EXTENDED TASKS FOR GCSE MATHEMATICS

A series of modules to support school-based assessment

MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

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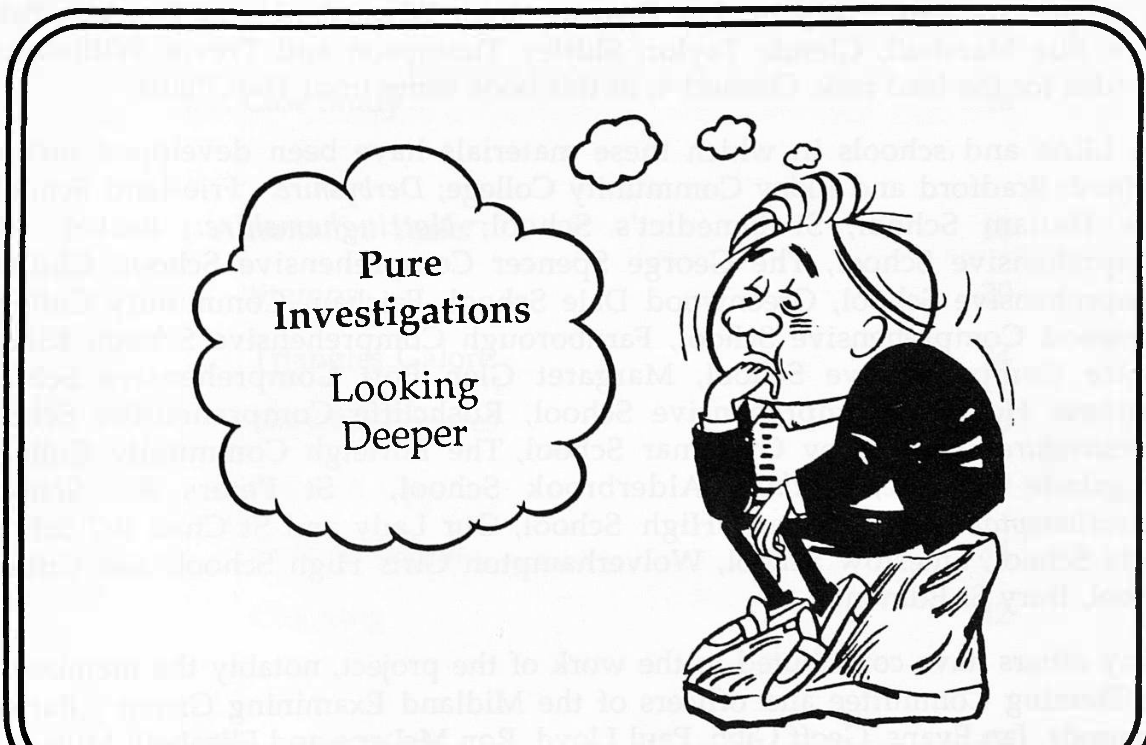


MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

Authors

This book is one of a series forming a support package for GCSE coursework in mathematics. It has been developed as part of a joint project by the Shell Centre for Mathematical Education and the Midland Examining Group.

The books were written by

Steve Maddern and Rita Crust

working with the Shell Centre team, including Alan Bell, Barbara Binns, Hugh Burkhardt, Rosemary Fraser, John Gillespie, Richard Phillips, Malcolm Swan and Diana Wharmby.

The project was directed by Hugh Burkhardt.

A large number of teachers and their students have contributed to this work through a continuing process of trialling and observation in their classrooms. We are grateful to them all for their help and for their comments. Among the teachers to whom we are particularly indebted for their contributions at various stages of the project are Paul Davison, Ray Downes, John Edwards, Harry Gordon, Peter Jones, Sue Marshall, Glenda Taylor, Shirley Thompson and Trevor Williamson. The idea for the lead task, Connect 4, in this book came from Tom Platts.

The LEAs and schools in which these materials have been developed include *Bradford:* Bradford and Ilkley Community College; *Derbyshire:* Friesland School, Kirk Hallam School, St Benedict's School; *Nottinghamshire:* Becket RC Comprehensive School, The George Spencer Comprehensive School, Chilwell Comprehensive School, Greenwood Dale School, Fairham Community College, Haywood Comprehensive School, Farnborough Comprehensive School, Kirkby Centre Comprehensive School, Margaret Glen Bott Comprehensive School, Matthew Holland Comprehensive School, Rushcliffe Comprehensive School; *Leicestershire:* The Ashby Grammar School, The Burleigh Community College, Longslade College; *Solihull:* Alderbrook School, St Peters RC School; *Wolverhampton:* Heath Park High School, Our Lady and St Chad RC School, Regis School, Smestow School, Wolverhampton Girls High School; and Culford School, Bury St Edmonds.

Many others have contributed to the work of the project, notably the members of the Steering Committee and officers of the Midland Examining Group - Barbara Edmonds, Ian Evans, Geoff Gibb, Paul Lloyd, Ron McLone and Elizabeth Mills.

Jenny Payne has typed the manuscript in its development stages with help from Judith Rowlands and Mark Stocks. The final version has been prepared by Susan Hatfield.

Contents

1 *Introduction*

LOOKING DEEPER is one of eight such 'cluster books' each offering a lead task which is fully supported by detailed teacher's notes, a student's introduction to the problem, a case study, samples of student work which demonstrate achievement at a variety of levels, together with six alternative tasks of a similar nature. The alternative tasks simply comprise the student's introduction to the problem and some brief teacher's notes. It is intended that these alternative tasks should be used in a similar manner to the lead task and hence only the lead task has been fully supported with more detailed teacher's notes and samples of student work.

The eight cluster books fall into four pairs, one for each of the general categories: Pure Investigations, Statistics and Probability, Practical Geometry and Applications. This series of cluster books is further supported by an overall teacher's guide and a departmental development programme, IMPACT, to enable teacher, student and departmental experience to be gained with this type of work.

The material is available in two parts

Part One		The Teacher's Guide
		IMPACT
	Pure Investigations	I1 - Looking Deeper
		I2 - Making The Most Of It
	Statistics and Probability	S1 - Take a Chance
		S2 - Finding Out
Part Two	Practical Geometry	G1 - Pack It In
		G2 - Construct It Right
	Applications	A1 - Plan It
		A2 - Where There's Life, There's Maths

This particular 'cluster book', LOOKING DEEPER, offers a range of support material to encourage students to undertake a pure investigation within any GCSE mathematics scheme. The material has been designed and tested, as extended tasks, in a range of classrooms. A total of about twelve to fifteen hours study time, usually over a period of two to three weeks, was spent on each task. Many of the ideas have been used to stimulate work for a longer period of time than this, but any period which is significantly shorter has proved to be rather unsatisfactory. The pure investigation tasks are, perhaps, rather different from the other two main types of extended task, those of a practical nature and those of an applied nature, in the sense that they allow students to seek out the pattern and beauty of mathematics without being constrained by real life.

It is important that students should experience a variety of different types of extended task work in mathematics if they are to fully understand the depth, breadth and value of the subject. Having emphasised the pure aspect of this cluster of ideas, it is interesting to note that the lead task within it does in fact start with a real situation, that of the commercially available game of Connect 4. The common element amongst all the items within this cluster is the idea that they may be used to stimulate generalisations or optimisations according to the individual need and ability of each student, hence the title of the cluster, LOOKING DEEPER.

Clearly, there are many styles of classroom operation for GCSE extended task work and it is intended that this pack will support most, if not all, approaches. All the tasks outlined within the cluster books may be used with students of all abilities within the GCSE range. The lead task of Connect 4 may be used with a whole class of students, each naturally developing their own lines of enquiry. It is intended that all the tasks within the cluster may be used in this manner. However, an alternative classroom approach may be to use a selection, or even all, of the ideas within the cluster at one time, thus allowing students to choose their preferred context for their pure investigative study. There is, however, a further more general classroom approach which may be adopted. This would be one that does not even restrict the task to that of a pure investigative nature. In this case some, or all, of the items within this cluster may be used in conjunction with those from one or more of the other cluster books, or indeed any other resource. The idea is that this support material should allow individual teacher and class style to determine the mode of operation, and should not be restrictive in any way.

Teachers who are new to this type of activity are strongly advised to use the lead tasks.

These introductory notes should be read in conjunction with the general teacher's guide for the whole pack of support material. Many of the issues implied or hinted at within the cluster books are discussed in greater detail in The Teacher's Guide.

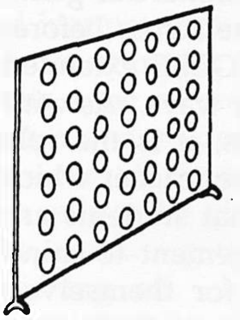
2 Connect 4™

The lead task in this book is called Connect 4. It is based on a readily available commercial game, and provides a rich and tractable environment for a pure investigation at GCSE level.

The task is set out on the next page in a form that is suitable for photocopying for students.

The Teacher's Notes begin on page 8. These pages contain space for comments based on the school's own classroom experiences.

CONNECT 4 ™



The game of Connect 4 is a very popular one with all age groups.

There are six rows (lines going across) and seven columns (lines going down) in a normal Connect 4 game.

This is a game for two players who take turns to drop their own coloured counters into the Connect 4 framework. The counter drops vertically down the column in which it is placed, and as far down as possible. The final position of each counter will of course depend upon how many others are already in that column. The winner is the first player to get four of their coloured counters in a line; this line may be horizontal, vertical or diagonal. When the game is over all the counters are taken out by releasing a holding mechanism at the bottom of the framework.

Try to find out how many different winning lines of 4 there are in this game?

What else can you find out about this game?

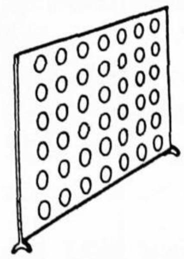
How could you change the game?

Find out as much as you can, but remember to keep some notes of your investigations as you go along so that you can write up your report when you have finished. Your report will be a very important part of the assessment so it must show everything that you did and thought about.

Investigate The Problem

Connect 4 -Teacher's Notes

Connect 4 is a readily available commercial game which students may well have come across before. It is an ideal starting point for a GCSE extended task and may be developed in many ways. As with many mathematical starting points, it is the role taken and activity stimulated by the teacher which are so important. It is essential that students are given both the time and encouragement to think about and consider the problem for themselves. They need to be encouraged to ask their own questions about the game since only in this way can we achieve a genuine individual student contribution to the task.



Connect 4 is a game for two players who take turns to drop a coloured counter into the Connect 4 framework. The counter drops vertically down the column in which it is placed, and as far as possible. The final position of each counter will of course depend upon how many others are already in that column. The winner is the first player to get four of their coloured counters in a line; this line may be horizontal, vertical or diagonal. When the game is over all the counters are taken out by releasing a holding mechanism at the bottom of the framework.

Many students will already know this game and its rules. It is quite useful, and certainly recommended, to have the actual game in the classroom when this work is being carried out. Students may well bring along their own Connect 4 games if requested to do so. There are also other very similar games on the market together with a few computer versions of the game. The most fruitful area for developing this task seems to be within general pure investigative mathematics. However, the work could well be carried into a more probability or permutations and combinations based study. Some may argue that it is in fact an application of mathematics to a student's everyday experience. It is accepted that all of these arguments hold true. However, it is the looking for patterns and moving towards generalisations that has dominated the classroom trials and hence the proposed category of pure investigative mathematics.

Clearly, it would be difficult, and inappropriate, to set out a two to three week lesson structure for this task, or indeed generally for this type of work. Much will depend upon the teacher and students, and how the work develops. However, it must be appreciated that sufficient time needs to be given to each aspect of the student's work: getting started, keeping going and finishing off. It is envisaged that time should be spent both in class and outside during all stages. Taking account of the above discussion then, it may be beneficial to outline one possible approach.

Understanding and Exploring the Problem

The ideal introduction to this task may well be to let the students play the game a few times against each other to get a general feel for it. They may then start on their own investigation by considering an initial problem such as

"How many different winning Connect 4 lines are there on this board?"

The game should be introduced within the context of GCSE mathematics and school-based assessment. Students will need an opportunity to discuss the rules and to play the game, even if they have done so before. Perhaps the initial question regarding the number of winning lines could be posed before this playing stage, since this will then allow the students' attention to be directed towards solving a problem from the outset. Students ought to be encouraged to discuss the game with each other, say within groups of four, as they play in pairs.

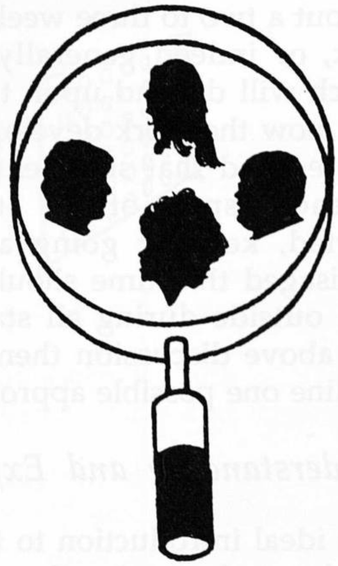
Questions such as

* Which positions are best?

* What direction is best for trying to win?

* Why is someone good at the game?

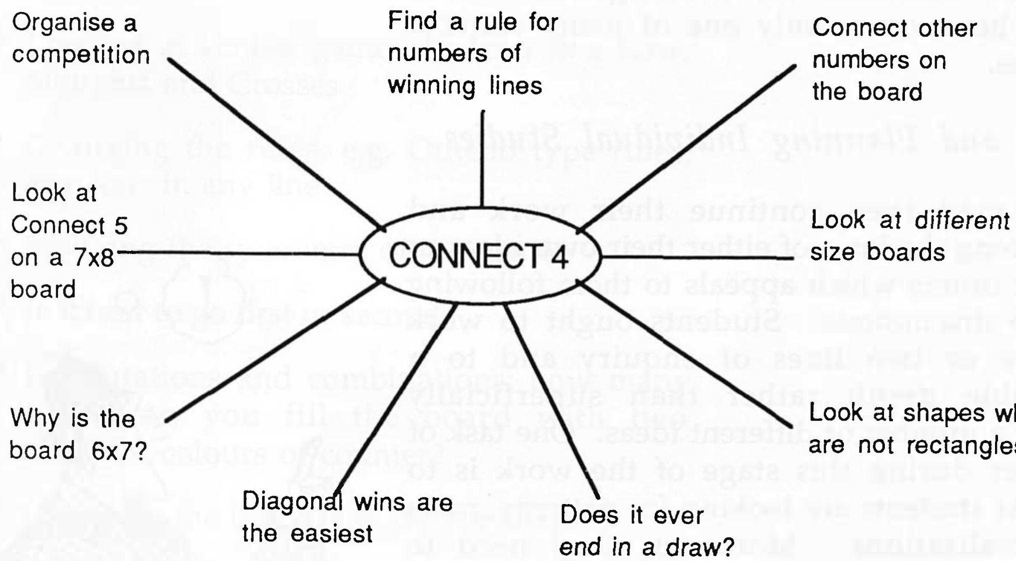
are all useful for aiding a broader understanding of the game and for encouraging students to identify avenues to be explored in more detail later on in the work. These questions are likely to come from the students during this playing and discussing stage. However, on hearing such questions it is worthwhile for the teacher to ask for the question to be repeated, and for further thoughts on this question within the group. The whole of the first week may well be taken up by this type of informal consideration, plus a more formal approach to the initial question relating to the number of winning lines on the standard Connect 4 board. Clearly, progress depends very much upon the individual situation.



During the trialling of this material, it proved to be essential that the teacher should continually emphasise that students should keep a note of their ideas and questions as they progress through the task. What is not recorded during one lesson is often forgotten before the next lesson. Students should be encouraged to try to use diagrams which they find helpful as they explain to each other the things they have discovered.

When the majority of students are getting into the problem, it may be a good idea to have a class discussion on what other things could be investigated relating to this game. Initially this is probably best done in small groups, but it is often profitable if this is followed by representatives from each group presenting their ideas to the class as a whole. Students may suggest changes to the game itself or further studies of the actual Connect 4 game as it stands. This will probably take at least half an hour although it is only an 'ideas stage' and the actual work need not be carried out immediately following the discussion.

The teacher or one of the students could act as a scribe by putting the ideas on the blackboard. A sunray or stick diagram like the one on the next page is quite useful for this purpose.



Each idea could then be discussed as a class with the teacher asking for further ideas. This really is an ideas stage and each idea need only be given a short time. The teacher here has to play a neutral role and allow all ideas to be put forward. However, some will naturally lead more easily to a pure investigation than others. It is here that the teacher may have to remind the group of the need to carry out such a piece of work and what this entails. Each student should create, in words, symbols or pictures, some mathematics that is new to herself. It is often easy for students to get involved in other types of work of a more practical or everyday nature or even ignore mathematics altogether in their study.

There are a number of closed questions which could be used to get the whole thing going. However, it would not be a suitable approach to simply issue students with a list of such closed questions and ask them to find the answers to these. What is being proposed here is an approach which offers a single question simply as a starting point and one which acts as an early focus for understanding the problem. While tackling this problem, students have time to consider what aspects are available for investigation. This approach however is only one of many suitable alternatives.

Devising and Planning Individual Studies

Students may then continue their work and develop along the lines of either their own ideas or one of the others which appeals to them following the group discussions. Students ought to work along one or two lines of enquiry and to a considerable depth rather than superficially looking at a number of different ideas. One task of the teacher during this stage of the work is to ensure that students are looking for patterns, rules and generalisations. Moreover, they need to communicate their conjectures and discoveries to others, using words, formulae and diagrams. As mentioned previously, it would be undesirable if a student wandered from this point and did not really experience a piece of pure investigative mathematics. At this stage in the task, the range of work may be vast both in ideas, quality and quantity. This can only be experienced by referring to examples of student's work, and not by simple description.

The initial investigation will then, hopefully, lead into at least one of a series of directions.

These may well include

* Generalisations to an NXM board
* Working in three dimensions (or more?)
* Devising a strategy for the game
* What are the best positions on the board?
* Investigating the probabilities

* Considering the general game Connect N. What values of N are commercially realistic?

* Looking at similar games, i.e.Four in a Row, Noughts and Crosses

* Changing the rules, e.g. Othello type rules, any four in any line

* Studying the symmetry of the game

* Is it best to go first or second?

* Permutations and combinations; how many ways can you fill the board with two different colours of counter?

* Changing the board size and/or shape

* Further generalisations

The above diversions and extensions may well be added to by your own students and this is something to be encouraged. Again, it would not be suitable to issue the above list of ideas on a printed sheet, or place them on a poster or blackboard.

However, if individual students cannot think of any questions to ask themselves, then the list may help you to formulate questions such as

* Can you think of any similar games to CONNECT 4?

* What if you change the rules?

* Do you think CONNECT 3 would be any good?

* What if the board was a different size?

* What is the best way to play Connect 4?

* Where would you put your counters? Why?

Implementing Plans and Pursuing Ideas

Students ought to be asking their own questions, at a suitable level and be seeking solutions to these themselves. This style of work needs a lot of encouragement and confidence.

It must be appreciated that with this type of task, students of all abilities can respond, but each at their own level. It is important therefore that the teacher encourages each student to achieve her full potential during the work. This involves each student stretching themselves using the mathematics that they already know, together with some further learning within the task. For some students this achievement may well be to consider the initial problem together with, say, similar problems relating to Connect 3 and 5 before writing a few comments about what happened and why. For others much more may be expected, with very able students possibly even obtaining a formula for Connect N within a three dimensional structure of size m x n x p, together with further considerations, some of which may surprise the teacher, and why not?

Reviewing and Communicating Findings

An important feature of the later stage of this work is the writing, or pulling together, of a report. This ought not to be in the form of an essay stating '... and then I decided to draw a diagram of ...' but a report describing and showing the work and mathematics carried out during the two to three week period. This ought to include mathematical pictures, diagrams, tables, algebra, calculations, descriptions and conclusions as appropriate to the individual student, task and work carried out. It is a good idea for teachers to outline this requirement at the start of the task and to continually remind the students of this throughout their work. Again this is not an easy task and it takes a considerable time. The report need not be a lengthy document.

Often this type of report writing is set as an exercise, to be completed for, or during, private study time. However, whilst accepting that a certain amount of this may be carried out in this way, the student may need, and benefit from, the opportunity to discuss their work during the time in which they are producing this report. Such discussion may well be with fellow students or the teacher.

The assessment, as with any pure mathematical investigation, will be based on a final report. This ought to be fully detailed showing all the stages through which the student progressed. It is often best to encourage students to keep a record or log in a rough note book as the investigation progresses. This note book will then be a great help in the writing of the final report or, as is often preferred, the notebook itself may be kept in such a way that it forms at least part of the final report. Naturally, you may well want the opportunity to take account of your individual discussions with the student when assessing the work and this can be included with relative ease. With extended task work it is often a pleasant surprise to find that the teacher can spend a fair amount of time talking to each student informally about their work. This allows the teacher to gain a greater overall understanding of what the student has achieved during her study and hence contributes significantly to the assessment of the student's work.

3 A Case Study

Fourth Year

Intermediate Level GCSE Group

This was the first time that these pupils had attempted any extended project of this nature in mathematics. I had some limited experience in project work for GCSE mathematics and so I was reasonably confident in attempting such a project with this group.

The "Student's Notes" were given to each pupil with little discussion and the pupils actually played Connect 4 for one lesson and in some cases two lessons, each lesson being one hour. The computer version was used by a small group and this was set up in one corner of the room. It was then that the suggested starter problem was set and the pupils began tackling the problem.

Following this, the majority of pupils quickly established the total number of winning lines and I was quite pleased with this. The difficulty came with where to take the problem. I asked them to think, as a class, about what they had done in the first part of the work. I then explained that they now had to extend their work in some way and that they had to decide. Initially there were not very many ideas and so I made a suggestion, a big error on my part I now feel. This was meant just as an example but it became the question that Sir had set. I suggested that they may like to look at Connect 5 on a 7x8 board and their study centred mainly around this, together with the initial problem in the student's notes. I shall avoid this approach next time, I think that they just need a little longer to think of their own ideas. The pupils then spent a lot of time drawing and explaining the horizontal, vertical and diagonal combinations and several of the group did a terrific job at this.

Offering avenues to explore was not easy for me as I found myself directing the work too rigidly with pupils who had no idea as to how to develop the theme. When I spoke to them on an individual basis, or to them as a four or three, they usually had several ideas on what they could do and I then felt it important to ask them about these in more detail. This usually got some spark into them and I was trying to get them to travel a long way along one direction rather than just doing lots of little things. I think that is important with investigation work.

The final written up projects were very varied in standard and generally I was pleased with the results for a first time full blown investigation. Many pupils, however, treated the project like they would a traditional project - pretty diagrams and neat paragraphs. Many of the projects were the same. This was due to possible teacher influence as I have previously explained. Few pupils really explored one avenue effectively and most pupils neglected an introduction to the problem. No pupils really set their own questions and were content to follow my suggestion. This was naturally a little disappointing particularly because I felt that I could have done things differently. However, I feel that the pupils got a lot out of this work and certainly the pupils and I now know what we need to do to be successful with this sort of work. It is much easier to explain to a group how to tackle an extended task, and what is expected, after you have completed at least one piece.

In a discussion following the project, pupils said how much they enjoyed the work and offered no criticism. However, when asked to write down their views some were more negative in their comments. Many pupils were disappointed with their marks and this perhaps again reflected their lack of experience.

Looking back upon the work there would be several points that I would raise.

a) Sufficent time is needed to really do a good job, I perhaps cut them short of a twelve hour target and this showed in their work. They had only really just got going when I wanted them to stop and write up.

b) Real Connect 4 games or the computer software is an essential motivator although as a teacher this needs careful directing.

c) Lack of experience on the pupils' part shows itself. It is important to develop the necessary skills before they reach the GCSE years.

d) There is a constant need for the teacher to emphasise to the students the need for them to look into their own ideas.

e) Teacher direction can be negative rather than positive. I certainly found this in the work of my pupils. They all followed the idea which I suggested as one possible avenue for investigation. I suppose this is lack of experience on my part.

f) Overall, I was quite satisfied with the work but I would hope for improvement the next time that I use it even if the pupils have little or no more experience than this group, I have moved forward myself.

4 *Alternative Tasks*

Stamps

Triangles Galore

Drawing Squares

Shape It Up

Dotty Polygons

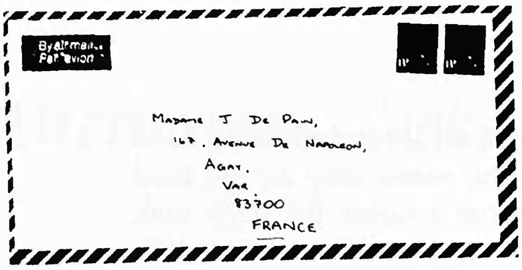
Chaining

Alternative Tasks

General Notes

The six alternative tasks are all intended to be used as pure investigations in the same way as the lead task, Connect 4. The teacher's notes for each task are brief and should be read and considered in conjunction with those for Connect 4. However, the student's notes are in the same form as those for Connect 4. The student's notes offered for the six alternative tasks in this cluster book are all written in a similar style. They outline the context of study to the student and offer one or two problems to be considered. This offers the student the opportunity to consider the problem and gain some understanding of it. Students are then invited to investigate the problem in any way they wish. However, there are further extension ideas which may be used if the teacher feels this is appropriate to any individual student, group or class. These suggest areas for investigation without prescribing exactly what should happen.

STAMPS



Stamps are used throughout the world. They are used as payment for sending things from one place to another. Stamps may be purchased either as singles or in books. There are many different values of stamp and these change regularly. The value of individual stamps often depends on the range of services offered by the postal service in that particular country and the cost of these services.

How much are stamps from your local post office?

Where else can you buy them from?

How many can you buy for a pound?

Suppose you could buy only 3p stamps and 5p stamps?

How would you send a first class letter?

A parcel?

Investigate The Problem

STAMPS : continued

You may like to consider investigating some of the following ideas

What about other total values using the 3p and 5p stamps?

How would you send a 32p airmail letter to Australia?

What if you had some other value stamps available? Try some.

What about using only the actual first and second class stamps as they are at the moment.

What total values can you make?

What total values are impossible with each set of stamps?

Can you spot any rules and patterns. If so try to write them down.

Find out as much as you can about this problem?

Can you think of any other situations in real life where a similar problem occurs?

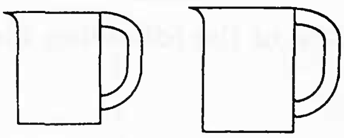
Try some of your own ideas.

Don't forget to keep a note of your work as you go because you will have to hand in a written report on your investigation when you have completed it.

Stamps -Teacher's Notes

Example

2 litres 3 litres



How do you measure 8 litres?

What about 23 litres?

This pure investigation is one which is reasonably well known in a variety of forms. Such investigation often develops from jugs of different sizes, weights or cuisenaire rods.

Such practical experiences offer an interesting starting point. If possible, some books of stamps could be made available to students.

The student's notes offer a few questions about the postage system for initial discussion before moving towards the more mathematical ideas of combinations of stamps. This initial discussion could be enhanced by the question 'Where else in life do we find this type of situation?' Clearly, the key feature of any such situation is a limited number of units to be combined. Apart from those already mentioned further studies could include: monetary systems, counting systems, scoring in darts, scoring in any other sport or game, buying kitchen units to complete a run along a whole wall etc.

One natural development for all these problems would be for students to consider what values can be made up from combinations of the basic units.

$$\mathbf{T = an + bm}$$

where a, b are integers

n,m are the unit values

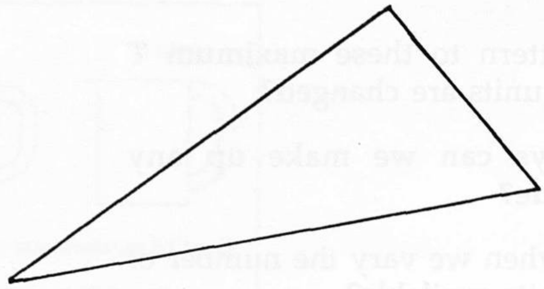
T is the total

Further questions such as

* What is the highest T value that cannot be made?
* Is there any pattern to these maximum T values when the units are changed?
* How many ways can we make up any individual T value?
* What happens when we vary the number of different basic units available?

This particular starting point is perhaps one which quickly leads on to individual students working on the same problem but using entirely different contexts. It is an ideal situation therefore for regular review sessions to take place in small groups. These could occur, say, two or three times during the study. Students simply explain what they are exploring or have discovered to other members of the group, who will be working on different contexts. For many students a different context will mean a different problem.

TRIANGLES GALORE



This triangle is known as a SCALENE triangle, this means that all three sides are of different lengths. All the sides of this triangle are a whole number of centimetres. Whole numbers are sometimes called INTEGERS. The largest side of the triangle is 7cm long. Can you find other triangles whose sides are integers, with the longest one being 7cm?

Investigate The Problem

You may like to try drawing some different triangles which fit the set of rules given, or you may like to think of another way to tackle this problem.

You may like to look at other longest lengths for your triangles.

You ought to state clearly any rules that you make up yourself.

Try to spot any patterns which allow you to predict the number of different triangles for any longest length without having to draw them all.

What about other shapes?

What about other rules?

Try any ideas which interest you.

Triangles Galore -Teacher's Notes

It would be easy for this problem to develop into a piece of practical geometry. However, here it is intended to be a pure investigation although naturally the first stage, understanding the problem, will probably involve some construction work to ensure students get a feel for the problem, only then will the pure aspect evolve. This work could also lead on to pure investigations of angle properties of polygons.

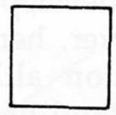
An interesting starting activity for a class could be to all attempt to draw a scalene triangle with integer lengths. Perhaps a natural triangle for students who are familiar with Pythagoras' Theorem would be the 3, 4, 5 right angled triangle. For students such as these, maybe a non right angled rule should be stated. A comparison of different looking triangles could then be made before the students move on to look at the initial problem on an individual basis.

This particular investigation may well involve students in drawing a large number or triangles before getting to grips with a systematic way of generating such triangles. For some students this stage may not be reached and their search for a pattern will be based solely upon empirical data. Questions like 'Which triangles are the same?' and 'How do I know that I have all possibilities?' will need to be considered.

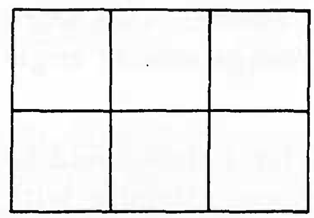
Further areas for investigation include

* A similar study of quadrilaterals
* Generating Pythagorean Triples
* A similar study with two fixed lengths
* Loci
* Similarity

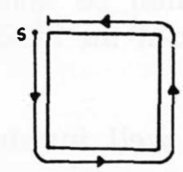
DRAWING SQUARES



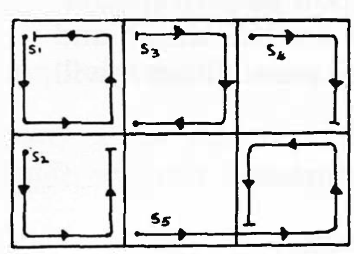
This shape is called a UNIT SQUARE. Each side may be considered as being one unit in length.



This shape is made up of six unit squares. The overall shape is a rectangle which is two units high and three units wide. This is sometimes called a 2 by 3 rectangle.



This unit square has been drawn using just one stroke. That is, you can draw the whole shape without lifting your pen or pencil off the paper. The stroke starts at the point S and then follows the arrows.



This 2 by 3 rectangle has been drawn using five strokes. When you draw shapes like this, you are not allowed to go over any line which has already been drawn. When you lift your pen or pencil up from the paper, you can start again at any point which you like.

Investigate The Problem

DRAWING SQUARES : continued

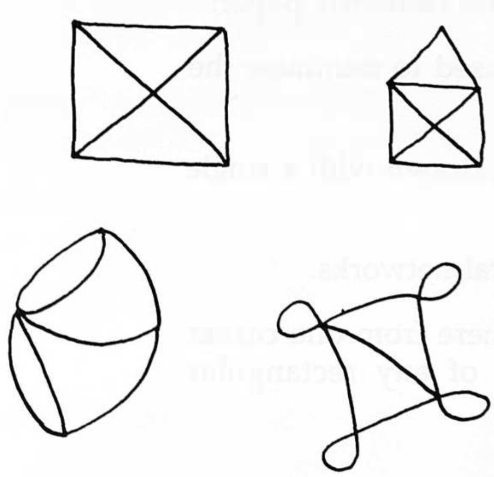
Investigate how different rectangles may be built up.

Investigate the minimum number of strokes which are needed to draw different sized rectangles.

What different shapes can you get if the stroke is 3 units long?

What about other length strokes.

What about the shapes drawn below? These are called networks. Which of these can be drawn with one stroke?



Can you work out when a network can be drawn with one stroke?

Investigate any related ideas which interest you or which you think may be important.

Try to find some rules and/or patterns.

Write them down.

Drawing Squares - Teacher's Notes

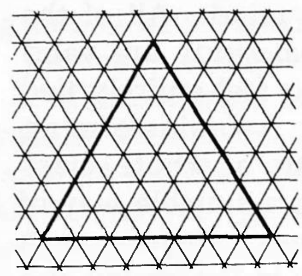
Drawing Squares is an investigation closely linked with ideas on traversability. Initially, students should be encouraged to look at different ways of drawing a relatively simple structure such as the 2 by 3 rectangle shown in the student's notes. By considering this simple case, students gain valuable insight into broader and deeper problems. Hence, this forms a suitable first stage activity.

A natural problem for investigation is 'What is the least number of traces needed for a given rectangular structure?' Following on from this would be the search for generalisations. However, many other ideas have arisen during our classroom trials and these include

* What is the longest single trace that can be drawn on a given rectangular structure?
* A similar investigation on isometric paper.
* What strategies can be used to minimise the number of traces?
* When can a structure be drawn with a single trace?
* Traversability and general networks.
* How many routes are there from one corner to the opposite corner of any rectangular structure?

Clearly, these ideas will only come from a class discussion if the students have successfully completed the first stage and then consider the second with a completely open mind. All of the above suggestions for individual study are valid and hence students ought not to feel entirely tied down by the initial activity. A brainstorming activity in small groups, with a whole class reporting back session would be a suitable way of generating some of these ideas, before each student completes her own work in the second and subsequent stages.

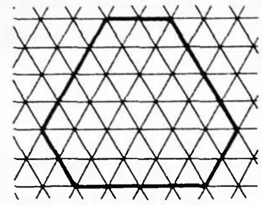
SHAPE IT UP



This shape is called an EQUILATERAL TRIANGLE. It has been drawn on ISOMETRIC GRID PAPER.

Perhaps you may like to find out what is special about this type of shape.

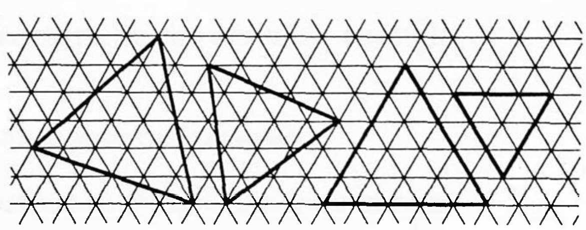
This shape could be called a TRUNCATED EQUILATERAL TRIANGLE.



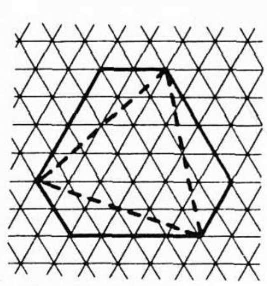
How many sides has this shape got?

Find out the usual name for a shape with this number of sides.

Here are some more equilateral triangles drawn on isometric lined paper.



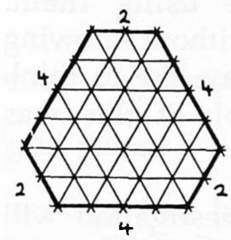
This equilateral triangle has been surrounded by a truncated equilateral triangle, but in a special way.



SHAPE IT UP : continued

Using one small equilateral triangle as a unit for measuring area, try to work out the areas of the two shapes in the above diagram.

The truncated equilateral triangle drawn in the previous diagram has sides of length 2 units and 4 units.



Investigate The Problem

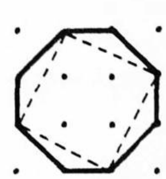
Investigate further cases and try to discover some rules if possible.

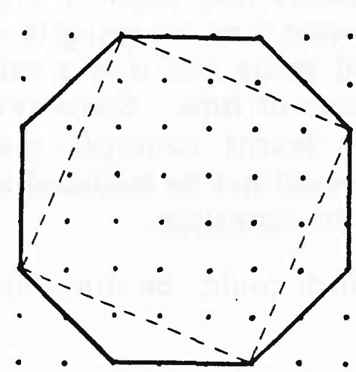
You may like to look at a relationship between the lengths of the truncated equilateral triangle and its area.

Another relationship to investigate could be the lengths of the truncated equilateral triangle and the area of the equilateral triangle inside it.

How about the lengths of the truncated equilateral triangle and the lengths of the sides of the equilateral triangle?

How about SQUARES inside TRUNCATED SQUARES (normally called Octagons)?





HOW ABOUT LOOKING INTO SOME OF YOUR OWN IDEAS?

Shape It Up - Teacher's Notes

Shape It Up is a straightforward starting point based on finding some form of generalisation between lengths and area. It is not a common or generally well known investigation, but that should not put you off using it in your classroom. Some teachers prefer to try out such new investigations themselves before using them. Others often use this type of idea without knowing what may come out of it. You may like to think where you would feel comfortable if this was considered as a continuum.

What emerges from the initial investigation will depend upon the ability of individual students. This, therefore, places even more emphasis on the first two stages, understanding the problem and devising a strategy to solve an individual problem. One way in which this first stage may be handled is to ask students to list, either orally or on paper, some of the key aspects of the diagrams provided in the student's notes. What aspects could be counted, compared, measured, calculated etc. For some students, individual lengths and perimeter may be more appropriate than area or ratio. However, it is best if the choices and ideas come from students, rather than the teacher deciding upon the range of concepts to be used or studied.

Following this type of introduction to the problem students will need a considerable time to look at some of these ideas before they begin to organise their thinking about what they are going to study. This may take several hours and it is a valuable experience, not a waste of time. Some revision work on previously learnt concepts may be necessary, but this would not be included in any written report on the investigation.

Some of the areas which could be considered by students include

* How many unit triangles in larger triangles?
* How many triangles altogether in any triangle?

* How many unit triangles point up and how many point down in a triangle?
* What is the maximum area on isometric paper given a fixed perimeter?
* What is the relationship between various lengths?
* What is the relationship between various areas and lengths?
* Looking at squares and truncated squares.
* Routes within shapes and grid papers.

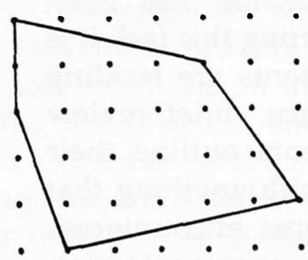
Although a class discussion session has been suggested as a suitable way of starting this task it is often quite fruitful, once all students are tackling their own problems, to introduce brief review sessions. In these, a few students outline their own line of approach, together with anything that they have discovered or problems encountered. This encourages mathematical communication and establishes a supportive environment in the classroom. This task is particularly suited to this type of informal discussion between students.

DOTTY POLYGONS

For this extended task you are offered the opportunity to investigate dotty polygons.

What do you mean, never heard of them?

Polygons drawn on dotty paper of course! You will be able to investigate anything that you want to relating to these shapes. You may not be able to think of anything to investigate at the moment but once you have had time to think and talk to the other members of your group about this, then you will soon get going.



DON'T FORGET: It is important to ask your own questions about your work as you go along. Always ask yourself things like

WHAT IF ..... I do this to my polygon?

..... I change this?

..... I look at this?

It is also important to keep a detailed record of what you are doing and why. Include all your drawings, ideas and questions, together with any answers which you come up with, even if your answer is 'Well, I couldn't really find out anything by changing this bit but it made me think about .....'.

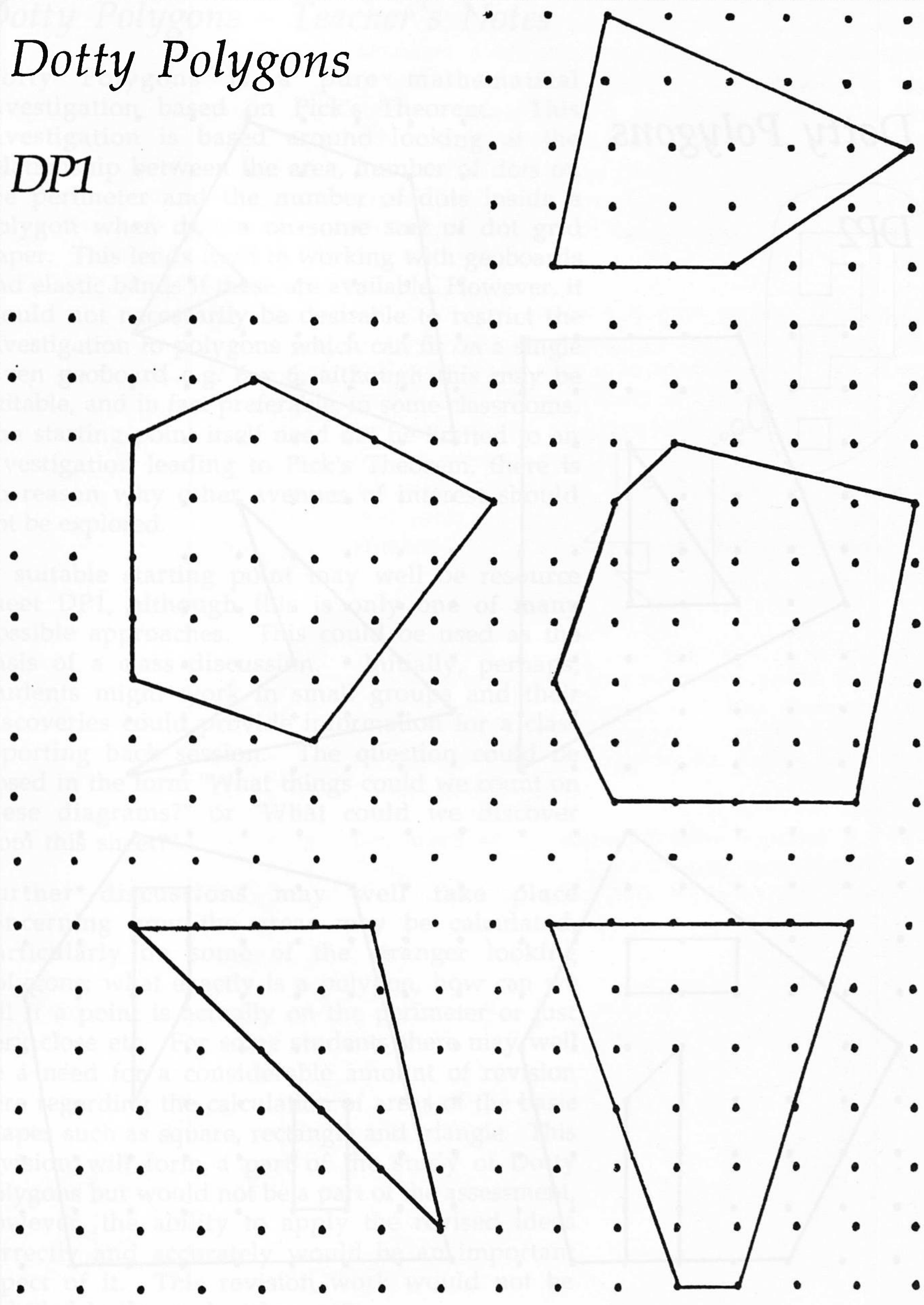
You will need to write a full report of your investigations to hand in as a part of your assessment. You may like to do this towards the end of your work or as you go along. Discuss it with your teacher and see how it goes.

Whatever happens, don't go DOTTY!

Investigate The Problem

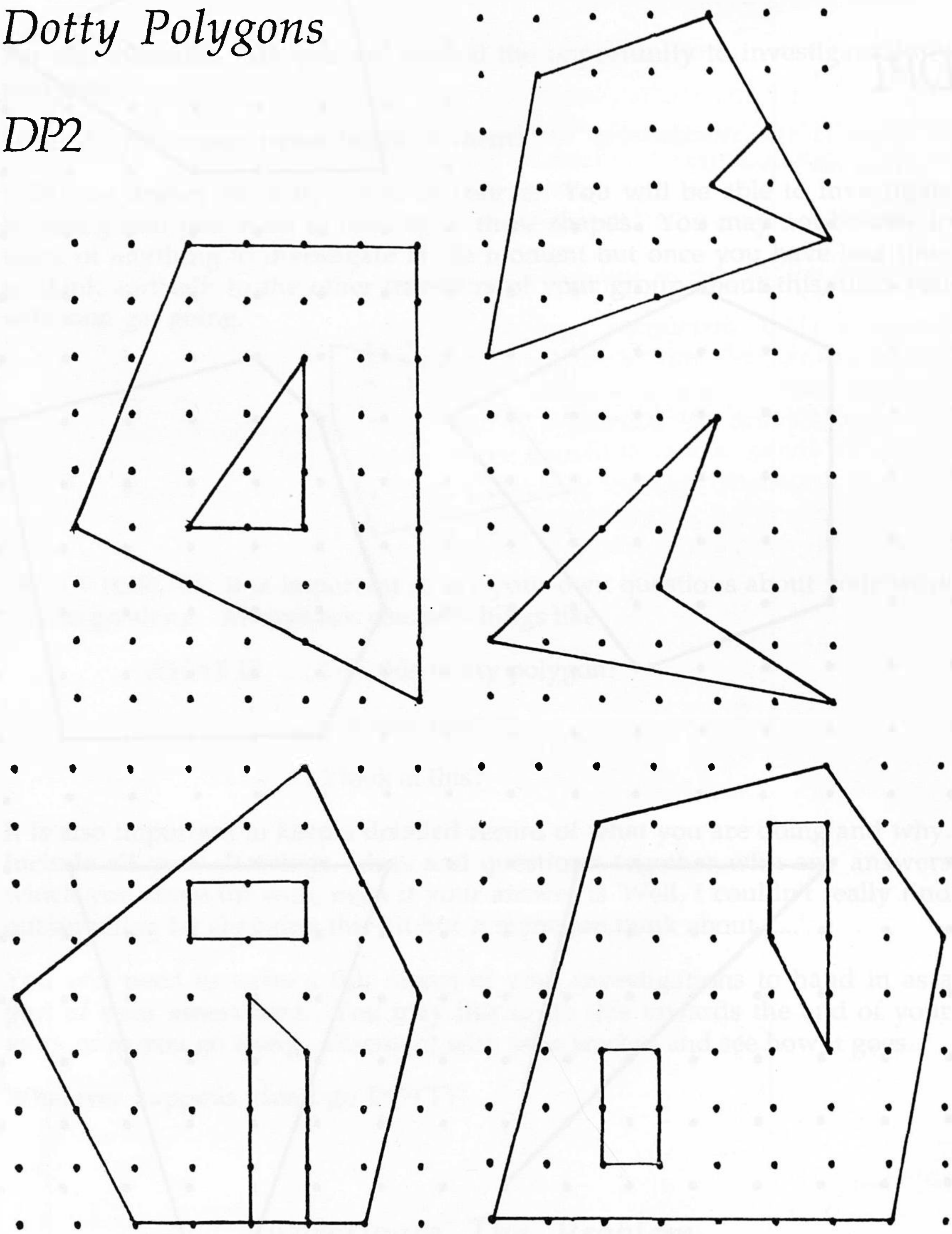
Dotty Polygons

DP1



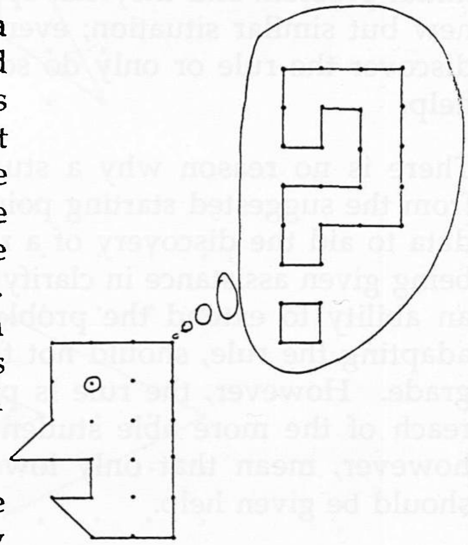
Dotty Polygons

DP2



Dotty Polygons - Teacher's Notes

Dotty Polygons is a pure mathematical investigation based on Pick's Theorem. This investigation is based around looking at the relationship between the area, number of dots on the perimeter and the number of dots inside a polygon when drawn on some sort of dot grid paper. This lends itself to working with geoboards and elastic bands if these are available. However, it would not necessarily be desirable to restrict the investigation to polygons which can fit on a single given geoboard e.g. 6 x 6, although this may be suitable, and in fact preferable, in some classrooms. The starting point itself need not be limited to an investigation leading to Pick's Theorem, there is no reason why other avenues of interest should not be explored.



A suitable starting point may well be resource sheet DP1, although this is only one of many possible approaches. This could be used as the basis of a class discussion. Initially, perhaps, students might work in small groups and their discoveries could provide information for a class reporting back session. The question could be posed in the form "What things could we count on these diagrams?" or "What could we discover from this sheet?".

Further discussions may well take place concerning how the areas may be calculated, particularly on some of the stranger looking polygons: what exactly is a polygon, how can we tell if a point is actually on the perimeter or just very close etc. For some students there may well be a need for a considerable amount of revision here regarding the calculation of areas of the basic shapes such as square, rectangle and triangle. This revision will form a part of the study of Dotty Polygons but would not be a part of the assessment, however, the ability to apply the revised ideas correctly and accurately would be an important aspect of it. This revision work would not be included in the students' reports.

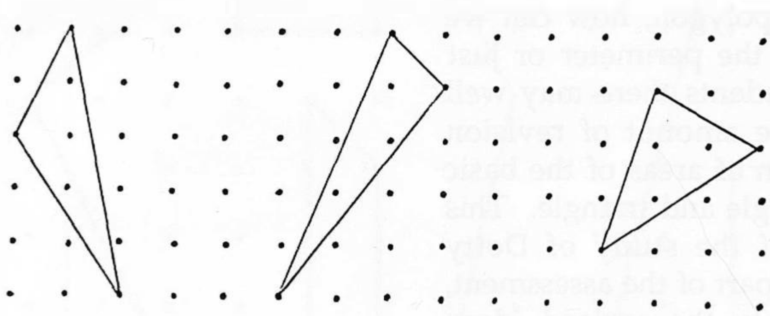
The relationship involved in this investigation is, perhaps, not an easy one to discover. However, it is possible for a student to complete a valuable piece of extended task work; ie. they contribute to their own learning, they demonstrate positive achievement, they gain an understanding of the initial problem and they can apply their ideas to a new but similar situation; even if they do not re-discover the rule or only do so with considerable help.

There is no reason why a student who sets out from the suggested starting point, collects relevant data to aid the discovery of a rule, and then after being given assistance in clarifying the rule, shows an ability to extend the problem by checking or adapting the rule, should not then achieve a high grade. However, the rule is probably within the reach of the more able students. This does not, however, mean that only lower ability students should be given help.

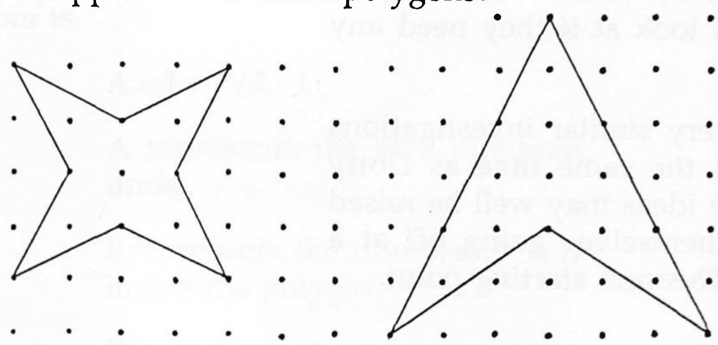
There are many extensions of the ideas involved with this work and they include

* What types of polygons is it possible to draw?

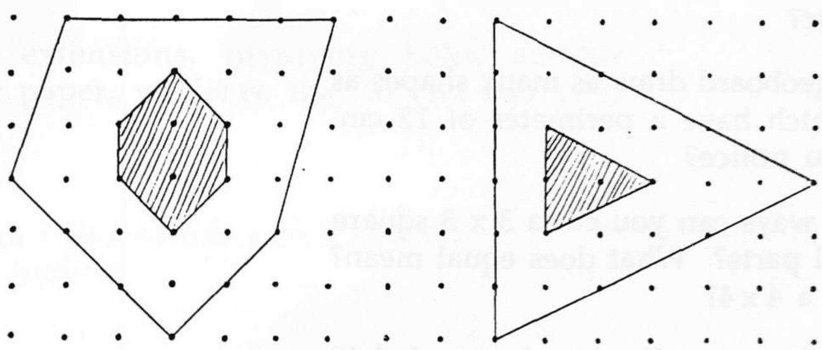
  e.g. triangle with 3 points on the perimeter and 3 inside, etc.



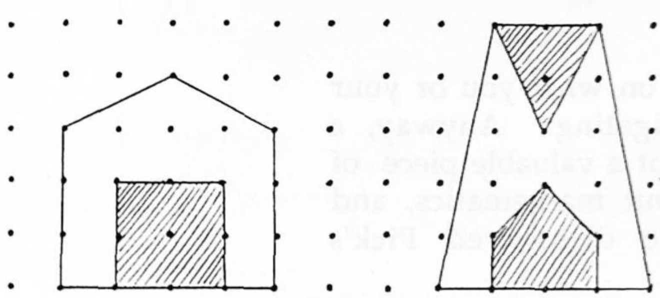
* What happens with reflex polygons?



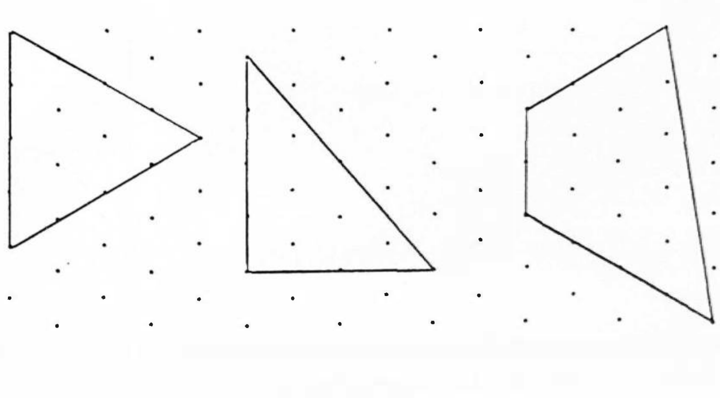
* What happens with 'polo-polygons' - polygons with holes in them?



* What happens with polygons with half holes - pieces cut out from the edge?



* What if isometric dotty paper is used?



Resource sheet DP2 offers some further dotty polygons for students to look at if they need any ideas for extensions.

There are many other very similar investigations which could be used at the same time as Dotty Polygons. Some of these ideas may well be raised by students who find themselves going off at a tangent from the Pick's Theorem starting point.

These may include

* How many triangles, or any other shapes, are there on a 3 x 3 geoboard? What about a 2 x 2, 4 x 4 etc?
* On a 3 x 3 geoboard draw as many shapes as you can which have a perimeter of 12 cm. What do you notice?
* How many ways can you cut a 3 x 3 square into 2 equal parts? What does equal mean? What about a 4 x 4?

All of these may be generalised and extended in their own right.

What is the result of this investigation, did I hear you say?

Well, I suppose it depends on what you or your students ended up investigating. Anyway, a student may have carried out a valuable piece of work and learnt a lot about mathematics, and themselves, but not have discovered Pick's Theorem.

However, if you really want to know, then look at the next page; but you have the option not to do so.

Pick's Theorem for any polygon on square dot lattice paper is

$$A = I + P/2 - 1$$

where A represents the area in square units

I represents the number of dots inside the polygon

and P represents the number of dots on the polygon's perimeter.

For the extensions, involving holes and/or isometric paper, we leave that to you and your students.

CHAINING

Numbers may be chained together in many ways. We simply make up a rule and see what happens.

EXAMPLES

A Helen: My rule is add the digits

I stop when I get a single digit

I could start with 47

What happens?

Let's see!

$$47 \twoheadrightarrow 11 \twoheadrightarrow 2 \twoheadrightarrow 2 \twoheadrightarrow 2$$

$$(4+7) \quad (1+1) \quad (0+2) \quad (0+2)$$

The number 2 is the end of the chain.

B Baljit: My rule is twice the units digit plus the tens i.e. 2U + T

$$54 \twoheadrightarrow 13 \twoheadrightarrow 7 \twoheadrightarrow 14 \twoheadrightarrow 9 \twoheadrightarrow 18$$

$$\twoheadrightarrow 17 \twoheadrightarrow 15 \twoheadrightarrow 11 \twoheadrightarrow 3 \twoheadrightarrow 6$$

$$\twoheadrightarrow 12 \twoheadrightarrow 5 \twoheadrightarrow 10 \twoheadrightarrow 1 \twoheadrightarrow 2$$

$$\twoheadrightarrow 4 \twoheadrightarrow 8 \twoheadrightarrow 16 \twoheadrightarrow 13 \twoheadrightarrow 7$$

$$\twoheadrightarrow 14 \twoheadrightarrow 9 \twoheadrightarrow 18$$

and so on.

What do you notice about this second example?

Investigate The Problem

CHAINING : continued

You may like to start by looking at one or both of the rules shown in the examples, but in much greater detail.

Try using small numbers to start with.

Try starting with different numbers

Try starting with all numbers up to 100.

Try making up your own rule.

You may like to use a rule to form chains which start with more than a single number

$$2 \rightarrow\rightarrow 7 \rightarrow\rightarrow 9 \rightarrow\rightarrow 6 \rightarrow\rightarrow 5 \rightarrow\rightarrow 1 \rightarrow\rightarrow 6 \rightarrow\rightarrow 7$$

$$\rightarrow\rightarrow 3 \rightarrow\rightarrow 0 \rightarrow\rightarrow 3$$

etc.

Can you see the rule here?

What happens if you continue this chain?

Try some ideas of your own.

Chaining - Teacher's Notes

Combining numbers to form new numbers is an interesting and fruitful way of generating an extended task. What happens, and why, is the natural thing to investigate. There are many well known investigations based on this chaining idea. Perhaps the two most famous chainings are HAPPY NUMBERS and ALL THE WAY TO ONE.

HAPPY NUMBERS

$52 \rightarrow\rightarrow 5^2 + 2^2 = 25 + 4 = 29$

$29 \rightarrow\rightarrow 2^2 + 9^2 = 4 + 81 = 85$

$85 \rightarrow\rightarrow 8^2 + 5^2 = 64 + 25 = 89$

etc.

ALL THE WAY TO ONE

If a number is even, halve it

If a number is odd, treble it and add one

$10 \rightarrow\rightarrow 5 \rightarrow\rightarrow 16 \rightarrow\rightarrow 8 \rightarrow\rightarrow 4 \rightarrow\rightarrow 2 \rightarrow\rightarrow 1$

All sorts of rules can be made up and investigated. The beauty of this piece of work is that a class of students can all make up their own rules to investigate, looking for rules, patterns and generalisations. Following this, they may well find through discussion that they can draw broad generalisations from each of their individual generalisations.

The examples given in the student's notes are just two to provoke discussion and an initial small group activity. This could be followed by a discussion about the types of chainings which could be used. These suggestions then could be written on a board or poster and possibly classified. Some students may investigate a single chaining, others a full classification to make broader generalisations.

The checking of such work by the teacher could prove to be an absolute nightmare. It is helpful therefore to use class time to talk to the students about accuracy, how can they be sure that they have not made errors, is there a pattern that seems reasonable but does not fit with some particular part of the results. It is also useful to check, some of this work when a spare moment arises. It is also worthwhile encouraging students to use calculators within their work on this task. It is the search for patterns and rules which form a key feature of this work, not necessarily basic arithmetic.

Other types of chainings include

$34 \to 3 + 4^2$

$28 \to (2 + 8)^2$

$(72, 30) \to (42, 30) \to (12, 30) \to$

$(12, 18) \to (12, 6) \to (6, 6)$

5 *Students' Work*

These six pieces of work broadly cover the full range of achievement. Two pieces of work are offered at each of the three levels of GCSE study; Foundation, Intermediate and Higher. These three levels are common to all GCSE schemes although the level titles differ.

The six pieces are in rank order of attainment and finish with the piece which is considered the best from the set. In Chapter 6, you will find detailed comments made on each piece by the Midland Examining Group Chief Coursework Moderator. We recommend that you should consider each piece of work in detail, make a few written comments and attempt to grade each student's work, before you read the moderator's comments.

For indentification purposes, the six student scripts are labelled *I1/1* to *I1/6*. Because of financial constraints the project team felt it desirable to reduce the size of the students' scripts, in order to facilitate the inclusion of a wide range of student achievement. In addition to loss of quality through a reduction in size, some student scripts suffer from the loss of colour which originally added emphasis and clarity to the arguments presented. Nevertheless, we are hopeful that many of the strengths inherent in the original scripts will become apparent as you read the following pages.

I1/1

Connect IV.

Connect 4

Object of the Game

Be the first player to get 4 of your counters in a row horizontally vertically or diagonally.

The Rules

1. Choose who plays first. The player starting the first game will play second in the next game.
2. Each player in his turn drops one of his coloured counters down any of the slots
3. The play alternates until one of the players gets FOUR counters of his colour in a row. The row may be horizontal, vertical or diagonal
4. The first player to get four in a row wins.

See figures 1-3 for examples of winning lines.

The Problem

The problem is to find out how many possible winning lines there are. You must find all the horizontal, vertical and diagonal lines there are. Then extend the concept of Connect 4 in some way to try to find some formulae or strategies.

Solving the Problem

Obviously, simply playing the game itself would not be enough to solve the problem. Therefore we must take an in-depth look at the grid and how you can win. Every possible winning line must be accounted for. You must be careful not to miss any. Later on we will look at how the number of winning lines can be found easily after extensive research into all the different possibilities. To find the winning lines you must draw them in - one by one. So find every winning line and you will discover that there are 69. This has took us some time - as we do not yet have resources to refer to suitable table or graph.

Extending Connect 4

Perhaps we should change the grid size or the win line size. Basically what we can say is that if the grid is largened and the length of the win line is shortened, then there will be more win lines. But this is more information than we need. We want to know how many possible win lines (decreasing or increasing the length) there are on a 6 and 7 grid. The smallest winning line we can have will be 3 counters - as it is impossible for the starter in either connect one or two to lose. The largest win line we can have is of seven - as eight will not fit on the grid. So apart from Connect 4 we must examine all the possible winning lines for the other sizes being

Connect 3 . three counters
Connect 5 : five counters
Connect 6 : six counters
Connect 7 . seven counters

Refer to figures 5-17 for examples of possible win lines for each game. Then refer to figure 4 for the table of totals.

Possible Formulae

There are a number of progressively harder formulae we can find and utilise. The easiest are for finding horizontal and vertical win lines. Starting with horizontal win lines we can use the following -

Going DOWN the table you will notice that the amount of horizontal win lines decreases by SIX each time, therefore:-

(Where P is previous) $H = P+6$ (going from Connect 7 to Connect 3)
or $H = P-6$ (going from Connect 3 to Connect 7)

Now going to vertical win lines you will notice that the amount decreases by SEVEN each time, therefore:-

$V = P+6$ (going from connect 7 to Connect 3)
or $V = P-6$ (going from Connect 3 to Connect 7)

If we now turn to diagonal win lines you will notice that the amount decreases by MULTIPLES OF FOUR, starting with 4 at Connect 6. But you can see that at Connect 7 you either add 4 if you are going UP to Connect 6 or subtract 4 if you are coming DOWN from Connect 6. Therefore

$D = P+(N\times 4)$ (Going Connect 7 to connect 3)
or $D = P-(N\times 4)$ (Going Connect 3 to Connect 7)

Finally to find the number of total winning lines you will notice that the amount decreases by SEVENTEEN PLUS MULTIPLES OF FOUR (starting at zero between 6 and 23)

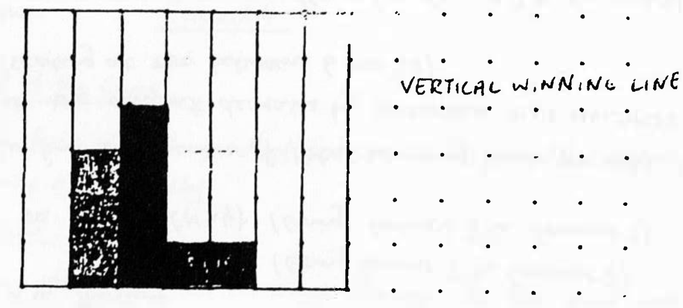
Therefore:

$T = P+17+(N\times 4)$ (Going from Connect 7 to Connect 3)

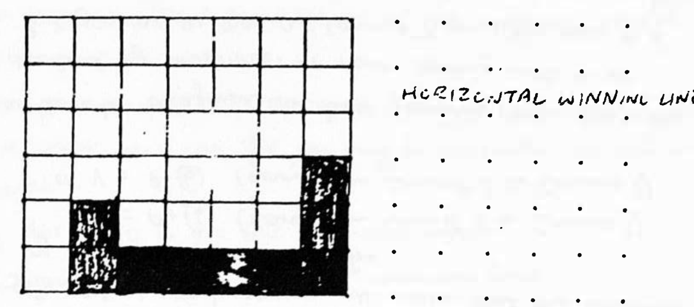
Conclusion

I can conclude that there are no easily identifiable strategies for the game of Connect 4. It is certainly untrue to say that the person who goes first will win. Either player could win by persisting with their tactical and strategic skills. The formulae I have presented here are only a handful of the possible ones which could be found through extensive research. The formulae cannot give any advantage to the players. Two good players would not consider luck an important factor in determining the outcome of the game. The investigation presented here only really scratches the surface. There is a wealth of research into the game which could be done. Changing the grid size will alter the number of winning lines and therefore the game play. Also, there are few (but possibly some) patterns that can be found and used to the players advantage. Summing up, I can say that this is a most interesting and detailed problem in which one can discover some interesting facts about the game of Connect 4.

(Fig 1)



(Fig 2)



(Fig 3)

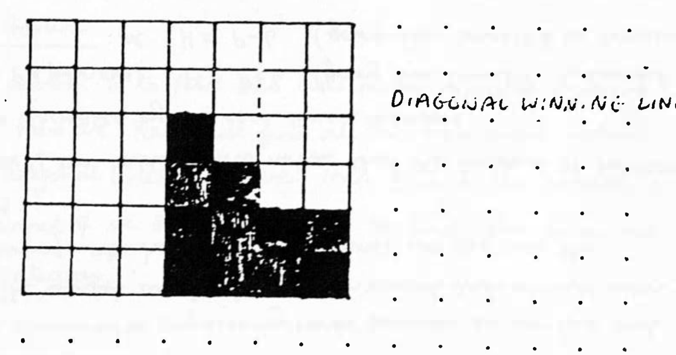
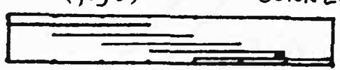


TABLE OF WIN LINES (fig 4)

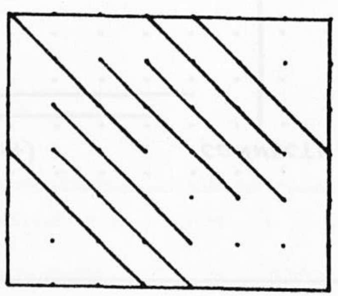
	Diagonal	Horizontal	Vertical	Total
Connect 3	40	30	28	98
Connect 4	24	24	21	69
Connect 5	12	18	14	44
Connect 6	4	12	7	23
Connect 7	0	6	0	6

(fig 5) CONNECT 3

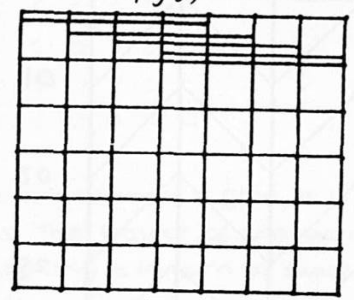


(fig 6)

(fig 7)

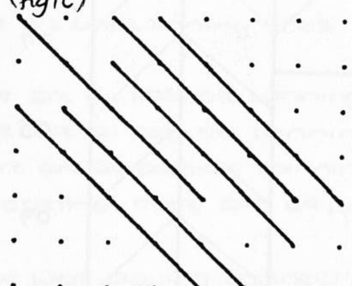


(fig 8) CONNECT 4

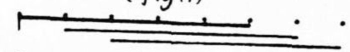


(fig 9)

(fig 10)

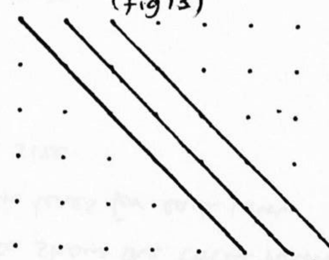


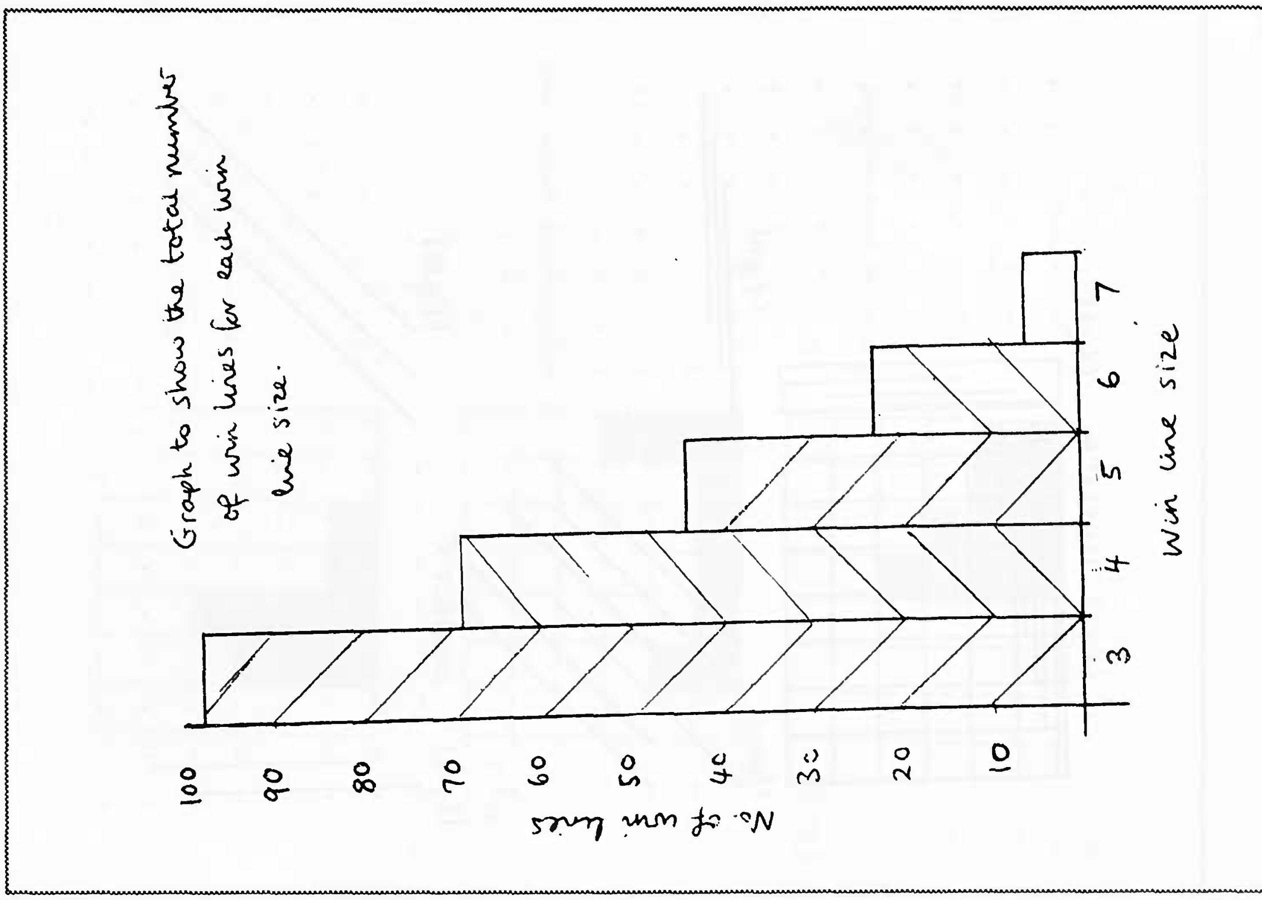
(fig 11)

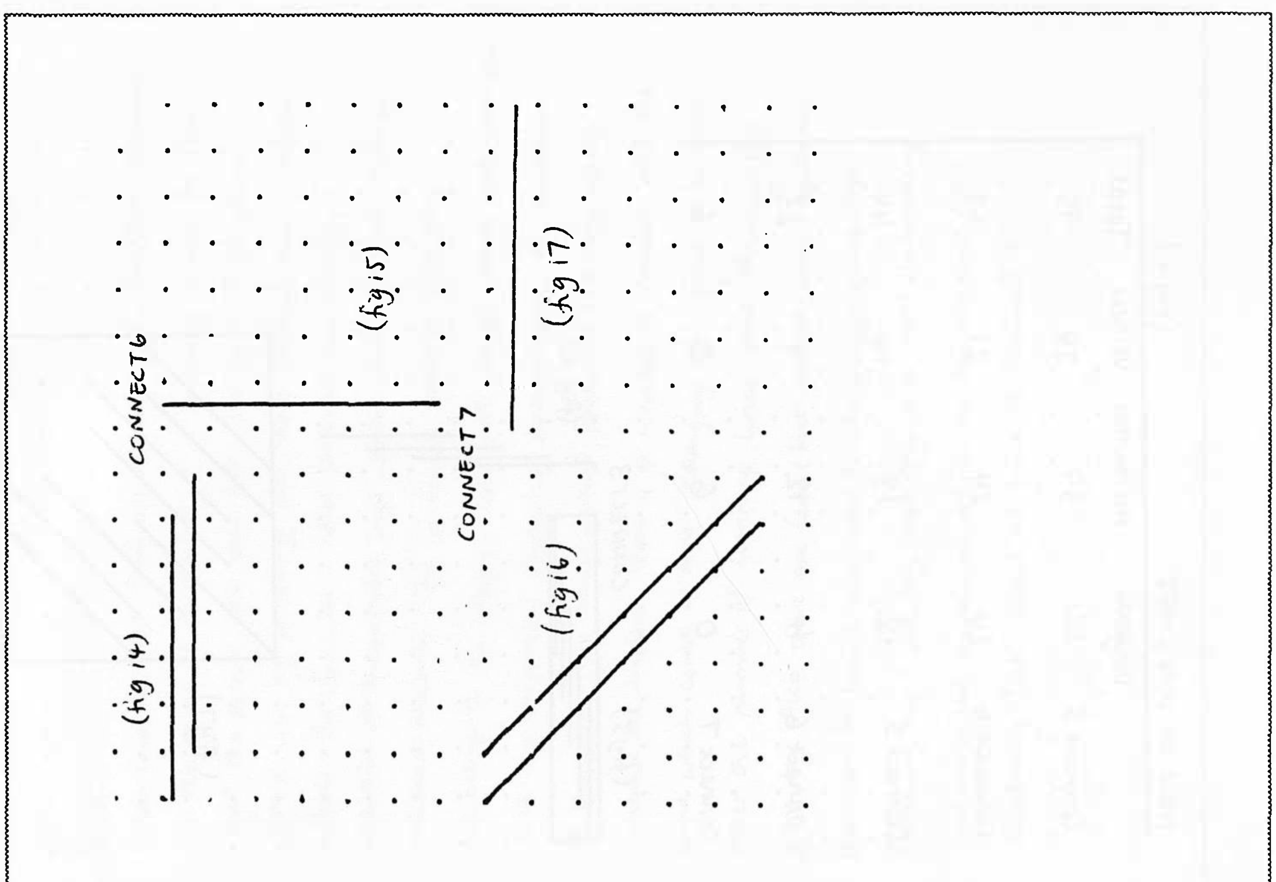


(fig 12) CONNECT 5

(fig 13)



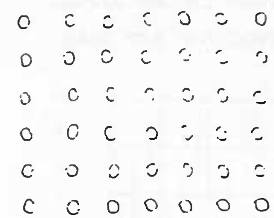

Graph to show the total number of win lines for each win line size.
No. of win lines
100
90
80
70
60
50
40
30
20
10
Win line size
3
4
5
6
7


CONNECT 6
CONNECT 7
(fig 14)
(fig 15)
(fig 16)
(fig 17)

I1/2

Connect 4 project

Connect 4

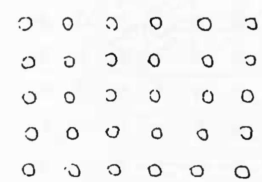


This is the connect 4 grid, It is a grid of 6 rows and 7 col umns. The object of the game is to try and get 4 in a like, taking it in turn to place the counters.

These are the winning lines

There are 24 possible winning diagonal lines
There are 21 possible winning vertical lines.
There are 24 possible winning horizontal lines.
Altogether there are 69 possible winning lines of 4.

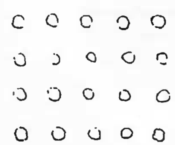
If we were playing connect 3 we would take 1 off each of the vertical and horizontal lines.



There are 24 possible winning diagonal lines
There are 18 possible winning vertical lines
There are 20 possible winning horizontal lines
Altogether there are 62 possible winning lines.

At the moment there is no obvious number pattern.

If we were playing connect 2 we would again take 1 off each of the vertical and horizontal lines.



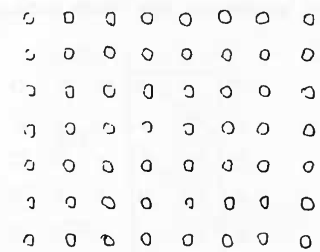
There are 24 possible winning diagonal lines
There are 15 possible winning vertical lines
There are 18 horizontal winning lines.
Altogether there are 55 possible winning lines.

Results.

number	vertical	diagonal	horizontal	total
2	15	24	16	55
3	18	24	20	62
4	21	24	24	69
5	24	24	28	76
6	27	24	32	83.

The pattern seems to be obvious the total seems to be rising by 7 each time, the horizontal total seems to be rising by 4 each time, the diagonal total stays the same everytime and the vertical total seems to be going up by 3 every time.

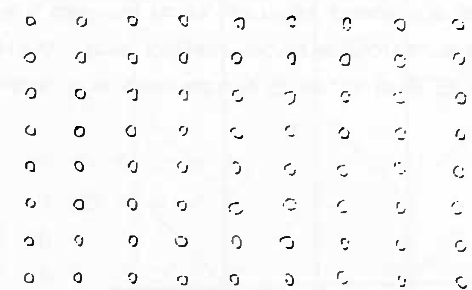
Now if we extend the project to connect 5 then we find that the same theory still works.



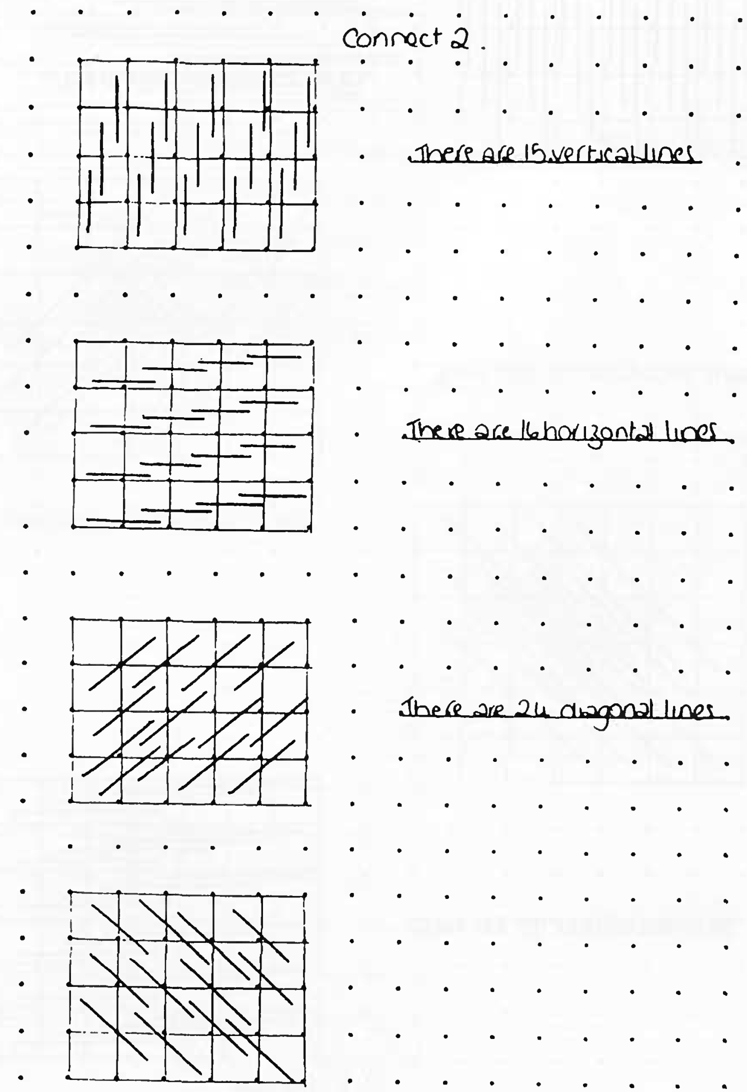
There are 24 possible winning diagonal lines
There are 24 possible winning vertical lines
There are 28 possible winning horizontal lines.
Altogether there are 76 possible winning lines.

Connect 6.

If we extend the theory to connect 6 we find that the theory continues, but only if we extend the grid size as well



There are 24 possible winning diagonal lines
There are 27 possible winning vertical lines
There are 32 possible winning horizontal lines
Altogether there are 83 possible winning lines.

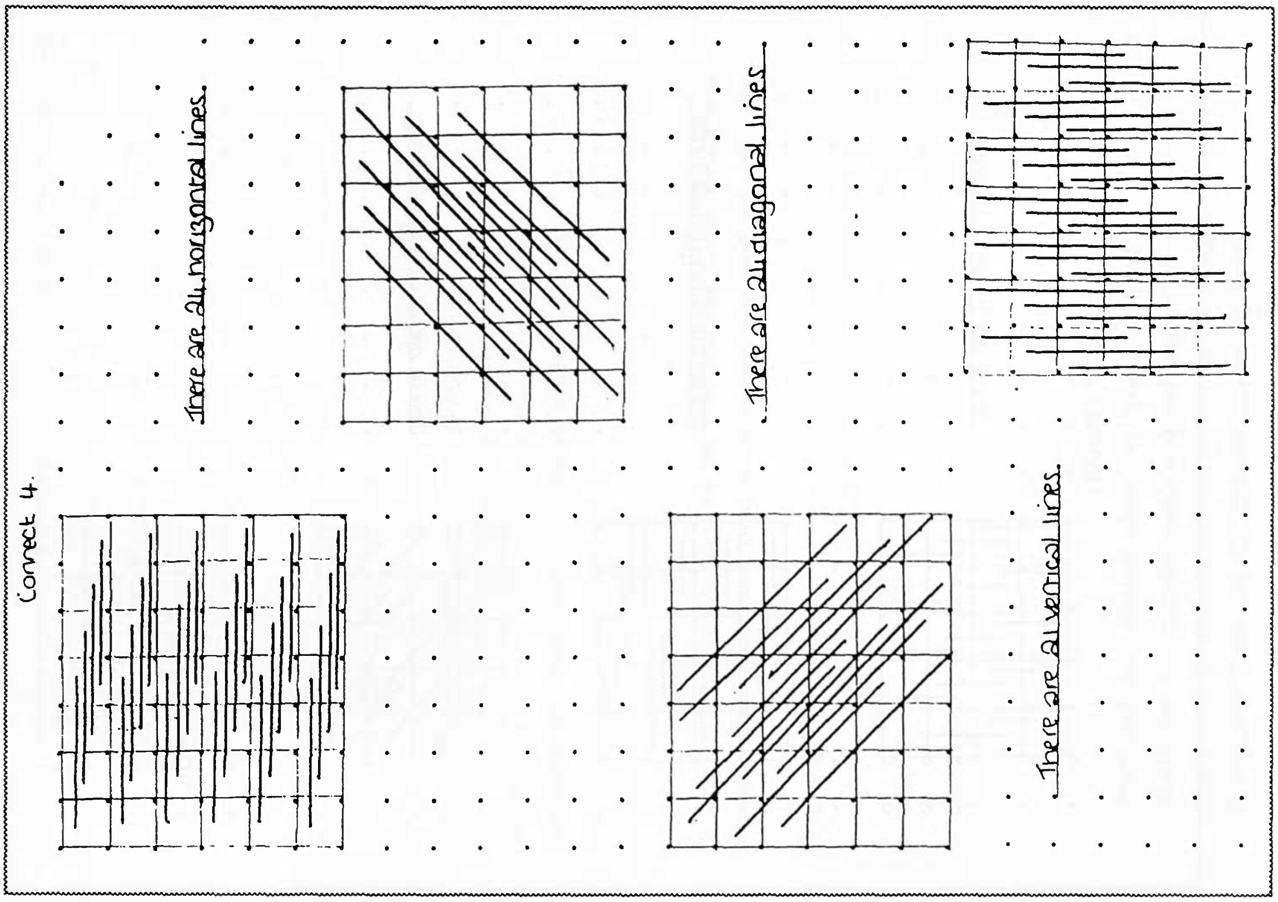


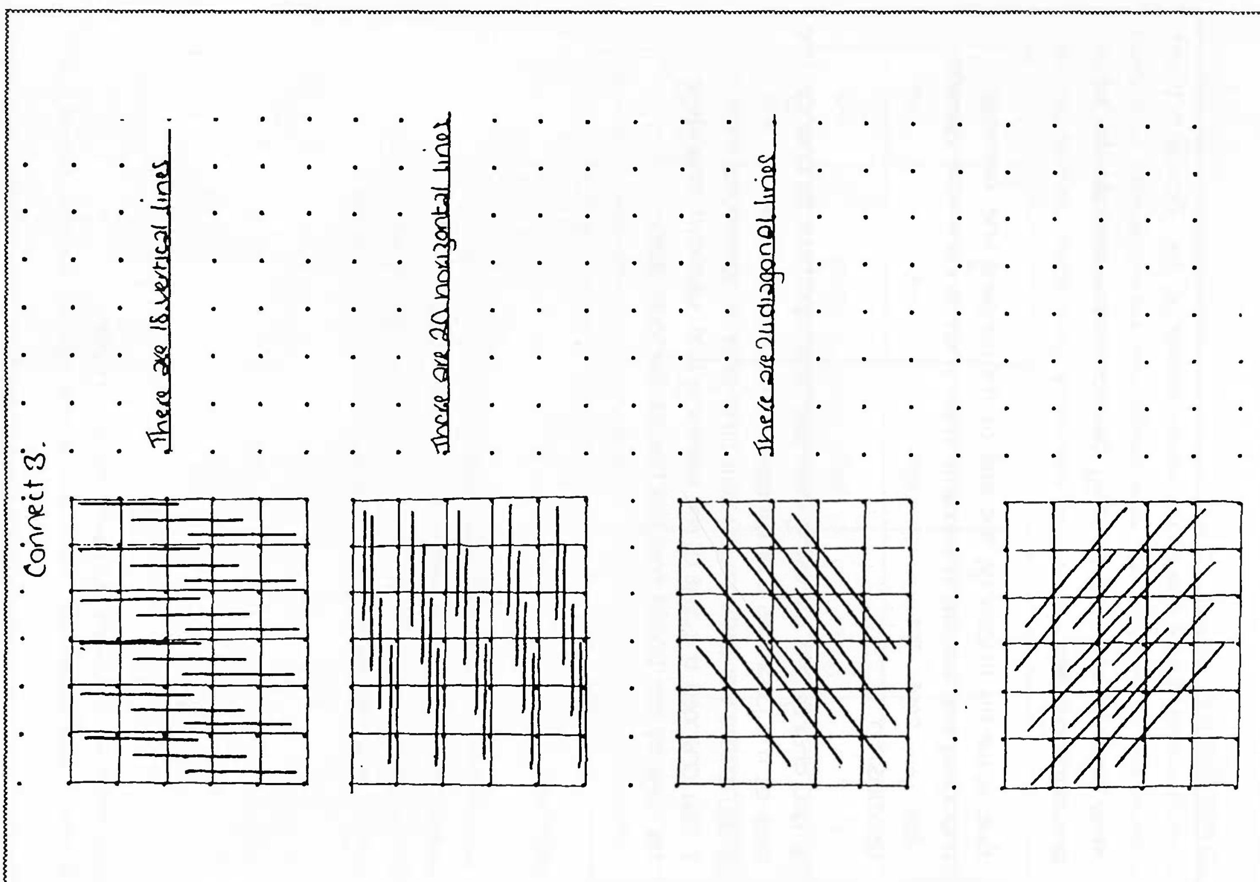
This pattern extends to any number you choose to be a winning line.

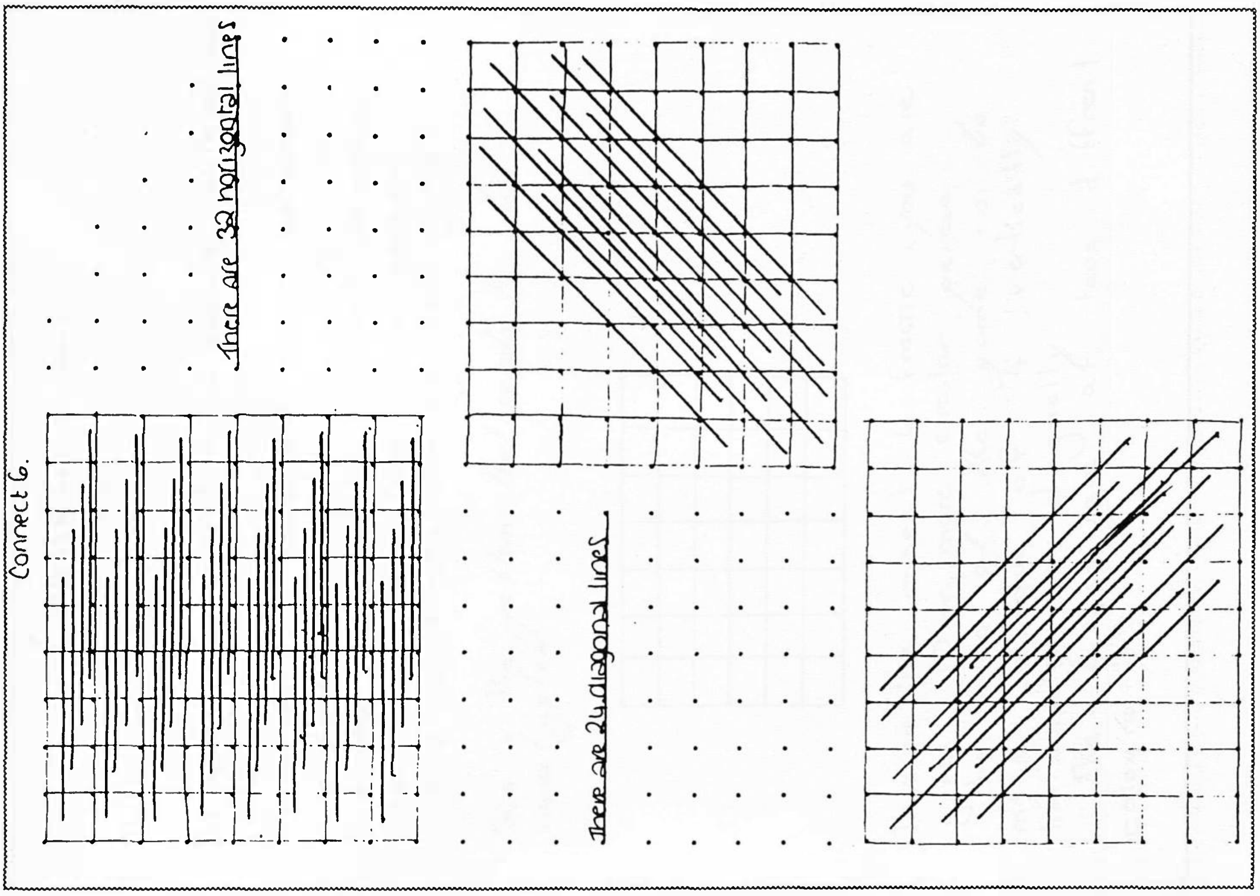
But if we try to change the size of the winning line without changing the size of the board then there is no visible pattern.
Have you tried?

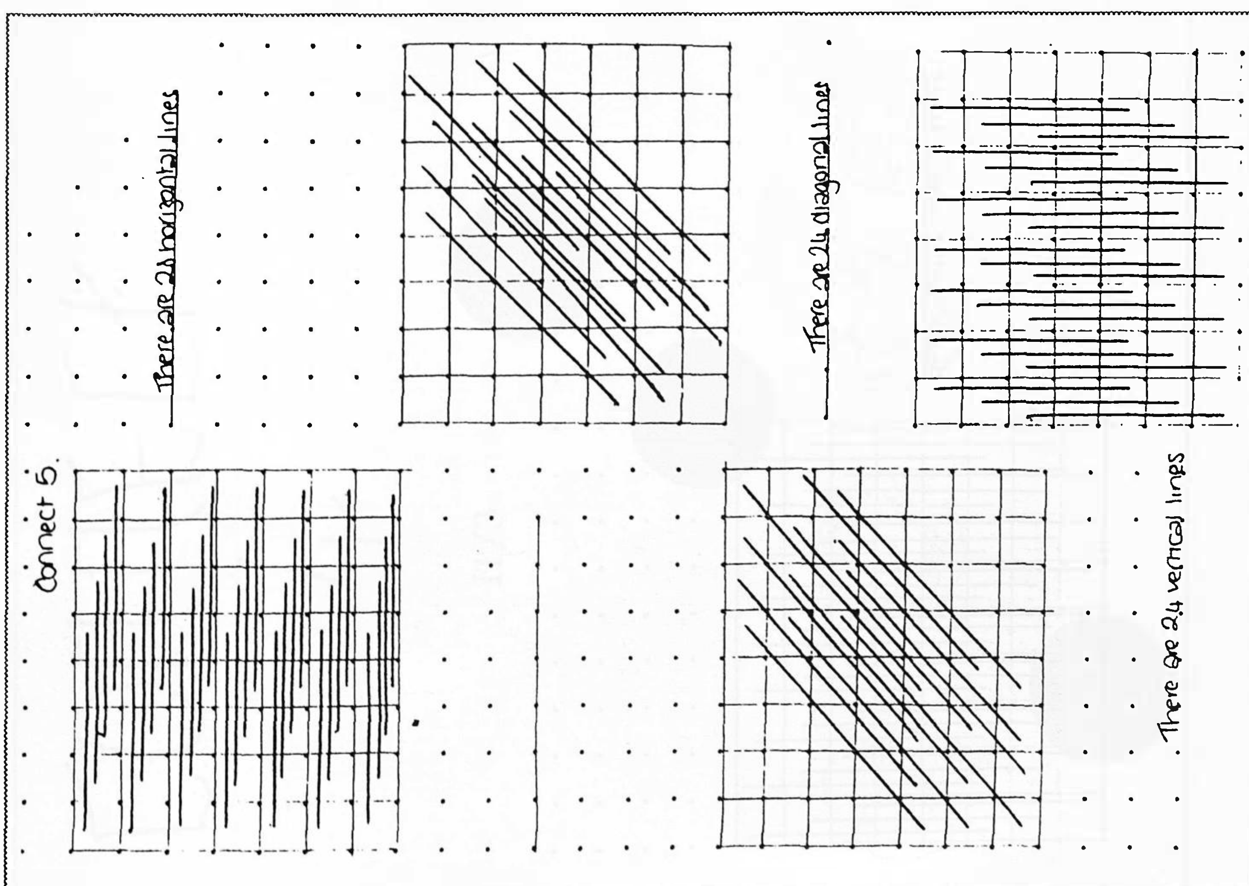
Conclusion

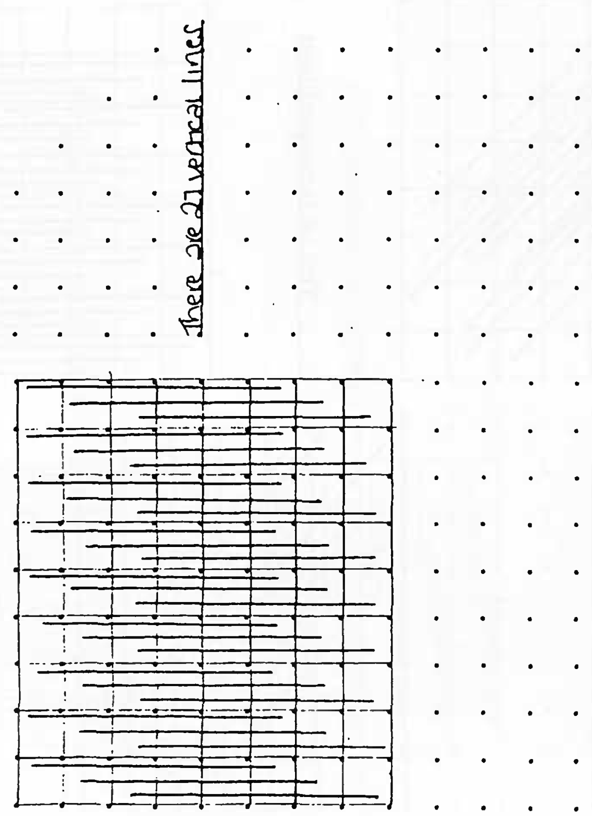
If you change the winning number then you have to change the size of the board as well otherwise there is no pattern.
If you extend the pattern you will find that it continues but
if you changed the size of the winning line without changing the size of the board this particular pattern does


Connect 4.
There are 24 horizontal lines.
There are 24 diagonal lines.
There are 21 vertical lines.


Connect 3.
There are 18 vertical lines
There are 20 horizontal lines
There are 24 diagonal lines

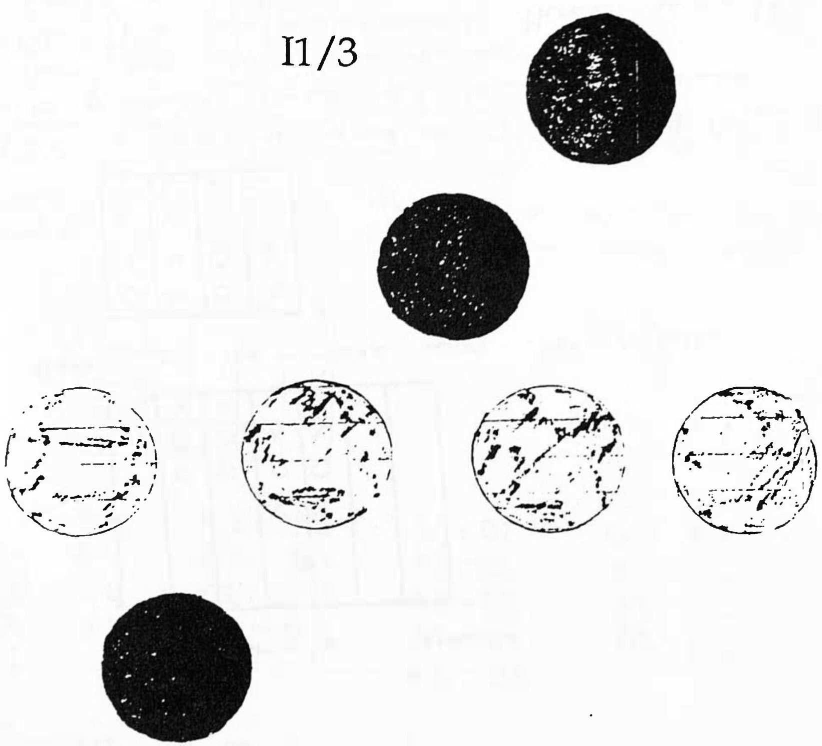

Connect 6.
There are 32 horizontal lines
There are 24 diagonal lines


Connect 5.
There are 24 horizontal lines
There are 24 diagonal lines
There are 24 vertical lines


There are 21 vertical lines

CONNECT 4

I1/3

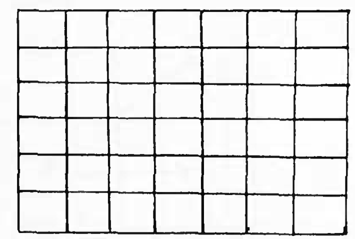


Connect 4.

1. Maths investigation.

The game of connect 4 is popular with all age groups.
There are six rows and seven columns. in a normal connect 4 game.
Find out how many diffrent winning lines of 4 there are in a game?
How could you change the game?

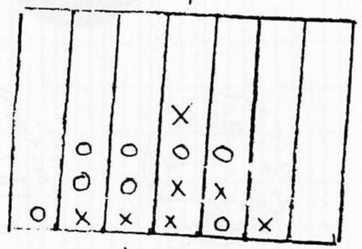
This is the problem that needs to be investigated.



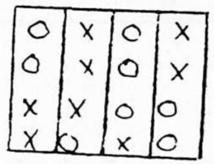
This is the connect 4 frame, you are given 10 or more circular peices
the idea of the game is to make a row of 4 vertically horizontally or diagnolly.
The pieces are of two diffrent colours.

2. Simple cases.

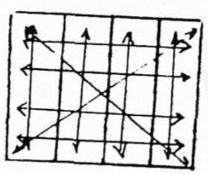
1)



Here circles have won horizontally

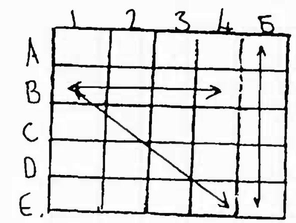


on a smaller grid it is more difficult to produce lines of four, because there are far less routes
On this scale lines of three would become better.



Here are all the possible routes on a four by four grid.
There is only one diagnol line of four there are ten routes.

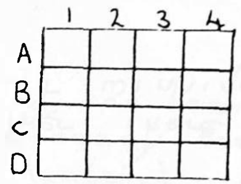
On a four by five grid there are 17 diffrent routes
Using the arrows method.
But there becomes to many lines upon the grid so a grid reference may be a good idea



This is the new grid references the lines would be written as followed

B1 - B4
A5 - E5
B1 - E4

3. using the Grid reference 2.

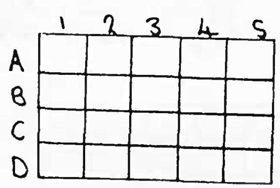


A1 - D1
A2 - D2
A3 - D3
A4 - D4

A1 - A4
B1 - B4
C1 - C4
D1 - D4

D1 - A4.

These are the routes of a four by four grid, with references. you have to do this in a systematic order or else it becomes complicated



A1 - A4
B1 - B4
C1 - C4
D1 - D4

A2 - A5
B2 - B5
C2 - C5
D2 - D5

A1 - D1
A2 - D2
A3 - D3
A4 - D4
A5 - D5

D1 - A4
A1 - D4
A2 - D5
A5 - D2.

There are ~~14~~ 17 routes above is a systematic order.

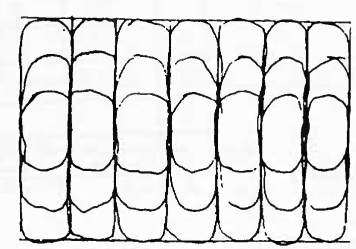
4. To investigate whether there are any patterns the best and simplist idea is to produce a table of results.

Grid size	routes found.
4 x 4	There are ~~nine~~ ten routes
5 x 4	There are ~~fourteen~~ seventeen routes
5 x 5	There are twenty ~~three~~ eight routes
5 x 6	There are thirty nine routes
6 x 6	There are fifty four routes
6 x 7	There are sixty nine routes

The 4x4 grid is the smallest to be able to observe a line of 4. and the 6x7 grid is the largest that needs to be produced.

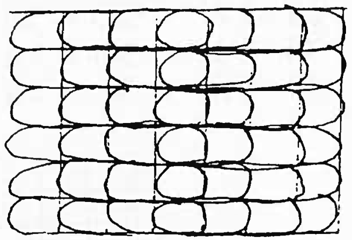
The 6x7 grid. This is the grid that is used in the connect 4 game.

Verticals.



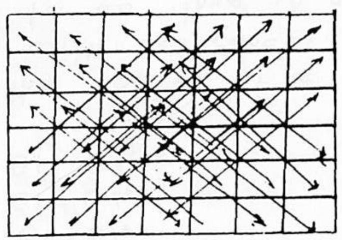
There are twenty one verticals.

Horizontals.



There are 24 Horizontals on a 6 × 7 grid.

Diagnols.

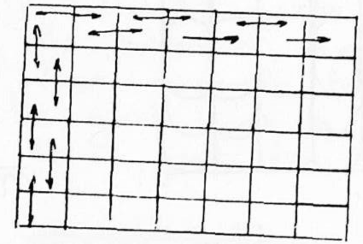


There are 24 diagnols in a 6 × 7 grid.

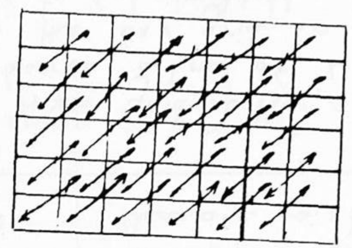
Altogether there are sixty nine ways of winning a connect 4 game.

6. Trying diffrent connects.

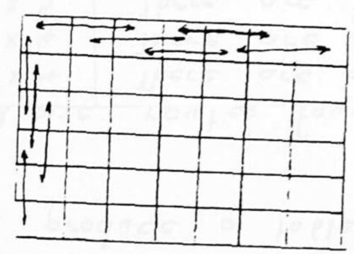
connect 2



horizontals = 36
verticals = 35
Diagnols = 60



connect 3

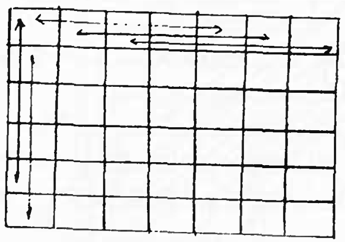


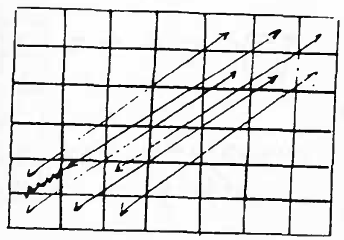
horizontals = 30
verticals = 28
Diagnols = 38

diffrent connects 2.

connect 5

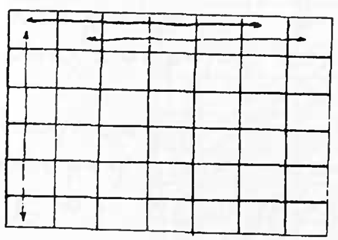
Connect 4 has been missed out becaus it has already been completed On page numbered 4.

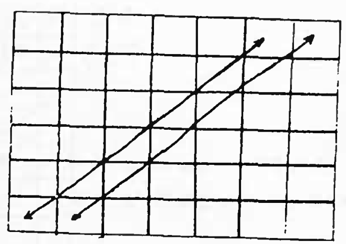




Horizontals = 18
verticals = 14
Diagnols = 12.

connect 6.

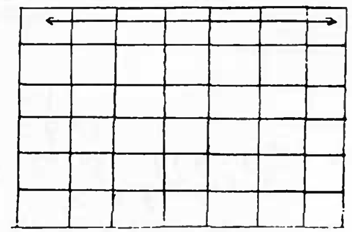




Horizontals = 12.
verticals = 7
Diagnols = 4

Trying diffrent connects 3

Connect 7



Horizontals = 6
verticals = 0
Diagnols = 0

connects	horizontal	vertical	Diagnol
connect 2	36	35	60
connect 3	30	28	40 ~~38~~
connect 4	24	21	24
connect 5	18	14	12 ~~18~~
connect 6	12	7	4
connect 7	6	0	0

horizontals have gone up in 6s
verticals have gone up in 7s

Trying diffrent connects
(finding the formula)

First of all the connects were worked out and a pattern was produced

horizontals go up in 6's
verticals go up in 7s
Diagnols the gap goes up in 4's.

From this I began to see where there could be a possible formula

For the horizontals you × 6 by whatever the number in the horizontal column is.

$6 \times \boxed{6} = 36$ $30 = 6 \times \boxed{5}$ $24 = 6 \times \boxed{4}$

Then add the number in the boxes above to the connect.
$6 + 2 = 8$ $5 + 3 = 8$ $4 + 4 = 8$
you will notice that they all add up to 8.

Then take away the connect from 8 and × by 6.

The formula becomes $6 \times (8 - x)$

For the vertical formula you do the same thing except using 7's and not 8 so the formula becomes $7 \times (7 - x)$

The formula for the diagnols is

$D = 2(8 - x)(7 - x)$

Checking the formula. connect 6

$x = 6$ $8-6$ $7-6$

$\cancel{2 \times 2 = 4 \times 1 = 4}$

Horizontals.
$6(8 - x)$
when $x = 6$
no. of wins $6(8-6)$
$= 6 \times 2$
$= 12$.

verticals.
$7(7 - x)$
when $x = 6$
no. of wins $7(7-6)$
$= 7 \times 1$
$= 7$

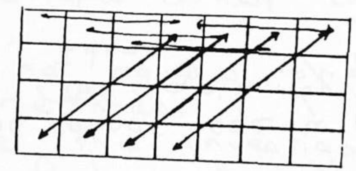
Diagnols.
$D = 2(8 - x)(7 - x)$
when $x = 6$
then $8-6$ $7-6$
$= 2$ and 1
then $2 \times 2 = 4$
$4 \times 1 = 4$

7. Altering the columns.

3x7

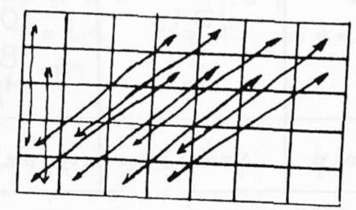
12 horizontals
0 verticals
0 Diagnols.

4x7



16 horizontals
7 verticals
8 Diagnols

5x7



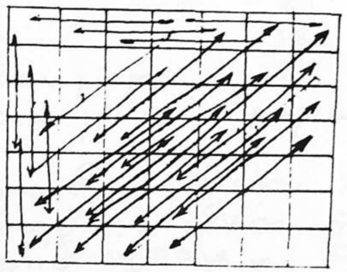
20 horizontals
14 verticals
16 Diagnols

Altering the columns 2.

6 x 7 There are 24 horizontals
21 verticals
24 Diagnols

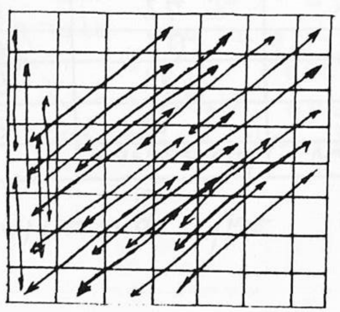
This is shown on section 4.

7x7



28 horizontals
28 verticals
32 Diagnols.

8x7



32 horizontals 36 Diagnols.
35 verticals

5. Finding Patterns.

Grid size	horizontal	vertical	diagnol	Total
4x4	4	4	2	10
5x4	8	5	4	17
5x5	10	10	8	28
5x6	15	12	12	39
6x6	18	18	18	54
6x7	24	21	24	69

From here you cannot see a pattern so altering the grid by less is the next Idea.

5x7	20	14	8	42
4x7	16	7	4	27
3x7	12	0	0	12
2x7	8	0	0	8

There is the beggining of a pattern for the horizontals they Go up in fours the other two are doubled.
This only counts when using the number seven.

Altering the columns 3.

Grid	Horizontal	vertical	Diagnol
3x7	12	0	0
4x7	16	7	8
5x7	20	14	16
6x7	24	21	24
7x7	28	28	32
8x7	32	35	40 ~~36~~

The pattern for the horizontals is gdeoup in 4's verticals it goes up in 7's Diagnals goes up in 8's.

The formula is column × 4 = Horizontals.

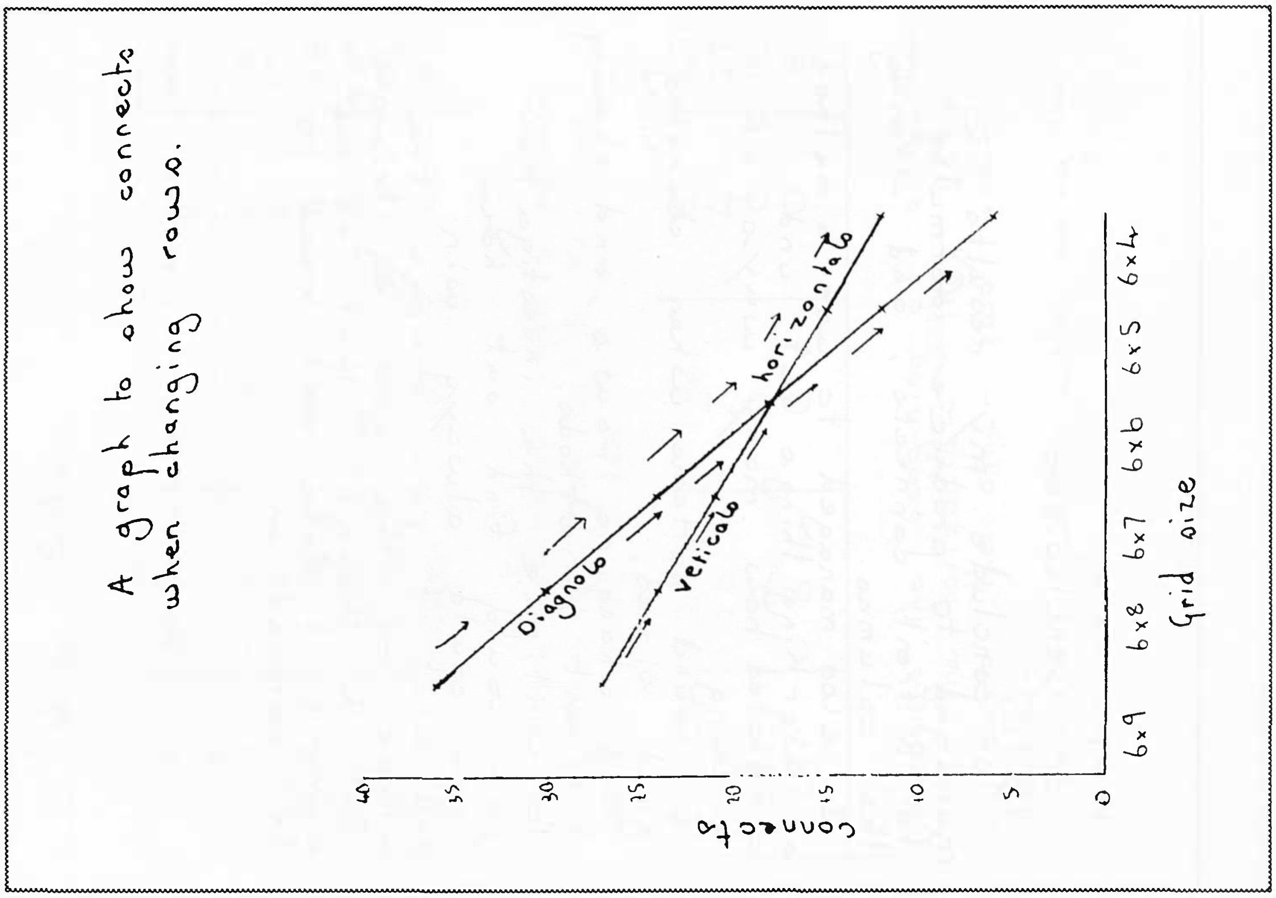
	Horizontals	verticals	Diagnols
x9	36	27	36
x8	30	24	30
x7	24	21	24
x6	18	18	18
x5	15	12	12
x4	12	6	6

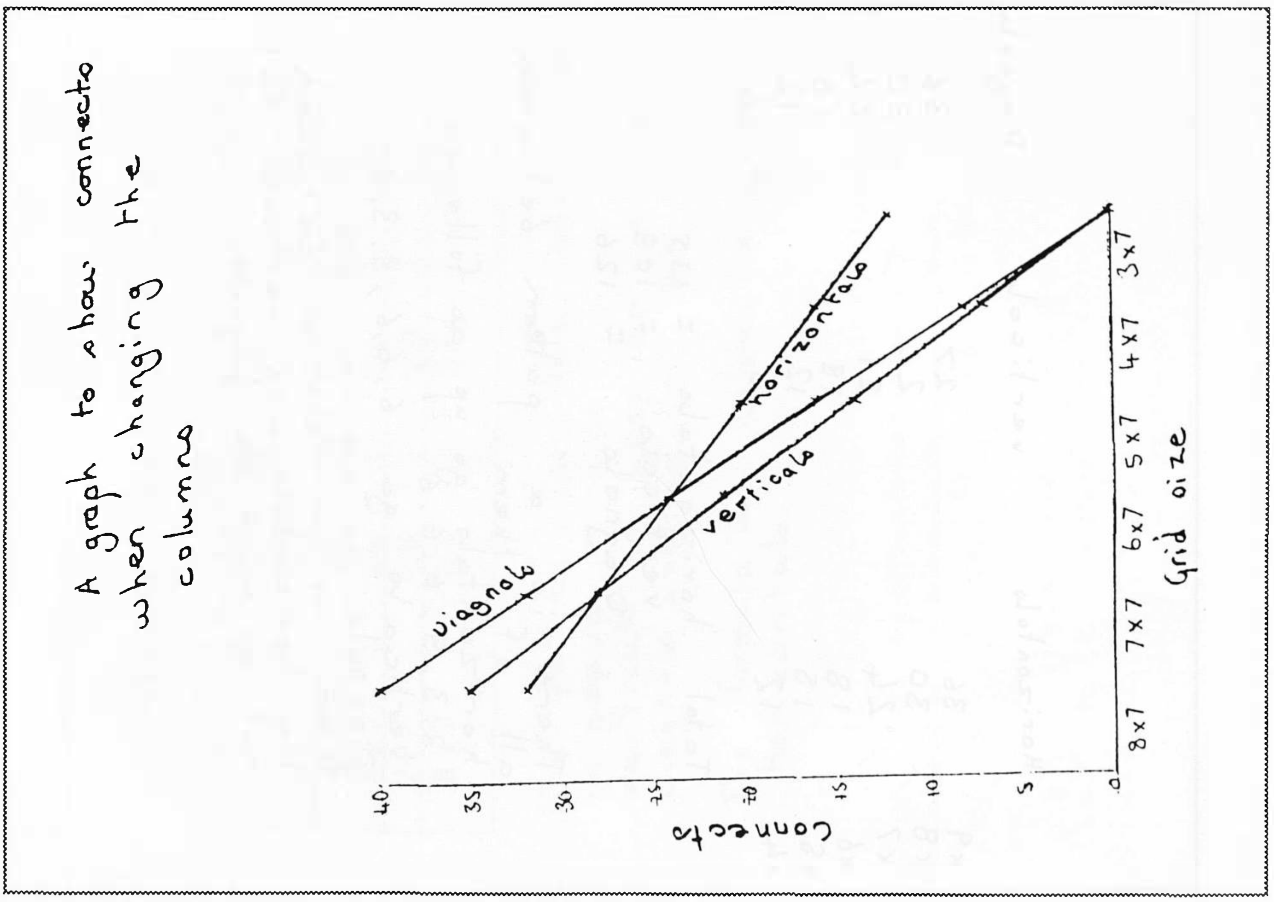
Total horizontals = 135
verticals = 108
Diagnols = 126

There is a pattern between all of them
horizontals go up as follows 3, 3, 3, 6, 6, 6.
verticals go 6, 6, 6, 3, 3, 3.
Diagnols go up in 6's.
There is a relation between the horizontals and verticals but not so much the diagnols.

Conclusions.

To conclude my results I managed to produce formulas for diffrent connects, and altering the columns.
I also managed to use a method of working things out and completed how many ways of winning.
I found patterns when changing grid sizes, and changing rows, and showing diffrent methods.
To continue the investigation you could find out how you could always win and what if you change the shape of the grid eg: triangle
But I thought that at the moment it does not need to be carried on.


A graph to show connects when changing rows.
Connects
Grid size
6x9
6x8
6x7
6x6
6x5
6x4
0
5
10
15
20
25
30
35
40
Diagnols
verticals
horizontals


A graph to show connects when changing the column
Connects
Grid size
8x7
7x7
6x7
5x7
4x7
3x7
0
5
10
15
20
25
30
35
40
Diagnols
verticals
horizontals

I1/4

Connect 4 Project

Contents Page of Connect 3, 4, 5 & 6.

Connect 4 - Project.

Introduction to the game.
The game is a board with ~~42~~ holes in it there are red and yellow counters; the object is to get a line of 4 either vertically, horizontally, or diagonally.

			o			
			X	o		
			X	X	o	
		o	X	o	X	o

a diagonal win

X						X
X						o
X	o					o
X	X				o	o

a vertical win

X	X	o	o	o	o	X

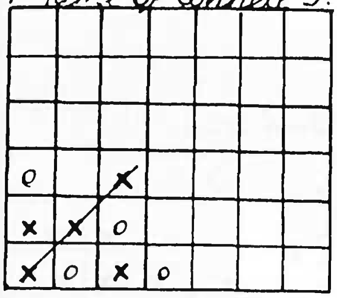
a horizontal win

Skill- Concentration is what you really need for this game concentrating on you opponents counters as well as your own.

X	o	o	o	X	X	o
o	X	o	X	X	X	o
X	o	o	X	o	X	X
X	o	X	o	X	o	X
X	o	X	o	o	o	X
o	X	o	o	X	X	o

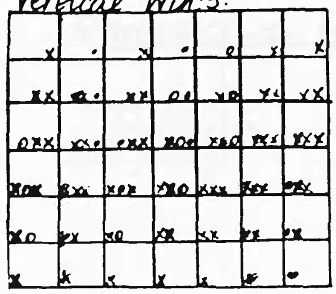
Nobody wins- this shows concentration on your opponents counters.

A Game of Connect 3.



Ways Of Winning Connect 3

Vertical Win's.

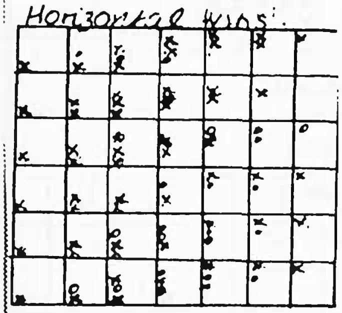


there are 4 vertical win's on each line 4 x 7 = 28.
There are 28 vertical wins of connect 3.

28 + 30 + 40 = 98

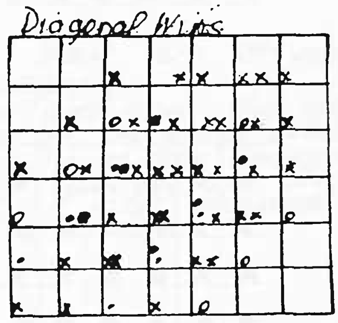
* Better put later after connect 4 has been investigated.

Horizontal Wins:

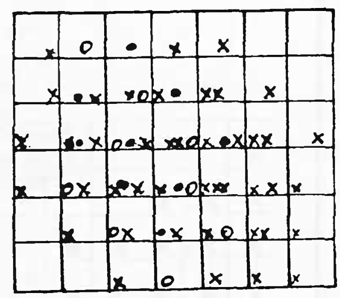


there are 5 horizontal wins on each line
5 x 6 = 30.
There are 30 horizontal wins of connect 3.

Diagonal Wins



There are 20 ways ↗ this way of connect 3.

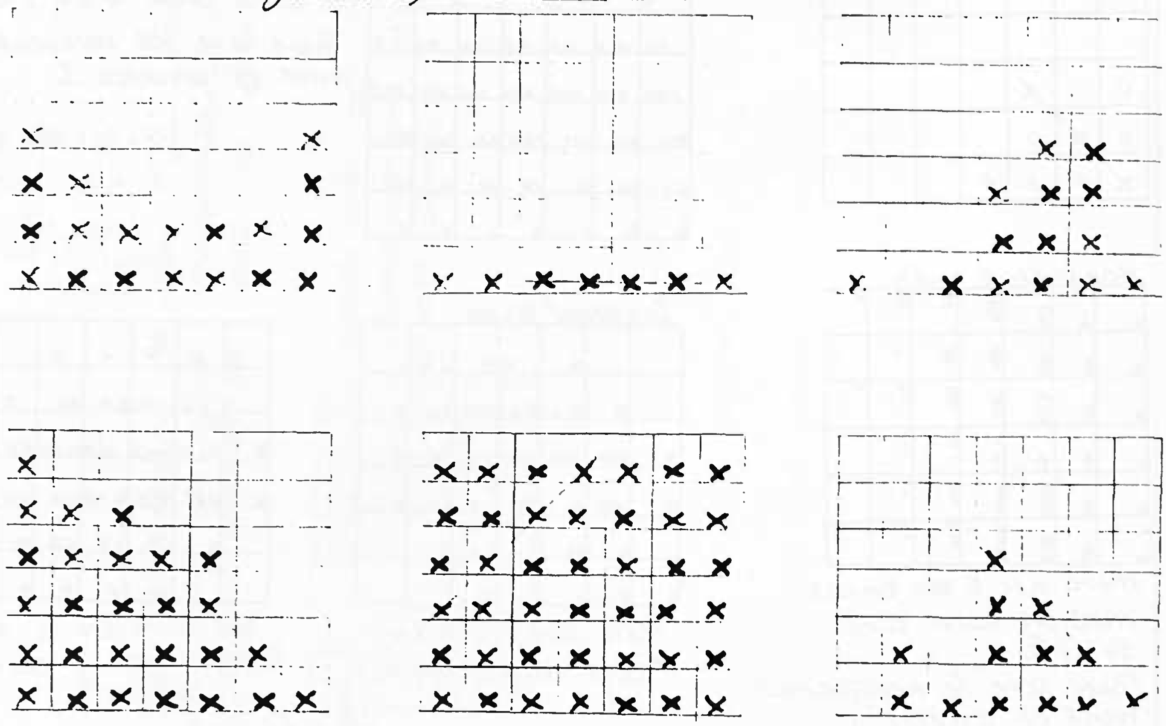


And there are 20 ways ↖ this way of getting connect 3

20 + 20 = 40.

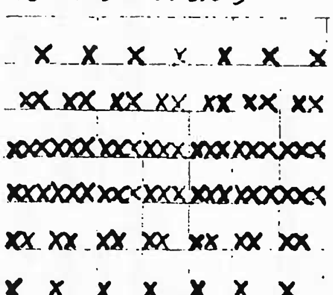
Connect 4.

Game's of Connect 4.



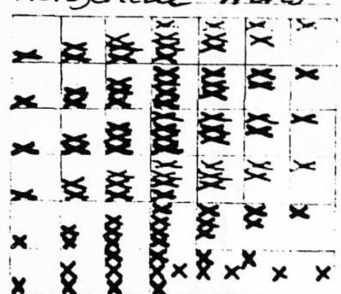
Ways of Winning conn-ect 4.

Vertical Wins



There are 21 ways of winning vertically 3 on each of the 7 lines.

Horizontal Wins



There are 24 ways of winning horizontally 4 on each of the 7 lines.

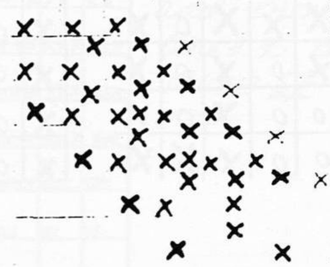
Altogether with the diagnol, vertical and horizontal there are (24 + 21 + 24 =) 69. ways of winning!

Connect 4.

12 win's

ways of winning diagnoly

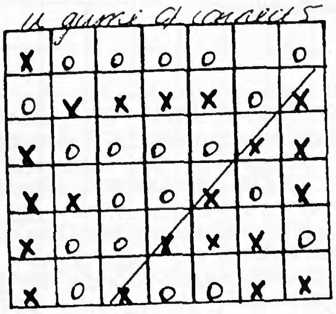
12 win's



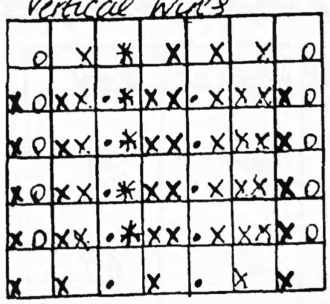
altogether there are 24 ways of winning diagnoly.
The pattern goes from the edge of the board to the middle like this
1 win, 2 win's, 3 win's, then 3 wins 2 + 2 wins.
So the further you get to the centre of the board, the more chances there are of wins in a line. ✓

Ways of Winning Connect 5

a game of connect 5

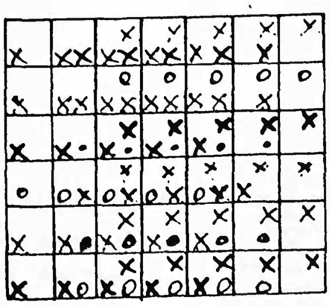


Vertical win's

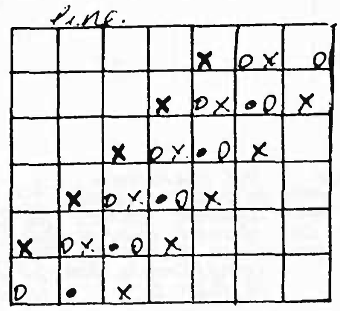


There are 14 vertical win's 2 on each line.

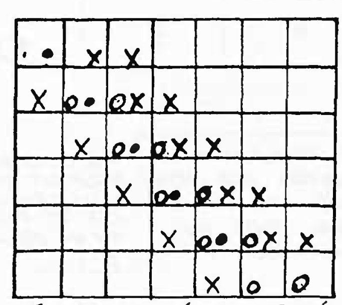
altogether there are 44 ways of getting connect 5.
It is quite hard to get connect 5 on a 7 by 6 board, most of the time nobody wins.



There are 28 horizontal win's of connect 5. 3 on each line.



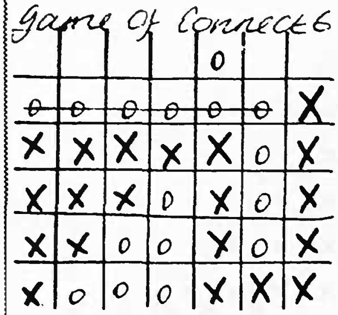
There are 6 ways of getting connect 5 ↗ this way.



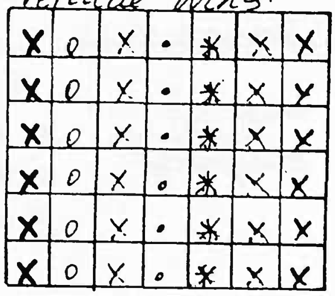
There are 6 ways of getting connect 5 ↖ this way.

Ways Of Winning Connect 6

Game of Connect 6.



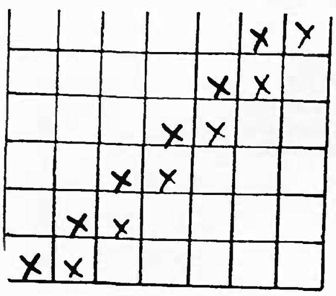
Vertical wins.

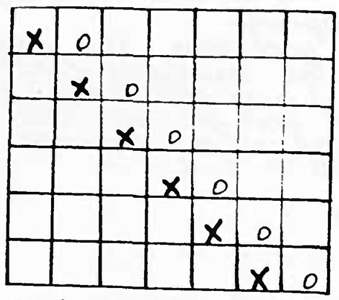


There are 7 vertical wins 1 on each line.

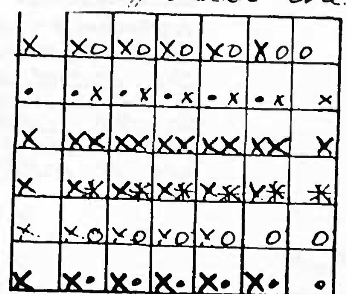
altogether there are 23 ways you can get connect 6.

It is just about impossible to get connect 6, unless your partner dosent know a thing about the game.





There are 4 horizontal wins altogether. 2 going ↗ way + 2 going ↖ way.



There are 12 horizontal wins. 2 on each line

Total ways of Winning Connect 3,4,5 + 6.

	Vertical	Horizontal	Diagonal	Total
connect 3	28	30	40	98
connect 4	21	24	24	69
connect 5	14	18	12	44
connect 6	7	~~4~~ 12	4	23

29

25

21

From these sets of results we can predict what will happen in connect 2 ~ this is what would happen ~ vertical wins = 35. (goes up 7 every time) Horizontal wins would be 36 and diaganol wins would be 60 (Horizontals going up in 6's + diaganols go in 4's)
35 + 36 + 60 = 131

The connection between the vertical results is that they go down in 7's.

The connection between the vert + horizontal results is that they go down in 6's.

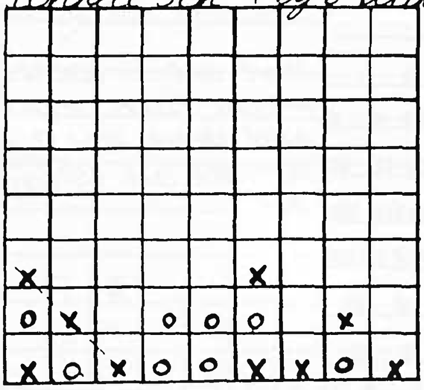
The connection between the diaganol results is that they go down in 4's starting from 16 then 12. then 8.

The connection between the totals is that when 98 is subtracted from 69 it leaves 29, and when 69-44 = 25 and when 44-23 = 21 the differences are 4. ✓

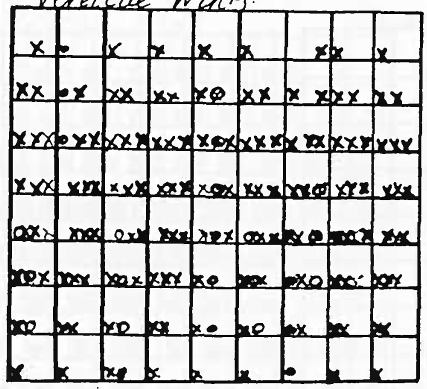
? no teacher assistance here. Sort out the chart and pattern.

Connect 3 on Different Size Boards

Connect 3 on 9 by 8 board

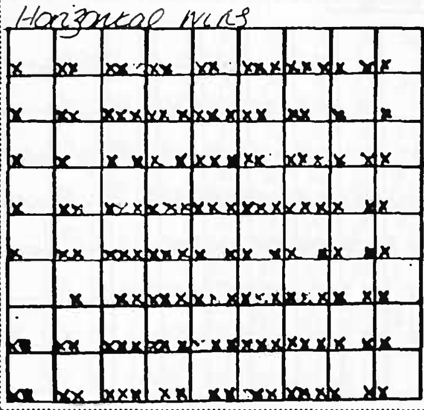


Vertical win's

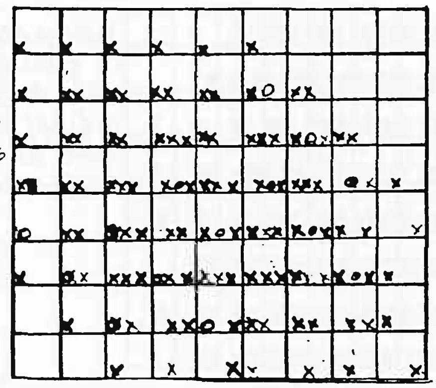


There are 6 win's on each line and there are 9 line's so that's makes 54 vertical win's.

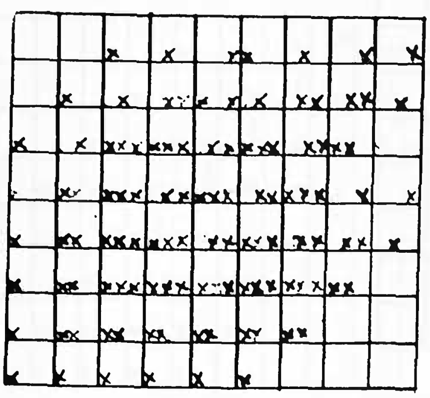
Horizontal wins



There are 7 win's on each line and there are 8 line's so that make's 64 horizontal win's.



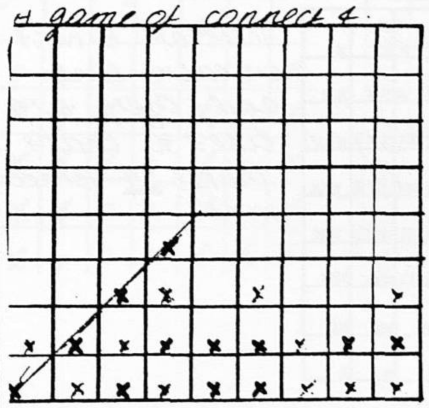
There are 43 ways of winning diagonaly ↖ this way

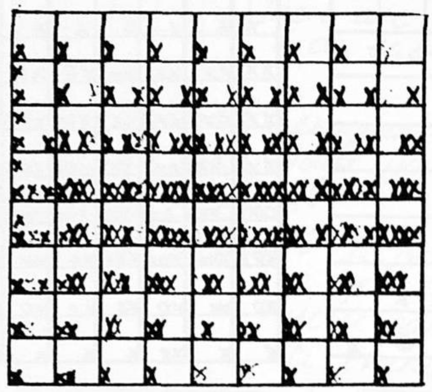


There are 48 ways of winning connect 3 / way. So altogether (54,64,43,48) there are 211 ways of winning connect 3 on a 9 by 8 board

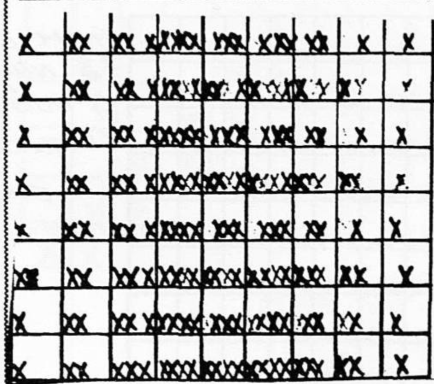
Ways of winning connect 4 on a 9 by 8 board

A game of connect 4.

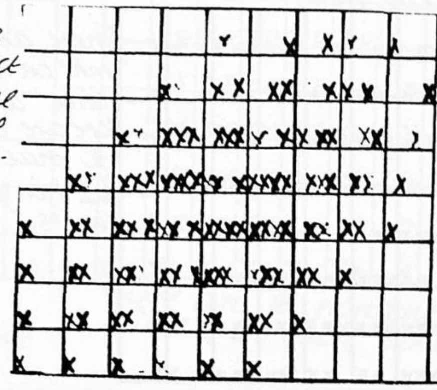




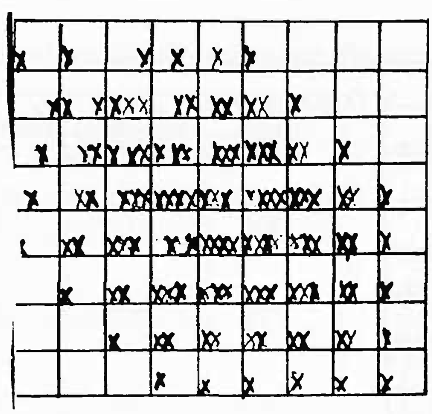
There are 5 connect 4's on each line. and there are 9 lines so there are 45 vertical connect 4's.



There are 6 ways of getting connect 3 on each of the 8 lines so there are 48 horizontal wins.



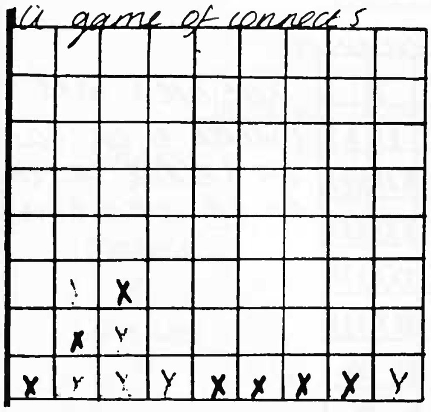
There are 29 ways of getting connect 4 ↗ way



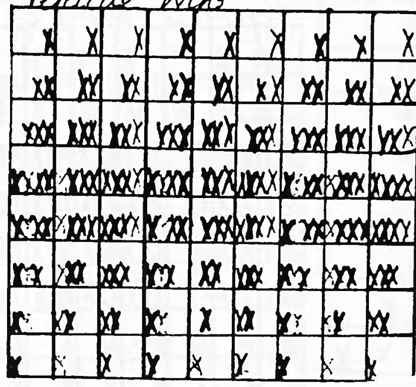
There are 30 connect 4's going ↖ way.
So altogether (45,48,29+30) there are 152 ways of getting connect 4 on a 9 by 8 board

Ways of winnig connect 5 on a 9 by 8 board.

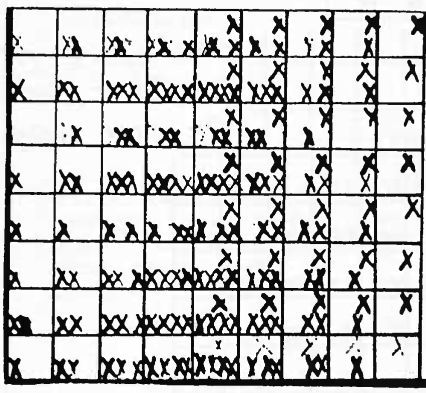
a game of connect 5



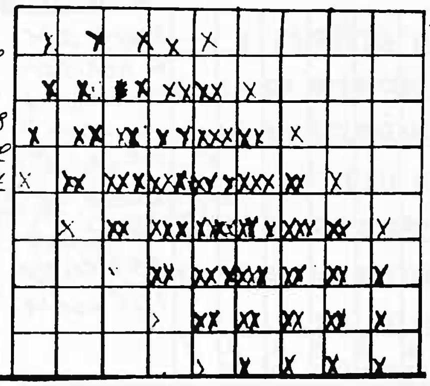
Vertical wins



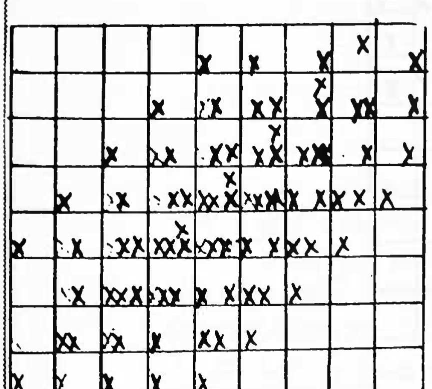
There are 4 connect 5's on each of the 9 lines, so there are 36 vertical connect 5's



There are 5 connect 5's of each of the 8 lines so there are 40 horizontal connect 5's



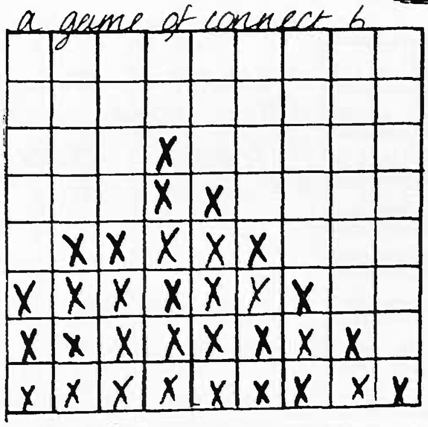
There are 20 ways of winning ↖ this way



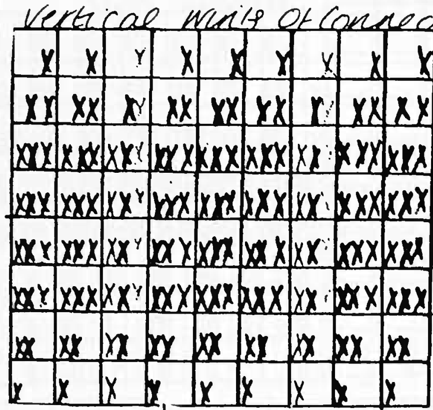
There are 17 ways of gettig connect 5 ↗ way.

Altogether there are (36,40,20,17) 113 ways of gettig connect 5 on a 9 by 8 board ways of winning connect 6 on a 9 by 8 board.

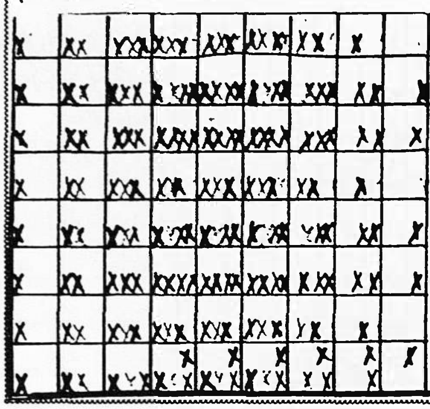
a game of connect 6



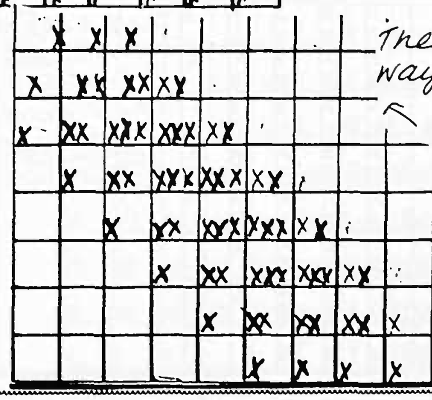
Vertical wins of connect 6.



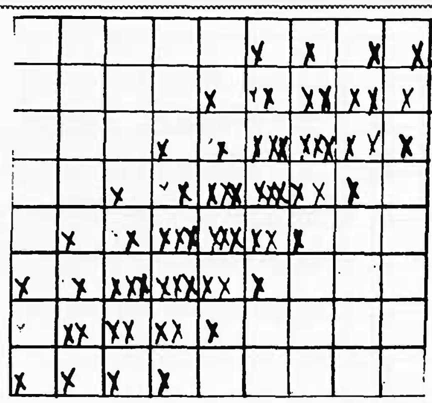
There are 3 wins of connect 6 on each of the 9 lines, so there are 27 vertical wins



there are 4 wins of connect 6 on each of the 8 lines, so there are 32 horizontal wins.



There are 12 ways of winning this way.



There are 12 ways of getting connect 6 ↗ way.
So altogether (27, 32, 12, 12) there are 83 ways of getting connect 6 on a 9 by 8 board

Total winning ways of connect 3, 4, 5 & 6 on a 9 by 8 board.

	vertical	diagonal	horizontal	Total
connect 3	54	91	64	211
connect 4	45	59	48	152
connect 5	36	37	40	113
connect 6	27	24	32	83

From the vertical sets of resuits the numbers go down in one more number than is in a vertical line, ie there are 8 holes in a 9 by 8 board vertical and these resuits go down in 9, same with the normal 7 by 6 board the vertical resuits go down in 7's.

The connection between the vertical numbers is that they come down in 9's.

I can see no connection here as the numbers go down in 32, 22 and 13.

There is a bit of a connection here as the numbers go 16, 8 & 8.

I can see no connection here as the numbers go down 49, 39 & 30.

Conclusions to Connect 3, 4, 5 & 6.

Now you know quite a lot about connect 3-6 how many ways there are of winning explained in diagram, words and tables. Obviosly there are more ways to win connect 3 than connect 6, now all you need to do is win the games you play!

I1/5

CONNECT 4 INVESTIGATION

INTRODUCTION

What I already know about Connect 4 :

It is a game for two players compeating against each other to attempt to make a sequence of 4 by displacing the coloured counters onto the board. It is adapted from the old game of Noughts and Crosses and is now a glorified version of it.

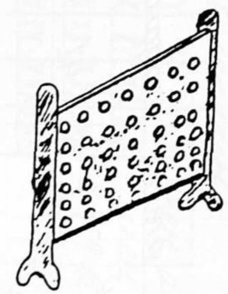
INVESTIGATION :

CONNECT 4

Student Notes

The game of CONNECT 4 is a very popular one with all age groups.

There are six rows (lines going across) and seven columns (lines going down) in a normal Connect 4 game.



What can you find out about this game?

Find out as much as you can but remember to keep some notes of your investigation as you go along so that you can write up your report when you have finished. Your report will be a very important part of the assessment so it must show everything that you did and thought about.

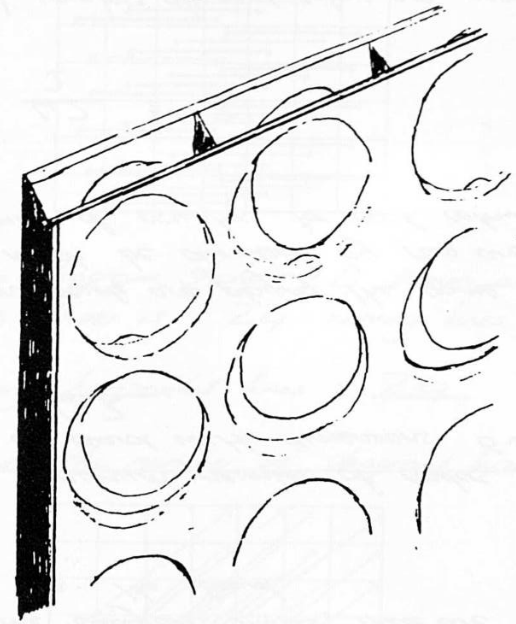
IDEAS

Before actually beginning to investigate the problem I decided to list some of the possible areas of research and some of the first ideas which sprung to mind.

- Find the number of winning lines on a 6 × 7 board
- Compare Connect 4 with Noughts and Crosses
- Investigate various tactics behind the game.
- Examine the situation with boards of different dimensions.
- Instead of connecting 4-in-a-row try using more than 4 and attempt to find any relevant equation
- Investigate changing the rules in various ways and examine the outcome
- Investigate bending around corners to win instead of connecting 4 in a straight line.
- Attempt to find the most useful place to go on the board on the first move.
- Consider ways in which more than 2 people can play

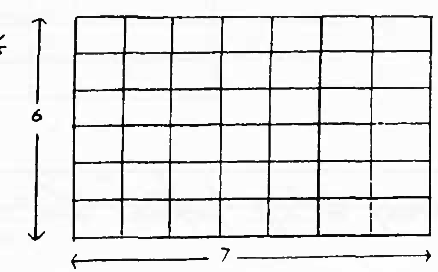
Obviously some ideas above prove little use whilst others seem to be very positive paths to explore into. Being at the beginning of the investigation I thought it best to start with very basic ideas using the Connect 4 system on a typical 6 × 7 board. Once becoming more knowledgable over the basic rules, strategy, and the game in general, then I would explore into more complex areas and attempt to achieve greater values. Extension activities would be considered throughout.

Below is a drawing of part of the Connect 4 board



For this investigation I thought it would be far more practical for a diagram to be used, so it would be easier to draw and would stand out more clearly

The diagram represents a typical 6 × 7 board.



INVESTIGATION

What are the minimum dimensions needed to play Connect 4 ?

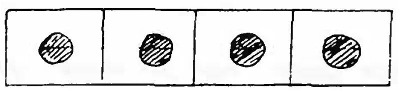
Since 4 must be connected in a straight line the minimum number for one of the dimensions must be at least 4.

∴ Minimum dimensions are 4 × ?

Since the dimension of 4 is sufficient for one line to be made the second dimension must be as small as possible, but greater than 0.

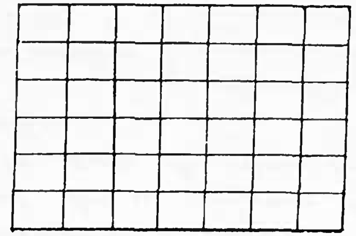
∴ Minimum dimension needed to play Connect 4 → $\underline{\underline{4 \times 1}}$

The diagram shows this to be correct as only one winning line can be made.



However, this is obviously impossible to happen in any game so the board is too small to experiment on. Therefore I decided to investigate various problems on the typical 6 × 7 board.

How many counters are needed for a board with dimensions 6 × 7 ?



As the diagram shows, there are 42 spaces available on the board.

∴ The minimum number of counters needed for a board with dimensions 6 × 7 is $\underline{42}$

As two people are playing the game this number must be divided by two and the total number of counters for each colour would be found.

$$\frac{42}{2} = 21$$

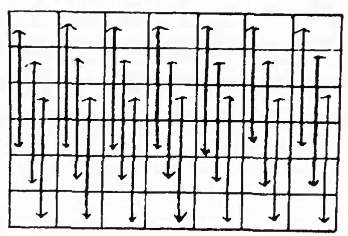
∴ $\underline{21}$ counters of each colour are needed

Playing Connect 4 on a 6 × 7 board what is the maximum number of winning lines?

To work this out the problems had to first be broken down and then a diagram be drawn.

Number of winning lines = No. of Vertical lines + No of Horizontal lines + No of Diagonal line

To work out the number of vertical lines

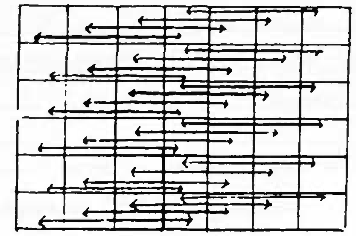


Each brown arrow represents one possible winning line and there are 21 lines altogether.

∴ Playing Connect 4 on a 6 × 7 board the number of vertically winning lines is

21

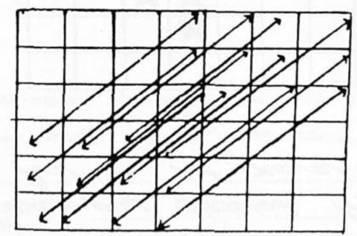
To find the number of horizontal lines



The diagram shows 24 brown arrows
Therefore there is 24 winning lines

Winning horizontal lines = 24

To find the number of diagonal lines



There are 12 brown arrows in the diagram. This has to be doubled since there must the same amount of arrows pointing in the opposite direction.

× 12

Therefore, the number of diagonally winning lines is 12 × 2.
12 × 2 = 24

Winning diagonal lines = 24

Winning Vertical lines = 21
Winning Horizontal lines = 24
Winning Diagonal lines = 24

∴ Number of Winning lines = 21 + 24 + 24

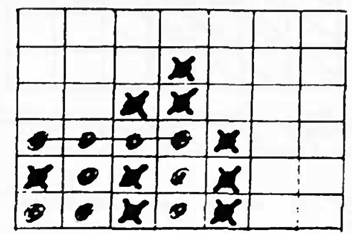
= 69

∴ The total number of winning lines using the Connect 4 system on a 6 x 7 board is 69

Is the person who goes first guaranteed to win?

The odds are undoubtedly stacked against this as the game would not really be worth playing.

Test examples showed that the person who went first is not guaranteed to win.



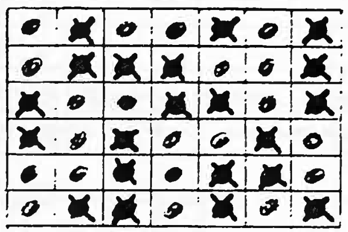
played first

Although brown played first the orange connected 4-in-a-row first and so won the game.

When playing Connect 4 on a 6 x 7 board is any player guaranteed to win?

This is not quite so obvious and would need further examination.

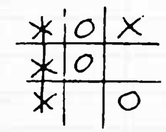
After a considerably large amount of games played, a situation was reached where in one game no player could win as all the spaces were filled.



∴ Either player is not guaranteed to win

Are the 6 x 7 dimensions the best choice for the dimensions, or should these be smaller to shorten the game, or larger to lengthen the game?

We know the original game of connect 4 is adapted from the old Noughts and Crosses and so we can compare the shortened Connect 4 to Noughts and Crosses, as these are basically the same.



An example of 'Noughts and Crosses'

After observing some games of 'Noughts and Crosses' the following table could be drawn.

Game	Winner		Draw
	Player 1	Player 2	
1	✓		
2			✓
3			✓
4		✓	
5			✓
6			✓
7			✓
8			✓
9			✓
10			✓

<u>Playing Connect 4 on a 6 x 7 board is it such a big advantage to go first ?</u>

Realistically, I believed the only way to solve this was to study some sample games and record the results. The games played are displayed on the separate sheet.

The table shows the results gained after 10 games were played

Game	Person who went 1st	Person who went 2nd	Draw
1		WON	
2	WON		
3		WON	
4		WON	
5	WON		
6		WON	
7	WON		
8	WON		
9	WON		
10	WON		

It is seen that the person who went first won 6 times, and the person who went second 4.

6 out of 10 (x100) = <u>60%</u>
4 out of 10 (x100) = <u>40%</u>

Despite the sample being relatively small the results obtained were very general but, were sufficient to draw conclusions from them

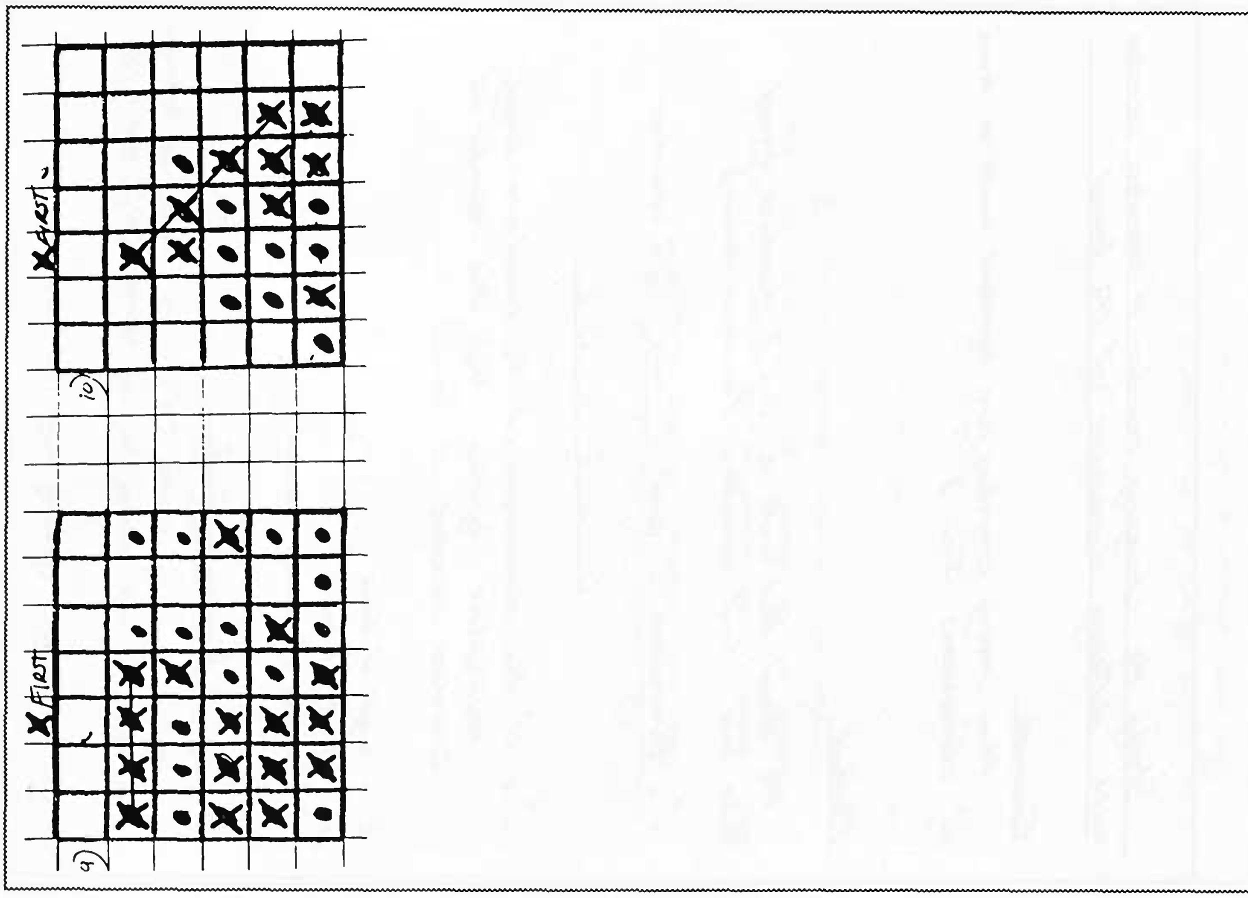
The table shows only a rough guide and is not accurate statistics as only 10 games were played with the same people. If accurate results were required then I would have used a considerably larger amount of games played with a random sample of people. However, I found it sufficient for my requirements.

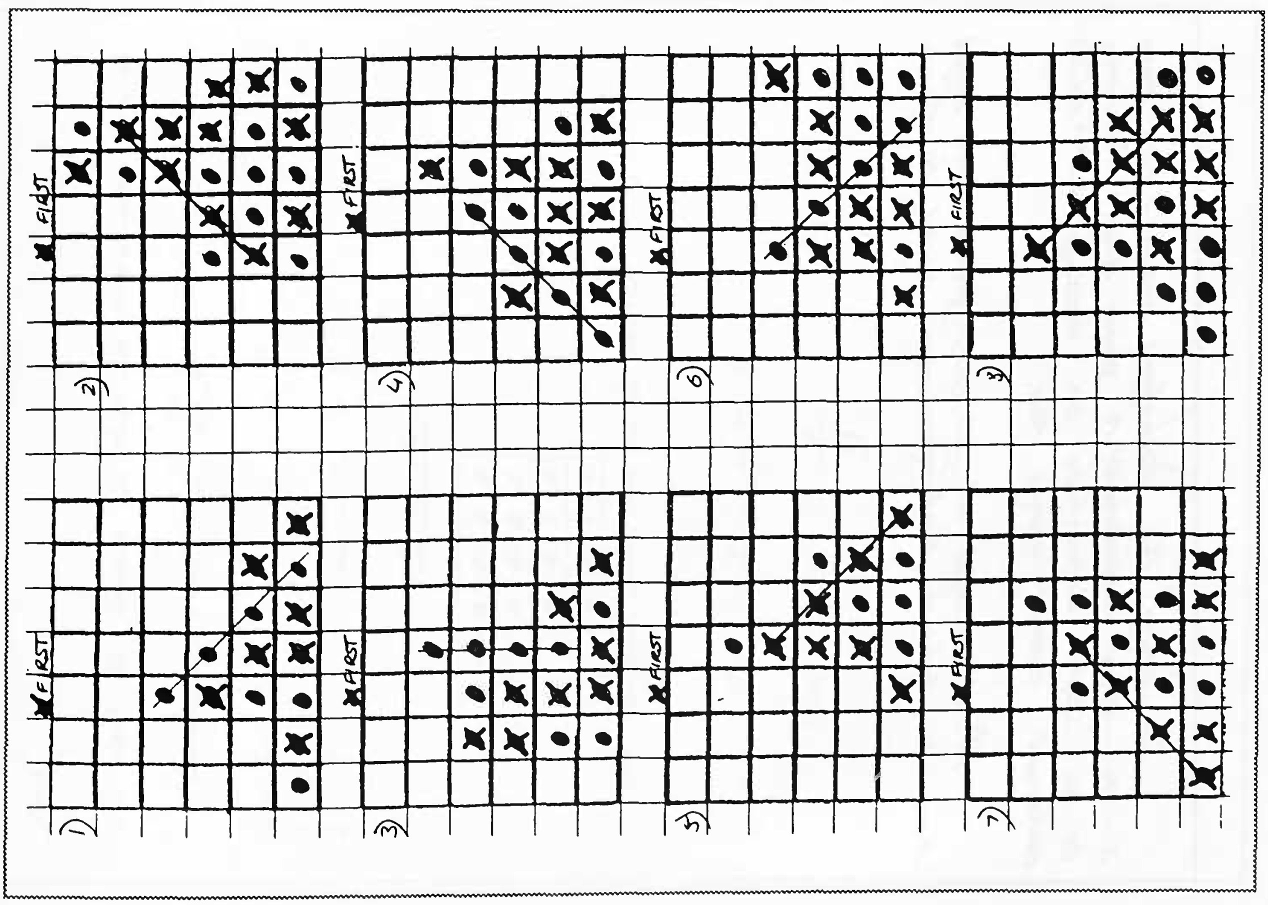
The fact that out of 10 games 8 were resulted in a draw, proves that with smaller dimensions, in this case 3 x 3, a draw is much more likely to occur.

Therefore the dimensions should not be as small as this

With the larger dimensions I feel this complicates the game to an even greater extent.

$\therefore$ <u>I believe the 7 x 6 dimensions is most probably the best choice and most realistic choice for the size of the board</u>





Considering the percentages shown I feel there is no great advantage to going first. There may be a slight advantage in leading but the result obtained cannot verify this.

If a sample of 10,000 games were played then I believe that conclusions could be drawn.

Study the changing number of counters needed with differing dimensions of the board

Example

How many counters are needed with a board of dimensions 5×3 ?

Method

We know that with a 6×7 standard board there are 42 counters (as shown earlier).

$\therefore$ Dimensions 6 and 7 have 42 counters.

$$6 \times 7 = 42$$

$\therefore$ If the dimensions of the board are simply multiplied together then the number of counters needed is found

x = length of board
y = width of board
z = Number of counters needed.

$$x \times y = z$$

<u>Test</u>

To test the formula we shall use the example 4×4

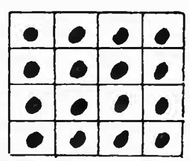
Dimensions are 4 by 4

$x = 4$
$y = 4$
$z = ?$

$x \times y = z$

$4 \times 4 = z$

$\underline{z = 16}$

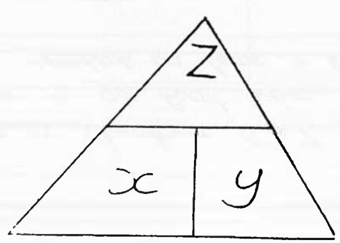


The diagram shows this to be correct.

$\therefore$ If, $x \times y = z$

then, $x = \frac{z}{y}$

$y = \frac{z}{x}$



To test the formula other examples had to be shown.

Q1) What is the width of the board (y), when the length of the board (x) is 5, and the number of counters needed (z) is 25.

$x = 5$
$y = ?$
$z = 25$

$x \times y = z$

$\therefore$ $y = \frac{z}{x}$

$y = \frac{25}{5}$

$y = 5$

$\therefore$ <u>The width of the board is 5</u>

This was found to be correct.

Q2. What is the length of the board (x), when the width of the board (y) is 8, and the total number of counters needed (z) is 72.

$x = ?$
$y = 8$
$z = 72$

$x \times y = z$

$\therefore$ $x = \frac{z}{y}$

$x = \frac{72}{8} = \underline{9}$

$\therefore$ <u>The length of the board is 9</u>

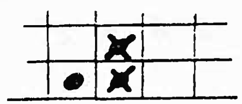
This was found to be correct.

On a standard 6 × 7 board is it such an advantage to go first when varying the number of counters needed in line to win?

Connecting 1

Inevitably the person who goes first by displaying their counter wins the game.

Connecting 2



By looking at the diagram it is seen that the person who goes first is guaranteed to win, when he plays his counter the automatically has 3 winning places. As only one can be blocked by his opponent he has a choice of 2 places in which to win the game.

Connecting 3

Is the person who goes first guaranted to win?

Out of 10 games, the person who went first won 8, and the person who went second won 2.

∴ The person who goes first has an overall advantage over the opponent but is not guaranted to win.

Connect 4

The results from previous problems show that the person who goes first has, if any, an extremely minor advantage over his opponent. In general, anyone could win.

Connect 5 onwards.......

This is found to have limited the advantage of the person who goes first winning. This shows to have a very balanced chance of either player winning.

Conclusion

By examining the results from connecting the various numbers, I have discovered that the person who goes first has a very great advantage when connecting smaller numbers (1-3) and very little advantage on the greater numbers (4+).

This can be displayed on a graph with the probability of winning against the amount of counters needed in line. However, this is definitly not an accurate graph since only one correct point can be plotted (probability 1), therefore it is only estimated and should not be used to claim accurate results.

Continuing with the 6 × 7 board study the change from 4-in-a-row to more than 4-in-a-row

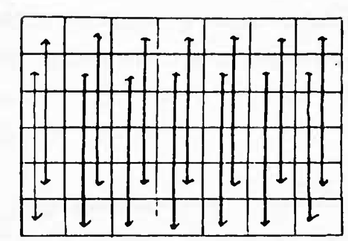
To do this I decided to start with the 5-in-a-row system and then work upwards if necessary.

5-in-a-row

Areas of interest – to find the number of winning lines.

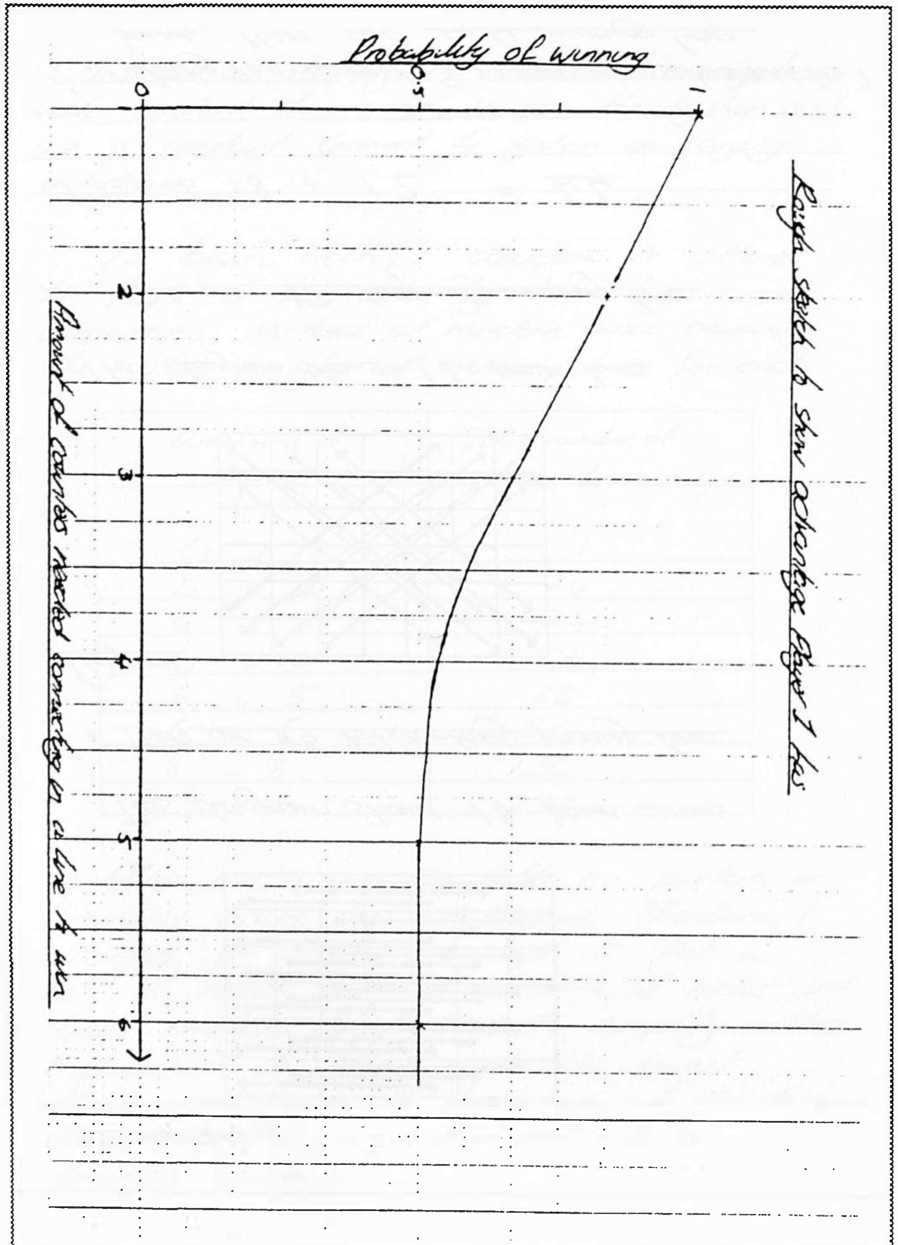
Since there are 69 winning lines when connecting 4-in-a-row, I predicted that there would be slightlest less when connecting 5. Drawing all possible winning lines on one diagram could become overcrouded and so I decided to separate them in vertical, horizontal and diagonal.

Vertical

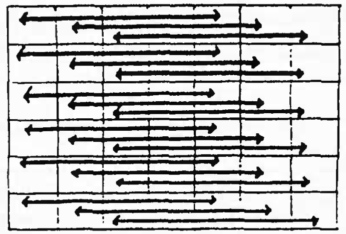


The diagram shows 14 brown arrows

∴ There are 14 vertically winning lines



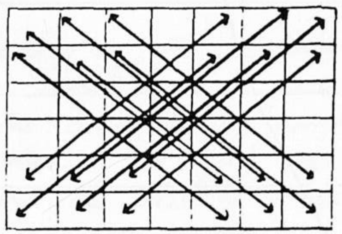
Horizontal



The diagram shows 18 brown arrows

∴ There are 18 horizontally winning lines

Diagonal



The diagram shows 12 brown lines

∴ There are 12 diagonally winning lines

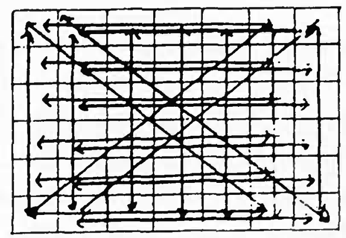
14 + 18 + 12 = 44

∴ Using the Connect 5 system on a standard 6 x 7 board there are 44 winning lines.

Connect 4 = 69 winning lines
Connect 5 = 44 winning lines

There seems to be no immediate relationship between Connect 4 and Connect 5 so I decided to work out the number of winning lines for Connect 6

As there will be even less lines on this, the total number of winning lines can be displayed on one diagram.



The diagram shows 23 brown lines

∴ There are 23 winning lines altogether

Connect 4 - 69
" 5 - 44
" 6 - 23

After examining the results no definite pattern could be found. Therefore I decided to resort back to the Connect 4 system and attempt to work out a formula for finding the number of winning lines

<u>Instead of studying the number of winning lines with constant dimensions, inverse the system and use a constant Connect 4 system on varying dimensions.</u>

To begin with, I decided to attempt to find a formula by tabulating various dimensions picked at random and examining the total number of winning lines for each.

Dimensions of board	Total number of winning lines
4 × 1	1
4 × 4	10
4 × 6	24
4 × 10	52
5 × 5	28
6 × 6	54
8 × 8	130
10 × 10	238

After examining the table no pattern or formulas could be established. Therefore, I decided it would be best to put a set of results into some form of order and then examine it further. To simplify matters I decided to start with the lowest dimensions and to keep one of the dimensions constant. These were then put into a suitable table.

Dimensions of board	Total number of winning lines
4 × 1	1
4 × 2	2
4 × 3	3 +7
4 × 4	10
4 × 5	17 +7
4 × 6	24
4 × 7	31
4 × 8	38
4 × 9	45
4 × 10	52

The table shows no significant patterns or formulas after examination. Although there is an increase in 7 after 2 winning lines, this is not terribly helpful as no equations can be found using them.

After having no success on the attempt to find a major formula I decided to try and break the problem into stages less complicated.

Keeping the dimensions similar to the above table; I decided to split the total number of winning lines into :-

Number of vertical lines + Number of Diagonal lines + Number of Horizontal lines

Therefore, the following table could be completed.

x	y	H	V	D	TOTAL LINES
4	1	1	0	0	1
4	2	2	0	0	2
4	3	3	0	0	3
4	4	4	4	2	10
4	5	5	8	4	17
4	6	6	12	6	24
4	7	7	16	8	31
4	8	8	20	10	38
4	9	9	24	12	45
4	10	10	28	14	52

x = constant dimension (rows)
y = variable dimension (columns)
H = total horizontal winning lines
V = total vertical winning lines
D = total diagonal winning lines.

By examining the table it was seen that various patterns were produced. From then on it was merely a question of finding vague formulas and testing them with several easy examples.

After a considerable amount of time testing with trial and error I came up with the following formulas.

$$H = y\,(x-3)$$

$$V = x\,(y-3)$$

$$D = (x-2)\,(y-3)$$

Obviously these had to be checked before being confirmed.

We know a board with dimensions 4 × 4 has 4 winning horizontal lines, 4 winning vertical lines and 2 winning diagonal lines.

<u>Testing using formulas</u>

<u>Horizontal</u>

$x = 4$
$y = 4$

$H = y(x-3)$

$H = 4(4-3)$

$H = 4 \times 1$

<u>$H = 4$</u>

<u>Vertical</u>

$x = 4$
$y = 4$

$V = x(y-3)$

$V = 4(4-3)$

$V = 4 \times 1$

$\underline{V = 4}$

<u>Diagonal</u>

$x = 4$
$y = 4$

$D = (x-2)(y-3)$

$D = (4-2)(4-3)$

$D = 2 \times 1$

$\underline{D = 2}$

Referring back to the table these answers were found to be correct. To test them again we know a standard 6 × 7 board has a total of 69 winning lines. Therefore, if we used the formulas to find the horizontal, vertical and diagonal winning lines then the sum of these should be 69.

<u>To find the horizontal</u>

$x = 6$
$y = 7$

$H = y(x-3)$

$H = 7(6-3)$

$H = 7 \times 3$

$\underline{H = 21}$

<u>To find the vertical</u>

$x = 6$
$y = 7$

$V = x(y-3)$

$V = 6(7-3)$

$V = 6 \times 4$

$\underline{V = 24}$

<u>To find the diagonal</u>

$x = 6$
$y = 7$

$D = (x-2)(y-3)$

$D = (6-2)(7-3)$

$D = 4 \times 4$

$\underline{D = 16}$

After checking the answers I found both the vertical and horizontal to be correct, but the diagonal was incorrect. There are actually 24 diagonal lines on a 6 × 7 board

∴ Alterations had to be made.

By keeping the basic framework to the diagonal formula I decided to experiment from there.

Realizing that there were 2 diagonal directions, I decided to try and find a formula for just one direction, then this can simply be multiplied by 2. Making a slight adjustment to the old formula I came up with the following,

$$D = (x-3)(y-3)$$

This gave the total number of diagonally winning lines in only one direction. However, they could easily be altered.

$$\underline{D = 2((x-3)(y-3))}$$

This then had to be checked.

<u>Example</u>

We know that when $x = 4$ and $y = 10$ there are 14 diagonal lines. (referring to table)

$x = 4$
$y = 10$

$$D = 2((x-3)(y-3))$$

$$D = 2((4-3)(10-3))$$

$$D = 2((1)(7))$$

$$D = 2(7)$$

$$\underline{D = 14}$$

The formular proved to be correct.

<u>Testing with a standard 6 x 7 board</u>

$x = 6$
$y = 7$

$$D = 2((x-3)(y-3))$$

$$D = 2((6-3)(7-3))$$

$$D = 2((3)(4))$$

$$D = 2(12)$$

$$\underline{D = 24}$$

Again the formula was proved correct.

$\therefore$ Correct and tested formulas are :

$$\underline{H = y(x-3)}$$

$$\underline{V = x(y-3)}$$

$$\underline{D = 2((x-3)(y-3))}$$

$\therefore$ Total number of winning lines are

$$H + V + D$$

Now having the formulas to work out the total number of horizontally, vertically and diagonally winning lines, I decided to look back and attempt to find a major formula for all dimensions.

$$H = y(x-3)$$

$$V = x(y-3)$$

$$D = 2((x-3)(y-3))$$

Firstly, however, I had to attempt to combine the formulas to produce one major one.

Where T is the total number of winning lines:

$$T = y(x-3) + x(y-3) + 2((x-3)(y-3))$$

This, however, had to be tested.

Using a 6 x 7 board find the total number of winning lines.

$$T = y(x-3) + x(y-3) + 2((x-3)(y-3))$$

$x = 6$. $y = 7$

$$T = 7(6-3) + 6(7-3) + 2((6-3)(7-3))$$

$$T = 7(3) + 6(4) + 2((3)(4))$$

$$T = 21 + 24 + 2(12)$$

$$T = 21 + 24 + 24$$

$$T = 69$$

The formula was shown to be correct.

The next stage was to look at the situation without connecting 4-in-a-row. Hopefully so some of connection can be made.

5-in-a-row

x	y	H	V	D	TOTAL LINES
5	1	1	0	0	1
5	2	2	0	0	2
5	3	3	0	0	3
5	4	4	0	0	4
5	5	5	5	2	12
5	6	6	10	4	20
5	7	7	15	6	28
5	8	8	20	8	36
5	9	9	25	10	44
5	10	10	30	12	52

From examining the table it is clearly seen that similar patterns are produced from the connecting 4 table. Therefore the formula would have to be tested.

Testing with 5 rows, 5 columns

$x = 5$ $\quad T = y(x-3) + x(y-3) + 2((x-3)(y-3))$

$y = 5$

$T = 5(5-3) + 5(5-3) + 2((5-3)(5-3))$

$T = 5(2) + 5(2) + 2((2)(2))$

$T = 5(2) + 5(2) + 2(4)$

$T = 10 + 10 + 8$

$\underline{T = 28}$

Considering that the total number of winning lines is actually <u>12</u> the formula was proved incorrect.

Therefore, having to resort back to the horizontal, vertical and diagonal formulas it is seen on the table that they are all incorrect.
After examining the table it is seen that wherever (-3) occurs in the formulas, (-4) should be in that place. Therefore the formulas would be,

$H = y(x-4)$

$V = x(y-4)$

$D = 2((x-4)(y-4))$

After examining both formulas for connecting 4 and connecting 5 I found that the changing number could be found simply by using the x dimension or the number needed connecting, by the equation,

$(x-1)$

By inserting this into the major formula, it becomes

$T = x(y-(z-1)) + y(x-(z-1)) + 2[(x-(z-1))(y-(z-1))]$

where,
T = total number of winning lines
x = number of rows
y = number of columns
z = number needed connecting in a line

The formula had to be tested with an example and I decided to use a 6×7 board, connecting 4 in a row, as we know there are 69 winning lines

<u>EXAMPLE 1</u>

$x = 6$ $\quad T = x(y-(z-1)) + y(x-(z-1)) + 2[(x-(z-1))(y-(z-1))]$
$y = 7$
$z = 4$ $\quad T = 6(7-(4-1) + 7(6-(4-1) + 2[(6-(4-1))(7-(4-1))]$

$T = 6(7-3) + 7(6-3) + 2[(6-3)(7-3)]$

$T = 6(4) + 7(3) + 2[(3)(4)]$

$T = 6(4) + 7(3) + 2(12)$

$T = 24 + 21 + 24$

$\underline{T = 69}$

Notice that the numbers represent the horizontal, vertical and diagonal lines which show to be correct

∴ Final tested formula for working out how many winning lines there are is,

$$T = x(y-(z-1)) + y(x-(z-1)) + 2[(x-(z-1))(y-(z-1))]$$

Where,

T = total number of winning lines
x = total number of rows
y = total number of columns
z = number needed connecting in line to win

EXAMPLE 2.

On a board with 4 rows and 10 colums, connecting 5-in-a-row how many winning lines are there altogether?

$x = 4$ $\quad T = x(y-(z-1)) + y(x-(z-1)) + 2[(x-(z-1))(y-(z-1))]$
$y = 10$
$z = 5$ $\quad T = 4(10-(5-1)) + 10(4-(5-1)) + 2[(4-(5-1))(10-(5-1))]$

$T = 4(10-4) + 10(0) + 2[(0)(10-4)]$

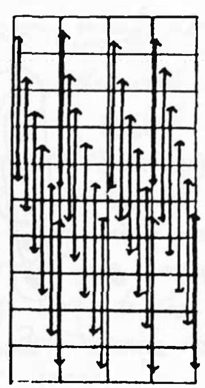
$T = 4(6) + 0 + 2[(0)(6)]$

$T = 24 + 0 + 2(0)$

$T = 24 + 0 + 0$

$\underline{T = 24}$

The diagram shows this to be correct.



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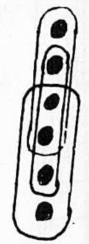
Connect Four

The game of connect 4 is a very popular one for all age groups

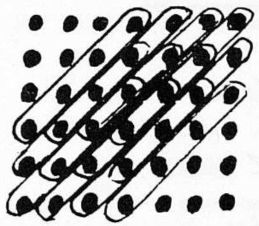
There are 21 winning moves in the vertical columns. 3 per column and 7 columns.

There are also 24 winning moves in the horizontal rows, 4 per row and 6 rows.



drawing to show winning moves.

drawing to show winning moves.

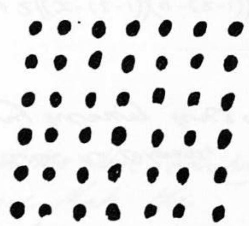


You can still win by having line of 4 in a diagonal direction There are 12 winning moves in this direction and 12 more in the other diagonal direction.

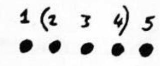
12 x 2 = 24 + 24 + 21 = 69 winning moves on a standard board

when you play connect four the first counter will go to the bottom row, lets say in column 4, the middle column The next counter that goes in this column will rest on the counter already there, so it will be in the second row up

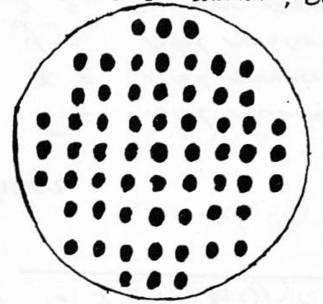
If red goes first in the middle at the bottom he has seven winning moves that he could use that is 9% of the total. If green goes next and puts his counter ontop of reds he eliminates one of reds moves, and takes up 13% of the posable winning moves. if red puts his second counter ontop of greens he eliminates one of greens moves and takes up 16%. From these results it looks like the nearer you get to the centre the more percentage of the winning moves you have.



If you can get three of your counters in a row with an empty space at both sides you know you have won because if your opponent puts there next counter on the left of your three (Position 1) you put your winning counter in Position 5, or vice versa. I think connect four is a larger game of noughts and crosses, and just like in noughts and crosses if you have two really good players the game will more often than not end in a draw.

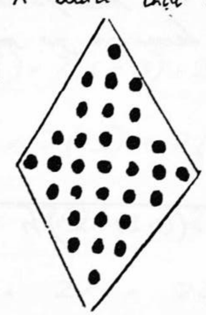


I could change the game by calling it connect 3, in this game you only have to get 3 counters in a row which makes the game alot easier to win, but alot harder to defend. OR I could do the opposite and call it connect 5 which would make it harder to win. I could change the height and length of the board by adding or taking away rows and columns. I could change the shape of the board to a circle or diamond, cross etc. Here are some examples



A board that is round.

or a diamond

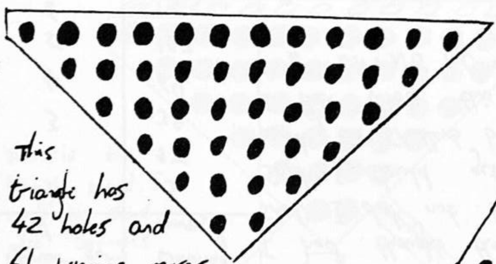


This round board has a total of 120 winning moves It has 61 holes to put the counters in, the normal connect four board has 42 holes

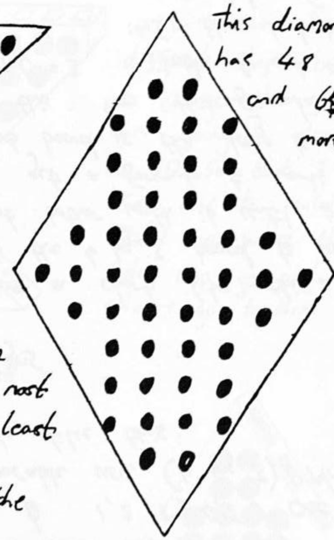
This board has 31 holes and a total of 34 winning moves

The method that I use to find the winning moves was the same method I used on the previous Page, I draw loops around rows of four and then add them all up

So that I can compare all the different shape boards with the Standard 7 by 6 board I am going to make all the new shape boards have roughly the Same number of holes as standard board which is 42. Then I shall compare all the boards and say which one will be easiest to win and easiest to defend



This triangle has 42 holes and 61 winning moves

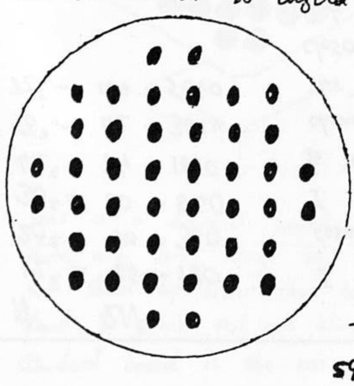


This diamond shap has 48 holes and 65 winning moves

From these results we can see that a Square board gives the most amount of winning moves for the least amount of holes, it is the most economical. But only just, all the other shapes are very close

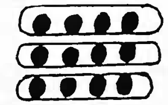
The Standard board is the board that is easiest to win on but the hardest to defend because there are the most winning moves. The circle is easiest to defend but the hardest to get a winning move on because it does not have as many winning positions

But the differences are very Small indeed, the only way to see if these predictions are correct is to Practise a few games



This circle has 44 holes and 59 winning moves

Now I am going to try and find a formula for connect four. I am going to use the standard board, which is Square. and one Side is one row longer than the other. The first thing I did was draw lots of different Sized Standard boards, e.g. a 4 by 3 board, this is the Smallest board you can posable use for connect 4 because there are only three winning moves. When I had done this several times with all the different Sized boards up to an 8 by 9 board I then made a difference chart like this.



only 3 winning moves

N =	12		20		30		42		56		72
$S(N)$ =	3		17		39		69		107		153.
1st Diff =		14		22		30		38		46	
2nd Diff =			8		8		8		8		

I made 'N' the number of holes in the board S(N) is the amount of winning moves for a board with 'N' amount of holes If when I get a difference table with the 2nd difference being the last I know I will have to use a formula like this, or it

$$N^2 + BN + C$$

would at least have to have a Square root in it

Number of holes	Winning moves
12	3
20	17
30	39
42	69
56	107
72	153

I then drew this chart. Now all I have to do is find B and C.

I made B 2 and left C for the moment. I also changed the first + to a minus in the formula

$$N^2 - BN + C$$

Because I thought it would make the formula have a better outcome.

N^- $2N$
$12^2 - 24 = 120$
$20^2 - 40 = 360$
$30^2 - 60 = 840$
$42^2 - 84 = 1680$
$56^2 - 112 = 3024$
$72^2 - 144 = 5040$

I did this formula and got these results. It is quiet along way out and I had a feeling it was not going to work so I had another go by changing B to $(N^2 - 2N)$. This also did not work but it was alot closer. Instead of being 120 out it was 2. But at the other end of the table it was -9 out so I had to take some away using C and add some on at the same time.

Next I changed the N to 1, 2, 3, 4, 5. One being the smallest board you can posable use (4 by 3). Mr. Gordon also told me to use a formula like this:

$$AN^2 + BN + C$$

+ signs can also be − signs

Number for N	Winning moves
1	3
2	17
3	39
4	69
5	107
6	153

I now have a chart like this. All along I had thought the 4 by 3 board to be so small I did not bother with it until Mr. Gordon said I should get a formula to work for this board, the smallest board is the key to the formula. I then wrote this: For this formula I make both A and B two

$N^2 = 1A + NB + C = 3$

$N^2 \times A + N \times B + C$

I then had to find C = lots of things!

$N = 1$ $c = -1$
$N = 2$ $c = +5$
$N = 3$ $c = +15$
$N = 4$ $c = +29$
$N = 5$ $c = +47$

If I make A abit bigger it will increase things abit, I made it 4 and left B as 2

I then did the formula again but left out C. In all cases I had to make C −3. So the formula was.

$N = 1$ $C = -3$
$N = 2$ $C = -3$
$N = 3$ $C = -3$
$N = 4$ $C = -3$
$N = 5$ $C = -3$

$$N^2 \times 4 + N2 - 3$$

Now I can work out how many winning moves there are in connect four.

A draw back of this formula is you have to work out what N is before you can do the formula. I have made a table down the left side of the page to help, but if I had a board 1000 by 1001 it because slightly hard. I am not very confident with formula and I now realise if I had worked abit more at the N = Number of holes in board formula it would have worked and got rid of the little draw back I have just mentioned.

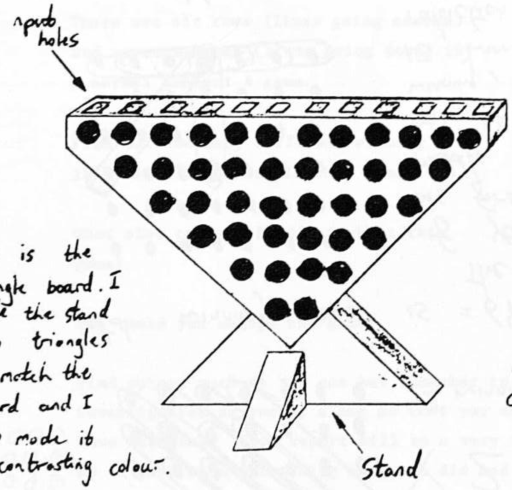
N =	X =	Y =
1	3	4
2	4	5
3	5	6
4	6	7
5	7	8
6	8	9
7	9	10
8	10	11
9	11	12
10	12	13
11	13	14
12	14	15
13	15	16
14	16	17
15	17	18
16	18	19
17	19	20
18	20	21

The formula to work out 'N'.
X = Rows Y = columns, there should be one more digit in Y, Y is always one larger. 'N' is two digits smaller than 'X'. So if I did have a board 1000 by 1001 'N' would be 998. The formula is this:

$X - 2 = N$ or: $Y - 3 = N$

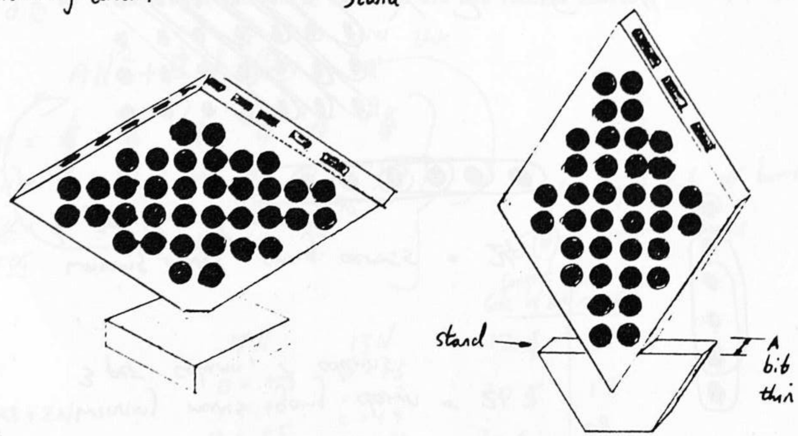
Since it took me four maths lessons and a few hours of homework to get this formula I am going to include the working out sheets.

New Design Connect 4 Board



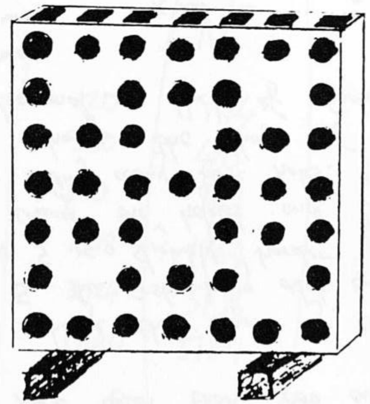
As I have come up with the idea of differed shape boards to change the game I am going to draw them in 3D so that I can see how to make them stand up, where the input holes can go, and which colours go together well.

This is the triangle board. I made the stand also triangles to match the board and I also made it a contrasting colour.



This is a diamond board, I have drawn it twice because there are two ways of having it up. The green and red one will stand up better than the yellow one because of the thin stand. I think red and blue are the best colours. The Square standard board is the easiest to make I think.

Modified connect 4

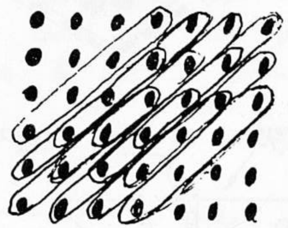


At a quick glance this looks like a standard board, but there are a few holes blanked off. The counters still can slide down the inside to get to the bottom holes, but to get to the holes above the blanked of hole you have to waste a counter to fill the space up that is blanked of. This changes the game and makes it more frustrating.

Connect Four (Rough Work)

winning moves going down = 21
3 per column, 7 columns

winning moves going across = 24
4 per Row, 6 rows

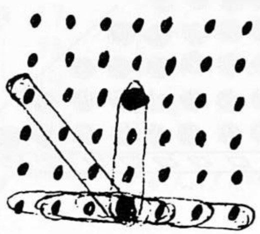


X 2 for other direction

12 winning moves x2 = 24

Total winning moves is = 69

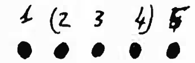
The Red dot
if your opponent Puts his first counter in the middle of the board he has 13 different ways of winning, and you have lost 13 winning places, thats a reduction of 18 %



13

The green dot
If your opponent puts his first counter in the middle of the bottom he only has six winning moves a reduction of 8 %

If you can get three in a Row with a space on both sides you know you have won because if your opponent puts there next counter on the left of the three (position one) you put the winning counter in position five. and vice versa.



connect four is like ~~noughts~~ naughts and crosses except alot bigger.

If you have played the game alot before and your opponent has not you always win until your opponent has got some experience of the game If the two players are really expert at connect 4 the game will more often than not end in a draw.

I could change the Game by calling it connect 3 or connect 5. The first being alot easier than connect 4 and connect 5 is a slightly harder game. Why

I could change the height and length of the board by adding or taking away some holes.

I could change the shape of the board to a circle, diamond. block of some holes, cover up

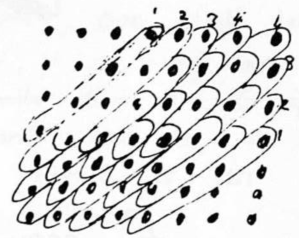
4 by 5 17 moves
5 by 6 39 moves
6 by 7 69 moves
7 by 8 107 moves

X row
Y columns
N winning moves
O Number dots

20 30 42 56 72
17 39 69 107 153
14 22 30 38 46
8 8 8 8
50

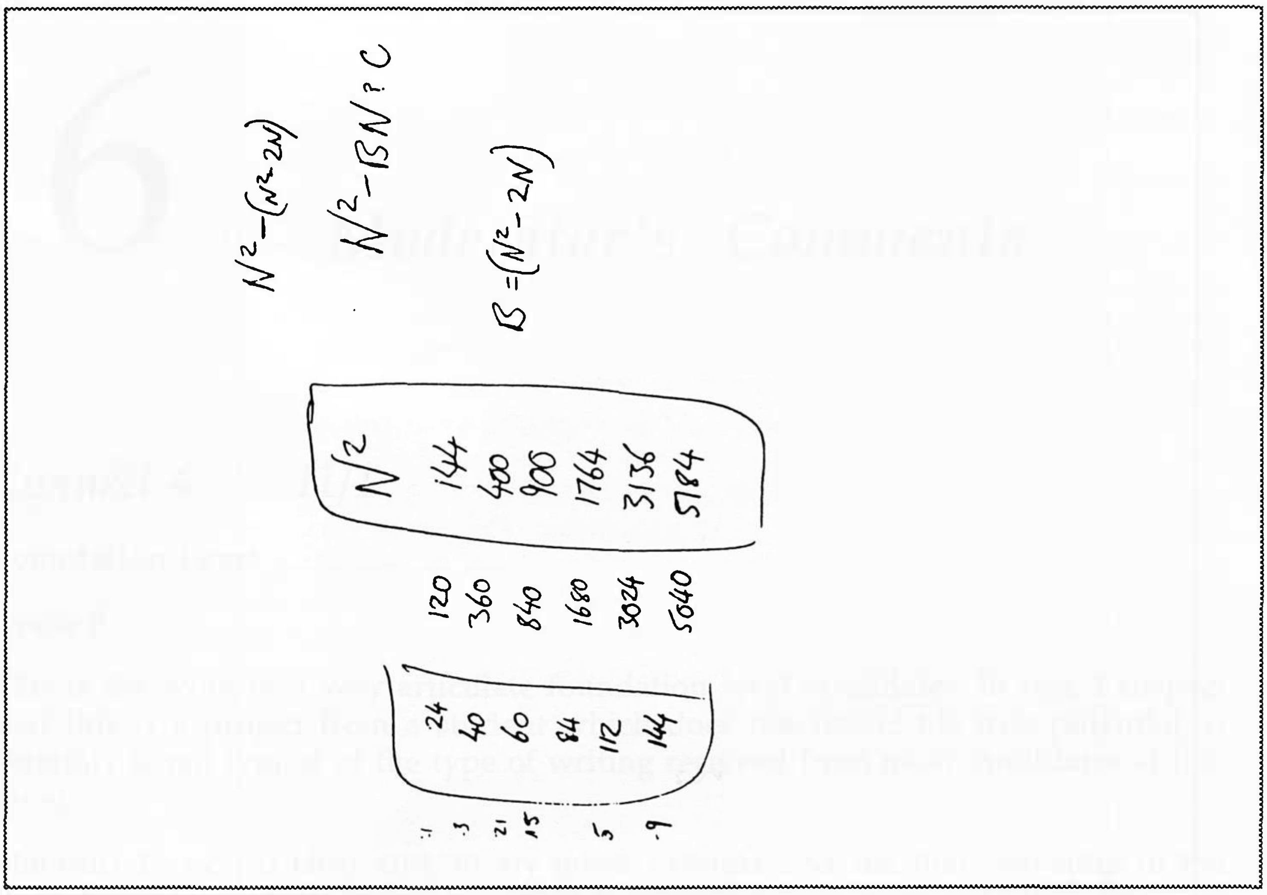
$4 \times 5 = 20$
$5 \times 6 = 30 + 9$
$6 \times 7 = 42 + 27$

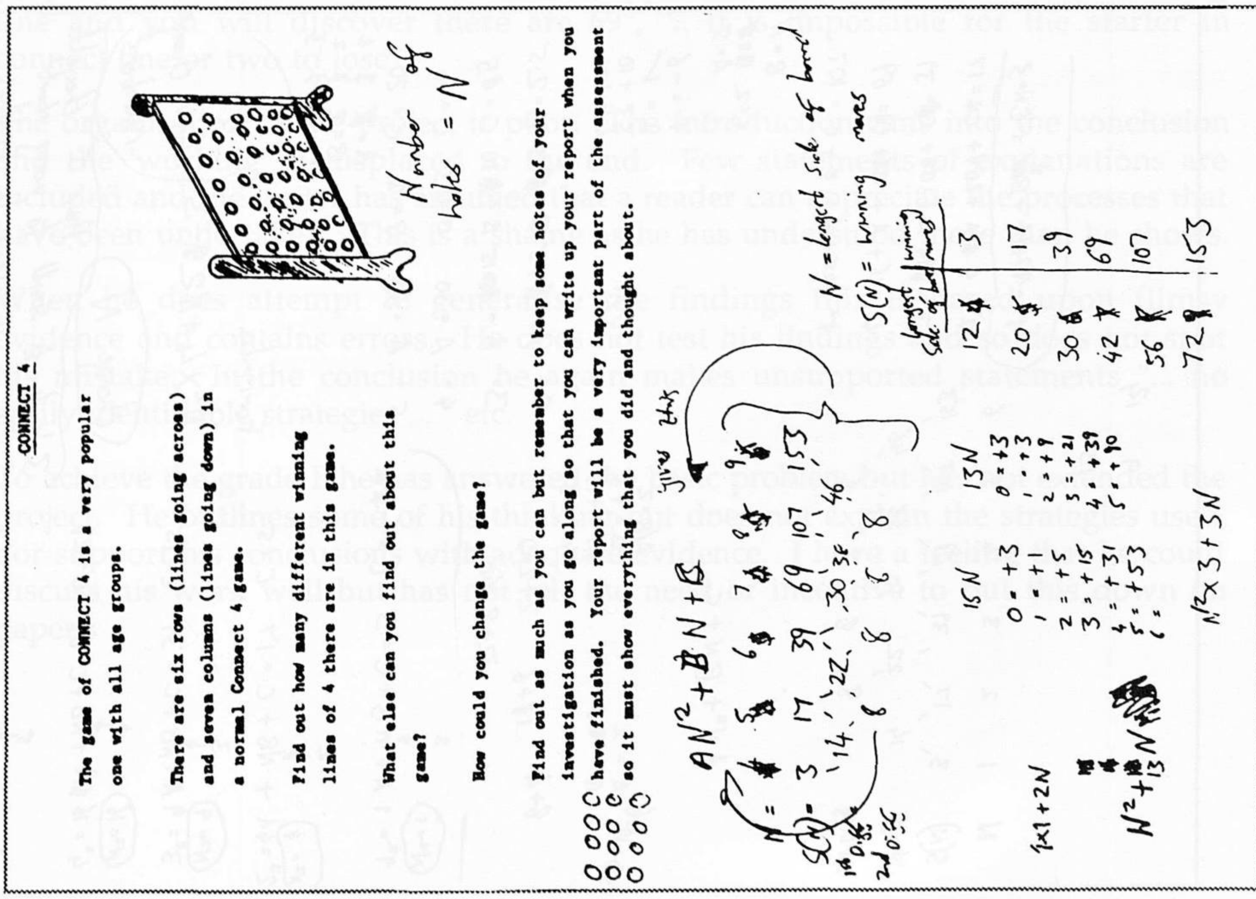
XY − 3



32 35 20 20
40
75
= N
$6^2 = 36$ $5^2 = 25$
$8^2 = 64$ $7^2 = 49$ −6 −16 −22

$X/2 \times Y - 3 \times 2 + 15$





CONNECT 4

The game of CONNECT 4 is a very popular one with all age groups.

There are six rows (lines going across) and seven columns (lines going down) in a normal Connect 4 game.

Find out how many different winning lines of 4 there are in this game.

What else can you find out about this game?

How could you change the game?

Find out as much as you can but remember to keep some notes of your investigation as you go along so that you can write up your report when you have finished. Your report will be a very important part of the assessment so it must show everything that you did and thought about.

9 14
13

N	1	2	3	4	5	6
S(N)	3	17	39	69	107	153
1st Diff		14	22	30	38	46
2nd Diff			8	8	8	8

$1^2 = 1A + BN + CN = 3$

$2^2 = 4A + 2BN + CN = 17$

$3^2 = 9A + 3BN + CN = 39$

$4^2 = 16 + A + BN + CN = 69$

$5^2 = 25 + A + 8 + C = 107$

B = N

C = 2

A = 5

$AN^2 + BN + C$

1 = −6
2 = ✓
3 = +10
4 = +24

50 + 10 = 60
A = 2
B = 5

A/B = 2
2 = +5
3 = +15
4 = +37

82

8+4   18+6   2+2=4
32+8 = 40

2 = A = 7.5 B = 3 C = 2.5
3 = A = 15 B = 10 C = 5
4 = A = 30 B = 13 C = 10
5 = A = 60 B = 2 C = 20

A = 3 4
B = 2 2
C = −7 −3

$N^2 = 1$
$1^2 = 1A + NB + C = 3$ C = −

$N^2 = 4$
$2^2 = 4A + NB + C = 17$ C = 5

$N^2 = 9$
$3^2 = 9A + NB + C = 39$ C = 15

$N^2 = 16$
$4^2 = 16A + NB + C = 69$ C = 29

$5^2 = 25A + NB + C = 107$ C = 47

Formula

$N^2 \times 4 + N2 - 3$

4
How change numbers

Change N

How I thought
3 by 4 to small to worry about but really key to problem

6 Moderator's Comments

Connect 4   I1/1

Foundation Level

Grade F

This is the work of a very articulate foundation level candidate. In fact, I suspect that this is a project from a student which does not reflect his true potential, it certainly is not typical of the type of writing received from most candidates at this level.

The introduction is clear and, in my mind, extends over the first two sides of the project. However, it is riddled with unsupported statements "Find every winning line and you will discover there are 69", ".. it is impossible for the starter in connect one or two to lose".

The organisation of the project is poor. The introduction runs into the conclusion and the 'working' is displaced to the end. Few statements of explanations are included and the writer has assumed that a reader can appreciate the processes that have been undertaken. This is a shame as he has understood more than he shows.

When he does attempt to generalise the findings this is based upon flimsy evidence and contains errors. He does not test his findings and so does not spot his mistake. In the conclusion he again makes unsupported statements "... no easily identifiable strategies ... " etc.

To achieve the grade F he has answered the basic problem but has not extended the project. He outlines some of his thinking but does not explain the strategies used, nor support his conclusions with adequate evidence. I have a feeling that he could discuss his work well but has not felt the need or incentive to put this down on paper.

Connect 4 I1/2

Foundation Level

Grade F

This is a good example of work by a foundation level student, probably on the E/F border.

The introduction to the project is brief but to the point. It outlines the salient points but indicates no directions the work might follow nor does it detail any strategies. The evidence for the numbers quoted is at the end of the project and not with the statements made. There is no reason given for reducing the size of the grid when reducing the number of counters to be connected, or the effect upon the results if this reduction does not take place.

In essence the work is a single stage investigation without development or refinement. One strategy has been applied and no real generalisation has taken place. There are some statements of conclusion but not all these are substantiated. There is no evidence of prediction of results but there must have been some checking as an error on the second side is corrected in the table that appears on the same page. Diagrams have been repeatedly drawn to obtain results without any evidence that she has understood or used the patterns observed in the diagrams.

This is a nice, tidy solution to one line of inquiry into the task with appropriate diagrams and use of 'equipment' in the form of dotty paper.

Connect 4 I1/3

Intermediate Level

Grade D

This is a good example of a piece of work on the C/D boundary.

The introduction sets the scene and asks a couple of questions though only drawn from the set task, but indicates little of the strategy to be applied to the task. The appearance is of a certain lack of precision and planning. A number of figures are quoted without substantiation and slightly unclear references made to diagrams.

When starting the project, I like the way a notation system has been invented, unfortunately it is a little unwieldy and has not been put to real purpose. In fact, throughout the project there is not a really clear method of notation or reference to diagrams. This is a weakness in the work which combines with other imprecise statements "... the 6 x 7 grid is the largest that needs to be produced.", "... I found pattern when changing grid sizes ...", "horizontals have gone up in 6's" when she meant "down in 6's" etc.

The overall impression of the work is that its scope is rather cramped and that the writer is prepared to make statements without sufficient support from experimental evidence. Attempts have been made to generalise the results but this is again a little clumsy, though certainly a reasonable application for an intermediate level pupil.

Once the writer moves out of the security of the 6 x 7 grid the organisation of the work is more haphazard and exhibits less forward planning. There is a clear attempt to generalise results, but in words rather than symbols, and the general formula is beyond the scope of the writer.

The conclusion is more of a written summary, a commonly found shortcoming, rather than a brief reorganisation and restatement of the findings of the work. In fact, I feel it to be rarely necessary to write a long conclusion if students have made careful statements of conclusion at the relevant point in the work.

I suspect, looking at the work from outside, that the student has worked very hard at this topic and produced a good and pleasing piece of work for their own level of mathematical attainment.

Connect 4 I1/4

Intermediate Level

Grade C

The appearance of this project is very pleasing. It is well presented with clearly drawn diagrams and well written comments. It is a good piece of work for an Intermediate candidate.

The introduction to the game is clear, if slightly deficient, in not mentioning that the counters are dropped into the top of the frame. What follows is a little confused in that she then mixes a new game (Connect 3) in with Connect 4.

The notation used in the project is worth noting in that she has used coloured crosses to indicate the location of the winning rows - this is, regrettably, not clear on the photocopies, of necessity, used here. This notation is unique in the group and so indicates some individuality in the project.

From this point on, the project is systematically put together to the first table of results where the findings are recorded clearly with, quite clear, comments on the results. She has noticed the numerical pattern in the results but has failed to make any attempt to generalise these. She has also used the results to deduce or predict the expected results for Connect 2. It is a pity she did not go on to test this prediction and demonstrate that it worked from first principles.

The latter part of the project is essentially a repeat of the first part and demonstrates that she is fully capable of following her own routines and lines of investigation in a slightly new situation. The project is an efficient commentary upon a line of enquiry into one aspect of the game, though it perhaps needed a little thought into the organisation of the evidence at the start of the write up.

Connect 4   I1/5

Higher Level

Grade B

If I were asked to grade this piece of work for G.C.S.E, I would place it very much on the A/B borderline. Were it to bear a grade A 'tag' I doubt I would argue too vehemently with the decision, however, there are good reasons for feeling it is just on the B side. The introduction is well written and shows strong personal involvement. It does not rely upon the restatement of the set task but contains a good breakdown of potential points to be investigated in the work. The introduction covers the essential points and begins to outline the strategy to be used in the task. In the investigation itself the author has clearly set himself questions to be answered. This begins very well with the establishment of the minimum necessary dimensions for a grid, though a more logical development would be to start with a minimum board and slowly expand this, as was achieved half way through the project.

In the first half of the investigation a lot of good groundwork is laid down in the analysis of the problem into vertical, horizontal and diagonal lines using diagrams, but this is not really picked up until much later in the task. The project loses direction until this point and deals with some fairly trivial areas. A number of assumptions are made when looking at the likelihood of winning - Are the same people playing? Does the same person go first? How adept were the players? What playing strategy was used? Although he did recognise the shortcomings of his work, I felt a number of statements of conclusion were made without sufficient evidence to support them. Having looked at the probability of winning on a 6 x 7 grid he then, quite justifiably, launches into a number of rather inconclusive 'blind alleys'. I felt he was looking for something to do at this point before he returned to an analysis of the game itself.

The project regained its direction when he returned to the winning lines on varying size grids. It took a while to get into his stride - the choice of 'random' grids did not seem very sensible but this was recognised and he, at last, got back to the formula: number vertical + number horizontal + number diagonal basis. From here the project contained a very worthwhile general study of the 'winning lines' in grids. However, the places which put doubt on the A grade are - confusion of x and y; horizontal and vertical lines and not picking this up when checking the work; a lack of 'construction' of the formulae from the analysis of the structure of the board i.e. why does y (x-3) give the number of 'horizontal' lines?; and a lack of simplification to the formulae. Having made these comments I do commend this piece of work as a thoroughly worthwhile investigation. It is, perhaps a little long, clearly taking more than three weeks and it embodies many of the desirable processes to be found in a good investigation from a very open starting point.

Connect 4 I1/6

Higher Level

Grade A

This coursework item contrasts well with the other Higher Level piece included. Whereas the other is extremely long and methodical, this goes straight to the point of what is to be done.

The introduction wastes no time on protracted, specific examples. He has clearly sorted out the method by which he is to attain his results and included this in the notation used on page 1. However, his unwillingness to include precise details of his reasoning at this stage means that the statements on winning lines on page 1 are vague and the percentages quoted are not clearly derived.

On page 2 the attempt to consider winning chances dies rather limply. On the same page he shows clearly that he has a grasp of possible future directions and has no difficulty in determining, logically, all the possible winning lines.

What distinguishes this piece is that he sets out clearly his intention is generalise the results of the Connect 4 board and achieves this. He does this in a methodical and somewhat pedantic way over pages 4, 5 and 6. He uses mathematical knowledge to achieve his ends (differencing and the an^2+bn+c) even though he is clearly familiarising himself with the techniques.

I approve of the acknowledgement of advice given to him during the development of the generalisation. This is entirely in keeping with the criteria for GCSE extended task assessment. He has taken the advice and applied it and understood the usage. The help has not directed the project, merely given him a tool to help the solution to the problem.

The drawings at the end are neatly drawn if lacking some technical expertise.

This is a neat, punchy project with comments very much to the point and illustrating the point that length is not necessary to achieve good marks.

On the debit side, I should like to see more inclusion of work in achieving results and more evidence of testing the formula in other situations. Rather than including his rough work he should have written this neatly into his final write up. The project is a little thin on evidence, if long on results. The conclusion is the achievement of the formula but it seems to fade away rather than conclude on a high note.

I should grade this as a good Higher Level piece ranking a little above the other, longer piece included (and hence placing it in the A grade bracket).

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