/4

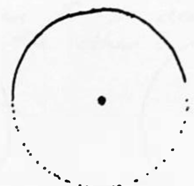
CONSTRAINTS

INTRODUCTION

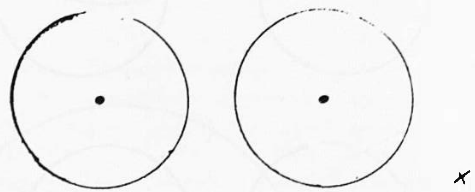
This piece of course work contains work on the following:

i) Locus
ii) 3-d Locus
iii) The Goat Problem

MOVING POINTS

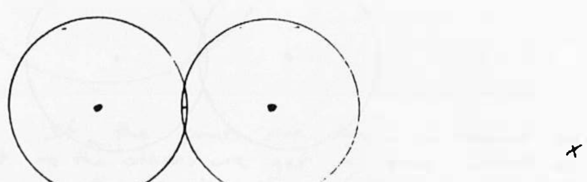


The problem was to draw a dot on a piece of paper, and then draw as many points as possible that are all the same distance from the dot. The shape I get (above) is a circle. This is called a LOCUS.



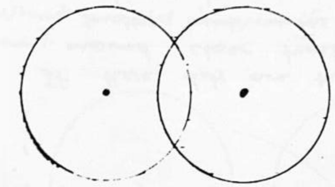
This time I started with two dots, and then draw the locus of all points which are the same distance from <u>both</u> of the dots. I ended up with two circles.

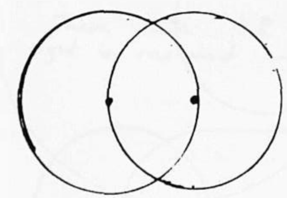
What would happen if the dots were closer together?



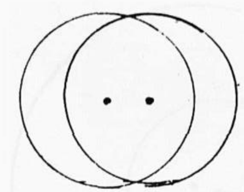
As the dots are moved closer together, the locus of each get closer together.

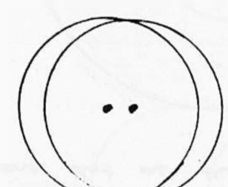
In this case they are touching.





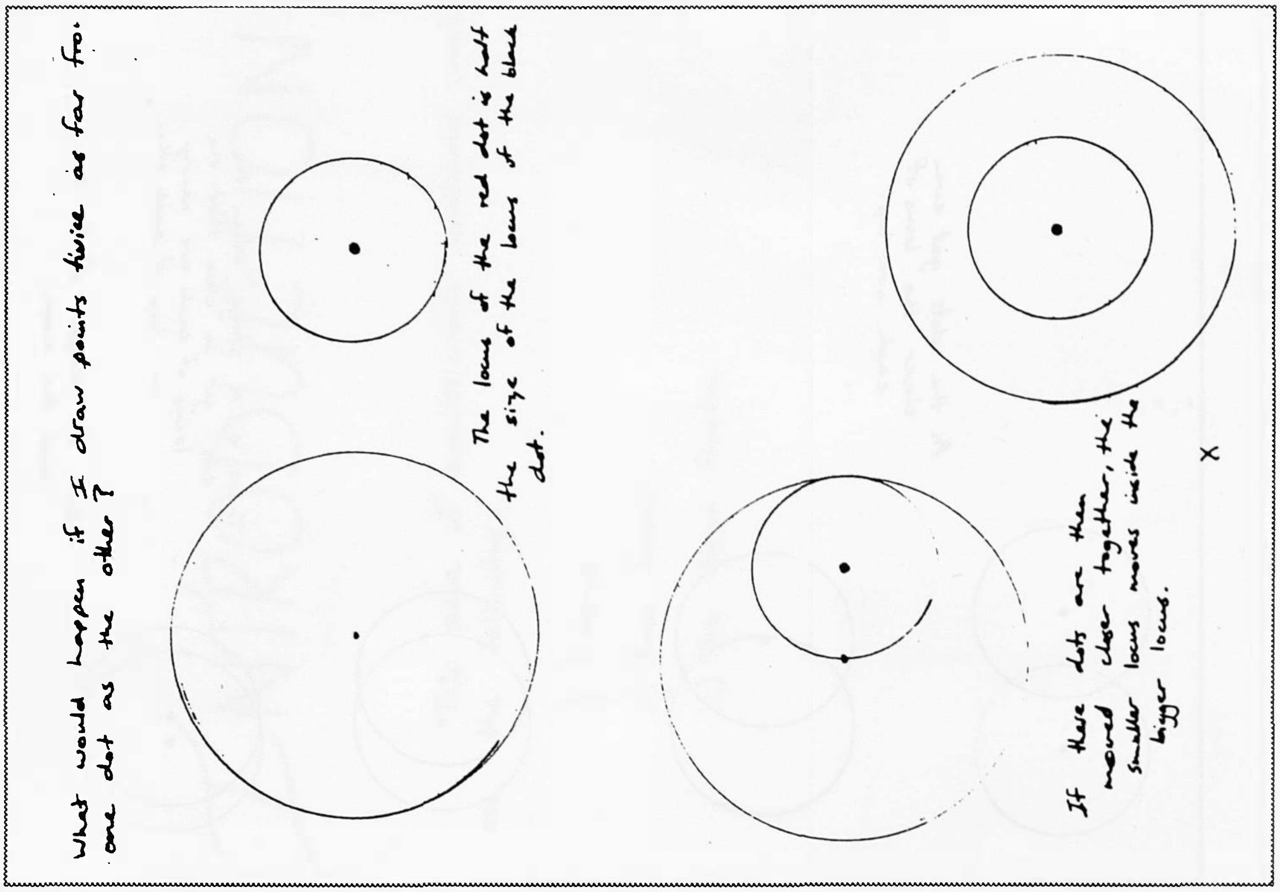
As the dots get even closer the locus of each over lap.

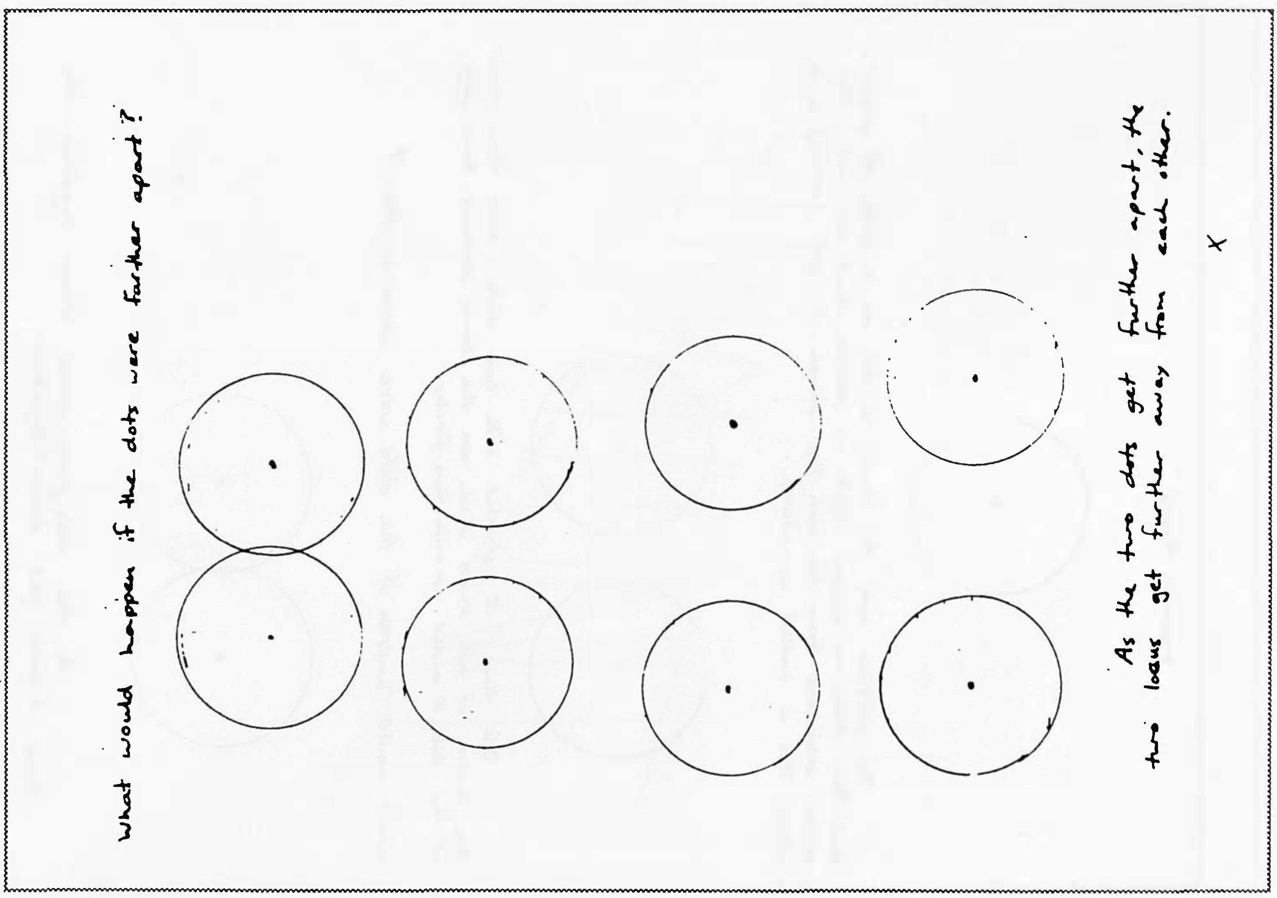




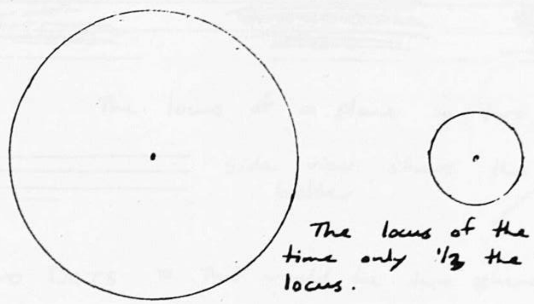
There is a stage when the dots get so close that the locus of each are nearly on top of each other

Like the eclipse of the sun and the moon.

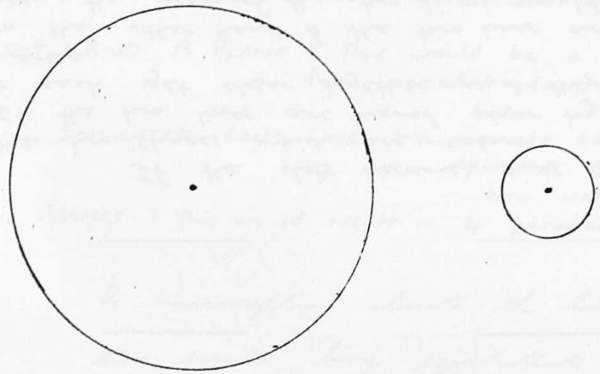

What would happen if I draw points twice as far fro. one dot as the other?
The locus of the red dot is half the size of the locus of the black dot.
If these dots are then moved closer together, the smaller locus moves inside the bigger locus.
X


What would happen if the dots were further apart?
As the two dots get further apart, the two locus get further away from each other.
X

What would happen if I draw points 3 times as far from one dot as the other?



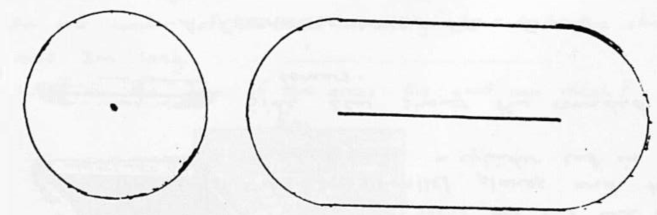
The locus of the red dot is this time only 1/3 the size of the bigger locus.



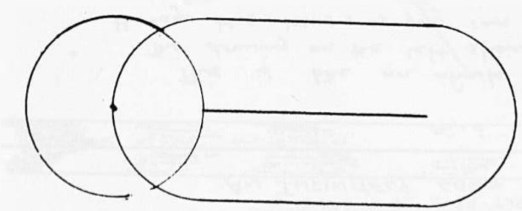
If the points are drawn 4 times as far from one dot as the other, we get a locus which is 4 times as big as the smaller locus (above).

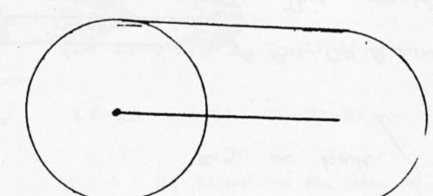
X

What would happen if one of the dots is replaced by a line?

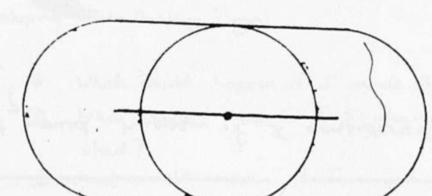


These are the two shapes we get if the dot on the right is replaced with a line.





As you can see in these drawings as the dot and the line move closer together the touch. Then as they get closer they overlap, and the locus of the dot goes inside the locus of the line.



X

What would happen it both dots became lines ?

If the dots became lines we would get the two shapes which are above.
If the two lines are moved closer together, the locus of each get closer together, and they too touch and overlap.
On the other hand if the two lines are moved futher away, the locus of the two lines would move further away from each other.

What would happen if I considered points in three dimensions ?

A DOT

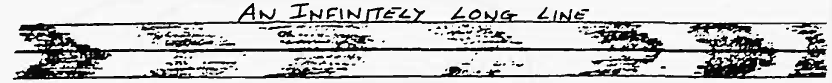
The 3-d locus of a dot would become a sphere.

e.g. a ball.

A BIT OF A LINE

This would be a medicine capsule a cylinder with hemi-spheres on the ends.

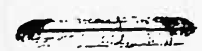
AN INFINITELY LONG LINE



This is like an infinite cylinder or a pipe. The drawing on the left shows a cut (vertical) sectio through this locus, as you can see it looks like a cylinder.

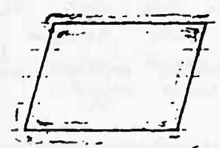
A PIECE OF PAPER

The 3-d locus for this one is hard to dra if has parallel planes and the sides are rounded (like a cylinder cut in half).



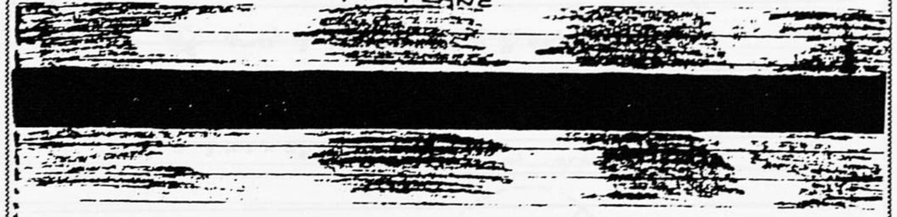
Side view shows the rounded sides and corners.

A RECTANGLE MADE FROM STICKS

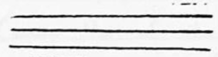


This locus is the same as the cylinde above but it is in the shape of a rectangle so the corners have the locus around them This reminds me off those straws which can bend (the bendy bit would go at each corner

A Plane



The locus of a plane is two parallel planes.



side view shows the parallel planes better.

Two Dots = This would be two spheres (or two balls)

A Dot and an Infinitely Long Line = This would be a sphere and a infinite cylinder.

A Dot and a Plane = This would be a sphere and two parallel planes.

Two Infinitely Long Lines = This would be two infinite cylinders. (A good example two water pipes).

Two Planes = This would result in 4 parallel planes.

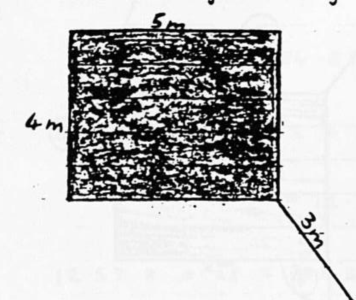
A misconception about the loci of two points being equidistant from both points.

Good 3-D work.

Goat Problems

Problem - Imagine a shed in a large field which is 4m by 5m. On one corner of the shed is a goat, tied to the shed by a rope 3m long.

What is the area of the grass the goat can reach?



The area of the circle is:

πr^2

$= 3.142 \times 3^2 = 28.278$

$= 28.28 m^2$

1/4 of the circle is covered by the shed so:

$(28.28 \div 4) \times 3 = 21.21 m^2$

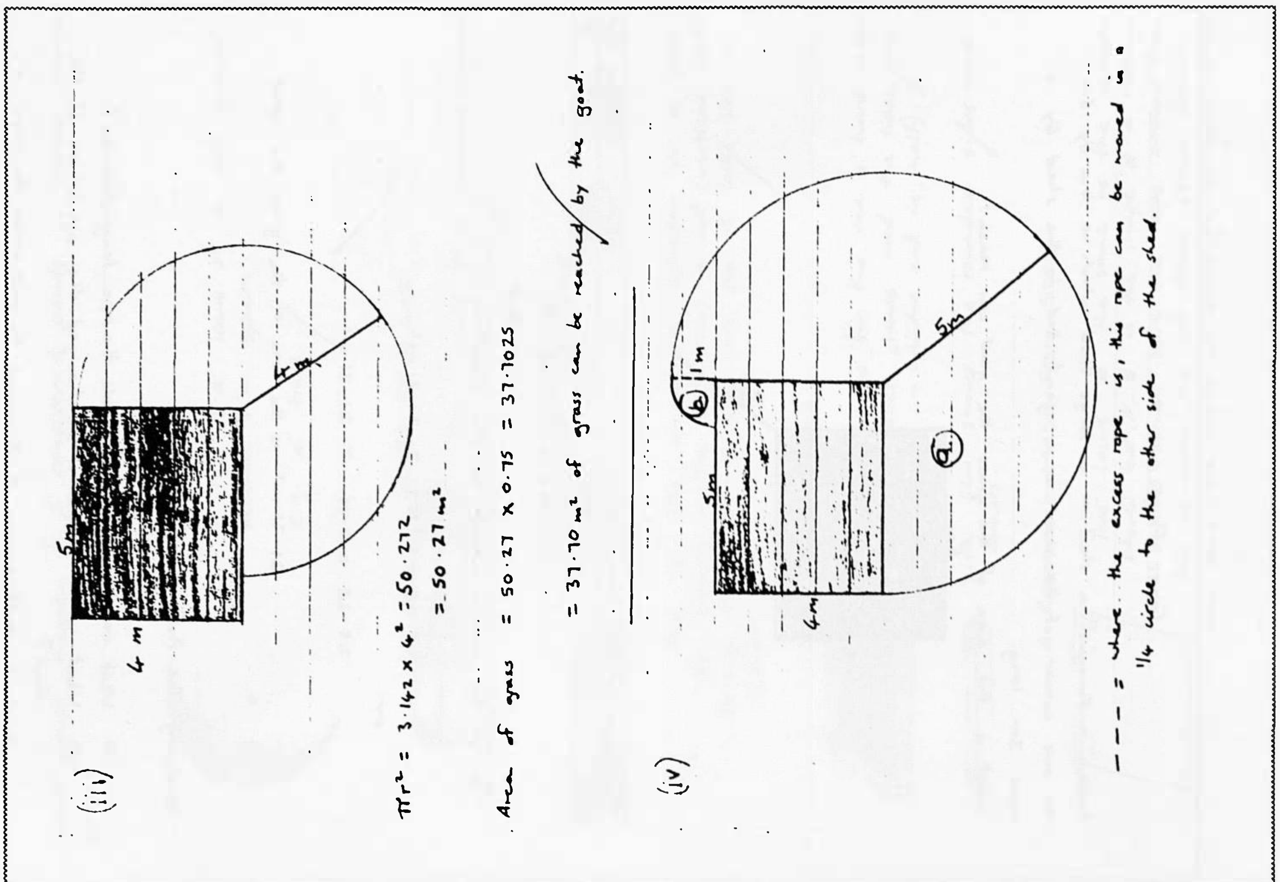
or

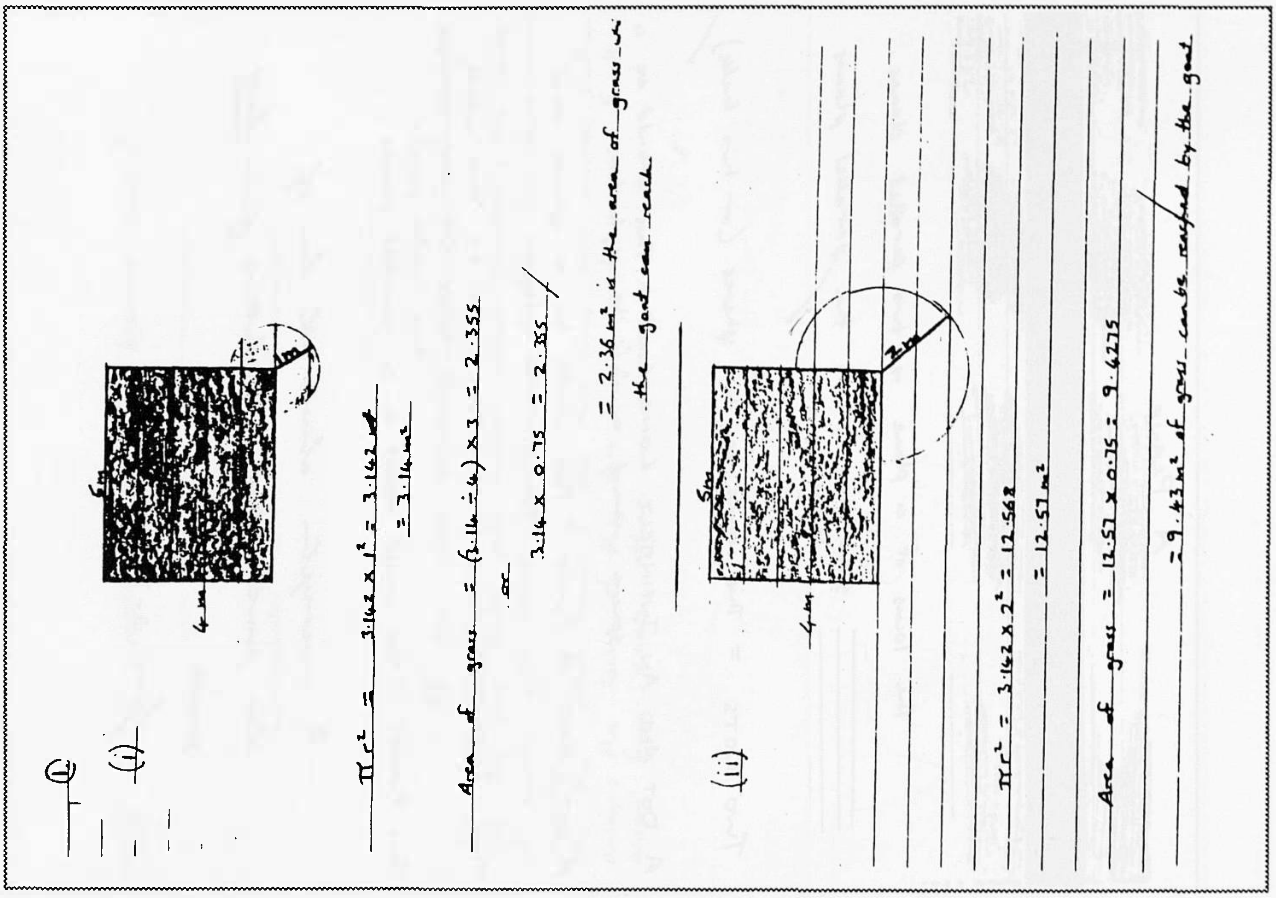
$28.28 \times 0.75 = 21.21 m^2$

$21.21 m^2$ is the area of the grass the goat can reach.

Developing The Problem:

1. What would happen if I made the rope longer/shorter?
2. What would happen if I changed the size/shape of the shed?
3. What would happen if I moved the point where the rope is.


(iii)
4 m
5m
4m
πr² = 3.142 x 4² = 50.272
= 50.27 m²
Area of grass = 50.27 x 0.75 = 37.7025
= 37.70 m² of grass can be reached by the goat.
(iv)
5m
1m
4m
5m
a
b
- - - = where the excess rope is, this rope can be moved in a ¼ circle to the other side of the shed.


(i)
4m
6m
1m
πr² = 3.142 x 1² = 3.142
= 3.14 m²
Area of grass = (3.14 ÷ 4) x 3 = 2.355
or
3.14 x 0.75 = 2.355
= 2.36 m² is the area of grass in the goat can reach
(ii)
5m
4m
2m
πr² = 3.142 x 2² = 12.568
= 12.57 m²
Area of grass = 12.57 x 0.75 = 9.4275
= 9.43 m² of grass can be reached by the goat

(iv)

Area of ⓐ = 3·142 × 5² = 78·55 m²

78·55 × 0·75 = 58·9125

= 58·91 m² of grass

Area of ⓑ = 3·142 × 1² = 3·142

= 3·14 m²

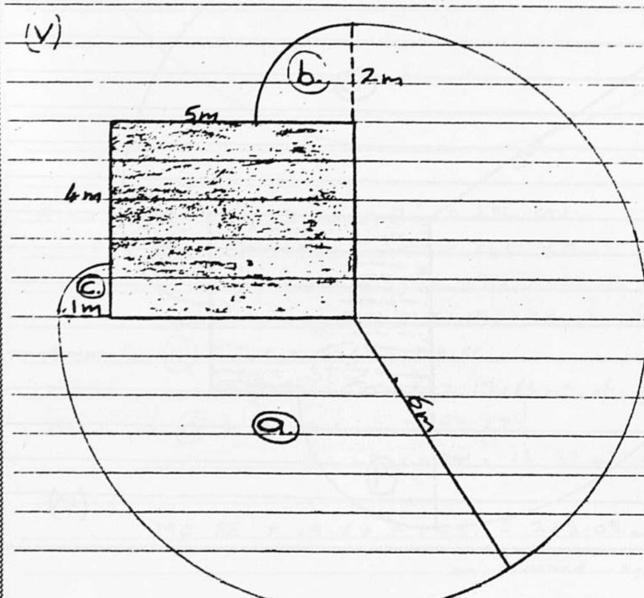
(only ¼ of the circle is grass)

3·14 × 0·25 = 0·785

= 0·79 m² of grass

58·91 + 0·79 = 59·70 m² of grass can be reached by the goat. ✓

(v)



(v)

Area of ⓐ = 3·142 × 6² = 113·112

= 113·11 m²

113·11 × 0·75 = 84·8325

= 84·83 m² of grass

Area of ⓑ = 3·142 × 2² = 12·568

= 12·57 m²

12·57 × 0·25 = 3·1425

= 3·14 m² of grass

Area of ⓒ = 3·142 × 1² = 3·142

= 3·14 m²

3·14 × 0·25 = 0·785

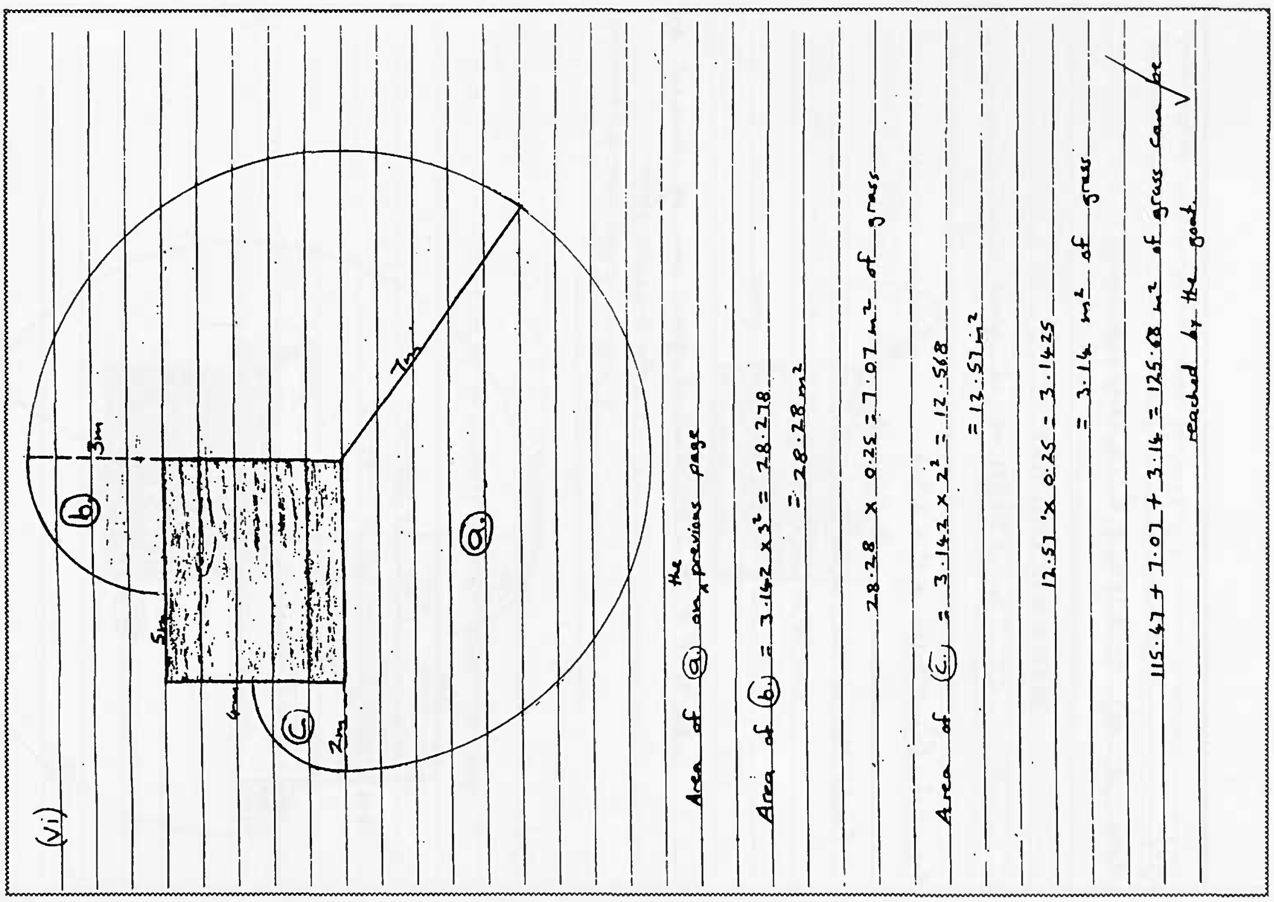
= 0·79 m² of grass

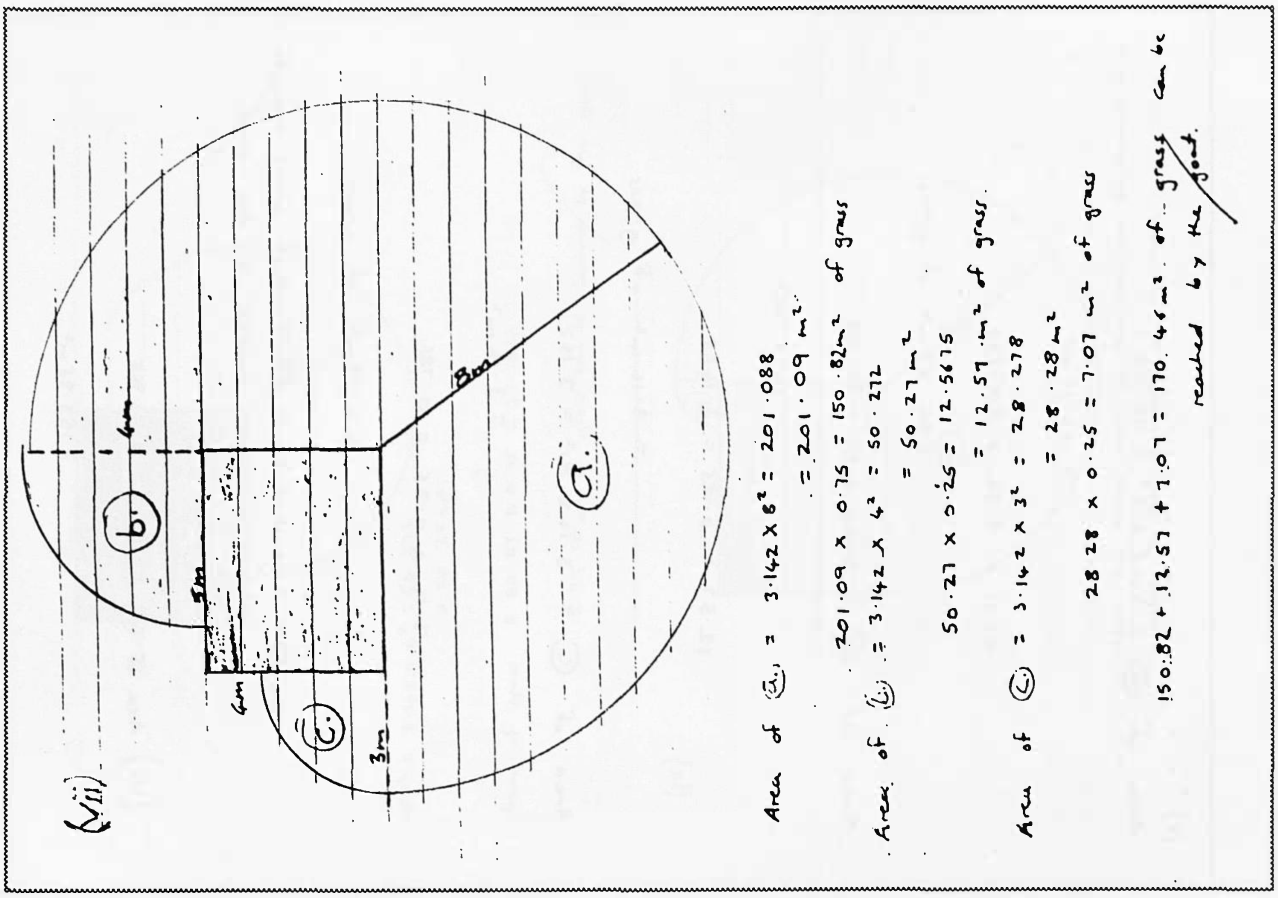
84·83 + 3·14 + 0·79 = 88·76 m² of grass can be reached by the goat. ✓

(vi) Area of ⓐ = 3·142 × 7² = 153·958

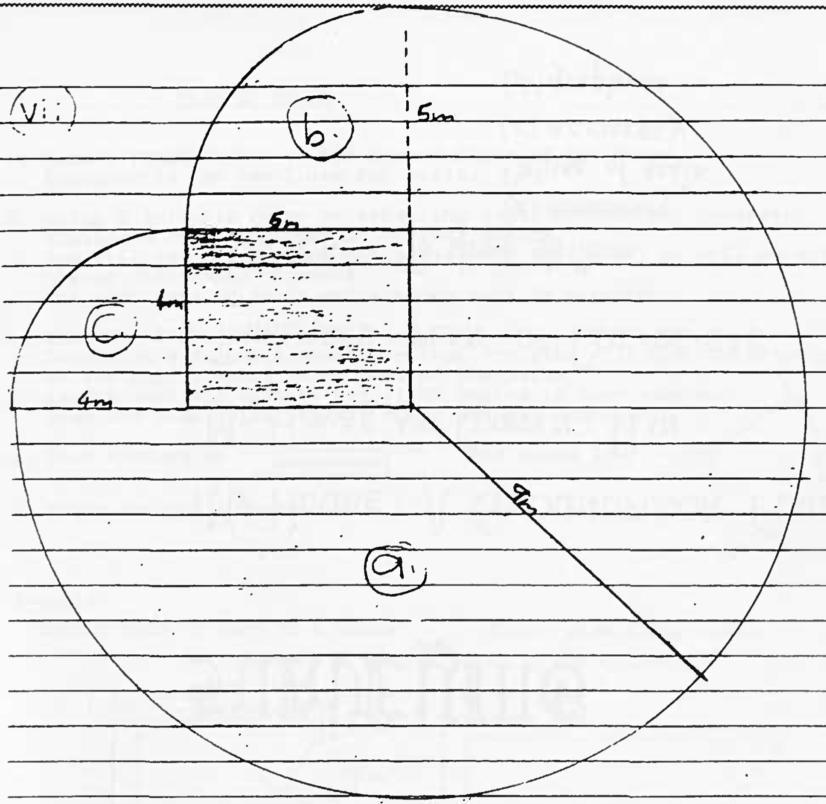
= 153·96 m²

153·96 × 0·75 = 115·47 m² of grass


(vi)
a
b
c
7m
3m
5m
4m
2m
Area of (a) on the previous page
Area of (b) = 3·142 × 3² = 28·278
= 28·28 m²
28·28 × 0·25 = 7·07 m² of grass
Area of (c) = 3·142 × 2² = 12·568
= 12·57 m²
12·57 × 0·25 = 3·1425
= 3·14 m² of grass
115·47 + 7·07 + 3·14 = 125·68 m² of grass can be reached by the goat.


(vii)
a
b
c
8m
4m
5m
4m
3m
Area of (a) = 3·142 × 8² = 201·088
= 201·09 m²
201·09 × 0·75 = 150·82 m² of grass
Area of (b) = 3·142 × 4² = 50·272
= 50·27 m²
50·27 × 0·25 = 12·5675
= 12·57 m² of grass
Area of (c) = 3·142 × 3² = 28·278
= 28·28 m²
28·28 × 0·25 = 7·07 m² of grass
150·82 + 12·57 + 7·07 = 170·46 m² of grass can be reached by the goat.

(vi)



Area of (a.) = 3.142 × 9² = 254.502
= 254.50m²
254.50 × 0.75 = 190.875
= 190.88 m² of grass

Area of (b.) = 3.142 × 5² = 78.55
78.55 × 0.25 = 19.64m² of grass

Area of (c.) = 3.142 × 4² = 50.27
50.27 × 0.25 = 12.57 m² of grass

190.88 + 19.64 + 12.57 = 223.09m² of grass can be reached by the goat. ✓

At this point I tried to find a formula to work out how much grass a goat can reach with any length of rope.

1m of rope = 2.36m²
2m of rope = 9.43m²
3m of rope = 21.21m²
4m of rope = 37.70m²
5m of rope = 59.70m²
6m of rope = 88.76m²
7m of rope = 125.68m²
8m of rope = 170.46m²
9m of rope = 223.09m²

2.36 9.43 21.21 37.70 59.70 88.76
7.01 11.78 16.49 22 29.06
4.71 4.71 5.51 7.06

As you can see from the formula, there isn't a rule. But then I noticed something, if we only used the first four results we could find a formula:

2.36 9.43 21.21 37.70
7.07 11.78 16.49
4.71 4.71

Then I looked back at my drawings and seen on these four the rope is just long enough to move just on two sides of the shed. As it gets longer, the rope itself is longer than the sides of the shed and starts to overlap onto the other sides of the shed.

e.g. The diagram on the previous page shows you what I mean. ✓

G2/5

TEACHER NOTE: This project was given in with rough plans for all the scale drawings and various photographs.

PRACTICAL GEOMETRY

SURVEYING

1. MY HOUSE: A GROUNDFLOOR PLAN

2. MY HOUSE AND GARDEN: PLAN

3. THIRLMERE: PLAN OF CUL-DE-SAC

PURPOSE: To show skills in

(a) measuring
(b) use of scale
(c) accuracy
(d) graphics

PRACTICAL GEOMETRY

SURVEYING

Aim of project

1. To produce a groundfloor plan of the house in which you live.
2. The presentation may be extended by adding-

   a plan of the garden,
   a plan of the street,
   any other extension of your own choosing.

The presentation

This should include-

1. A title page (see below)
2. A detailed account of how you went about the task
3. The rough plan(s) and measurements made
4. The final plan(s)
5. Any other material considered relevant

The title page

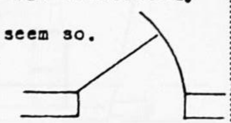
1. Your name
2. Main heading- Practical Geometry
3. Sub Heading- Surveying
4. Object- e.g. To produce a plan of my house
5. A reason for carrying out the work

PLAN OF HOUSE

1. Make a rough sketch of the ground floor of the house.
Remember to use two lines for walls.

2. Using a suitable ruler or measuring tape, measure all necessary distances and write them in on your sketch.
You will need to measure the outside of the house, as well as the inside dimensions of rooms.
The positions of doors and windows must be measured.

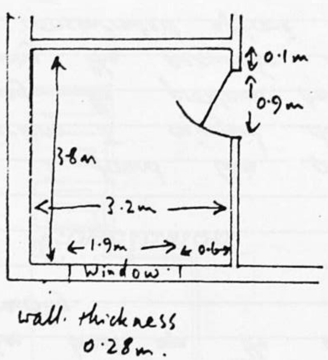
4. Make the final plan.
Decide on a suitable scale, so that the plan will fit comfortably on the paper, while using most of the paper.
Assume that all corners are right angles if they seem so.
Draw the plan using pencil, ruler and set square.

4. Show windows by  and doors by 

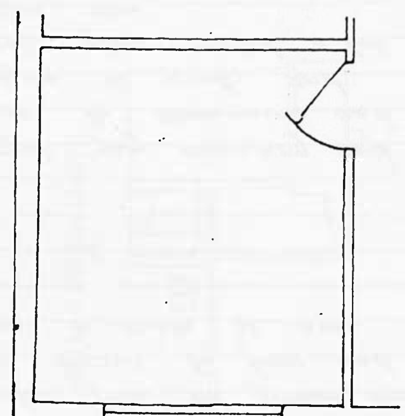
5. State the scale used.

Example

Sketch plan of part of a house



Final plan using scale
1cm = 0.5m



Method of Working.

In order to draw accurate plans of my house, garden and the cul-de-sac in which I live, I had to collect many measurements.

I did the plan of the house first, using a steel tape and a metre stick to measure the interior walls of all the rooms. As I knew the width of the walls, I found it unnecessary to take exterior measurements.

The plan was quite complicated, because of the angles, but, once I had discovered that the design is built up on 90° and 18° angles, it became more straight forward. Measuring the angle of the fire place, however, was quite difficult.

Using a scale of 1:40, I carefully transferred the measurements onto A3 paper.

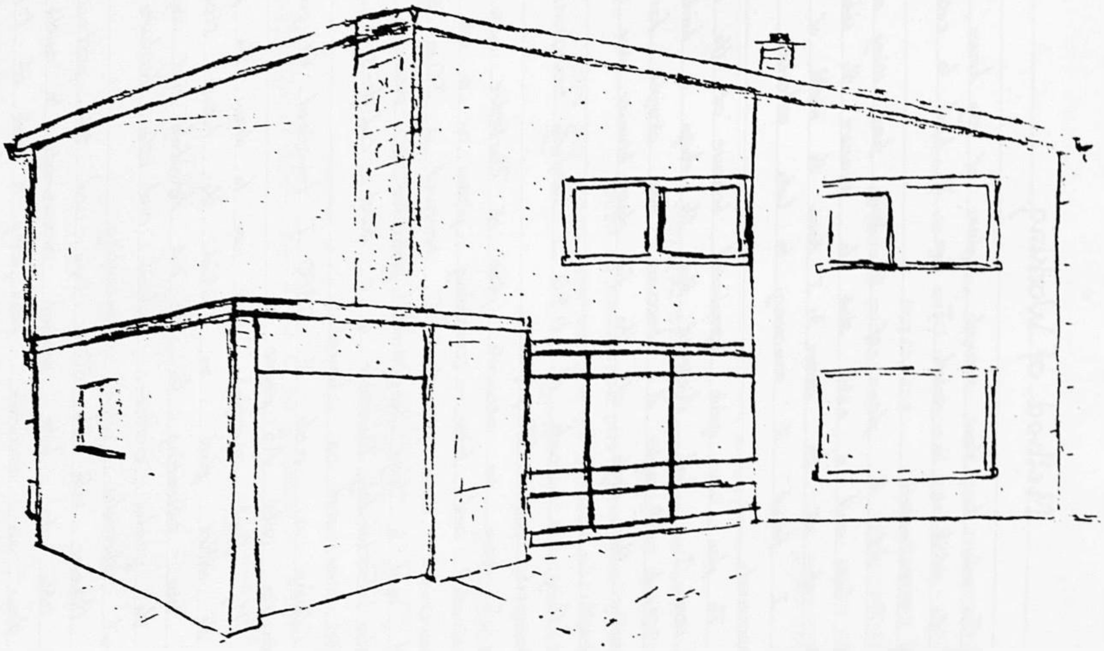
To draw an accurate plan of Thirlmere was a complicated and time-consuming process, as a lot of measurements were involved. I borrowed a 30m tape and used a large, home-made protractor to measure the angles. Fortunately, Thirlmere is a quiet cul-de-sac, so traffic was not a hazard.

Using a scale of 1:300 I transferred the gathered information onto A3 paper.

My final surveying task was to draw the plan of the entire plot on which the house stands. This was interesting to do, but definitely not simple, as the garden contains rockeries and other features which present obstacles when measuring.

However, with the 30m tape and the protractor, I was able to take enough measurements to enable me to draw an accurate plan, using a scale of 1:150.

I was surprised to see how elongated



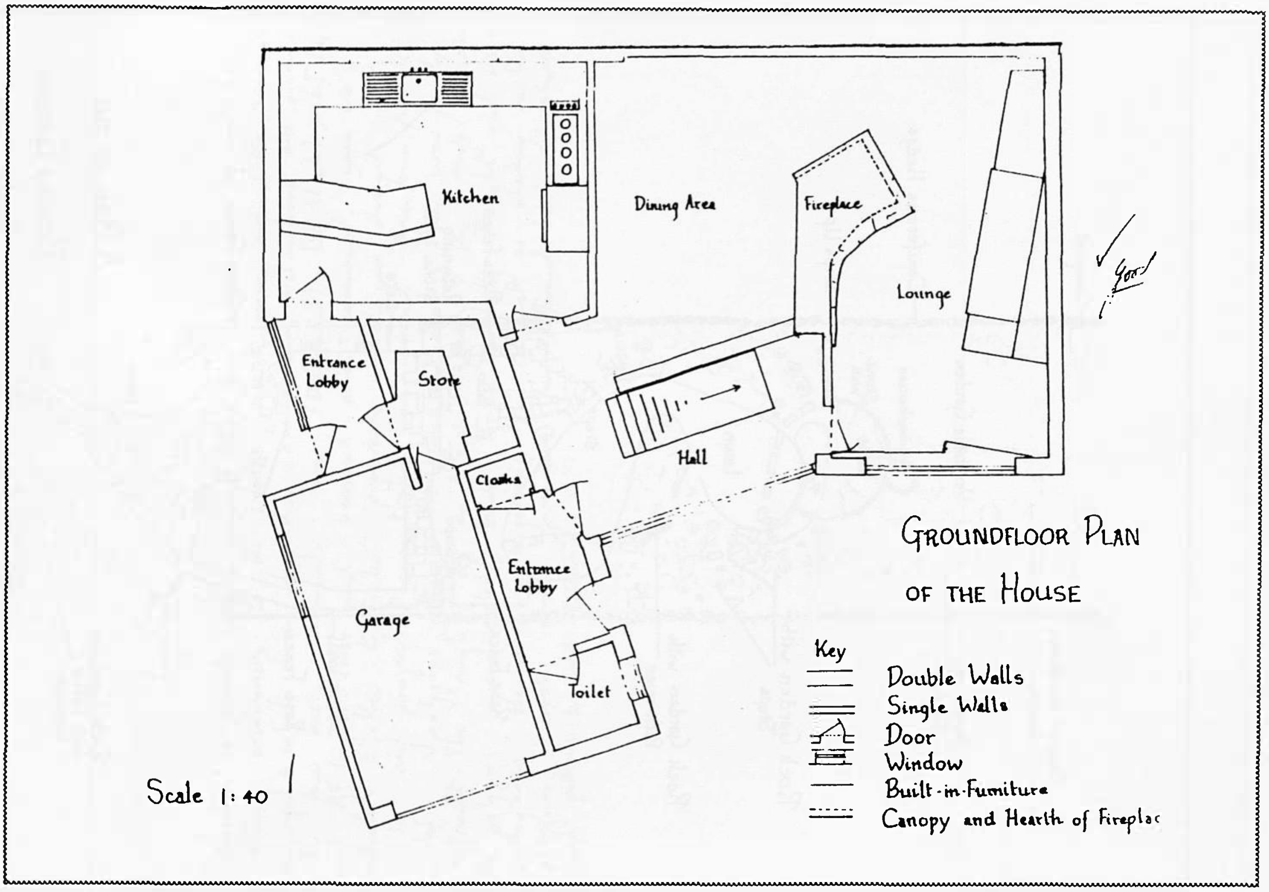
the plan is, as the garden from the ground, does not appear narrow. This is because the trees and bushes break up the view to make it more interesting.

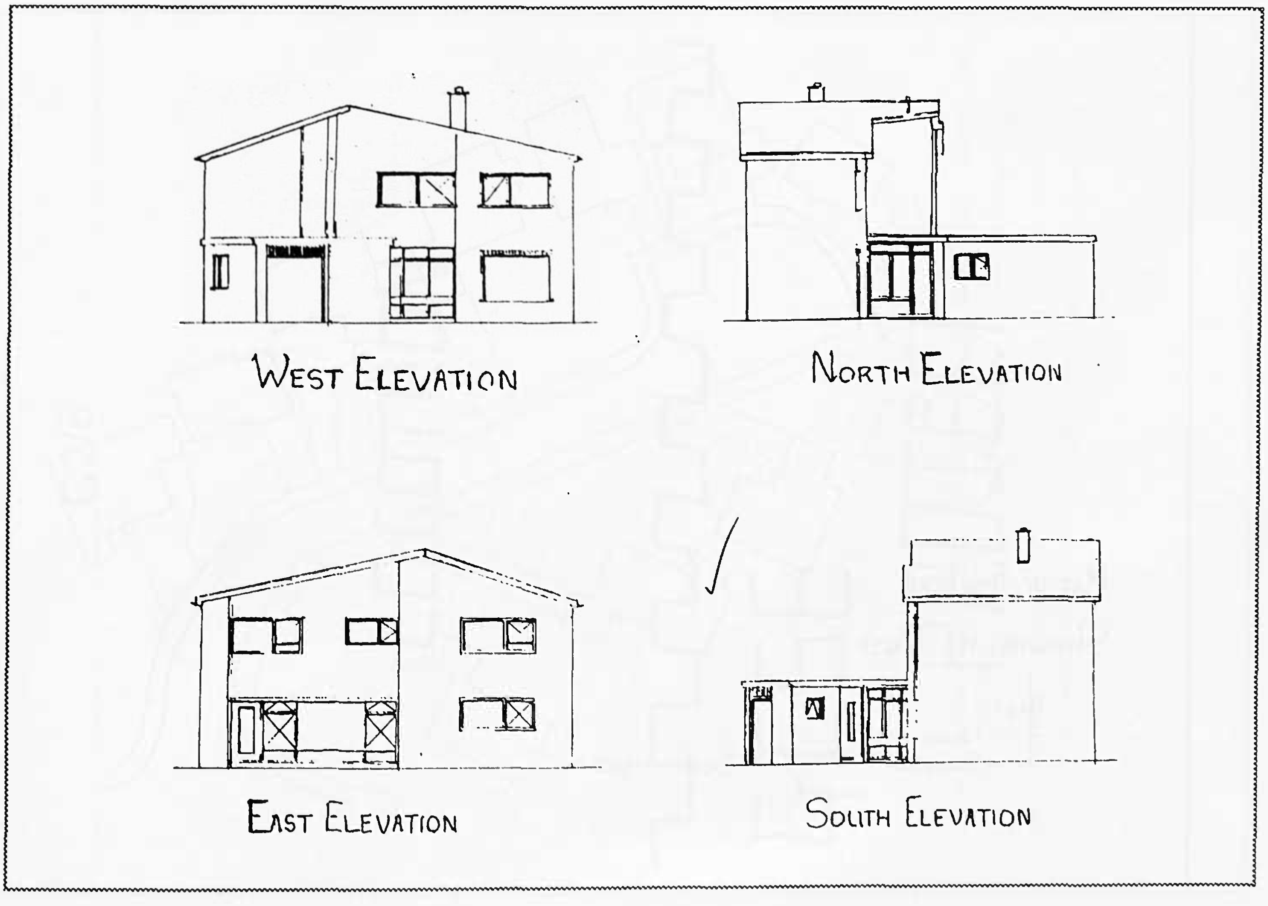
Conclusion

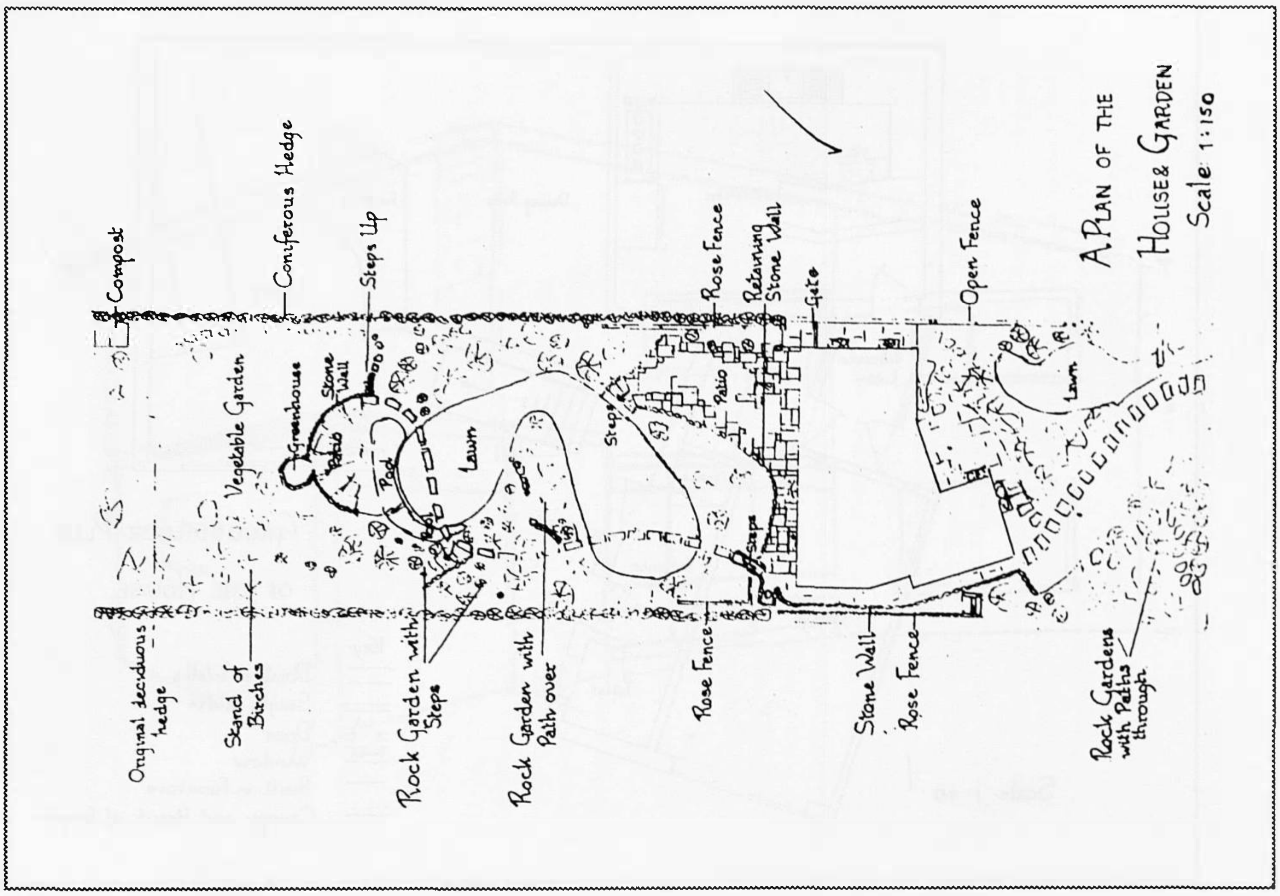
I found this project very interesting and absorbing. I enjoyed doing the measurements and solving the problems. Analysing the results and drawing the accurate scale plans needed a lot of concentrated effort and time.

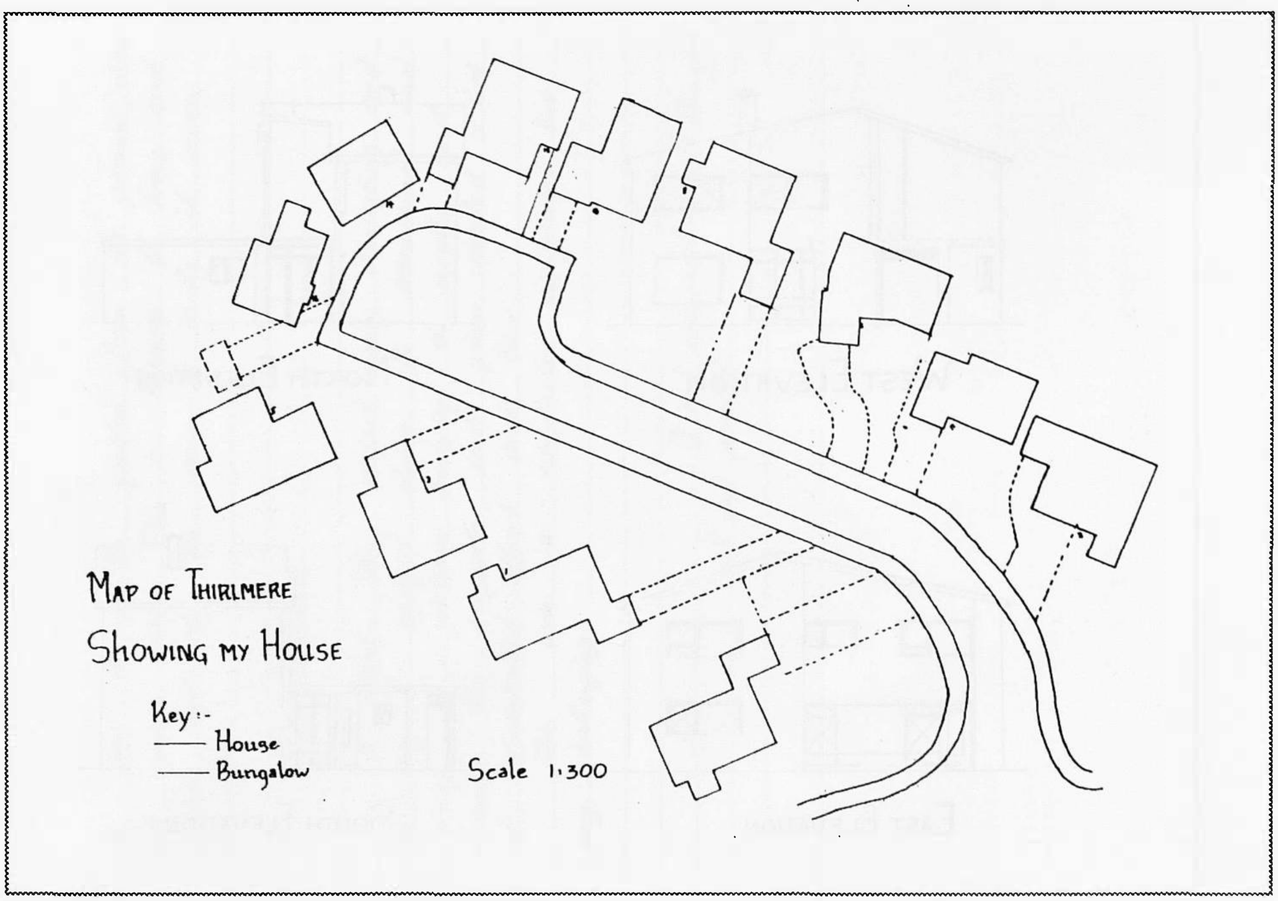
This was a complicated assignment, but very challenging.

A neat assignment – well thought out and presented


Kitchen
Dining Area
Fireplace
Lounge
Entrance Lobby
Store
Cloaks
Hall
Entrance Lobby
Garage
Toilet
Groundfloor Plan of the House
Key
Double Walls
Single Walls
Door
Window
Built-in-Furniture
Canopy and Hearth of Fireplac
Scale 1:40


West Elevation
North Elevation
East Elevation
South Elevation


Compost
Coniferous Hedge
Steps Up
Rose Fence
Retaining Stone Wall
Gate
Open Fence
A Plan of the House & Garden
Scale 1:150
Vegetable Garden
Greenhouse
Patio
Stone Wall
Pool
Lawn
Steps
Patio
Steps
Lawn
Original deciduous hedge
Stand of Birches
Rock Garden with Steps
Rock Garden with Path over
Rose Fence
Stone Wall
Rose Fence
Rock Gardens with Paths through


Map of Thirlmere
Showing my House
Key:-
House
Bungalow
Scale 1:300

G2/6

LOGOTRON

GCSE

PROJECT

Logotron

Logotron is geometry on a computer. Patterns and shapes can be made by giving commands to the "turtle", which moves around the screen, leaving a white line, depending on which instructions are given. The four basic commands are FD (forward), BK (back), LT (le and RT (right). These and several other commands can be collected together to make programs which can draw.

In my project I concentrated on nested three, four, five etc sided polygons because the process of nesting them was fairly difficult. The angle of each side and the angle at whi the polygons were nested had to be carefully thought out usin Pythagoras theorem, trigonometry etc. As well as making nested polygons I wrote some programs for different kinds of shapes and patterns to add interest.

Good

Logotron

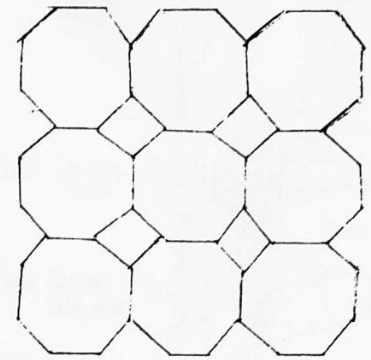
This project involves working out programs, on a computer, for drawing certain patterns and shapes.

The shapes I made are:-

Octagon and Square pattern
Spiral of Squares
Circles
Spiral
A Program to Make Many Shapes
Nested Three Sided Shapes
Nested Four Sided Shapes
Nested Five Sided Shapes
A Program to Make Any Nested Polygon
More Nested Three Sided Shapes

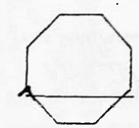
Logotron

A problem was set to produce one of a set of patterns, from which I chose the regular octagon and square pattern.



First I made a program to produce one octagon:

```
TO OCT
REPEAT 8 [FD 100 RT 45]
END
```



The turtle was in the wrong place to start the next octagon, so I changed the program slightly

```
TO OCT
REPEAT 13 [FD100 RT 45]
END
```

The turtle was now facing the wrong way so I added "Seth 0" to the program, which makes the turtle face upwards.

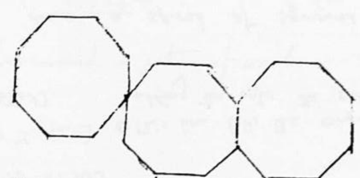
```
TO OCT
REPEAT 13 [FD 100 RT45] SETH 0
END
```

I needed more octagons so I added another program

```
TO ROW
REPEAT9[OCT]
END
```

but a 'long' line of octagons was printed.

This time the pattern turned out like this;



This time only one octagon was out of line. The reason for this was because the number of times I had to repeat the length of the side was wrong, but did not show up until the last octagon.
I changed the program again;

```
TO OCT2
RT 135
REPEAT 11 [FD 100 LT 45]
END

TO ROW2
REPEAT 3 [OCT2]
END
```

The octagons were now drawn properly, so I only had one more row to make.
I had to get into position for the last row so added

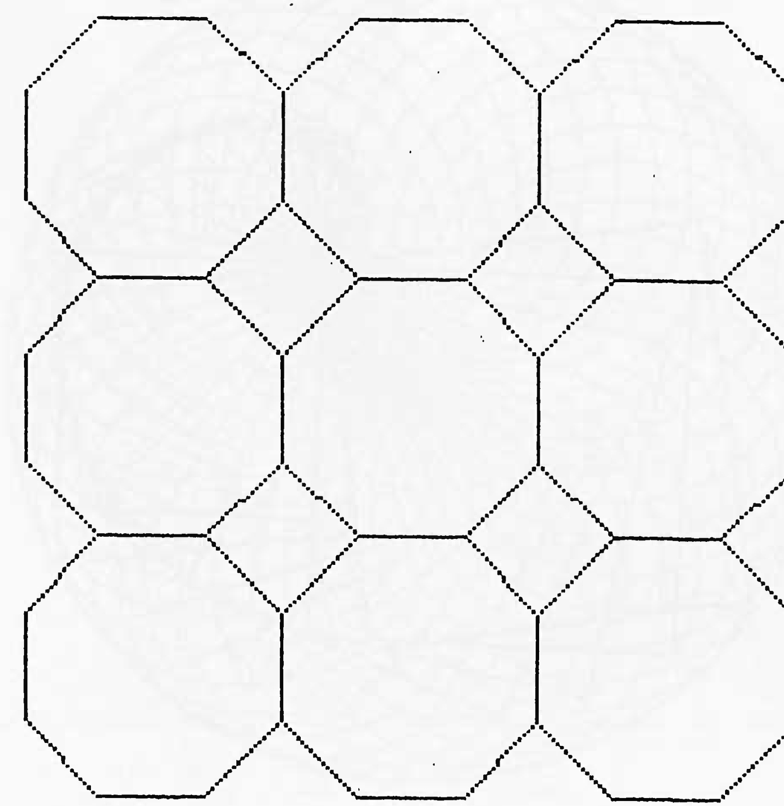
```
TO OCT2
RT 135
REPEAT 11 [FD 100 LT 45]
END

TO ROW2
REPEAT 3 [OCT2]
PU SETH180 FD 200+141.42 PD      (the diagonal of one square again)
END
```

Excellent.

To obtain the last row, the program for the first row was used again.
I made a short program to move the turtle to the top left hand corner of the screen

```
TO P
PU FD200   LT90 FD 200 PD
END
```



Squares

The next set problem was to chose between several patterns to make. I chose the spiral of squares.

(see printed sheet)

first I made a program to produce one square with a variable in so I could change the length of the sides:.

```
TO SQ :S
Repeat 4 [FD:S RT 90]
End
```

I then altered the program so a spiral of squares, gradually going up in size would be drawn:.

```
TO SQ :S
Repeat 4 [FD :S RT 90]
RT 10
End
```

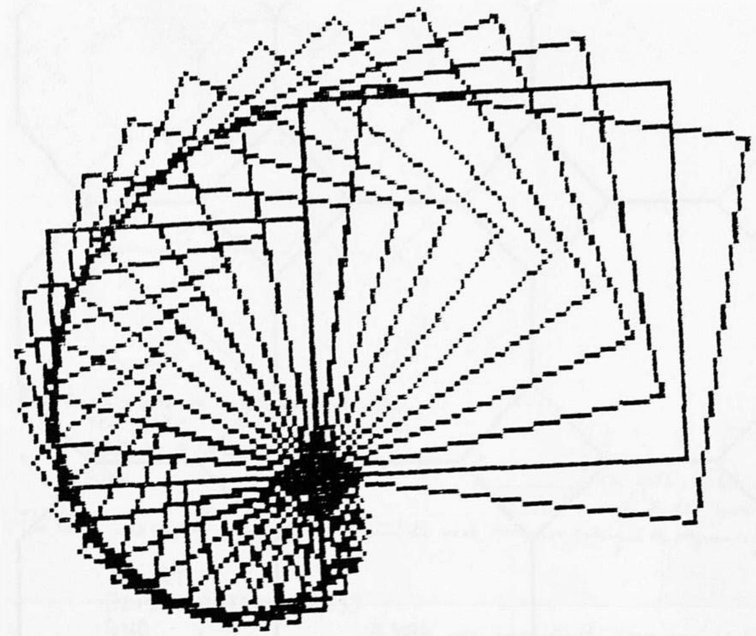
This worked but I had to type in the size of each square before it could be drawn.

Again I altered the program, this time I added a 'loop' to the program (when the computer reached a certain line it went back to the begining.).

```
TO SQ:S
Repeat 4 [FD :S RT90]
RT 10
SQ :S + 10          ← This line tells the computer to go back to the begining
IF :S = 500 [STOP]    adding ten onto the variable last used
END
```

The computer drew a spiral of squares, but did not stop when the variable was 500 (as line five tells it). This is because it is after the line telling the computer to go back to the begining, so every time the computer went through the program, it just missed it. Good point.

The line telling the computer when to stop had to be put nearer the begining, where it could be found, so I placed it between the first and second lines:.



SQ2 10 4 88

Circles

My next program involved a pattern of circles.

```
TO CIRC
REPEAT 360 [FD 2 RT 1]
RT 10
END
```

This program told the computer to draw a circle, then move the turtle right by ten degrees, ready to start the next one.

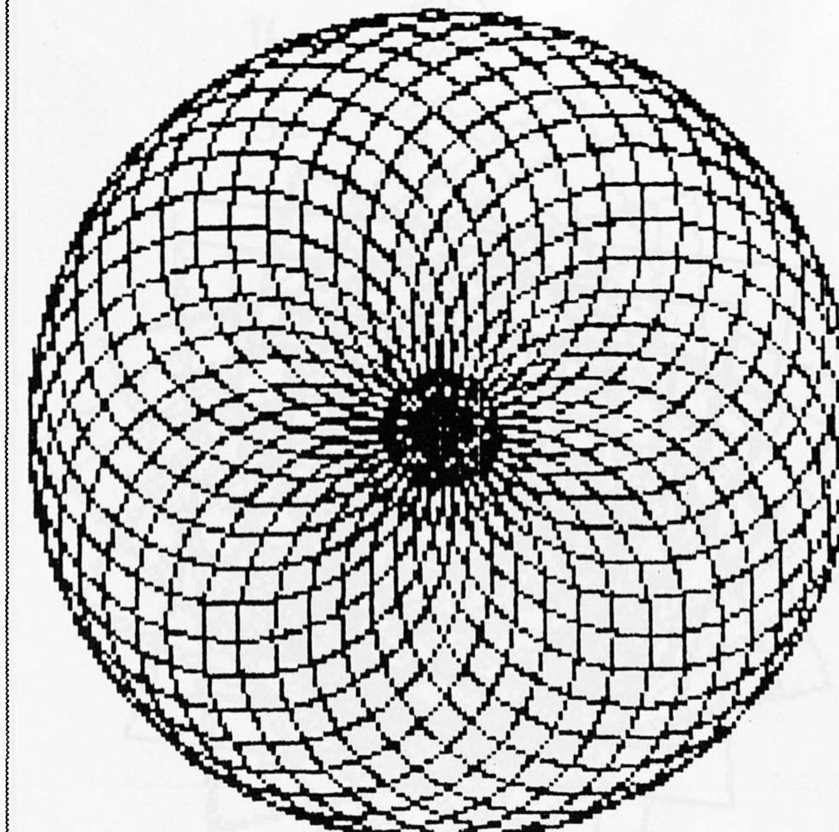
```
TO CIRCLE
REPEAT 50 [CIRC]
END
```

This told the computer to draw 50 of the circles in the "CIRC" program, moving ten degrees before starting the next one. A loop of circles was formed, but when the loop's end met the beginning, they over lapped, looking messy, so I knew less circles were needed. I worked out that if I moved ten degrees after every circle, and there were 360 degrees in a circle, 360 ÷ 10 = 36, which meant I only needed 36 circles in the loop.

Good

```
TO CIRCLE
REPEAT 36 [CIRC]
END
```

This time the circle pattern worked.

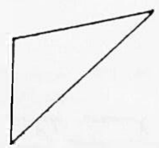


Spiral

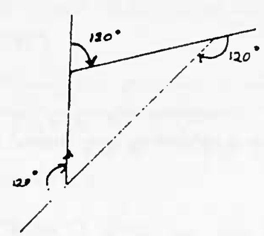
For the next program I decided to make a spiral.

```
TO SP :S
IF :S =500 [STOP]
REPEAT 50 [FD:S RT 120]
END
```

Once the program had run, I saw the turtle was just drawing a big triangle, over and over again.



I realised this was happening because I had told the computer to turn 120 which is the size of each external angle in a triangle.



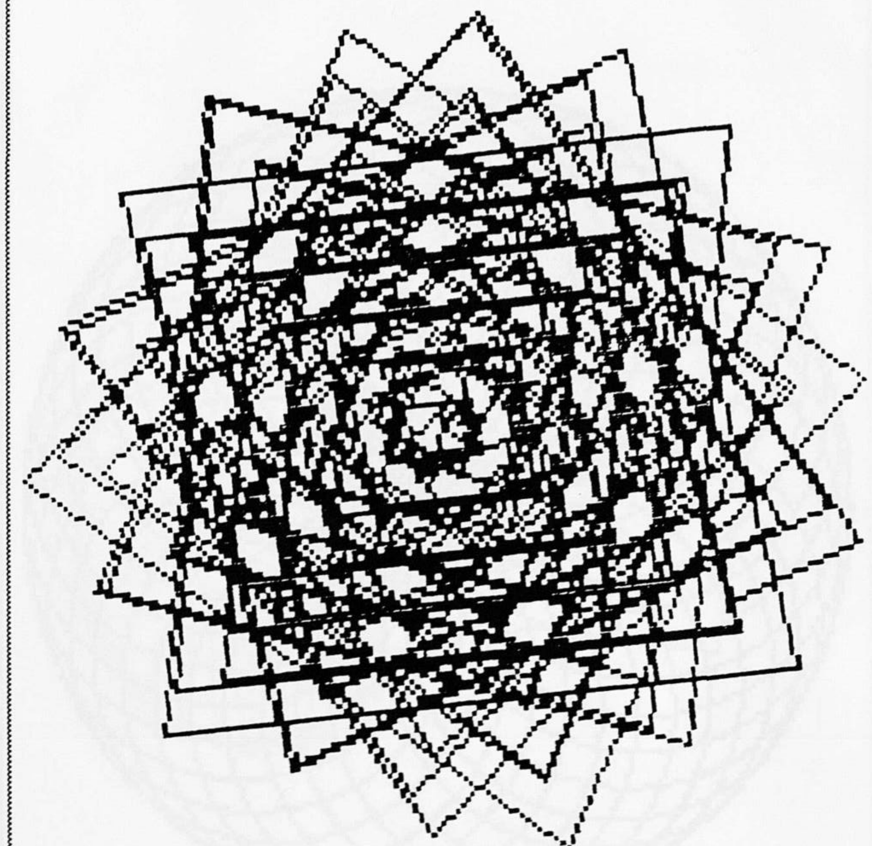
well thought out.

I changed the program slightly, providing a different angle; this time the spiral was drawn. I also added a line which changed the size of variable, making the lines longer after every run through the program.

```
TO SP :S
IF :S =500 [STOP]
REPEAT 50 [FD:S RT 120]
SP:S +10
END
```

I really like this design — it produces a marvellous effect.

This time the program worked



SP2

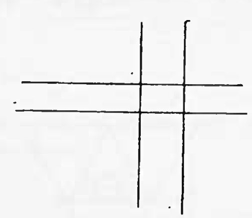
Shapes

I next wrote a program in which all the lengths and angles could be changed by a variable.

```
TO S :S
REPEAT :S [FD :S RT :S]
END
```

I typed in " S 239."
The turtle kept weaving in and out around itself untill a solid white ring was produced

When "S 9990" was typed in four lines were drawn again and again.



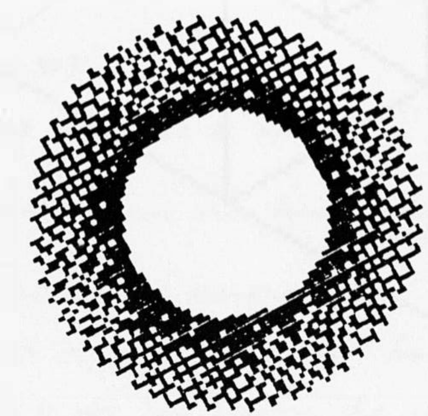
"S 246" was typed in, and, a circle of complicated lines was drawn

I altered the program slightly so three different variables could be given.

```
TO S :S :P :A
REPEAT :S [FD :P RT :A]
END
```

I typed in "S 50 200 100" and a circle of complicated lines appeared (as in the previous exercise) as did "S 50 300 297", but this circle had a larger

This was drawn when "S 246" was typed in

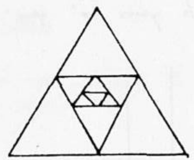


"S"

Three Sided Shapes

For my next investigation I decided to do three, four and five sided shapes (all regular polygons) which were nested inside one another.

My first problem :-



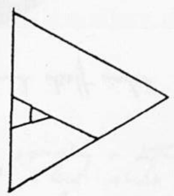
First I tried :

```
TO TRIANG :S
IF :S = 6.25 [STOP]
REPEAT 3 [FD :S RT 120]
FD :S/2 RT 120
TRIANG :S \2
```

(I decided on 6.25 because it is 100 divided by 2, divided by 2, divided by 2, divided by 2 (100 is the length of the longest side) which equals a length half of long as the smallest triangle, then fore stopping the program at the fourth trian

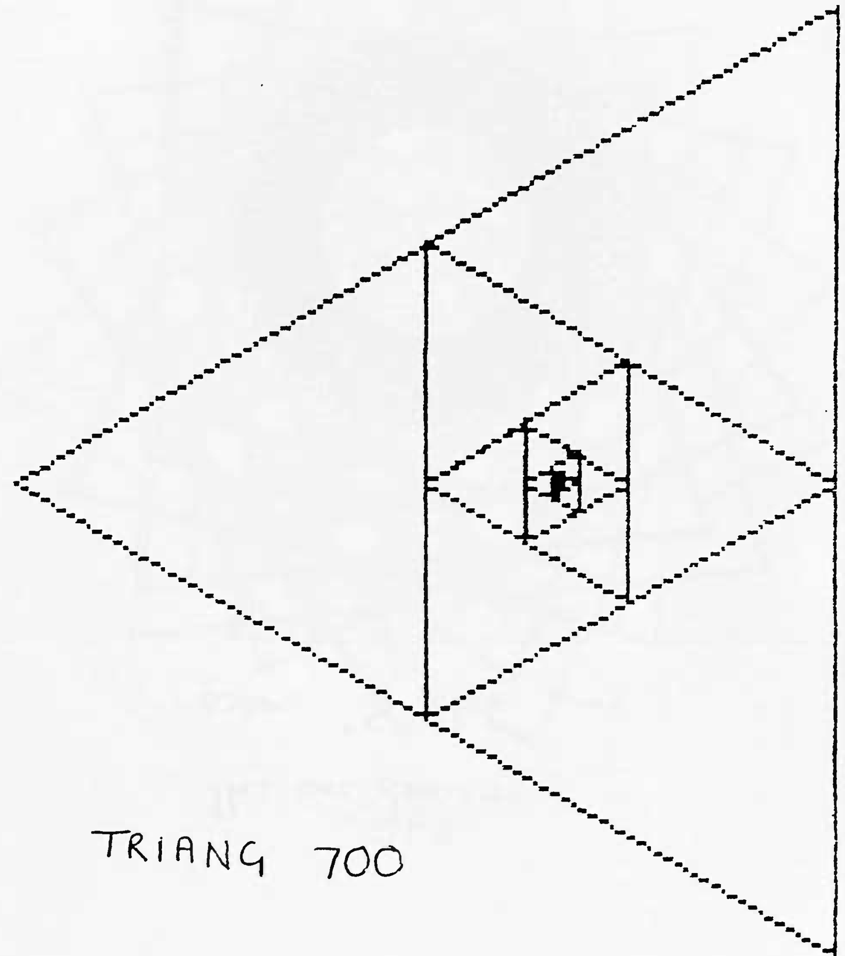
on the third and fourth line I decided to go right 120° because each angle side the triangle is 120°

This program produced :-



I then changed the direction of an angle...

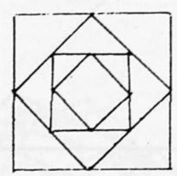
```
TO TRIANG :S
IF :S = 6.25 [STOP]
REPEAT 3 [FD :S LT 120]
FD :S/2 RT 120
TRIANG :S/2
END
```



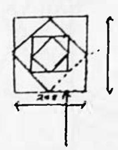
TRIANG 700

Four Sided Nested Shapes

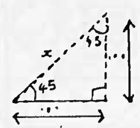
I decided to make a program which draws four sided nested shapes:-



to do this I had to work out the side lengths in relation to one another



This distance must be 100, because the [illegible] of the next nested square [illegible] touches the middle of the side of the square before ✓



(I knew the angles were 45° because a triangle's sum of internal angles = 180° 180 − 90 (the right angle) = 90. It was an isosceles triangle, so 90 ÷ 2 = each angle was 45°.) ✓

The length of the side of the second square can now be found by using the pythagoras theorem

$$100^2 + 100^2 = x^2$$
$$10000 + 10000 = x^2$$
$$20000 = x^2$$
$$\sqrt{20000} = x$$
$$\sqrt{20000} = 141.42136$$

Now the first two sides are known the connection between them can be found.

I tried many different ways to find the connection, and eventually found it. The side of the square had to be used. Its square root had to be found, which then had to be multiplied by 10. The answer was the length of the next square.

$$\sqrt{200} = 14.142136$$

✓ Excellent

$$14.142136 \times 10 = 141.42136$$

the pythagoras theory is:-

$$y^2 + z^2 = x^2$$
$$= y \times y + z \times z = x \times x$$

(to find x the square root of the answer has to be found.)

I tried:-

```
TO SQUA :S
REPEAT 4[FD :S RT 90]
FD :S/2  RT 45
SQUA SQRT (:S * :S + :S * :S)
END
```

Excellent.

The first square was drawn properly, but the rest just appeared everywhere because I had added the square of :s to the square of :s then found the square root of the answer, which was larger than :s, so the squares became bigger instead of smaller, so I divided each :s by 2. ✓

```
TO SQUA :S
REPEAT 4 [FD :S RT 90]
FD :S/2  RT 45
SQUA SQRT (:S/2 * S/2 + :S/2 * :S/2)
END
```

This time the nested squares were drawn properly.

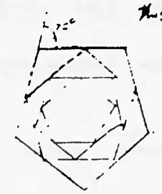
Extremely well thought out. With very little help (apart from your need to ask how Logotron did square roots) you carefully isolated each new problem and removed it.

Five Sided Nested Shapes

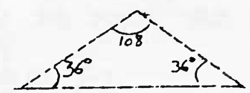
Before I could write the program for the five sided nested shapes I needed to find all the angles etc.

The total of the exterior angle in a polygon = 360

This is a regular shape each angle = 360 ÷ 5 = 72



72 ÷ 2 = 36



180 − (36 + 36) = 108

I made a guess that the pentagon was 3/4 the size of the one before

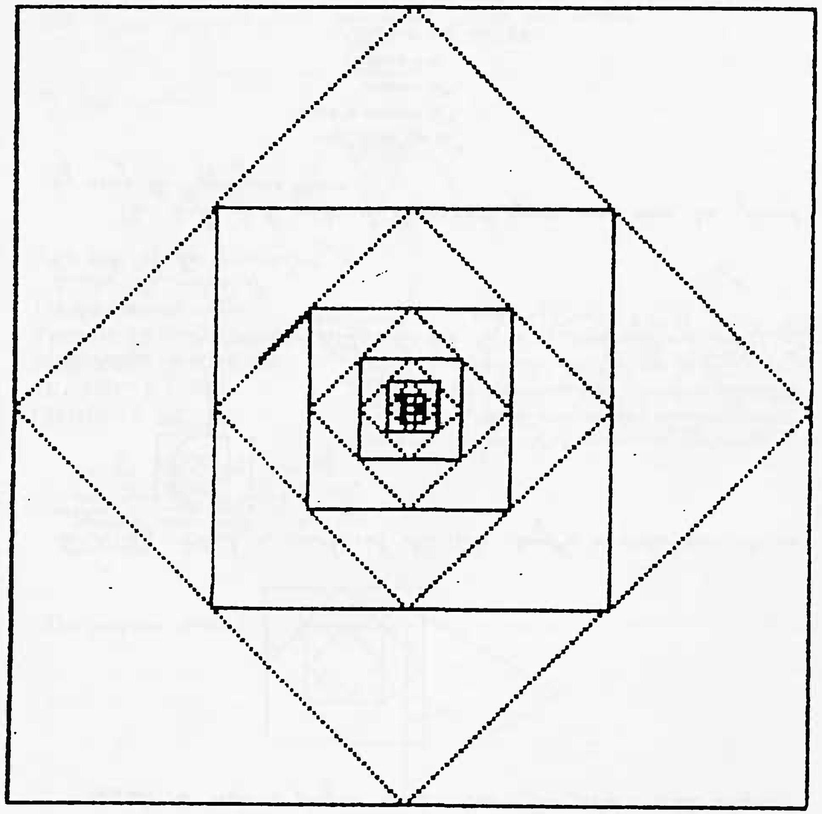
```
TO F :S
REPEAT 5 [FD :S RT 72]
FD :S/2  RT 32
F :S/4 * 3
END
```

All the pentagons were clustered to one side, so I tried again.

```
TO F :S
REPEAT 5 [FD :S RT 72]
FD :S/2  RT 32
F :S/4 * 4
END
```

This time, all the pentagons were the same size because in the fourth line I divided the variable by 4, then multiplied it by 4 leaving the number I started with, so the same pentagon was drawn again, but at a different angle.

I then realised I would have to work out all the angles and lengths exactly.



SQUA700

All Nested Shapes

Next I decided to write a program which, when the number of sides was typed in, would draw the nested shape of any polygon.

S = No. of sides
L = length of side

```
TO   G   :S  :L
REPEAT :L [FD :L RT 360/:L ]
FD :L/2  RT 360/:L/2
G :S :L* (COS 360/:L/2)
END
```

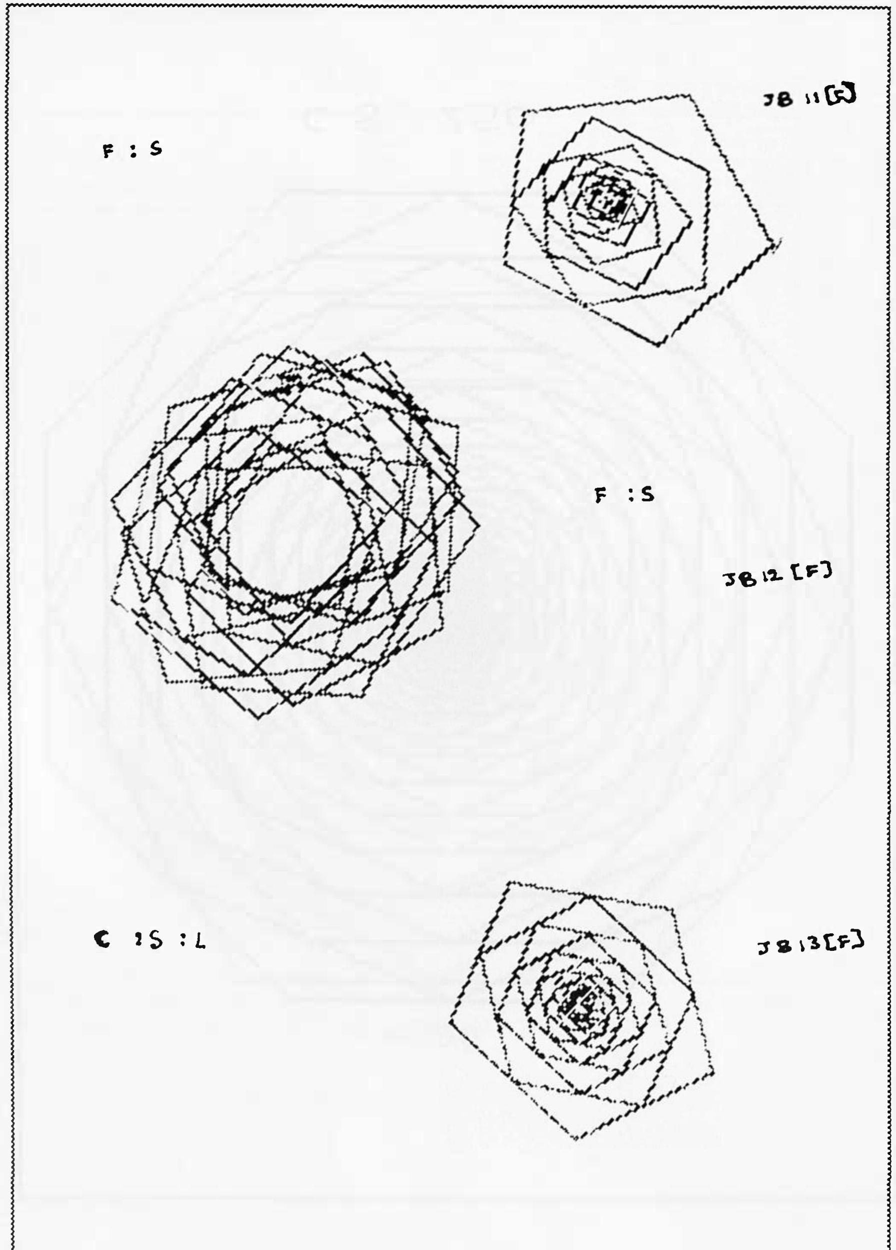
When I tried the program with a five sided shape two lines were drawn across the screen, so I checked my program. I discovered that I had mixed some of the variables around, which I changed.

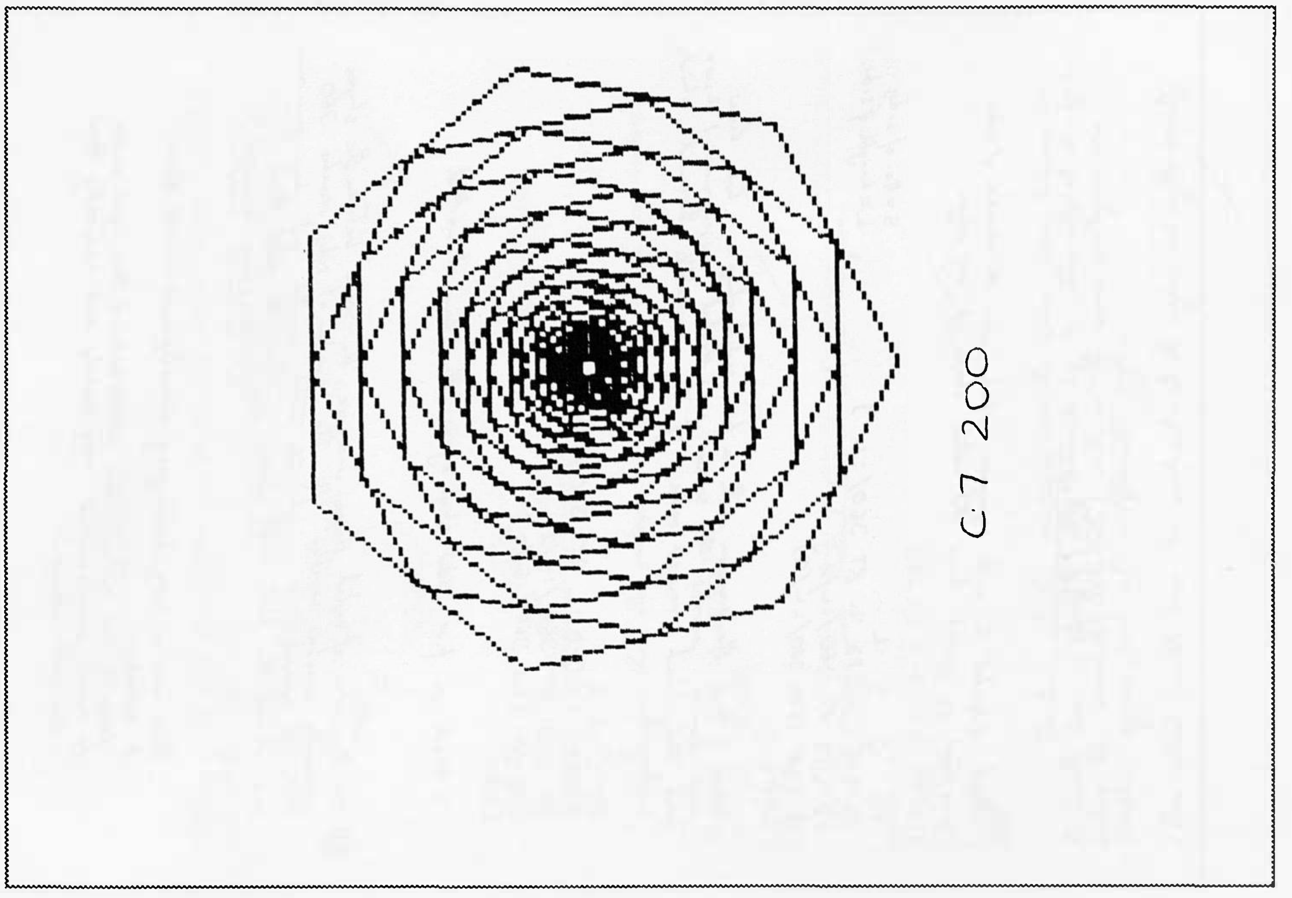
```
TO C :S :L
REPEAT :S [FD :L RT 360/:S]
FD :L/2  RT 360/:S/2
G :S :L* (COS 360/:S/2)
END
```

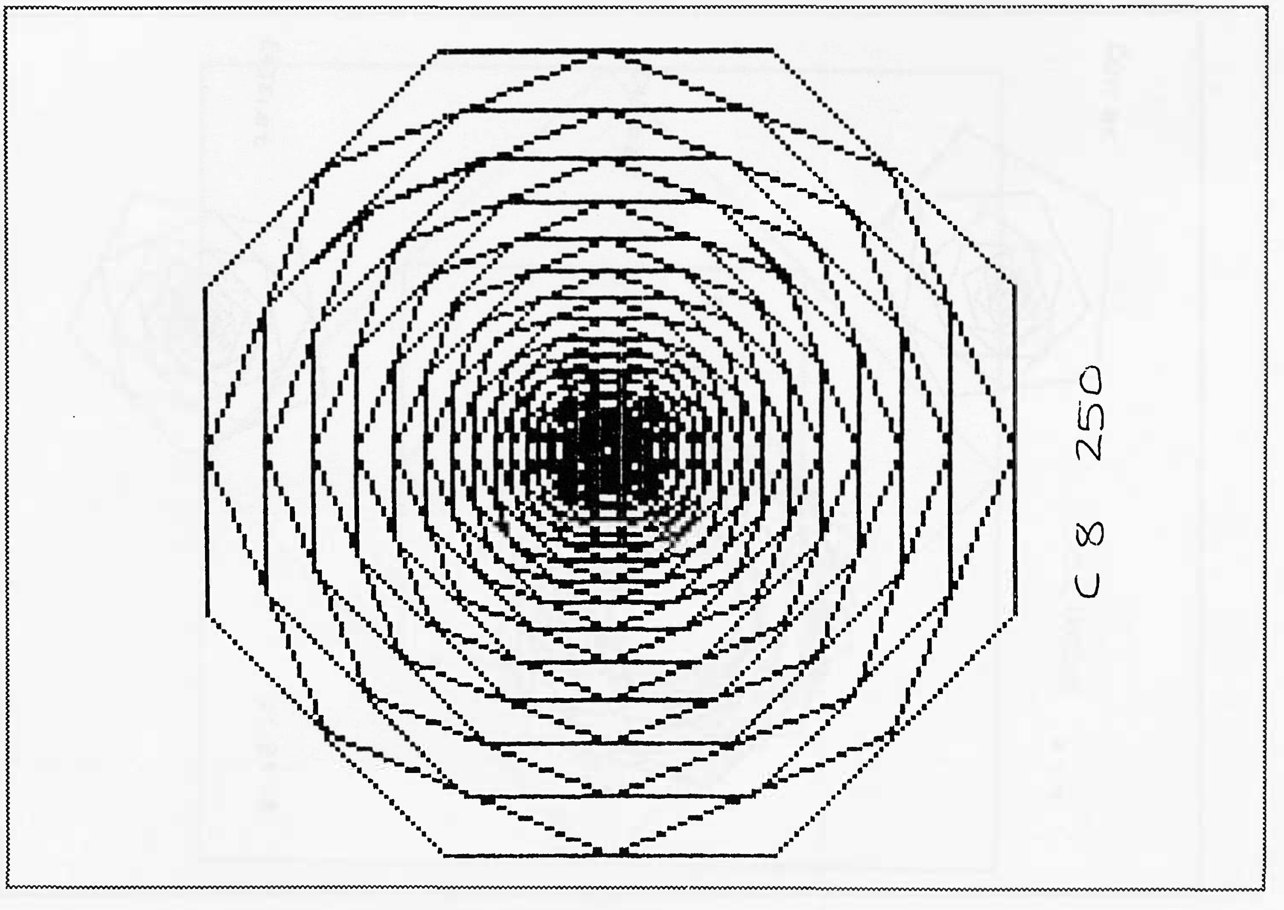
I tried a five sided shape again, this time it worked.

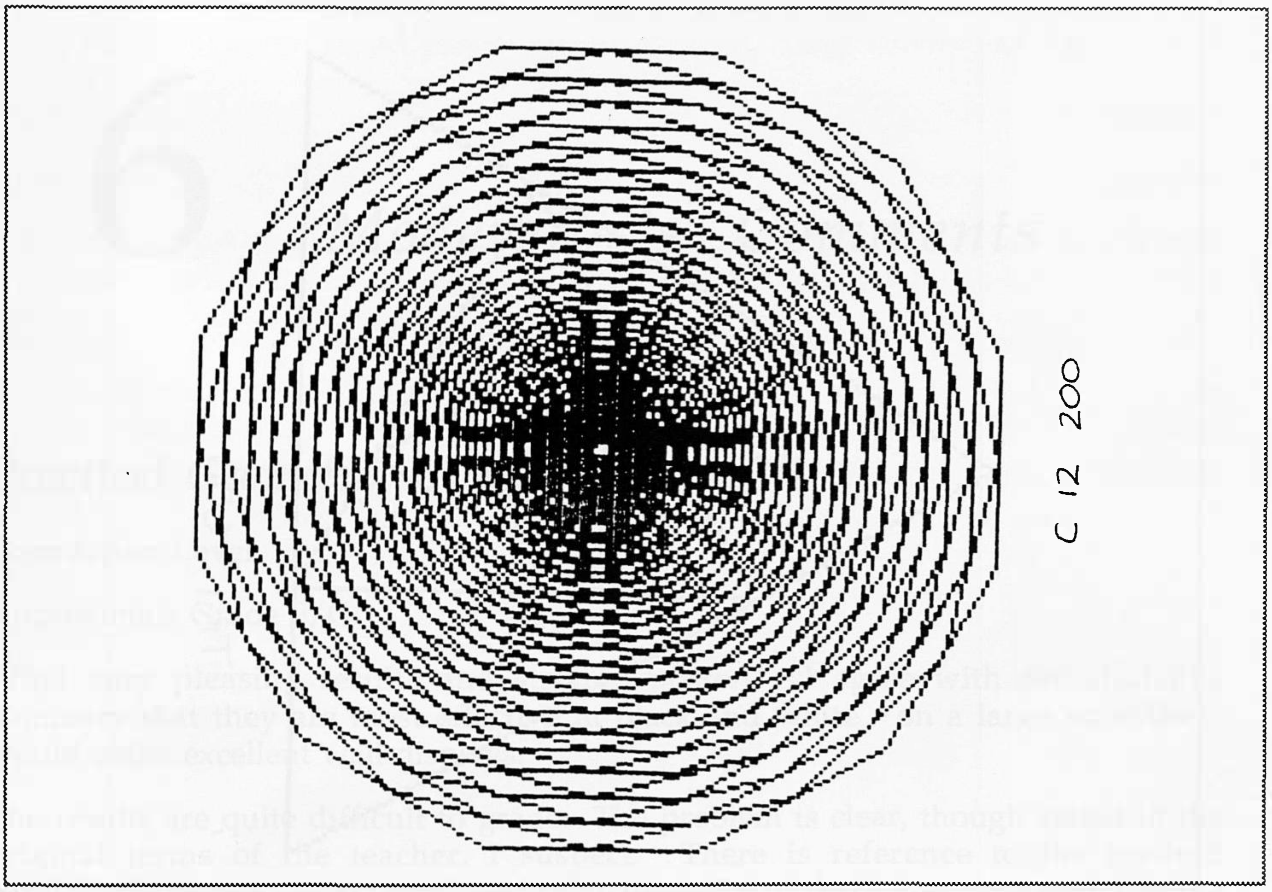
NB As the number of sides of nested shapes increase, the space between the shapes gradually becomes smaller. When the number of sides reaches 360 (the equivalent to a circle) the turtle would just keep going round and round the first circle drawn, unable to nest them.
(An interesting thought.)

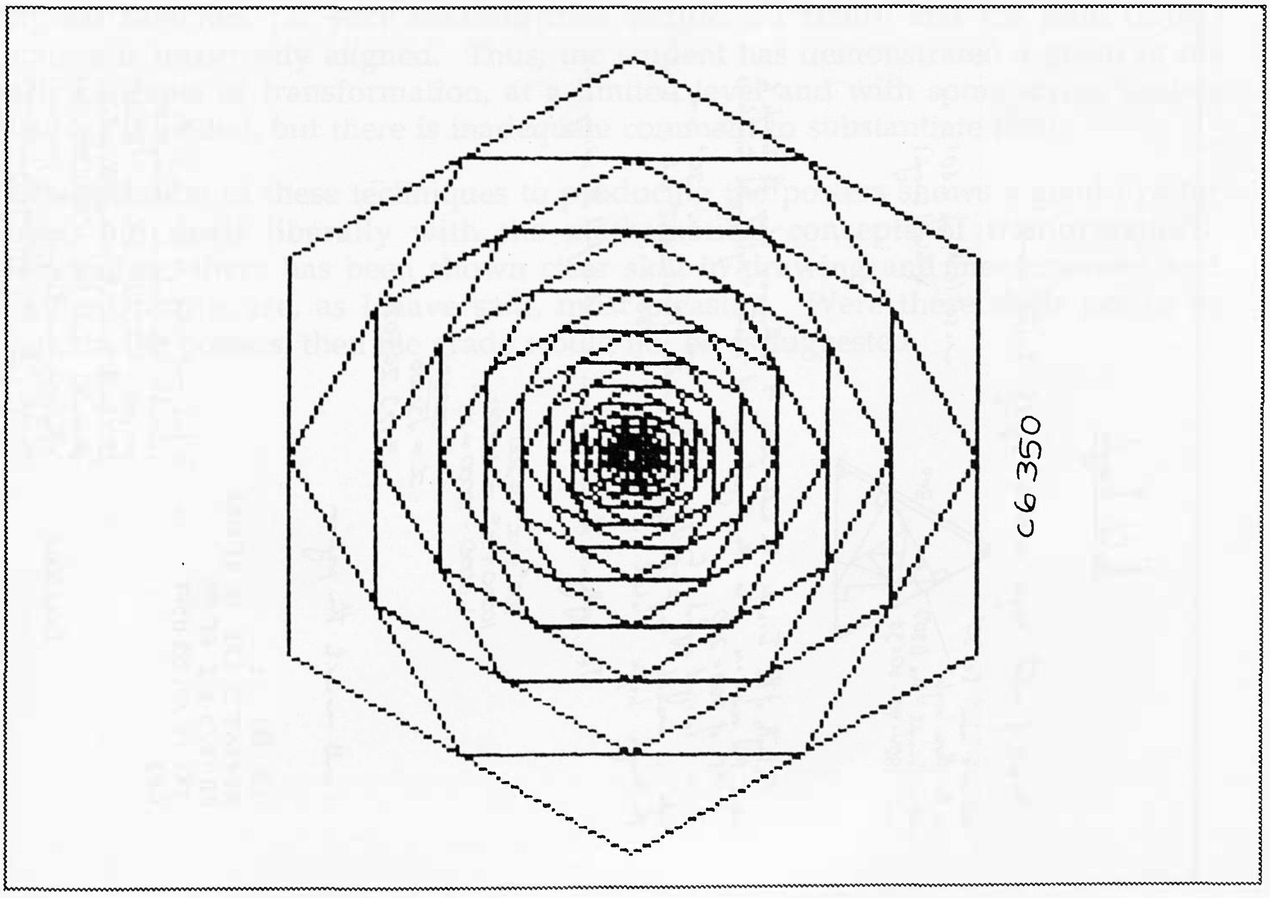
This was a very challenging development — well done.
~~seconds~~ -
Despite the sophisticated idea behind this, you seem to have developed it very easily and naturally from the last design. . . . . . .




C 7 200


C 8 250

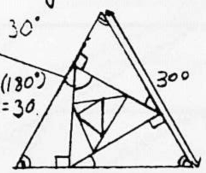

C 12 200


c6 350

Nested Triangles

This set of nested triangles were to be slightly twisted around.

This angle must be 30° as the three angles make a straight line (180°) and 180 − 90 − 60 = 30.



∴ 60° (each angle is 60° in an equilateral triangle)

The lengths of the triangle. The turtle moves 2/3 of the way along the triangle to begin the next one, so the longest side of each right angled triangle is 2/3 of 300 = 200

In a right angled triangle where one angle is 30°, one side is exactly half the length of another. The shortest side is 1/3 of the triangle before, which, in this case, is 100.

The pythagoras theorem can now be used to find the missing side.

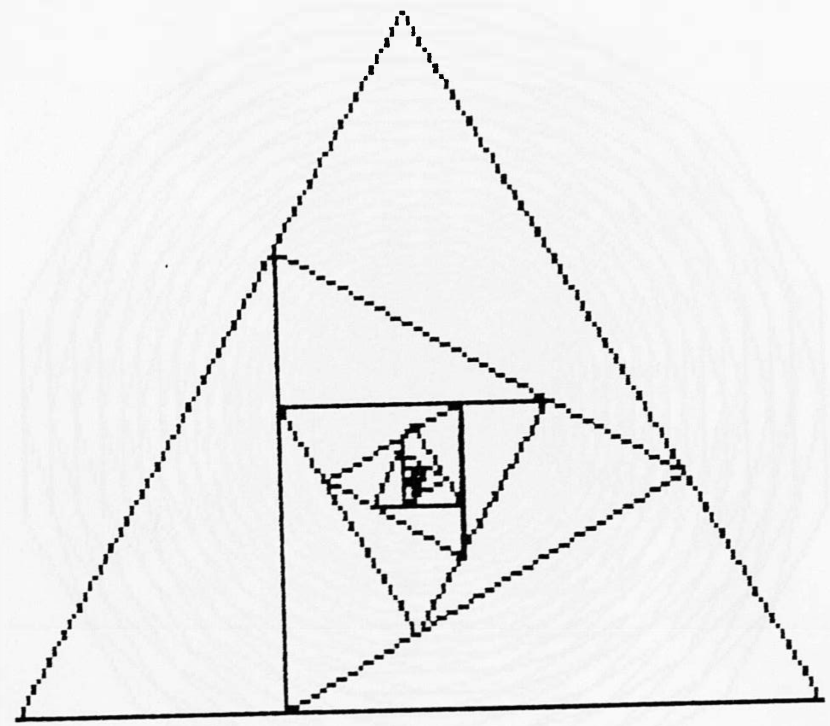
$100^2 + x^2 = 200^2$
$10000 + x^2 = 40000$
$40000 - 10000 = x^2$
$= 30000$
$x = \sqrt{30000}$
$= 173.20508$

I could now write the program

```
TO TRI :S
REPEAT 3 [FD :S RT 120]
FD :S/3 * 2 RT 90
TRI :S / 1.7320508
END
```

← To find out what to do with the variable after every run through the program I divided 300 by 173.20508 (the length found after using the pythagoras theorum) which was 1.7320508, and after every run through the program divided the variable by that number which left the length of the next triangle side

Excellent.



TRI 600

6 Moderator's Comments

Practical Geometry : Poster 88 G2/1

Foundation Level

Approximate Grade F/G

What very pleasing results and striking posters! I agree with the student's summary that they are most effective in black and white - on a large scale these would make excellent wall displays.

The results are quite difficult to grade. The problem is clear, though stated in the original terms of the teacher, I suspect. There is reference to the method employed, though this is not done in strict detail and there are some errors in the original sketches, i.e. each rotation uses a different centre and the final (270^0?) rotation is incorrectly aligned. Thus, the student has demonstrated a grasp of the basic concepts of transformation, at a limited level and with some errors (unless this was intended, but there is inadequate comment to substantiate this).

The appication of these techniques to producing the posters shows a good eye for effect, but deals liberally with the mathematical concepts of tranformations. Nonetheless, there has been shown clear skill in drawing and measurement and the final results are, as I have said, most pleasing. Were these skills not to be found in the posters, then the grade would not be as suggested.

Practical Geometry : Surveying G2/2

Foundation Level

Approximate Grade F

This represents a good attempt to answer the problem to a reasonable standard at this Level, and is certainly a good grade F piece.

There is little commentary except a few brief remarks about the superficial details of the survey. Although some problems are mentioned, he does not explain what he did to overcome them.

The solution is essentially an answer to a single stage problem, but the results are clearly drawn with some care. He has demonstrated skill in a limited range of techniques, and he has plainly enjoyed the task. I'm sure that this type of task, which involves pupils in a 'real' situation, is much more rewarding than a string of textbook situations. In the course of his work the student has also invented his own 'notation' for his drawings, and he has completed the task with some thought.

Practical Geometry : College Plan G2/3

Intermediate Level

Approximate Grade C/D

Despite the self confessed errors in the measurements, this piece demonstrates sufficient competence in a range of skills to place it at the higher end of the Intermediate Level spectrum. There are obvious errors in the execution of the work and the student's own comments highlight this very well. There is not a clear understanding of the techniques involved, as is freely admitted. Students should not feel they are 'giving themselves away' by being candid in their writings. Rather, they should be encouraged to explain clearly what they have done and what they have found difficult.

In the latter part of the project the student has applied trigonometric techniques, with reasonable accuracy - at Intermediate Level - to find the heights of the buildings. A possible improvement in this section would have been to apply simple checks on accuracy - something which does not seem to have been done anywhere. This could be accomplished using very trivial methods, e.g. counting bricks etc., but enough to support or refute the heights calculated.

Practical Geometry : Constraints G2/4

Intermediate Level

Approximate Grade C

What a most unfortunate beginning to this piece of work. The pupil has clearly worked hard at this only to receive a row of crosses here, which must be very crushing. I hope that it was pointed out to the pupils that not all this work is lost. Despite the misconception, the pupil shows a nice line in logical thought and develops his spurious argument in a sensible and reasoned way. One question is begged, what was the teacher doing while this misconception went uncorrected? It is a crucial part of our job to oversee and monitor our pupils' work. Where an obvious misconception has developed which could invalidate the entire work, then steps should be taken to remedy the situation. This could be done by judicious questioning, restating the case, or in the final analysis, telling a pupil the correct method. Marking should reflect this intervention but this is better than causing the entire project to founder.

Here, thankfully, the pupils has recovered the situation with some nice descriptive work on 3D loci and with a well developed analysis of the 'Goat Problem'. Had the early work been better done, the grade could have been higher.

Practical Geometry: Surveying : G2/5

Higher Level

Approx Grade C/B

This is an extremeley well presented and 'precise' piece of coursework, although it does present a peculiar problem. How can the authenticity of such a piece be verified, and how can the student's own participation in the original survey be judged? This, of course, is where the conversation between pupil and teacher is crucial and the clarifying of practical points is essential. The student could be asked to demonstrate some of the techniques involved at school.

Although the introduction and commentary are informed and lucid, they do not serve to explain any of the strategies employed to obtain the measurements, nor how the relative alignments of the buildings were checked. On the whole, the finished product is both clear and carefully drawn with measurements which appear to transfer tolerably well between the two drawings. Competence in a limited range of drawing skills is clearly demonstrated. The general attention to detail and tidiness, as well as the precision of the task, suggests a Higher Level piece.

Practical Geometry : Logotron : G2/6

Higher Level

Approximate Grade A

This candidate clearly deserves a high grade for the level of work and clarity of argument presented. The level of geometric understanding needed to produce the early shapes is quite low. However, the precision with which this understanding is applied, and shortcomings in the programs are corrected, indicates an excellent grasp of the problem. One of the key features that distinguish this project from more routine offerings, is that at all stages the writer makes clear and cogent comments upon the processes involved, and the likely implications of the work.

With the arrival of the National Curriculum and the growing need for cross-curricular work, this would seem to be an appropriate task to be assessed upon both Mathematical and other (perhaps Information Technology?) criteria.

Printed in England by Burgess & Son (Abingdon) Limited.

Published by the Shell Centre for Mathematical Education.

ISBN 0 906126 48 7

First published 1989.

9 780906 126486