EXTENDED TASKS FOR GCSE MATHEMATICS

A series of modules to support school-based assessment

MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

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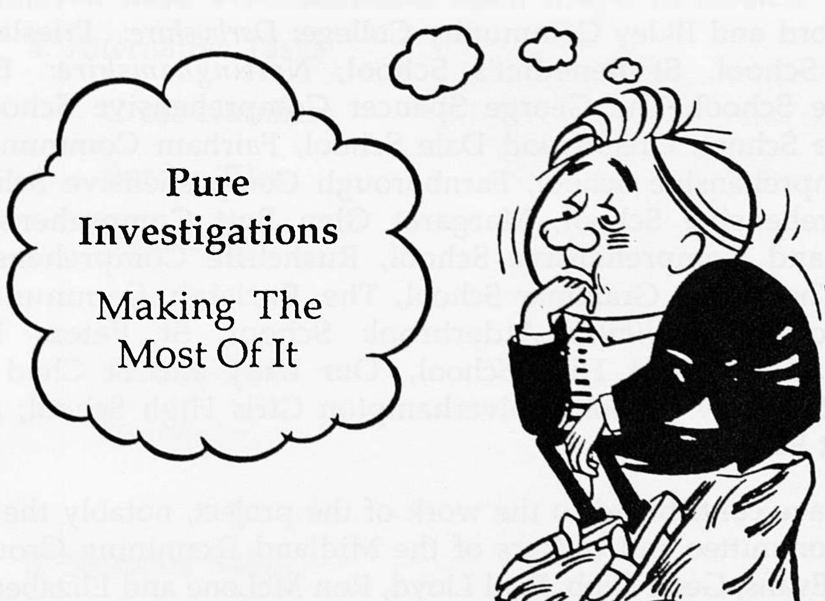


MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

Authors

This book is one of a series forming a support package for GCSE coursework in mathematics. It has been developed as part of a joint project by the Shell Centre for Mathematical Education and the Midland Examining Group.

The books were written by

Steve Maddern and Rita Crust

working with the Shell Centre team, including Alan Bell, Barbara Binns, Hugh Burkhardt, Rosemary Fraser, John Gillespie, Richard Phillips, Malcolm Swan and Diana Wharmby.

The project was directed by Hugh Burkhardt.

A large number of teachers and their students have contributed to this work through a continuing process of trialling and observation in their classrooms. We are grateful to them all for their help and for their comments. Among the teachers to whom we are particularly indebted for their contributions at various stages of the project are Paul Davison, Ray Downes, John Edwards, Harry Gordon, Peter Jones, Sue Marshall, Glenda Taylor, Shirley Thompson and Trevor Williamson.

The LEAs and schools in which these materials have been developed include *Bradford:* Bradford and Ilkley Community College; *Derbyshire:* Friesland School, Kirk Hallam School, St Benedict's School; *Nottinghamshire:* Becket RC Comprehensive School, The George Spencer Comprehensive School, Chilwell Comprehensive School, Greenwood Dale School, Fairham Community College, Haywood Comprehensive School, Farnborough Comprehensive School, Kirkby Centre Comprehensive School, Margaret Glen Bott Comprehensive School, Matthew Holland Comprehensive School, Rushcliffe Comprehensive School; *Leicestershire:* The Ashby Grammar School, The Burleigh Community College, Longslade College; *Solihull:* Alderbrook School, St Peters RC School; *Wolverhampton:* Heath Park High School, Our Lady and St Chad RC School, Regis School, Smestow School, Wolverhampton Girls High School; and Culford School, Bury St Edmonds.

Many others have contributed to the work of the project, notably the members of the Steering Committee and officers of the Midland Examining Group - Barbara Edmonds, Ian Evans, Geoff Gibb, Paul Lloyd, Ron McLone and Elizabeth Mills.

Jenny Payne has typed the manuscript in its development stages with help from Judith Rowlands and Mark Stocks. The final version has been prepared by Susan Hatfield.

Contents

1 *Introduction*

MAKING THE MOST OF IT is one of eight such 'cluster books' each offering a lead task which is fully supported by detailed teacher's notes, a student's introduction to the problem, a case study, examples of students' work which demonstrate achievement at a variety of levels, together with six alternative tasks of a similar nature. The alternative tasks simply comprise the student's introduction to the problem and some brief teacher's notes. It is intended that these alternative tasks should be used in a similar manner to the lead task and hence only the lead task has been fully supported with more detailed teacher's notes and examples of students' work.

The eight cluster books fall into four pairs, one for each of the general categories: Pure Investigations, Statistics and Probability, Practical Geometry and Applications. This series of cluster books is further supported by an overall teacher's guide and a departmental development programme, IMPACT, to enable teacher, student and departmental experience to be gained with this type of work.

The material is available in two parts

Part One		The Teacher's Guide
		IMPACT
	Pure Investigations	I1 - Looking Deeper
		I2 - Making The Most Of It
	Statistics and Probability	S1 - Take a Chance
		S2 - Finding Out
Part Two	Practical Geometry	G1 - Pack It In
		G2 - Construct It Right
	Applications	A1 - Plan It
		A2 - Where There's Life, There's Maths

This particular 'cluster book', MAKING THE MOST OF IT, offers a range of support material to encourage students to undertake a pure investigation within any GCSE mathematics scheme. The material has been designed and tested, as extended tasks, in a range of classrooms. A total of about twelve to fifteen hours study time, usually over a period of two to three weeks, was spent on each task. Many of the ideas have been used to stimulate work for a longer period of time than this, but any period which is significantly shorter has proved to be rather unsatisfactory. The pure investigation tasks are, perhaps, rather different from the other two main types of extended task, those of a practical nature and those of an applied nature, in the sense that they allow students to seek out the pattern and beauty of mathematics without being constrained by real life.

It is important that students should experience a variety of different types of extended task work in mathematics if they are to fully understand the depth, breadth and value of the subject. Having emphasised the pure aspect of this cluster of ideas, it is interesting to note that many of the tasks within it do in fact start with real contexts. The common element amongst all the items within this cluster is the idea that they may be used to stimulate generalisations or optimisations according to the individual need and ability of each student : hence the title of the cluster, MAKING THE MOST OF IT.

Clearly, there are many styles of classroom operation for GCSE extended task work and it is intended that this pack will support most, if not all, approaches. All the tasks outlined within the cluster books may be used with students of all abilities within the GCSE range. The lead task of Barriers may be used with a whole class of students, each naturally developing their own lines of enquiry. It is intended that all the tasks within the cluster may be used in this manner. However, an alternative classroom approach may be to use a selection, or even all, of the ideas within the cluster at one time, thus allowing students to choose their preferred context for their pure investigative study. There is, however, a further more general classroom approach which may be adopted. This would be one that does not even restrict the task to that of a pure investigative nature. In this case some, or all, of the items within this cluster may be used in conjunction with those from one or more of the other cluster books, or indeed any other resource. The idea is that this support material should allow individual teacher and class style to determine the mode of operation, and should not be restrictive in any way.

Teachers who are new to this type of activity are strongly advised to use the lead tasks.

These introductory notes should be read in conjunction with the general teacher's guide for the whole pack of support material. Many of the issues implied or hinted at within the cluster books are discussed in greater detail in The Teacher's Guide.

2 *Barriers*

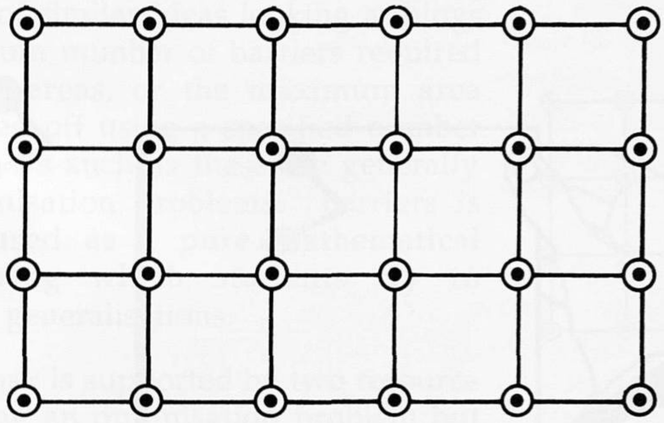
The lead task in this book is called Barriers. It is based on a real life situation and provides a rich and tractable environment for a pure investigation at GCSE level.

The task is set out on the next page in a form that is suitable for photocopying for students.

An alternative lead task, Pin Up, is provided on page 8.

The Teacher's Notes begin on page 9. These pages contain space for comments based on the school's own classroom experiences.

BARRIERS



The diagram above shows a barrier system made up of posts (⦿) and fences (—). The barrier system can be used to section off areas of land for many reasons; for example, to keep sheep in.

This barrier system is 5 units long and 3 units wide.

How many barriers and posts are needed?

What about other dimensions?

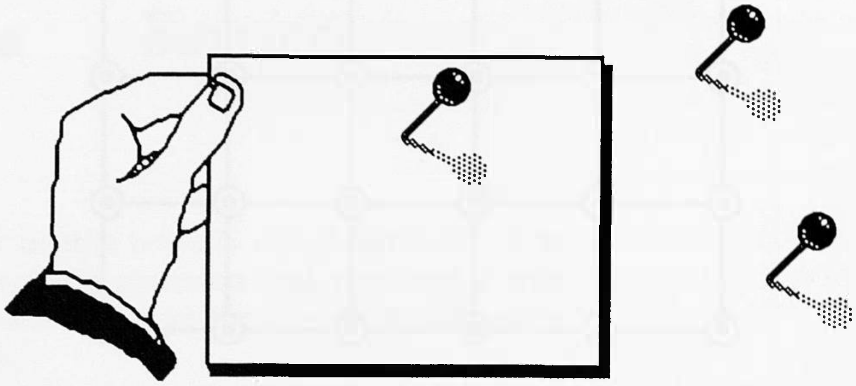
What about other ways of joining the barriers?

Now think of some other ideas to investigate using your mathematical knowledge.

Can you find any rules or patterns in your work?

Investigate The Problem

PIN UP



Bedroom and classroom walls are favourite places to pin up pictures of various types; for example, cars, fashion designs, bands, pop idols, teams, TV personalities etc.

There are many ways of pinning up these pictures including the use of drawing pins. The question is however, how many drawing pins do you actually need to pin up a given number of pictures?

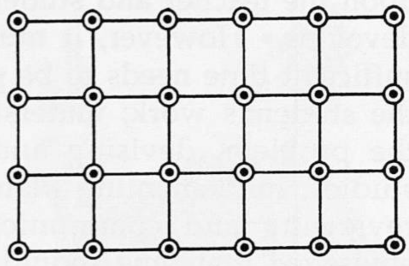
You are allowed to investigate anything you like concerning this problem.

You should keep a note of the problems you set yourself because this will form an important part of your report. You should also include in your report any diagrams, tables, graphs that you use, together with your ideas and discoveries however small you think they are at the time. It should be a complete record of everything you work on relating to this idea.

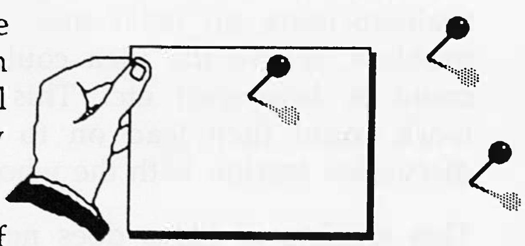
Investigate The Problem

Barriers - Teacher's Notes

Barriers is a pure mathematical investigation and is one of a family of similar ideas looking a things such as the minimum number of barriers required to fence off given areas, or the maximum area which can be fenced off using a specified number of barriers etc. Tasks such as these are generally classified as optimisation problems. Barriers is intended to be used as a pure mathematical investigation during which students try to discover rules and generalisations.



The work on this task is supported by two resource sheets, each offering an optimisation problem but within different contexts. These two resource sheets offer starting points within different everyday situations but, clearly, within the same mathematical theme. You may decide to use just one or both starting points within your classroom. Indeed, you may wish to consider more than the two suggested here. Using these alternative contexts allows a whole class to work within one particular area of mathematics. This enables the teacher to assess a similar set of skills for each student while creating a feeling of personal involvement in the problem, since only a few students are working within each context.



There are a variety of other contexts, many of which may already be familiar to you, which can fit easily into this work. However, it is important that you should ensure that the individual lines of enquiry pursued by your students should lead to a pure mathematical investigation, if this is the area of experience that you intend to assess. It is easy for students who use starting points written for this purpose to develop lines of enquiry which lead to the production of an assignment which is more appropriately assessed within some other area. Whilst this may be desirable for some teachers, it may not be suitable in all classrooms. Particular GCSE schemes may impose constraints which make this unacceptable. Areas of experience which are relevant to your particular scheme may need to be covered. Often a balance between pure investigations, practical in method and applications, practical in purpose, tasks is required.

It would be difficult, and inappropriate, to set out a detailed lesson structure for this task, or indeed generally for this type of work. Much will depend upon the teacher and students, and how the work develops. However, it must be appreciated that sufficient time needs to be given to each aspect of the student's work; understanding and exploring the problem, devising and planning individual studies, implementing plans and pursuing ideas, reviewing and communicating findings. It is envisaged that time should be spent both in class and outside during all stages. Taking account of the above discussion then, it may be helpful to outline one possible approach.

Understanding and Exploring The Problem

When using this type of starting point, it is often helpful to introduce the investigation using a discussion lesson. Initially, such a lesson may involve students working in small groups, brainstorming on what they could do with the problem, where the idea could be used, how it could be developed, etc. This initial small group work could then lead on to a report-back and discussion session with the whole class.

This sharing of ideas does not necessarily mean that all students then do exactly the same things during their investigation. Such introductions are intended to stimulate students to think about the problem in a broad sense, and let their minds wander around it before setting out and pursuing in depth one aspect of the problem. In the absence of these circumstances, students often produce a narrow solution to the initial problem, without considering alternative directions in which they might be able to make a genuine personal contribution. As a teacher, you may like to emphasise these points during the initial stages of the work.

Devising And Planning Individual Studies

There is no reason to restrict the range of contexts to be investigated; these may be thought of by either the teacher or the student. Although the general link between the two problems suggested is the feature that each may be developed into an optimisation problem, there is no reason why this should be the only direction of the work. It may well be that students decide to investigate other questions and not to consider the optimisation aspect at all. In fact, students should be encouraged to devise their own questions for investigation and their own methods to use within their investigation. During the classroom trials of this material, a broad range of problems arose within most classrooms.

Areas for investigation using the Barriers context could include

* The number of barriers and posts required
* The relationship between the number of barriers and posts
* Generalisations to N x M structures
* Maximum area for a given number of barriers.
* Minimum number of barriers for a given area.
* Using other structures to aid the problem; eg, a wall
* Alternative arrangements for given areas; ie, rectangle numbers and factors
* Other arrangements and patterns of posts and fences

and many more.

A similar list of extension ideas may be constructed for any starting point, and the above list probably applies equally well to the Pin Up problem on the second resource sheet.

Implementing Plans and Pursuing Ideas

These two starting points are suited to all abilities and an important role for the teacher is to ensure that all students reach their maximum level of achievement within their work on these tasks. The teacher's support is an important feature of all extended task work. Clearly, it is not suitable for an extended task to be reduced to the answering of a list of teacher directed questions. However, there is a place for carefully planned discussion between teacher and students. This should aid the students as they continue their work in new directions, or deeper into their own chosen direction.

Strategic questions such as

> "What have you tried?",
>
> "What have you found out so far?"
>
> "Have you tried some simpler cases?"
>
> "Well, what do you think?"
>
> "Have you checked if that works?"
>
> "Can you see any pattern?"
>
> "How can we organise this?"
>
> "Would a diagram help?"

encourage students to organise or re-organise their own thinking.

There is also often a need for a teacher to introduce new 'mathematical tools' to students in order to enable them to continue with their work. This type of support is very reasonable, and the assessment of students is then based upon their ability to use the advice and help given.

Reviewing and Communicating Findings

Many teachers have found that with pure mathematical investigations, it is both advantageous and beneficial for students to write their report as they go along. Sometimes this log of their work forms the basis of their final report, on other occasions it is handed in for assessment purposes.

When students have completed the tasks they have set themselves, it is important that they should be encouraged to look back over the path taken and what they have discovered. Students need to reflect upon why some avenues proved to be more profitable than others. Time spent on small group discussions about what each student has accomplished can be extremely useful at this stage. Such discussion can help to focus the students' thinking, prior to the completion of their written reports. It can also help students to organise their discoveries and explanations at this crucial stage.

The assessment of this type of work is based mainly upon the students' reports of their investigations. It is worthwhile emphasising this point, and perhaps outlining at the outset the type of approach which ought to be taken when writing up this work. Further discussion of 'write-ups' is contained in The Teacher's Guide. Many teachers find it useful to produce a checklist for their students, detailing what should be included in their final written report. Examples of such checklists can be found in The Teacher's Guide. Over a three week period, any teacher will obviously find out a great deal about each student's learning during the development of the problem, and this will naturally be taken into account during the assessment stage.

3 *A Case Study*

Fourth Year

Intermediate Level GCSE Group

"I tackled Barriers with my fourth year group which is an intermediate group, but possibly approaching the foundation level. This was their third piece of extended coursework within the MEG scheme. Our previous two experiences had been very pupil based, we simply outlined the general category, eg. Statistics and Probability, and told them to think of a piece of work and to carry it out. We felt that we had to do it this way in order to fulfil the requirements of GCSE coursework. This, we now realise, is not true and it caused a great deal of stress and worry for all concerned.

As a department we welcomed the guidance offered by this material and I decided to use this particular piece. Certainly, looking back, the biggest thing for me and my pupils was the idea of brainstorming. This is something which we were all totally new to, but it was such a positive thing that it is now a major feature of my general teaching style. It is a teaching strategy that I now just slip into with pupils of all ages and abilities when trying to get them to think for themselves. It seems to make them think in two dimensions rather than in just a linear way - this in my view, has great benefits for their work and progress. In general, it is a useful vehicle for all of my pupils now and it certainly helps them to make personal decisions relating to their work.

Throughout this work my pupils were very positive, although there were, of course, low points for each of them and for myself. Their individual work demonstrated enormous diversity even though the whole group were working from the same starting point. This wide range of approaches can be rather intimidating for the teacher and it makes it difficult to support the pupils individually. When I say support, I mean in a general way rather than a prescriptive way.

At times, I felt rather frustrated. This was due to the fact that I had 'completed' the problem myself in a very 'personally obvious way' in an extremely short time compared to the three weeks suggested within this material. The frustration was caused by the pupils not doing what I thought was obvious. I suppose this is something we are just going to have to put up with if we want our pupils to develop as thinking mathematicians, whatever their individual abilities. However, some pupils do spend too long on trivial ideas rather than the more mathematical ones. The teacher's notes do warn about this, but it is still difficult to decide how and when to intervene in order to support pupils. Some teacher guidance followed by sensible development is much better for the pupil than to spend the whole time doing their own 'trivial thing'. I do not think that this is cheating, although I probably would have a year ago.

I have already mentioned the low points. For me, it was at the end of the first week. Three one hour lessons had passed and the pupils seemed to have done nothing except experiment. This was very worrying but, looking back on it, very necessary, although the group's total inexperience of any investigative work of this type made things much worse. At the end of the first week, I was so worried that we had a single lesson in the microcomputer room using 'Sunflower'. This is a piece of software of an investigative nature which requires pupils to grow sunflowers using seeds, soil and, more importantly, three chemicals. The necessary amounts of each chemical have to be determined. There were two of us in the micro room with this class and we simply asked each pupil to work with one or two other pupils they had never previously worked with, and to have a go at the problem.

The only other comment we made was to ask pupils to think about why they were doing this task at this stage in their work, and what they felt they were learning. This was a terrific lesson, they got so much out of it, and during a ten minute feedback period at the end of the lesson, they could speak well about their work and experiences and what they felt they had learned. Their ideas included

* Keeping notes as you go
* Have a go and see what happens
* Important to take account of others' views
* Do not change everything at the same time
* Try one thing at a time
* Follow a pattern
* Talk about what you are doing.

Basically, we followed the teacher's notes quite carefully throughout this work, apart from the micro lesson. I would say that three weeks is an essential minimum, but not far off the maximum for this type of work. Initially, I thought that it would be no more than two weeks, but this would have been virtually useless. It is not filling time, but necessary time to do a good job. Given the overall situation, I am reasonably happy with the final write-ups, which were written up over a couple of lessons and homework.

We got involved in GCSE coursework very early on and it was totally new to us. I now realise that this was useful, but there were problems. Investigative and problem solving skills and techniques have to be taught, and this needs developing throughout the secondary age range, particularly in years one to three.

Pupils need to be encouraged to keep notes about what they do and anything that they notice; to investigate one or two ideas in depth rather than looking at many little bits. Teachers need to support their pupils, but they need to be careful about the type of support they give. If pupils are not prepared for this type of work before they get to the GCSE years, then it may be too late, or at least very difficult. We need to give opportunities to share ideas in small groups; do not make my mistake, however, and have groups of six or seven; three or four is much better. Stress independent thinking but also the sharing of ideas.

My final comment is, ' it was ok this time, but next time it will be good.' We have all come a long way in just three weeks."

4 *Alternative Tasks*

Cross Numbers

Border Tiles

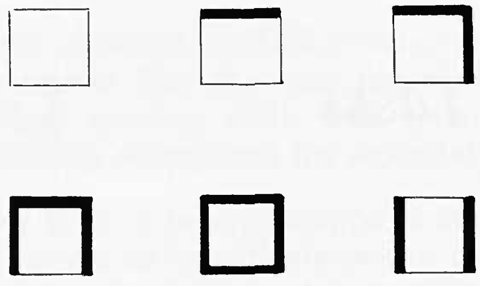
Dominoes

Stars

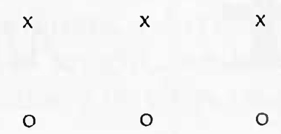
Joining Dots

Crossings

BORDER TILES

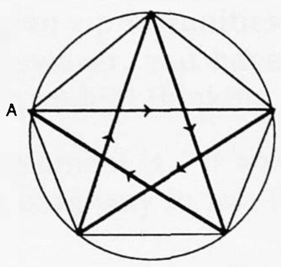


CROSSINGS

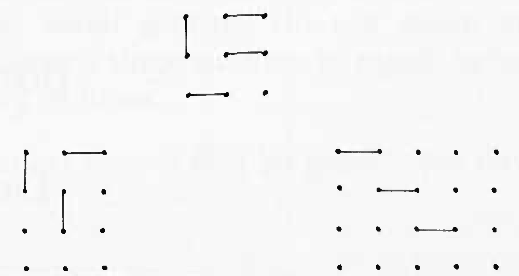


Here we have a diagram showing three noughts and three crosses.

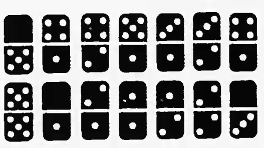
STARS



JOINING DOTS



DOMINOES



Dominoes is quite a popular family game.

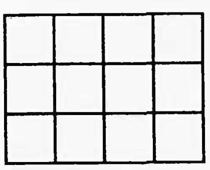
Do you know how many pieces there are in a normal set of 'Double Six' dominoes?

You can buy other sets of dominoes such as 'Double Nine'.

There are many ideas about dominoes which you could investigate if you wanted to.

Here is just one

How could you place dominoes on this grid board so as to cover it completely?



Investigate The Problem

CROSS NUMBERS

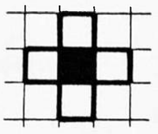
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Here is a number square.

It shows all the numbers from 1 to 100.

We can form Cross Numbers by placing a cross on top of this grid.

We could use a cross like the one shown below.



Here is one example.

1	2	3	4	5
11	12	13	14	15
21	22		24	25
31	32	33	34	35
41	42	43	44	45

Alternative Tasks

General Notes

The six alternative tasks are all intended to be used as pure investigations in the same way as the lead task, Barriers. The teacher's notes for each task are brief and should be read and considered in conjunction with those for Barriers. However, the student's notes are in the same form as those for Barriers. The student's notes offered for the six alternative tasks in this cluster book are all written in a similar style. They outline the context of study to the student and offer one or two problems to be considered. This offers the student the opportunity to consider the problem and gain some understanding of it. Students are then invited to investigate the problem in any way they wish. However, there are extension ideas which may be used if the teacher feels this is appropriate to any individual student, group or class. These extension ideas suggest further areas for investigation without prescribing exactly what should happen.

CROSS NUMBERS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Here is a number square.

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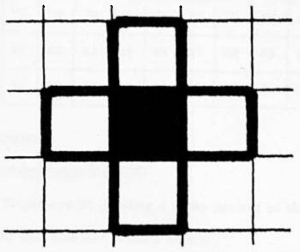
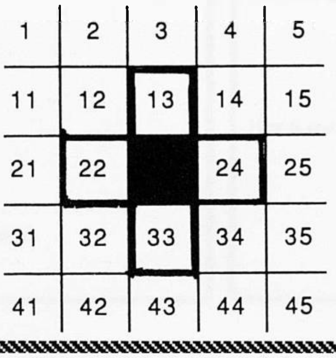


Here is one example.



CROSS NUMBERS : continued

Work out some of the following sums

(24 x 22) - (33 x 13)

(24 + 22) + (33 + 13)

(33 - 13) - (24 - 22)

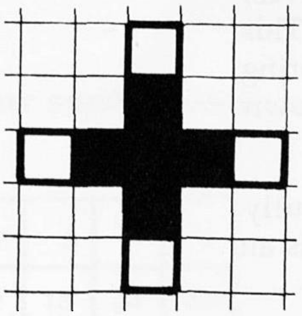
Investigate The Problem

What happens as the cross moves around the grid?

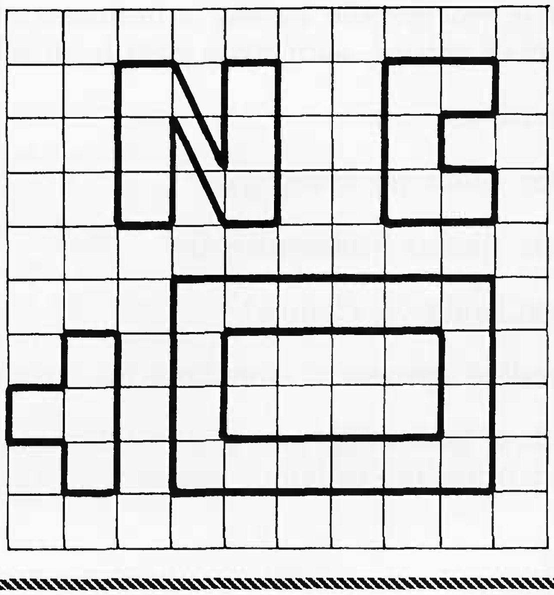
Can you find a rule or pattern?

Can you explain why this works?

Try other size cross numbers.



Try some other shapes.



Cross Numbers - Teacher's Notes

Cross numbers offers a rich starting point for students of all abilities. Hints, which indicate some possible directions, are provided in the form of calculations to be carried out. The initial activity is probably best introduced by giving the whole class the number grid and a piece of tracing paper on which they can draw their initial cross. After a short time questions such as

* Why do you think those results occurred?
* What do you think would happen if the cross was moved?
* Where can you fit the cross on and where can you not?

may be suggested. These questions, posed by the teacher, or better still by the students themselves, will set up the initial investigative situation. This will allow students to spend a little time exploring the problem before deciding upon their own direction.

Naturally, these problems may not be fully resolved. They are designed to give the students an entry point into their own work.

When students have spent some time on this problem, it is worthwhile having a brainstorming session in small groups, with some class feedback.

Questions such as

* What could we investigate?
* What similar problems exist?
* What could we change?

provide possible openers to stimulate this activity.

The strength of leadership provided by the teacher will depend upon the previous experiences of the students.

The extension activities, which could be pursued by some students at a later stage, offer some further ideas which could be considered by individual students. Other ideas which have emerged during classroom trials include

* Using tables of different widths

1	2	3	4	5	6	7
8	9					

* Using the multiplication square

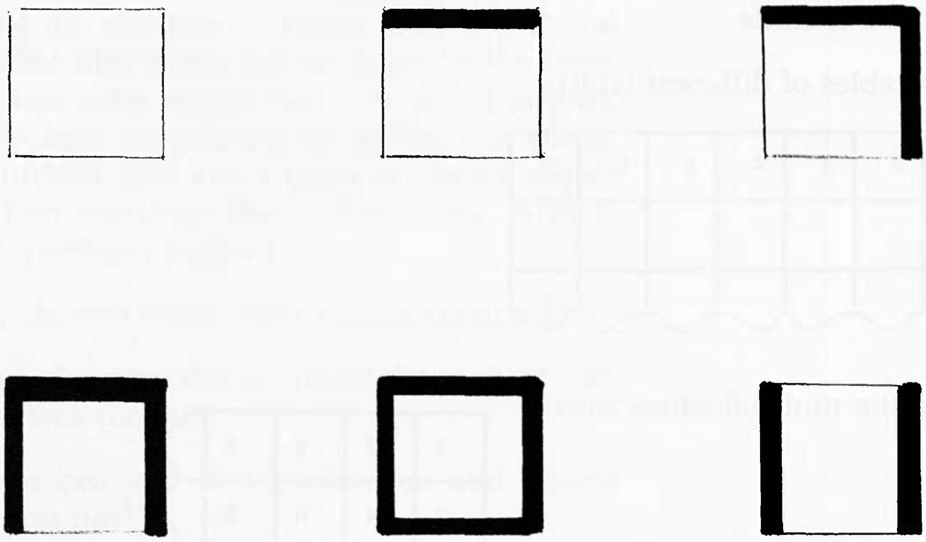
1	2	3	4
2	4	6	8
3	6	9	12
4	8	12	16

* Using other number bases

1	2	3	4	10
11	12	13	14	20

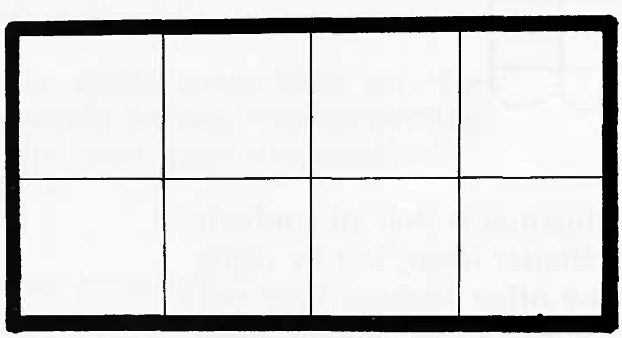
The beauty of this investigation is that all students may be looking at very similar ideas, but by using different shapes, or some other feature, they can work independently. Clearly, for higher level students a verbal explanation of what they discover and why it happens would be expected, together with an algebraic solution. However, foundation level students may achieve considerable success by simply extending the initial activity in their own way, and writing a conclusion concerning anything they notice and whether or not this seems reasonable to them.

BORDER TILES



Square border tiles can have borders on any number of edges. One possible set of border tiles is shown above.

These tiles can be placed together to give other shapes.



Investigate The Problem

BORDER TILES : continued

Try looking at other rectangles made up from these border tiles.

Try to find some rules.

Look at shapes other than rectangles.

What about working in three dimensions?

Try creating some of your own tiles and problems.

Try working with other types of patterns.

Border Tiles - Teacher's Notes

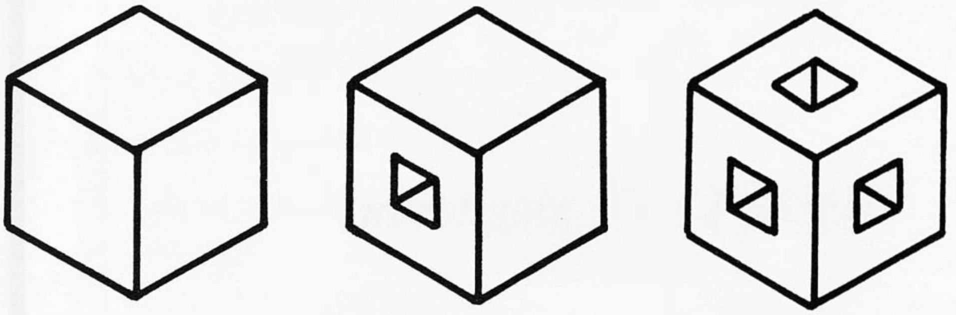
In many ways, this problem closely resembles the lead task of Barriers. Therefore, the teacher's notes for that task apply directly to this one. This pure investigation, like many others, starts from a simple idea but may be developed systematically through to some form of generalisation.

The initial set of tiles is made up of all possible combinations of edged tiles. They are not all useful, or needed, for any given line of investigation. During the initial stages of this work, it may be helpful if sheets of such tiles are printed so that the students can cut them up to experiment with.

An obvious line of enquiry for any student who has had any previous experience with this type of work would be to look at different rectangles. Questions which could be considered include

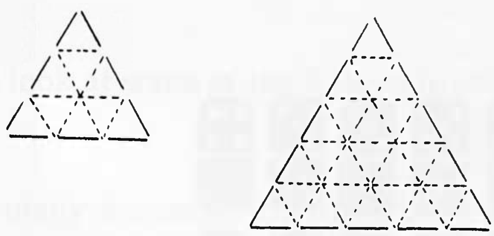
* How many of each tile do we need to make an N x M array?
* Can I predict how many tiles of each type are needed?
* Can I find a useful rule?
* What if I move into three dimensions?

The three dimensional border cubes situation is very similar to the better known painted cube or one of its many associated problems.

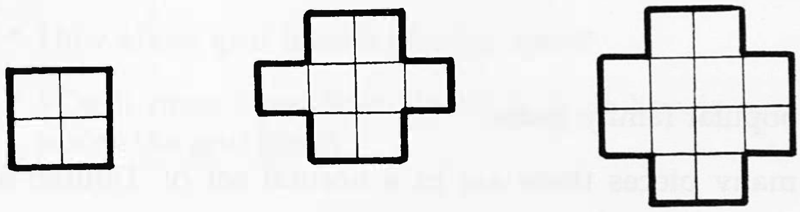


Other areas for investigation may well include

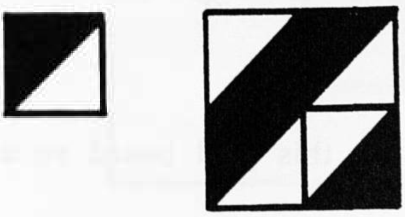
* Working with triangular tiles



* Working with shapes other than rectangles



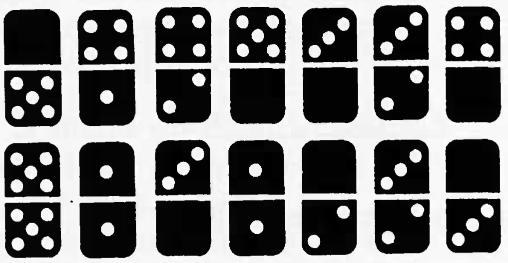
* Tiling patterns using a single tile



This last idea is, perhaps, more suited as an extended task within the practical geometry category rather than the pure investigation.

This particular investigation is more obvious and perhaps less variable than some. Therefore, you may decide that it is one which is particularly suited to a certain group which you teach, or useful as one of many being used simultaneously in a single classroom. The selection of tasks and styles of operating within the classroom need much careful thought in order to meet the individual needs of both teachers and students. Further guidance on these aspects of extended task work is offered in the Teacher's Guide.

DOMINOES



Dominoes is quite a popular family game.

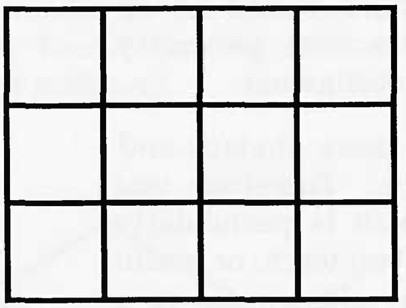
Do you know how many pieces there are in a normal set of 'Double Six' dominoes?

You can buy other sets of dominoes such as 'Double Nine'.

There are many ideas about dominoes which you could investigate if you wanted to.

Here is just one

How could you place dominoes on this grid board so as to cover it completely?

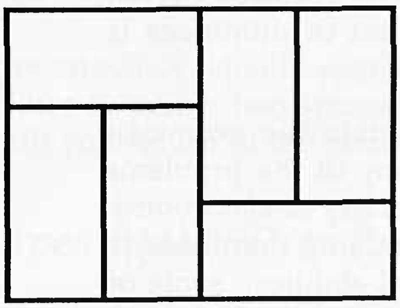


Investigate The Problem

DOMINOES : continued

You could look at some of the following ideas relating to dominoes

* How many dominoes are there in a normal set?
* How many dominoes are there in other size sets?
* How many ways can dominoes be placed on a chessboard so as to cover it totally?
* How about grid boards of other sizes?
* Which ones have fault lines? A fault line is a line which goes right across the grid board.



* What is the sum of the dots on a set of dominoes?
* What scores can you get when you play 5's and 3's?
* If you place the dominoes in a line in a single direction what is the last number?

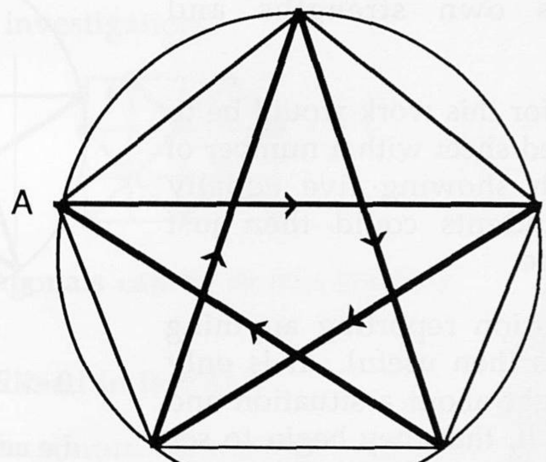
Dominoes - Teacher's Notes

It is essential that students should have the opportunity to experiment with a set of dominoes as they work on this task. Students could use a real set of dominoes, a printed set produced by the teacher, or they could draw a full set for themselves. Some teachers and students who have considerable experience of this type of work, have simply given out a set of dominoes and asked their students to carry out a pure mathematical investigation on some aspect of the set. However, this may not prove suitable in most classrooms; but it is an interesting approach for discussion.

The initial activity within this task could take the form of students looking at their set of dominoes and noting as many things as they can about them. This could be done either individually or in pairs. A short, whole class feedback session could identify both the obvious and less obvious features. What could we investigate within a set of dominoes is then a natural extension from this.

The list of questions set out within the extension ideas for students suggests many of the problems which have been tackled in a variety of classrooms. Although the whole idea of studying dominoes is suitable for GCSE students of all abilities, some of the ideas set out in this list may be more suited to particular levels of the full ability range. It is likely that the extension sheet may not be needed, because if students are given the opportunity they will usually generate their own list of suitable mathematical lines of enquiry. In this case, the list may be of greater use to the teacher than to the student. It may be used to provide support for particular students rather than for all students.

STARS



Can you draw this star?

The five points are approximately equally spaced around the circle. Starting at the point A, draw lines to points two spaces further round the circle in a clockwise direction until you return to the point A.

Investigate The Problem

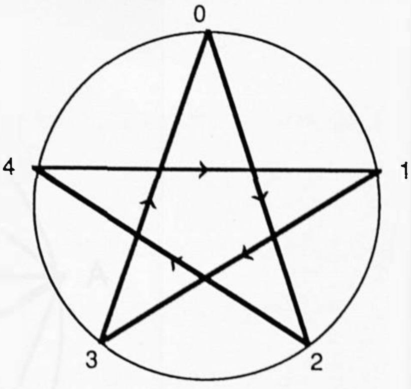
There are many things you could try using this idea including

* Moving a different number of spaces around the circle
* Using a different number of points on the circle
* What happens if you go anticlockwise?
* Investigate Spirograph, the shape drawing game.

You may like to choose one or two of these ideas or some of your own to look at in detail. Do not do just a little bit on each.

Stars - Teacher's Notes

Stars can be thought of either in various geometric terms or as a number pattern. Clearly each representation has its own strengths and weaknesses.



0 -> 2-> 4-> 1-> 3-> 0

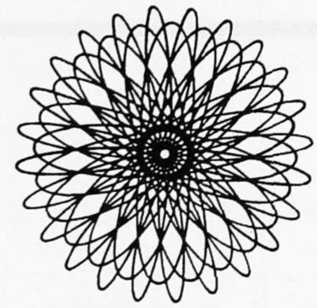
i.e add 2 mod 5

A suitable starting point for this work would be to offer the students a printed sheet with a number of identical diagrams, each showing five equally spaced points. The students could then just investigate this simple case.

A general discussion session reporting anything they notice or discover is then useful. It is only after students have thought about a situation and have actively worked on it, that they begin to see possible avenues for extension. During the trials of this material, this proved to be a useful approach. Students can write down a list of possible ideas for themselves, and then share a few of these ideas within their small group or with the whole class. They are then in a much better position to move forward to work on their individual tasks.

Within this particular problem and its follow-up activities, students of all abilities make the same initial moves but with very different discoveries, as one would expect. The least able student may possibly only discover one or more sets of special cases and attempt to explain why, or predict further cases. The most able could go way beyond this producing a complete generalisation.

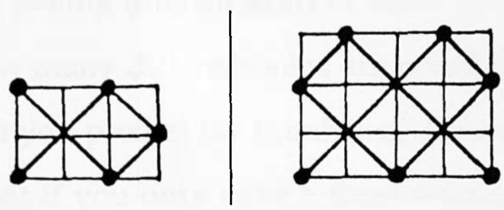
Students may decide to attempt to predict the number of complete rotations involved in producing each star. In this case, the computer program Circle, which is contained in the Shell Centre 'Blue Box', *Problems With Patterns and Numbers*, may be useful.

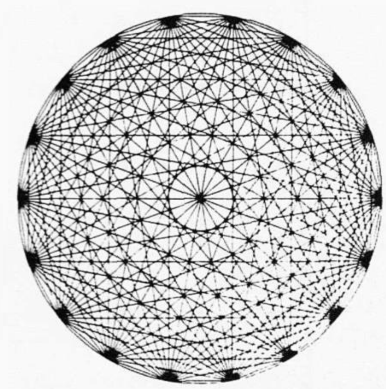


Further valid and useful directions which may well develop from this starting point may include

* Investigating spirograph either by drawing or on a microcomputer

* The 'snooker' investigation

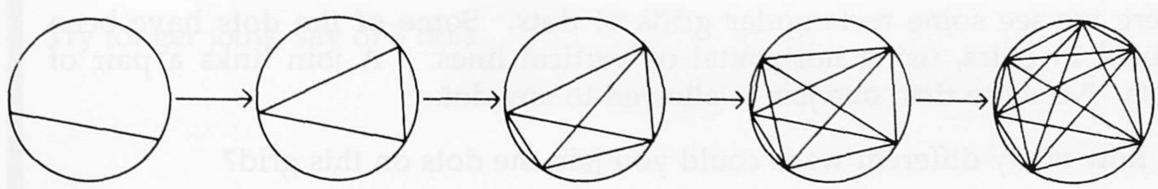




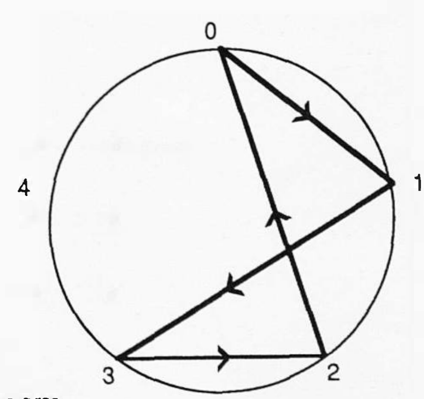
* How many diagonals can be drawn inside a polygon?

  This idea is outlined in IMPACT

* What is the maximum number of regions produced within a circle with a given number of points?

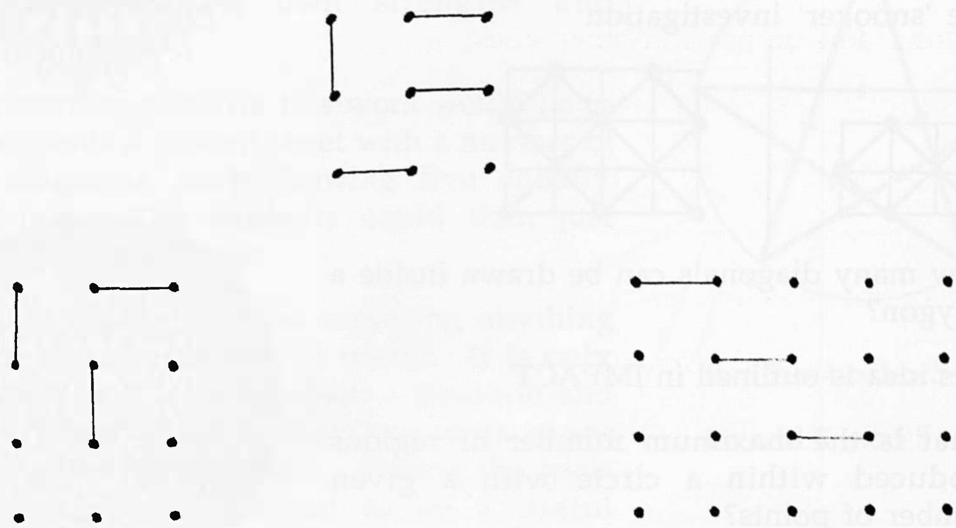


* Function jumps e.g. n -> 2n + 1



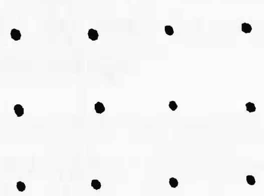
Although some of these ideas may not seem directly related to the suggested starting point, there is no reason why they cannot form the student's own starter and, indeed, they have been suggested and studied in some classrooms as GCSE extended tasks.

JOINING DOTS



Here we see some rectangular grids of dots. Some of the dots have been joined in pairs, using horizontal or vertical lines. A join links a pair of dots. No more than one join is allowed to any dot.

In how many different ways could you join the dots on this grid?



Investigate The Problem

JOINING DOTS : continued

Try joining dots on grids of other sizes.

How many different joins are possible on any grid?

Can you predict the number of different ways of joining a grid?

What if you only have a fixed number of joins?

How many dots are left over?

Can you find any rules or relationships?

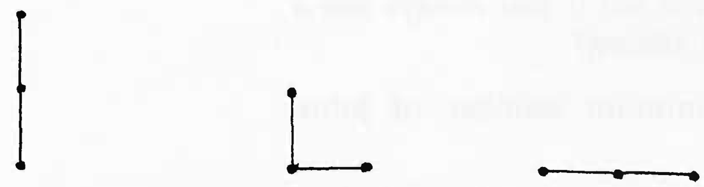
Can you make any predictions?

What is the maximum number of joins?

What is the minimum number of joins?

How many spare dots do you get?

Try longer joins, say of 3 dots.



Joining Dots - Teacher's Notes

This particular pure investigation has, perhaps, no obvious initial line of enquiry. Instead, there are a number of ideas which students may suggest.

A suitable starting point for Joining Dots is to ask students to join the dots on a three by four array and to follow this up by looking at the variety of ideas which students have used. You may let your students tackle this problem without any stated rules or, alternatively, by stating certain conditions such as those outlined within the student's notes i.e.

* A join links a pair of dots
* A join is either horizontal or vertical
* Any one dot can have only one join.

A suitable way forward from this initial activity is to organise a whole class discussion. Students could then be encouraged to suggest questions which may be posed about this starting point. Questions which have arisen during trials include

* Can you use all the dots?
* How many dots are left if you always use a certain pattern of joining?
* What is the minimum number of joins needed?
* What is the maximum number of dots which can be left unjoined?
* Does an array which is twice as big need twice as many joins?
* Does a particular joining rule give any maximum or minimum feature?

Within all of these questions, students can, of course, develop their own ideas about making predictions, checking these and developing general rules. It is likely that any particular chosen direction taken by an individual student will involve more than one of the above listed questions. However, it is essential to avoid a situation where such a list generated by the class during discussion, becomes the worksheet of questions to be answered. Certainly, from classroom trials of this material, there seems to be a delicate balance here, and it is useful if the teacher emphasises that these are merely some ideas which could be used, together with each student's own ideas.

CROSSINGS

X X X

O O O

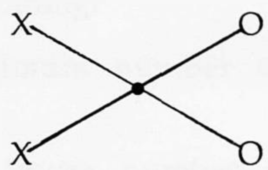
Here we have a diagram showing three noughts and three crosses.

Every nought has to be joined to every cross.

How many lines are needed?

Draw the diagram to check.

When two lines cross we have a crossing.



Here we have just one crossing.

How many crossings are there on your diagram?

Investigate The Problem

CROSSINGS : continued

Try a different number of noughts and crosses.

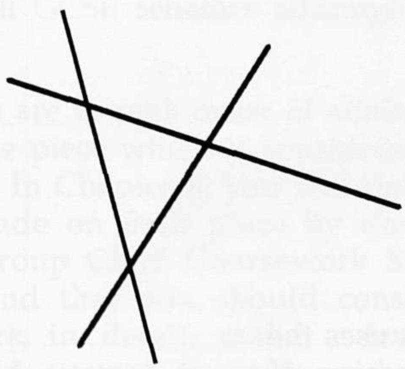
Try another row of symbols as well as noughts and crosses.

Can you find any rules?

Can you make predictions from your rules?

Look at a similar problem.

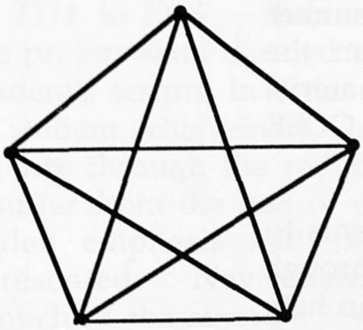
Eg



3 lines

7 spaces

3 crossings



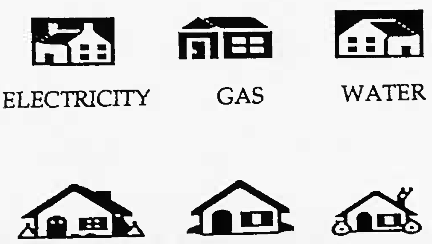
5 dots

10 lines

11 spaces

Crossings - Teacher's Notes

This task offers a pure investigation based on the age old problem of service supplies to houses. Perhaps the traditional problem based on supplying electricity, gas and water to three houses without the supplies crossing each other, would make a suitable and stimulating opener for this work.



How can this be done?

The problem posed in the student's notes uses the same initial situation but allows the crossing of lines and suggests an investigation into the number of lines and the number of crossings.

Again, the student extension sheet offers a number of related ideas. These have come from the classrooms involved in the trials of this material and have therefore all been used for GCSE extended tasks.

You may find it helpful to allow your students to use some computer software as they work through this task: Anita Straker's program Crosses can help students to organise their thinking around this task.

The two examples given as similar problems on the extension sheet are perhaps more difficult than the initial idea on crossings. Therefore, you may prefer to use them with the higher end of the ability range.

5 *Students' Work*

These six pieces of work cover a wide range of achievement. Two pieces of work are offered at each of the three levels of GCSE study; Foundation, Intermediate and Higher. These three levels are common to all GCSE schemes although the level titles differ.

The six pieces are in rank order of attainment and finish with the piece which is considered the best from the set. In Chapter 6, you will find detailed comments made on each piece by the Midland Examining Group Chief Coursework Moderator. We recommend that you should consider each piece of work in detail, make a few written comments and attempt to grade each student's work, before you read the moderator's comments.

For identification purposes, the six student's scripts are labelled *I2/1* to *I2/6*. Because of space constraints the project team decided to reduce the size of the student's scripts, in order to include a wide range of student achievement. In addition to the loss of quality through the reduction in size, some scripts suffer from the loss of colour which originally added emphasis and clarity to the arguments presented. Nevertheless, we are hopeful that much of the strength inherent in the original scripts will become apparent as you read through the following pages.

PIN UP **B4**

I2/1

Bedroom and classroom walls are a favourite place to pin up pictures of various types eg, cars, fashion designs, bands, pop idols, teams, TV personalities etc.

There are many ways of pinning these pictures up including the use of drawing pins. The question is however, how many drawing pins do you actually need to pin up a given number of pictures?

You are allowed to investigate anything you like to do with this problem. However, you ought to keep a note of the problems which you set yourself because this will form an important part of your report. You should also include in your report any diagrams, tables, graphs, etc that you use together with your ideas and discoveries however small you think they are at the time. It ought to be a complete record of everything you worked on relating to this idea.

Investigation

Young people today spend lots + lots of their 'pocket money' on posters. Mother's despair when they pay out money to have their child's bedroom decorated only to go in a few days later to see their heroes pinned up on their walls.

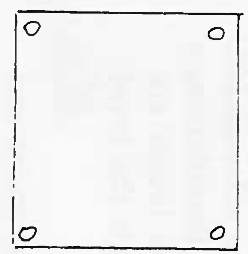
But have you ever thought, how the pictures are stuck up? By, quite often, drawing pins. Then have you carried on to think, how many drawing pins you actually need to pin up a given number of pictures?

To find out read on.

How many drawing pins do you actually need to pin up a given number of pictures?

1. One picture – there many ways of pinning it up lets investigate the best way.

Solution 1.

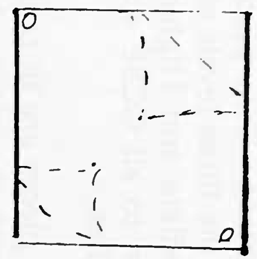


Is it a good way or a bad way. why?

good ✓
bad.
indifferent

This is a good way. The picture won't fall off and the sides won't crease.

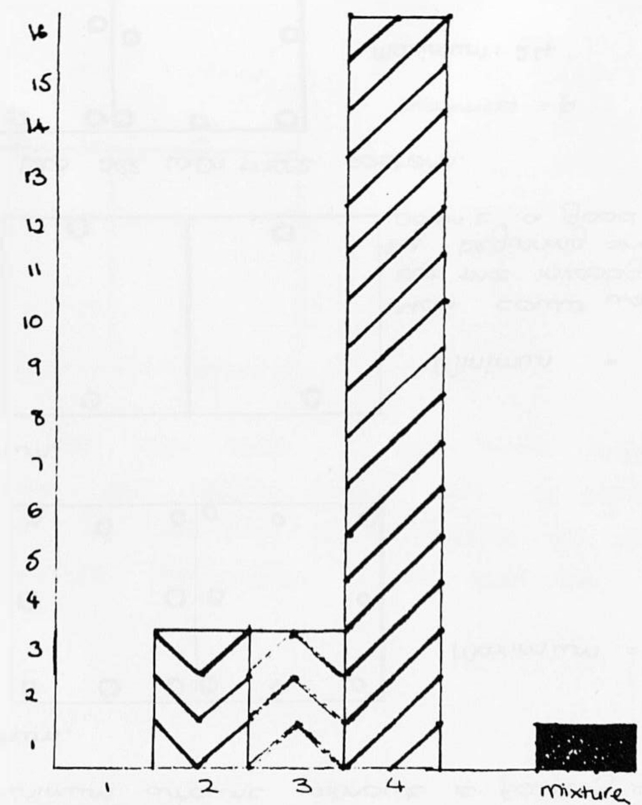
Solution 2.



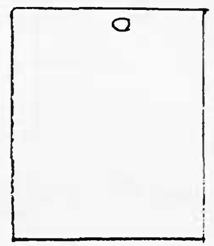
Is it a good way or a bad way? why?

bad. ✓
good
indifferent

It is a bad way because the two corners would curl up.



Solution 3.

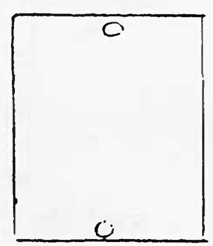


Is it good or bad?
Why?

good ☐
bad. ☑

It would hold the poster up but it could be blown about or be easily torn.

Solution. 4.



Is it good or bad?
Why?

good ☑
bad ☐
Indifferent ☐

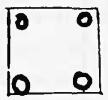
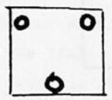
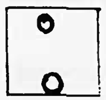
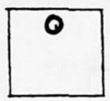
It uses the least number of pins but it holds up the posters well.

Solution 1 + 4 were the best.

How Many Pins?

Many people hang up posters but how many pins do they use look over the page a graph will show you.

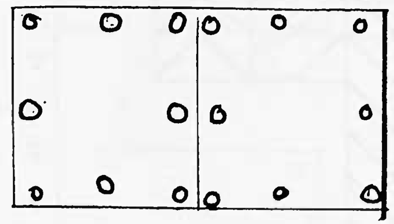
Table

Amount of pins		Amount of people.
	4	16
	3	3
	2	3
	1	0
mixture		1

Investigate.

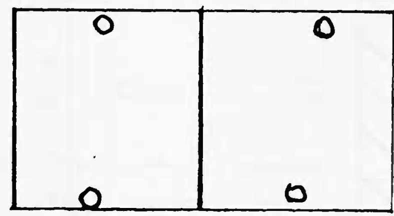
I have two posters. I want to find the maximum amount of pins I could use. Also to find the minimum amount without it falling down.

Maximum.



Maximum = 16

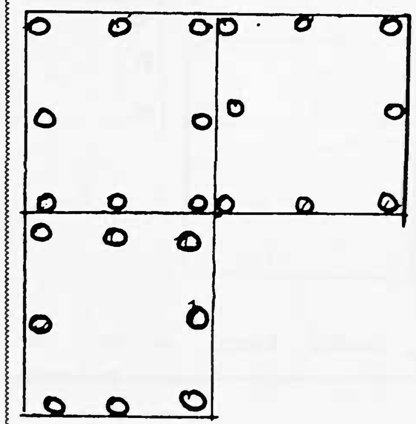
Minimum.



Minimum = 4

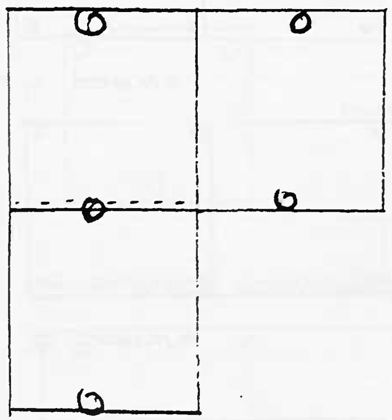
You could use ~~to~~ two but the investigation at the beginning showed it wasn't a good idea.

Now lets see with three posters.



Maximum = 24

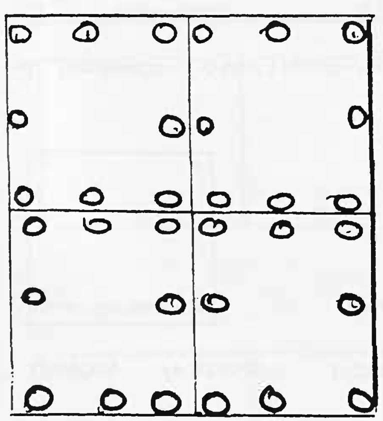
Minimum.



Minimum 5
The dotted line is where it overlaps slightly.

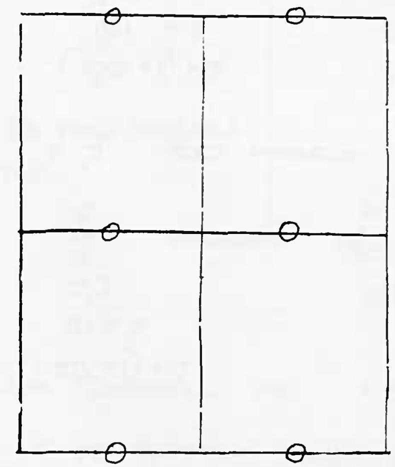
Now try four posters

Maximum



Maximum = 32

Minimum

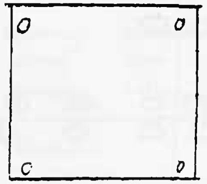


Minimum = 6.

You can see that if you think about it you can really save alot of safety pins instead of wasting them There is alot of difference between 6 pins and 32.

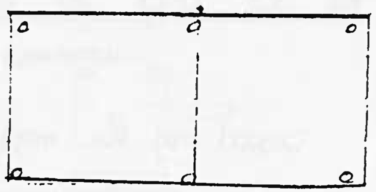
Lets find a formulae

Using 4. pins. 1 poster.



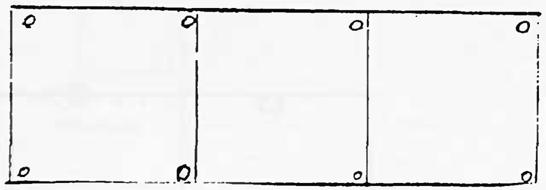
number of pins = 4

2. posters (overlapping slightly).



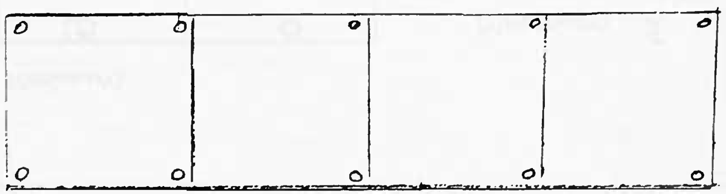
number of pins = 6

3 posters.



number of pins = 8.

4 posters.



number of pins = 10

pieces of Paper	Nº. of Pins	formulae
1	4	2×2
2	6	2×3
3	8	2×4
4	10	2×5
now logical that		
5	12	2×6
6	14	2×7
7	16	2×8

formulae we could use is $(n+1) \times 2$

a. 20. posters ...how many pins?

$(20+1)\times 2$

21×2

42

the number of pins would be 42.

b 2. 100 posters how many pins?

$(100+1)\times 2$

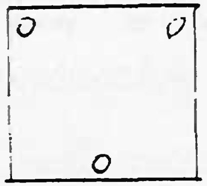
101×2

202

The number of pins needed would be 202

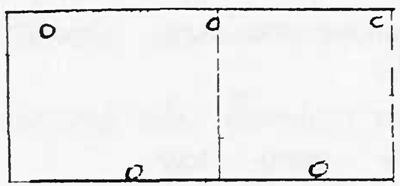
Lets use three pins.

1 poster.



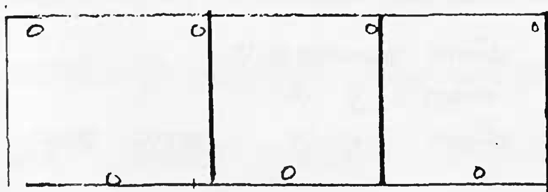
number of pins = 3

2 posters (overlapping slightly)



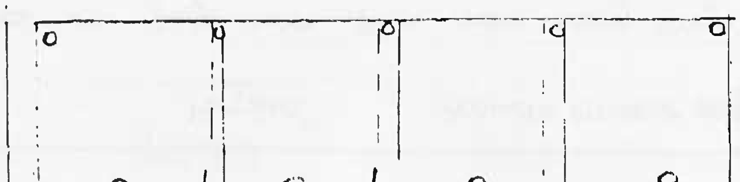
number of pins = 6

3 posters.



number of pins = 7

4 posters number of pins = 9



Nº of posters	Nº of pins	formulae.
1	3	1 + 1 × 2
2	5	1 + 2 × 2
3	7	1 + 3 × 2
4	9	1 + 4 × 2

Its now logical that :

5	11	1 + 5 × 2
6	13	1 + 6 × 2
7	15	1 + 7 × 2

the formulae is $1 + 2n$

Hw001. And how many pins you need for 12 posters

$1 + 2n$
$1 + 2 \times 12$
$1 + 24$
$= 25$

you need 25 pins for 12 posters

So 2. how many pins for 130 posters

$1 + 2n$
$1 + 2 \times 130$
$1 + 260$
$= 261$

You need 261 pins for 130 postess.

Puzzles! Answers on next page.

Like on page two find how many ways there are of pinning up two posters, successfully! Not overlapping!

Are there a) 4 ways
b) 5 ways
c) 6 or more ways.

2. Find out the maximum pins needed for 5 posters not overlapping!

Are there a) 30 pins
b) 40 pins
c) 50 pins.

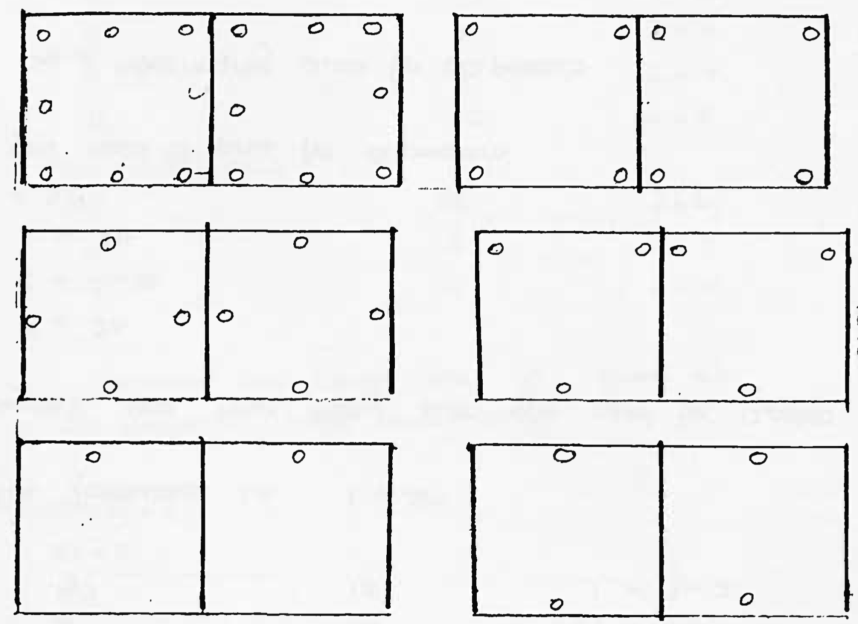
3 Now find the minimum (Not overlapping)

Are there a) 5 pins
b) 10 pins
c) 15 pins

4 Find a formulae using 2 pins (see pages 9-12), then calculate how many pins you need for
a) 12 posters
b) 19 posters
c) 109 posters
d) 325 posters.

Answers

1. Answer c) 6.



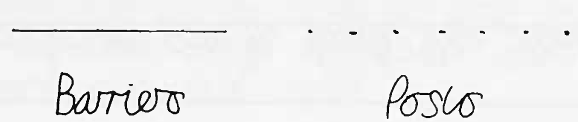
2. Answer b) 40 pins
3. Answer b) 10 pins.

4 formulae = 2n.

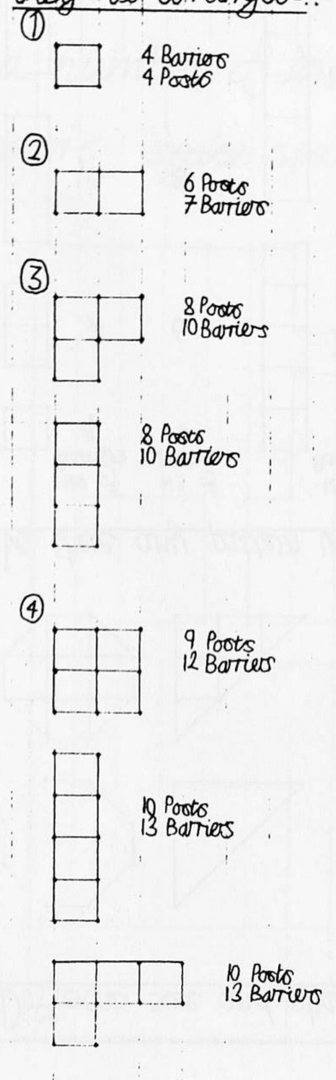
a) 2n = 2×12 = 24 pins.
b) 2n = 2×19 = 38 pins
c) 2n = 2×109 = 218 pins
d) 2n = 3×325 = 650 pins.

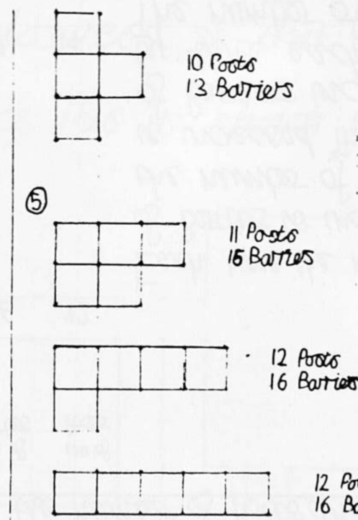
I2/2 INTRODUCTION

My Maths project on investigateon is all about investigating a system of Barriers and posts. I will investigate the construction of them. Also investigating the size and shapes of the enclowwes ~~will~~ which I will build using the Barriers and Posts. As I am doing this I will look for patterns and rules which I discover as I am working though my project. Below are the ~~simbles~~ symbols which I will use for the Barriers and the Posts.



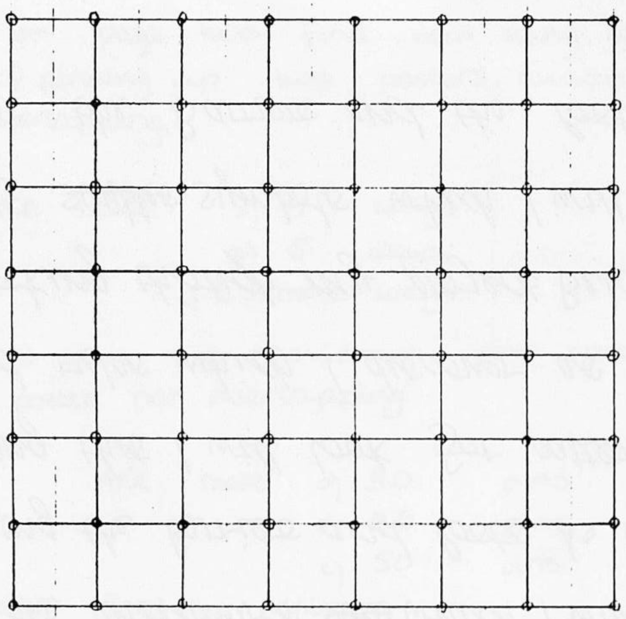
How many barriers and posts are need to construct a number of enclosures. Also how many different ways can they be arranged?





This shows that when the arrangement of the enclosures is changed it afects the number of posts and barriers used.

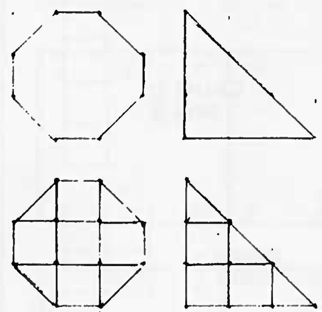
One enclosure can be divide into smaller sections.



This enlarges the number of sections. But they become smaller and more compacted.

These enclosures are useful for keeping things part such as animals. This would be expensive to ~~mad~~ make because it uses alot of Barriers and posts.

Different size and shaped enclosures can be made easily



These shapes look better but use alot more Barriers and Posts that the similar way using squares.

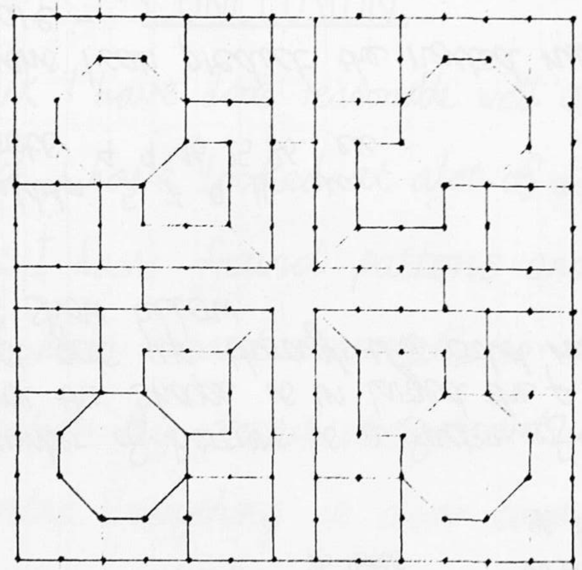
Different shaped enclosures can be put into a large enclosure.

Is there any pattern in the number of posts and barriers used.

No of Barriers	No of Posts
4	4
7	6
10	8
13	10
16	12
19	14

Each time the number of Barriers is increased the number of posts is increased. The number of Barriers increases by three each time. The number of posts increases by two each time. This show a clear pattern. From this I can estimate the outcome of more enclosures.

A large enclosure can be sectioned out in to a number of smaller enclosures as shown below.

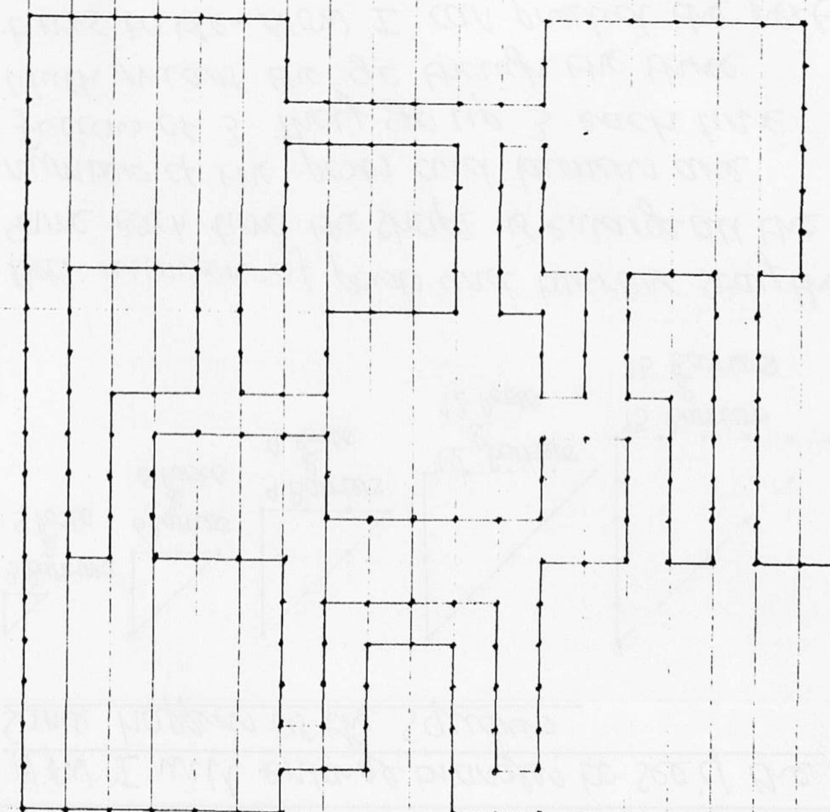


Also shown about is how you can create patterns using the enclosures. The patterns can be any of ~~th~~ your ~~chose~~ chose it is up to you how you arrange them. I have arranged them so that they are symmetrical.

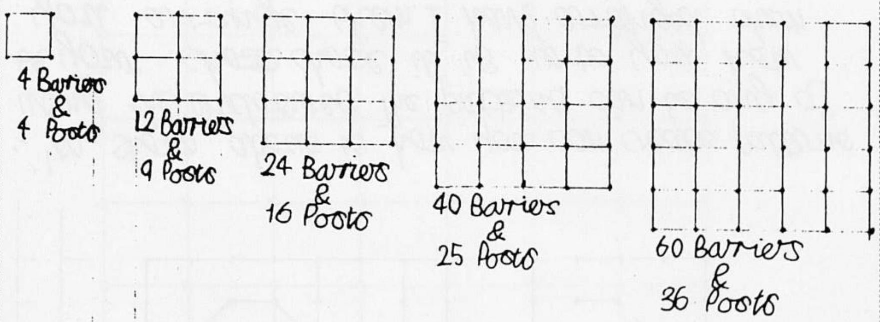
I have used the barries at only 90° and 45°, but ~~there~~ they can be used at any angle but this becomes very difficult and harde to work out and draw.

Routes can be mapped out using the enclosures

These ~~Fa~~ routes could be used to join up a number of enclosures so that there is more ~~space~~ space. This is shown below.



What happens to the number of Barriers and posts when a shape is enlarged?

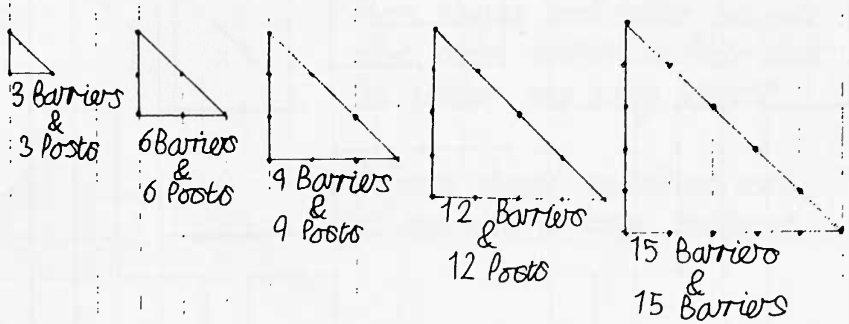


Each number of Barriers is a factor of 4.
Each time the shape is in larged the ~~n~~ number of post gos up though the odd numbers.
This is show below.

Nº that is added on 5 7 9 11.. etc
Nº of Posts 4 9 16 25 36.. etc

This means I can predict the nexted numbers.
They are:-
Nº that is added on 13 15 17 19... etc
Nº of Posts 36 49 64 81 100... etc

NOW I will enlarge triangles to see if the same happens as the squares.



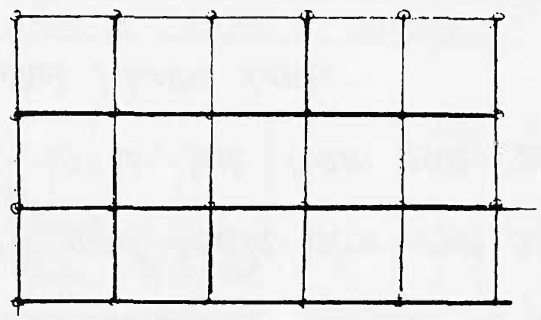
Both numbers of posts and barriers stay the same each time the shape is enlarge all the numbers of the posts and barriers are factors of 3. They go up 3 each time which means the go though the three times table. Now I can predict the next numbers.

No of Barriers 17 21 24 27 30 33 36 39 42...etc
No of Posts 18 21 24 27 30 33 36 39 42. etc

CONCLUSION

I think I have done reasonable well on my project I have looked at alot of different areas. I have found patterns and rules which has meant I am able to make prediction. My project is neat and well presented. Everything is clear and easy to understand. The project is a little short and could do with a bit more work in it, but I am quite satisfied with what I have done.

I2/3 Barriers! (B1)



Posts.
Barriers.

The diagram above shows a barrier system made up of posts (o) and fences (-). The barrier system can be used to section off areas of land for many reasons eg, to keep sheep in. !!!

firstly, we made a table relating to the dimensions of the fence and the number of posts and barries.

1×5 = 12 posts and 12 fences.
1×4 = 10 " " 10 "
1×3 = 8 " " 8 "
1×2 = 6 " " 6 "
1×1 = 4 " " 4 "

2×5 = 14 " " 14 "
2×4 = 12 " " 12 "
2×3 = 10 " " "
2×2 = 8 " " "

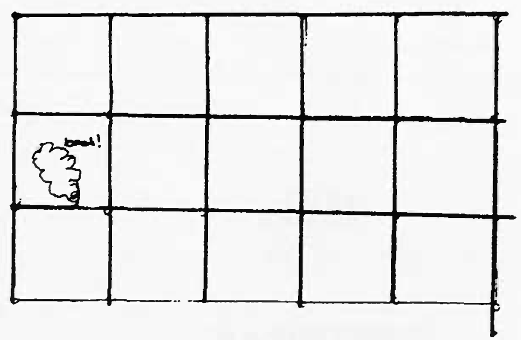
3×5 = 16 " " 16 "
3×4 = 14 " " 14 "
3×3 = 12 " " 12 "

we have worked out a formula to find the number of posts and fences needed if we know the dimensions of the field in units. (1 UNIT = 1 FENCE [—])

number of units across = n
number of units down = m

number of fences needed to make barrier = $2(n+m)$.
number of posts needed also = $\underline{\underline{2(n+m)}}$

Barriers ! (B1)



The diagram above shows a barrier system made up of posts (o) and fences (-). The barrier system can be used to section off areas of land for many reasons eg, to keep sheep in.

Secondly, we made another table! DA-DA-DA-DA This table related to the number of posts and fences needed to make the barrier with the pens inside.

1 × 5 =	16 fences,	12 posts.
1 × 4 =	13 "	10 "
1 × 3 =	10 "	8 "
1 × 2 =	7 "	6 "
1 × 1 =	4 "	4 "
2 × 5 =	27 fences	18 posts.
2 × 4 =	22 "	15 "
2 × 3 =	17 "	12 "
2 × 2 =	12 "	9 "
3 × 5 =	38 fences	24 posts
3 × 4 =	31 "	20 "
3 × 3 =	24 "	16 "

Using these results wh we have made a formula to find the no. of fences need

number of fences across : n
number of fences down : m

number of fences =

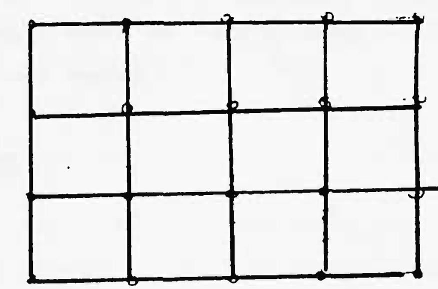
$(2m \times n) + (n + m)$.

It took us a little while longer to work out a formula for the number of posts needed !!

We began to work this out by trying to find a connections between the number of posts in a line up, and the number of posts in a line down.

4 × 3

4 posts



5 posts.

Dimensions = 4 × 3

Posts = 5 lots of 4 & 4 lots of 5

From this little discovery we made a formula to show the number of posts needed for any fence :

number of fences across = n
number of fences down = m

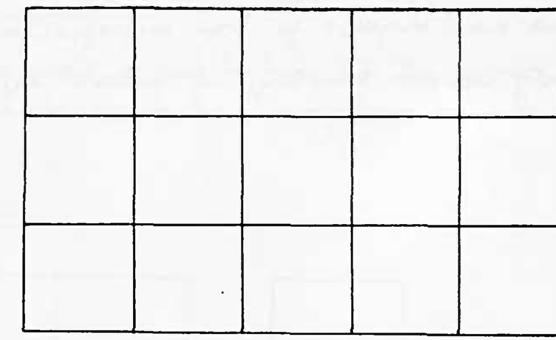
number of posts needed =

$= \underline{\underline{(n+1)(m+1)}} =$

I2/4

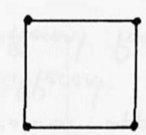
BARRIERS

Barriers

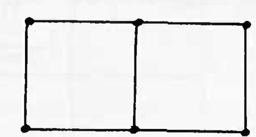


The above diagram shows a barrier system made up of posts (•) and fences (–). The barriers are used to section of land for many reasons such as keeping sheep in. How many barriers and posts are needed?

Some Simple Cases



4 posts 4 fences



6 posts 7 fences

Question

A barriers has 24 posts and 40 fences.
How many barriers can be built?
How much land is consumed by the barriers if they measure 4 meters by 8 meters?
Another piece of information is required to answer the question correctley.
How are the barriers Arranged?
Off the graph = 11 enclousures

Straight line Arrangment (Posts)

$(N-1\times 2)+4$ and $(N-1\times 3)+4$ are the two equasions found to find the number of posts and fences from number of enclousures.

∴ Using a straight line arrangement eleven enclousures can be built.

Block Arrangment

Posts

From my two graphs i can look up the answer to this kind of arrangment.

30 posts = 19 barriers

19x2 = 36 barriers

∴ 36 barriers from 60 posts.

Conclusion

As the enclousures - barriers vary so greatley in their different arrangments i cannot produce as set of common applicable rules - equasions to solve all the different problem arrangments as they vary in different formats. See (examples page 6).

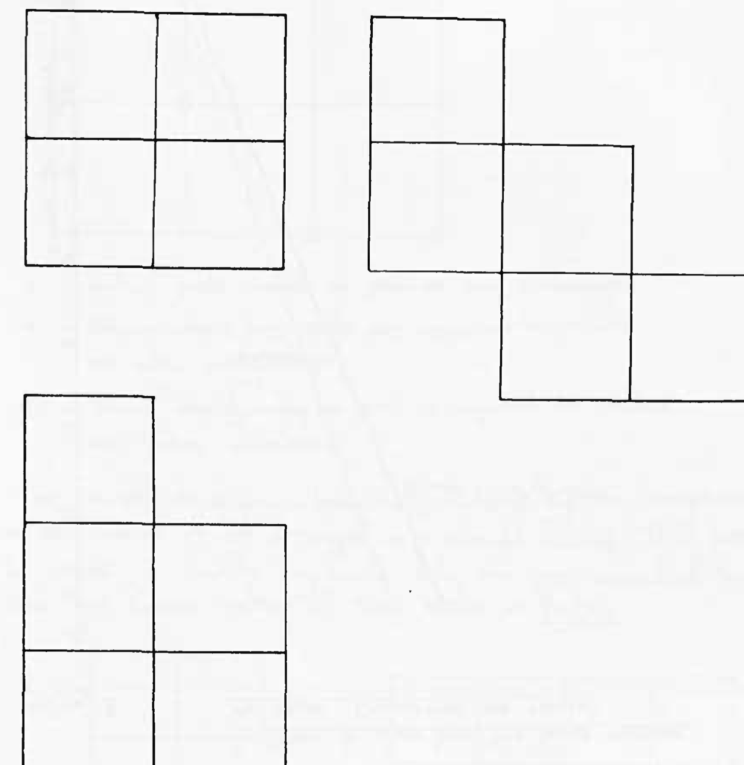
Although this may not be the case, i suggest that the graphs and the two straight line equasions should be adquate to solve most or all of the occuring arrangements (see page 5 ~~st~~ side b).

Using the graphs in this way you can find the most cost effective way of creating more enclousures.

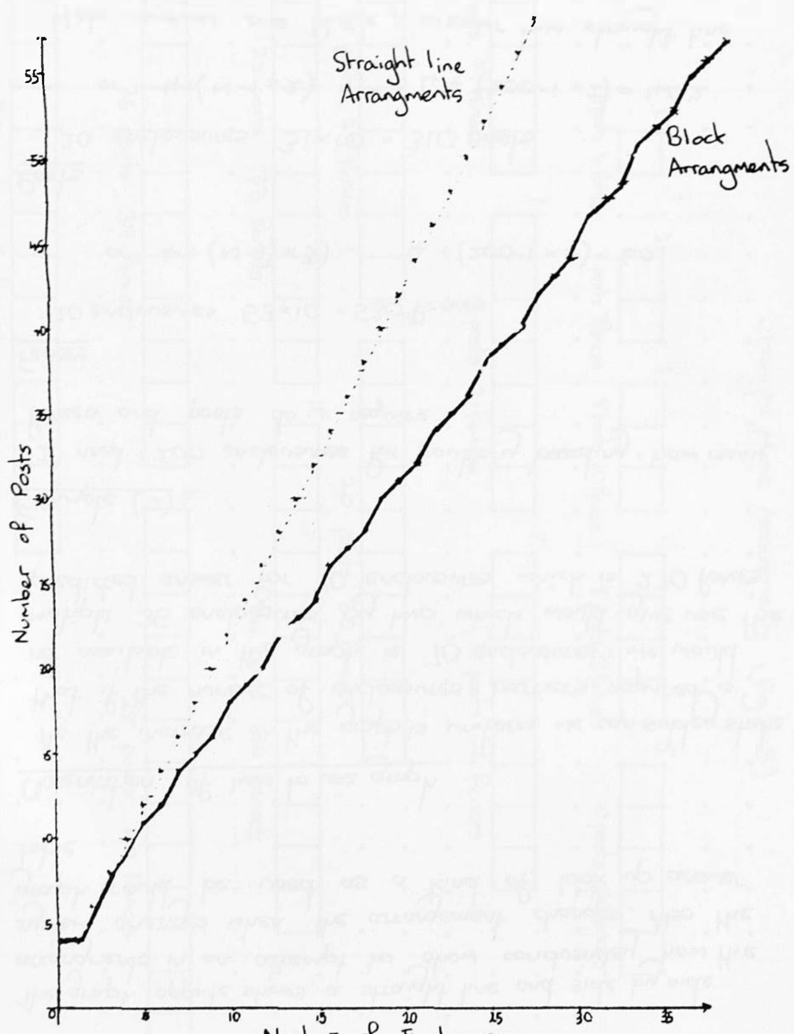
As shown clearley of the two graphs see fig 3 and 4 The block arrangment vairies greatley from that of the straight line arrangment

For this reason we need to know how the barriers are arranged to asserte how many fences or posts are required.

Example



A Graph to show the Amount of Posts Needed as the Enclousures Increase



The graph oppsite shows a straight line and side by side arrangments in an attempt to show condusivley how the system changes when the arrangement changes. Also the graph could be used as a kind of look up answer table.

Observation of how to use graph

As the increase on the graph is uniform we can subley state that if the number of enclousures - barriers required is not available in the graph ie 70 enclousures : we would multiple 35 enclousures by two which would give use the predicted answer for 70 enclousures which is 270 fences.

Example (2)

I need 200 enclousres for poultary breeding : how many fences and posts do i require

Fences

20 enclousres $53 \times 10 = 530$ fences

or $4 + (N-1 \times 3)$ ∴ $4 + (200-1 \times 3) = 601$

Posts

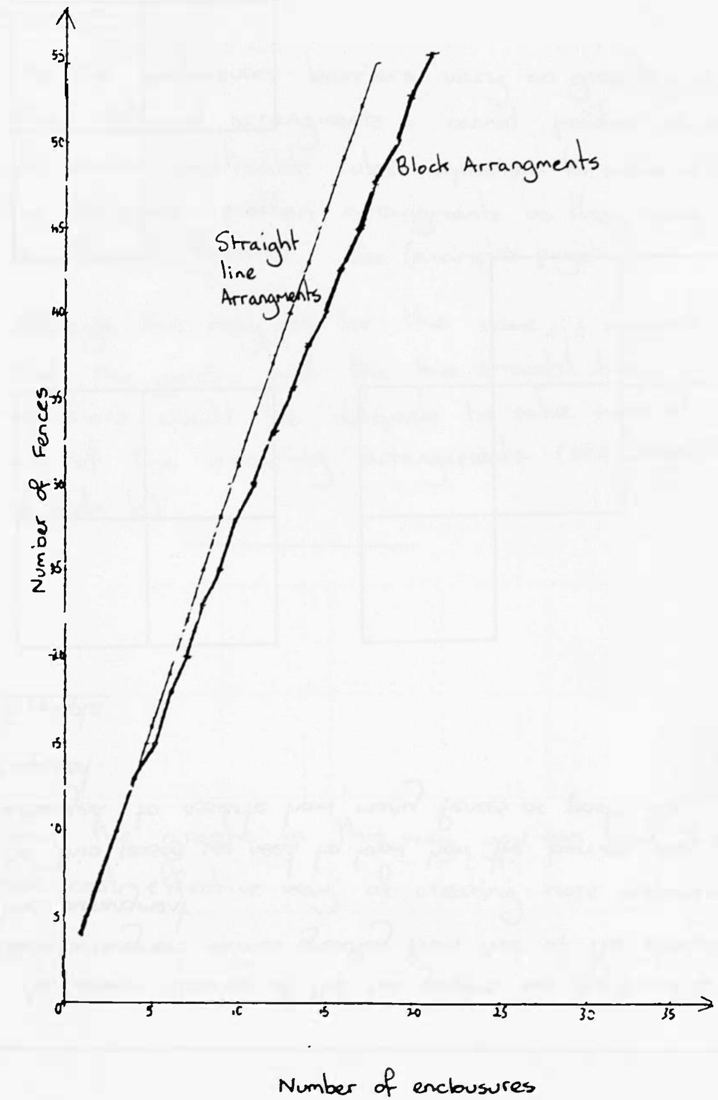
20 enclousures $31 \times 10 = 310$ posts

or $4 + (N-1 \times 2)$ ∴ $4 + (200-1 \times 2) = 402$

Note: answer one block, answer two straight line

Fig 3

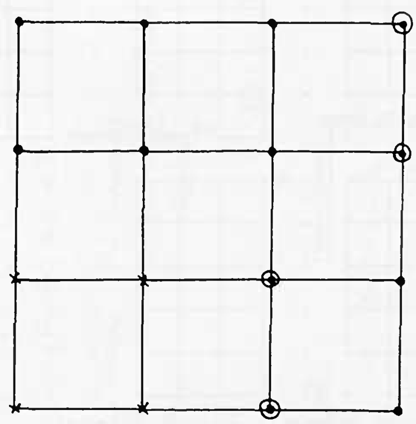
A Graph to show how The fences increase as More Enclousures are Needed.



Observation

As the barriers increased i noticed the following pattern

For example i am going to use a nine section grid.



X = Initial posts needed to produce one enclosure.

• = Shows where two posts are required to produce an extra enclosure.

⊙ = Shows where only one post is required to produce an extra enclosure.

The example above shows how the enclosure system changes over a set amount if not arranged in a straight fashion. It is possible to create a fourth by using only one post instead of two and two fences instead of three shown in fig(2)

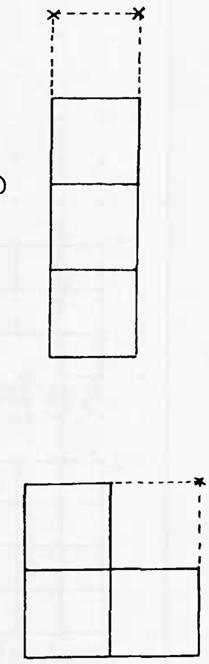
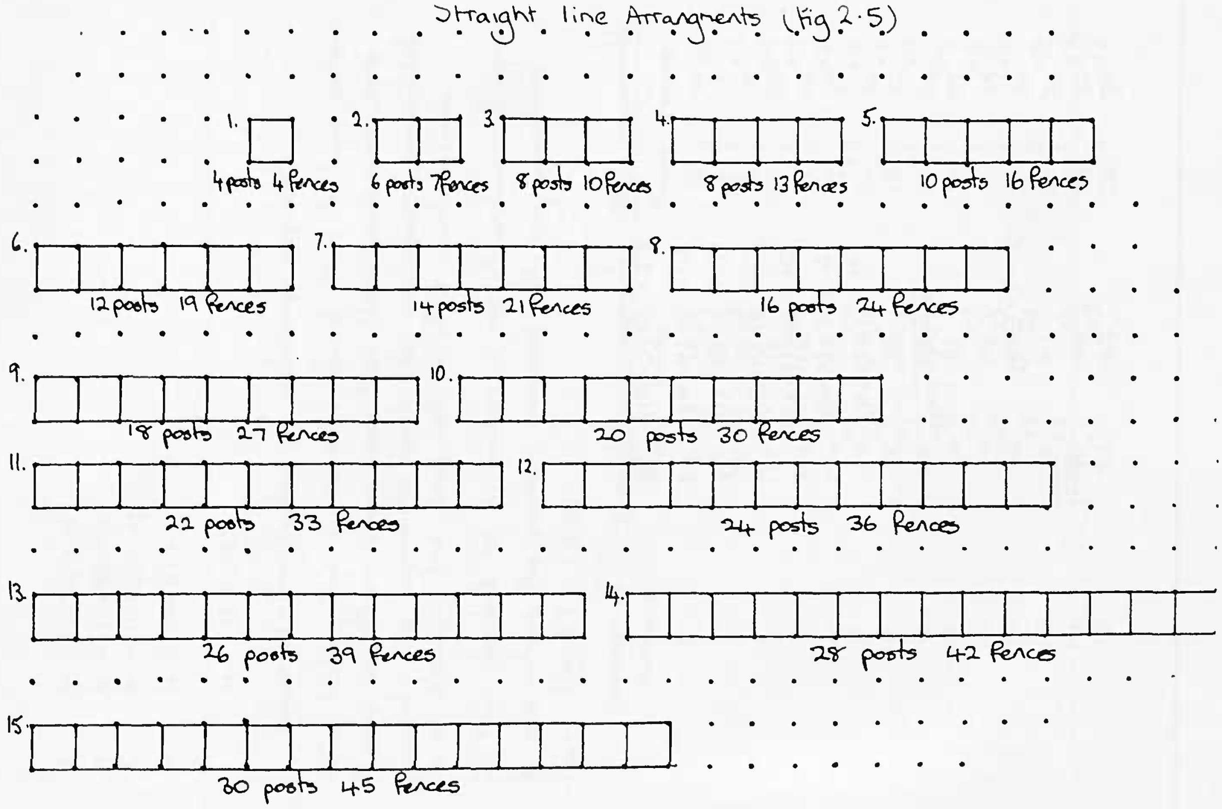
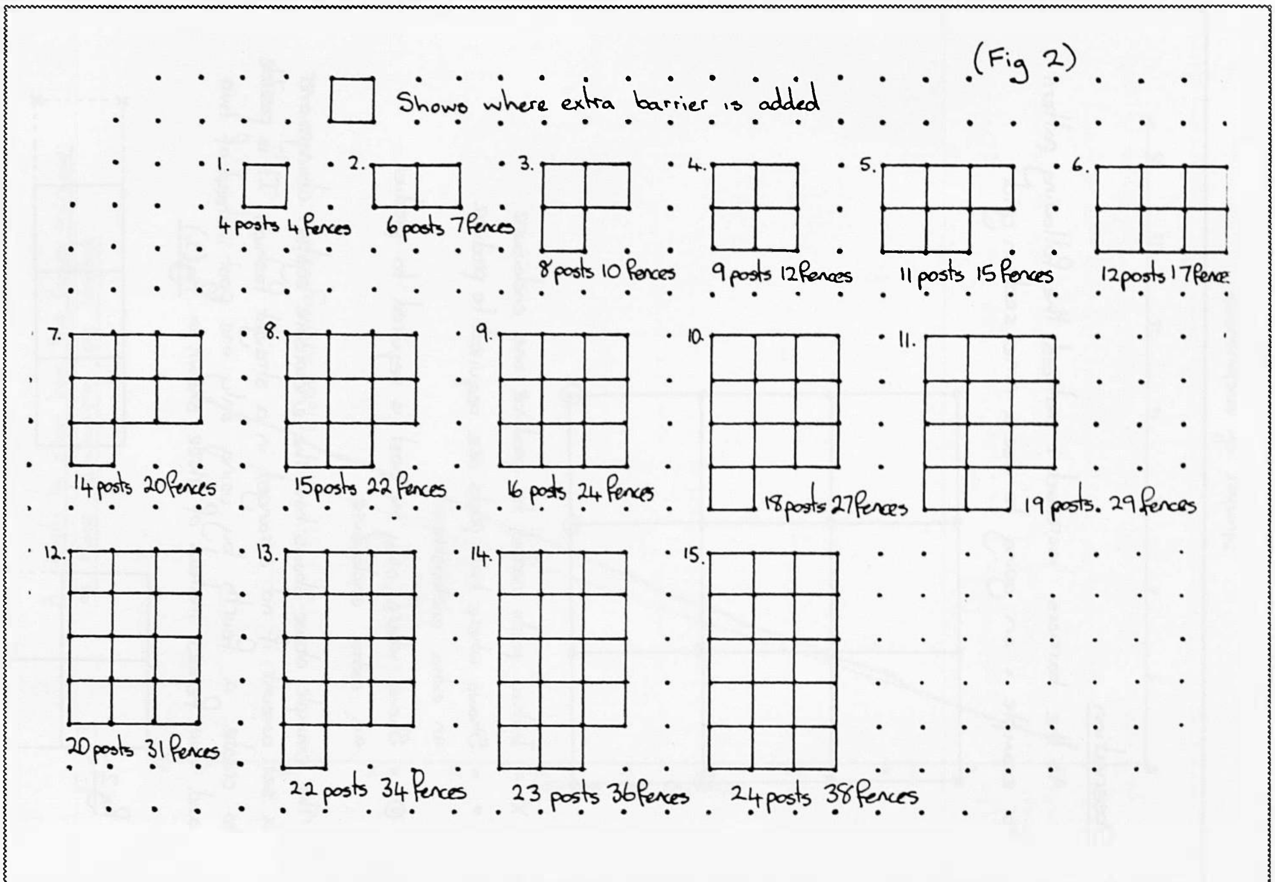


Fig 2

Straight line Arrangments (fig 2·5)





Test

∴ $P+B-1=F$

Predictions

1. $(4+1)-1 = 4$
2. $(6+2)-1 = 7$
3. $(8+3)-1 = 10$
4. $(10+4)-1 = 13$
5. $(12+5)-1 = 16$
6. $(14+6)-1 = 19$
7. $(16+7)-1 = 22$

3. $4 + (2\times2) = 8$
4. $4 + (2\times3) = 10$
5. $4 + (2\times4) = 12$
6. $4 + (2\times5) = 14$

$4 + (N-1 \times 2)$

Where N = The number of enclousures required.

After testing the equasions i found them to have certern floors in them when more than three enclousures were required. The equasions work for enclousures assembled in a straight line but not in other combinations.

I then decided to accumulate a set of enclousure diagrams to see where the problem occured with my equasions. (see fig 2) – (2.5)

Number of Barriers	Straight line Arrangements: Posts	Straight line Arrangements: Fences	Block Arrangements: Posts	Block Arrangements: Fences
1	4 +2	4 +3	4 +2	4 +3
2	6 +2	7 +3	6 +2	7 +3
3	8 +2	10 +3	8 +1	10 +2
4	10 +2	13 +3	9 +2	12 +3
5	12 +2	16 +3	11 +1	15 +2
6	14 +2	19 +3	12 +2	17 +3
7	16 +2	22 +3	14 +1	20 +2
8	18 +2	25 +3	15 +1	22 +2
9	20 +2	28 +3	16 +2	24 +3
10	22 +2	31 +3	18 +1	27 +2
11	24 +2	34 +3	19 +1	29 +2
12	26 +2	37 +3	20 +2	31 +3
13	28 +2	41 +3	22 +1	34 +2
14	30 +2	44 +3	23 +1	36 +2
15	32	47	24	38

There is another equasion for fences and posts.

By subtracting the number of barriers required from the number of fences and adding one we get the amount of posts required.

$\therefore (F-B)+1 = P$

Where F = Fences
B = Barriers required
P = Posts

$\therefore (F-B)+1 = P$

<u>Test</u>   <u>Predictions</u>

1. $(4-1)+1 = 4$
2. $(7-2)+1 = 6$
3. $(10-3)+1 = 8$
4. $(13-4)+1 = 10$
5. $(16-5)+1 = 12$
6. $(19-6)+1 = 14$
7. $(22-7)+1 = 16$

By adding the number of posts to the barriers required and subtract one, we get the number for how many fences are required.

$\therefore P+B-1 = F$

Where P = Posts
B = Barriers Required
F = Fences

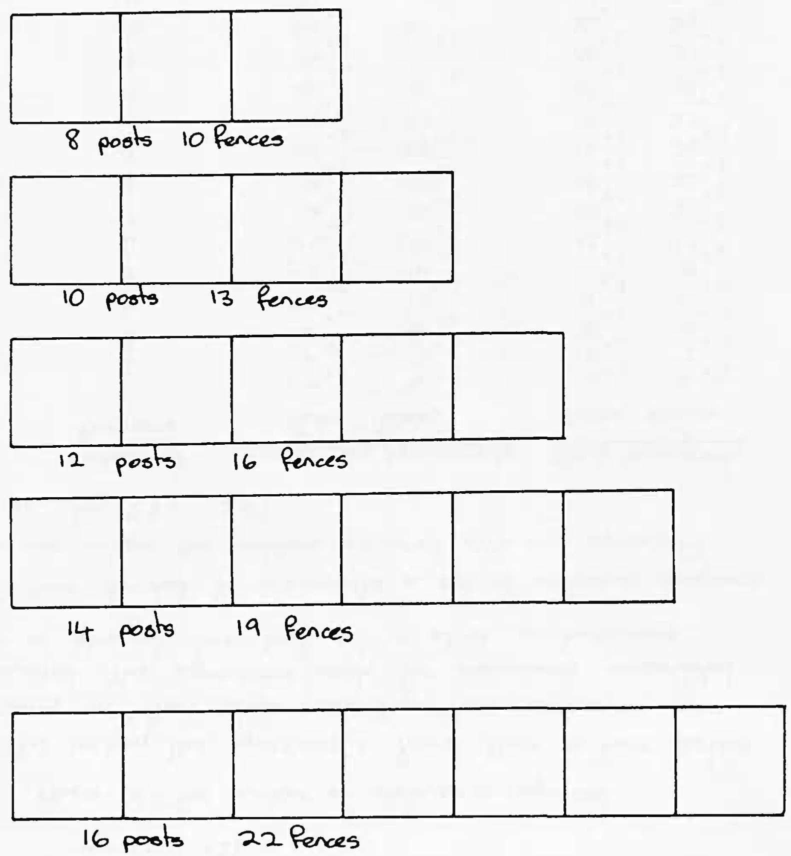


Table (fig 1)

Number of barriers	1	2	3	4	5	6	7
fences	4	7	10	13	16	19	22
Posts	4	6	8	10	12	14	16

Observation

Adding an extra barrier to an existing one involves adding <u>three fences</u> and <u>two posts</u>.

After studying the table i noticed some patterns shown see (fig 1.) and found i could create the equasions for fences and posts.

4 is a constant as it is the fundementary bases of one enclousure - barrier.

Fences

Barriers Test Production

1. 4
2. $4 + (3 \times 1) = 7$
3. $4 + (3 \times 2) = 10$
4. $4 + (3 \times 3) = 13$
5. $4 + (3 \times 4) = 16$
6. $4 + (3 \times 5) = 19$

Equasion

$\therefore\ 4 + (N - 1 \times 3)$ Where (N) is the number of extra enclousures required.

We subtract one from N as four is the number of posts for one enclousure and so if we want the amount of fences for five enclousures we would get the answer for six because we add the answer for one enclousure to the five.

Posts

Barriers Test Production

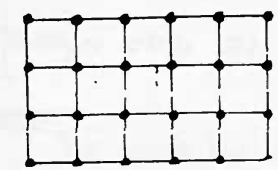
1. 4
2. $4 + (2 \times 1) = 6$

Barriers.

I2/5

BARRIERS.

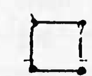
The problem that I am working on is called 'Barriers'. Here is a diagram of barriers.



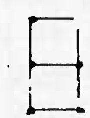
The system is made up of posts • and fences ——. The barriers are used to section off areas of land for many reasons e.g to keep sheep in.

For the diagram above you need 38 fences and 24 posts.

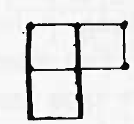
Here are some simple cases to show how many sheep fit into a barrier.



= 1 sheep



= 2 sheep



= 3 sheep

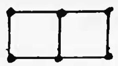
You could carry on doing this for: 4 sheep, 5 sheep, 6 sheep etc. You know that there is only one sheep per section, because each section are with 4 posts and 4 fences would just hold 1 sheep.

Here is a system I devised to tackle the problem of how many posts and fences are needed.

1 by 1 square

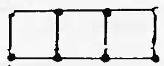
This square has 4 posts and 4 fences

1 by 2 square



This square has 6 posts, and 7 fences

1 by 3 square

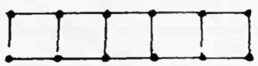


This square has x posts, and 10 fences.

1 by 4 square

This square has 10 posts and 13 fences.

1 by 5 square



This square has 12 posts, and 16 fences

When I had done these sets of squares, I found out that the number of posts go up in 2's every time. There is also a pattern for the fences they go up in 3's every time.

Here is a table to show the 1 by 1 squares.

	Nº OF POSTS	Nº OF FENCES
1 by 1	4	4
1 by 2	6	7
1 by 3	8	10
1 by 4	10	13
1 by 5	12	16

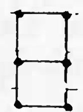
BARRIERS cont.....

When I had finished this table I found out that there was a pattern between the number of posts and the number of fences. Between the 1 by 1 posts and fences there was no difference, because they were both 4. Between the 1 by 2 posts and fences there was a difference of 1, because there were 6 posts and 7 fences. Between the 1 by 3 there was a difference of 2, because there were 8 posts and 10 fences. Between the 1 by 4 posts and fences there was a difference of 3, because there were 10 posts, and 13 fences. Between the 1 by 5 posts and fences there was a difference of 4, because there were 12 posts and 16 fences.

So the conclusion to this, is that the difference 'went up by 1 every time.

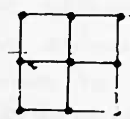
When I had finished all the 1 by 1 squares and so on, I went on to the 2 by 2 squares, and so on.

2 by 1 square



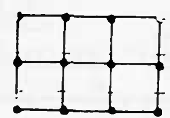
This square has 6 posts and 7 fences.

2 by 2 square



This square has 9 posts, and 12 fences.

2 by 3 square

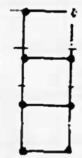


This square has 12 posts, and 17 fences.

Barriers cont..

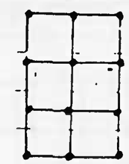
When I had finished all the 2 by 1, 2 by 2 etc. squares, I went on to do the 3 by 1, 3 by 2, 3 by 3 squares etc.

3 by 1 square



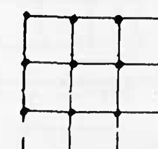
This square has 8 posts, and 10 fences.

3 by 2 square



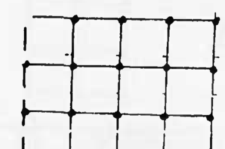
This square has 12 posts, and 17 fences.

3 by 3 square.



This square has 16 posts, and 24 fences.

3 by 4 square.

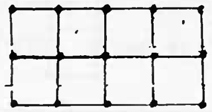


This square has 20 posts, and 31 fences.

3 by 5 square.

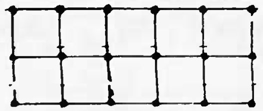
This square has 24 posts, and 38 fences

2 by 4 square.



This square has 15 posts, and 22 fences.

2 by 5 square



This square has 18 posts, and 27 fences

The pattern I found after doing these squares, was that the number of posts go up in 3's every time and the number of pefences go up in 5's every time.

Here is a table to show the 2 by 2 squares

	Nº OF POSTS.	Nº OF FENCES.
2 by 1	6	7
2 by 2	9	12
2 by 3	12	17
2 by 4	15	22
2 by 5.	18	27

When I had finished this table, I noticed another pattern, which was between the posts, and the fences. The difference was that it went up in odd numbers. The odd numbers went up in 2's.

6	7	difference 1
9	12	difference 3
12	17	difference 5
15	22	difference 7
18	27	difference 9

The pattern I found after doing these squares, was that the number of posts go up in 4's every time, and the number of fences go up in 7's every time

Here is a table to show the 3 by 3 squares

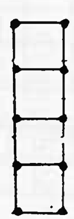
	Nº OF POSTS	Nº OF FENCES
3 by 1	8	10
3 by 2	12	17
3 by 3	16	24
3 by 4	20	31
3 by 5.	24	38

When I had finished this table, I noticed that there was a pattern, between the posts and the fences. The difference goes up in 3's every time.

8	10	difference 2
12	17	difference 5
16	24	difference 8
20	31	differense 11
24	38	difference 14

When I had finished all the 3 by 1, 3 by 2 squares etc. I went on to do all the 4 by 1, 4 by 2, 4 by 3 squares etc.

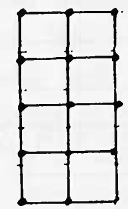
4 by 1 square



This square has 10 posts, and 13 fences.

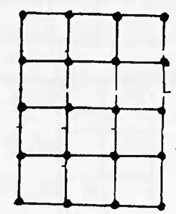
BARRIERS cont.

4 by 2 square.



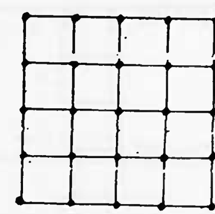
This square has 15 posts, and 22 fences.

4 by 3 square.



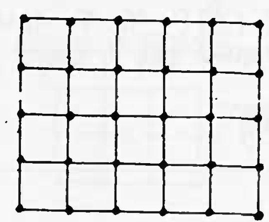
This square has 20 posts, and 31 fences

4 by 4 square.



This square has 25 posts, and 40 fences.

4 by 5 square.



This square has 30 posts, and 49 fence

The pattern I found after doing these squares, was that the number of posts go up in 5's every time, and the number of fences go up in 9's

Here is a table to show the 4 by 4 squares

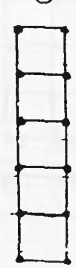
	Nº OF POSTS	Nº OF FENCES
4 by 1	10	13
4 by 2	15	22
4 by 3	20	31
4 by 4	25	40
4 by 5	30	49

When I had finished this table, I noticed that there was a pattern between the posts and the fences. The difference goes up in 4's every time

10	13	difference 3
15	22	difference 7
20	31	difference 11
25	40	difference 15
30	49	difference 19

When I had finished all the 4 by 1, 4 by 2 squares etc, I went on to do all the 5 by 1, 5 by 2, 5 by 3 squares etc.

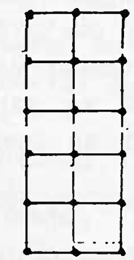
5 by 1 square.



This square has 12 posts, and 16 fences.

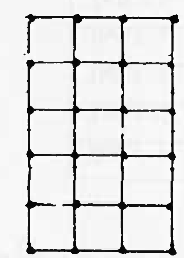
BARRIERS cont.....

5 by 2 square.



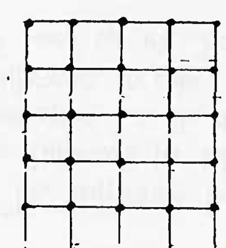
This square has 18 posts, and 27 fences

5 by 3 square.



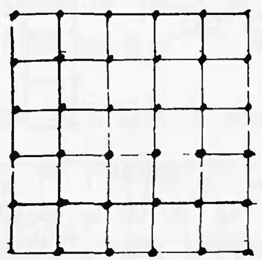
This square has 24 posts and 38 fences.

5 by 4 square.



This square has 30 posts, and 49 fences.

5 by 5 square



This square has 36 posts, and 60 fences

The pattern I found after doing these squares was that the number of posts go up in 6's every time, and the number of fences go up in 11's.

Here is a table to show the 5 by 5 squares

	Nº OF POSTS	Nº OF FENCES
5 by 1	12	16
5 by 2	18	27
5 by 3	24	38
5 by 4	30	49
5 by 5	36	60

When I had finished this table, I noticed that there was a pattern between the posts and the fences. The difference goes up in 5's every time.

12 16 difference 4
18 27 difference 9
24 38 difference 14
30 49 difference 19
36 60 difference 24

Eventually I had finished all the different sizes of squares

BARRIERS cont...

The differences between the tables go up by 1 everytime. The differences for table 1 went up in 1's The differences for table 2 went up in 2's. The differences for table 3 went up in 3's. The differences for table 4 went up in 4's. The difference for table 5 went up in 5's.

Here is a table to show those results above more clearly.

	POSTS	FENCES
TABLE 1	2	3
TABLE 2	3	5
TABLE 3	4	7
TABLE 4	5	9
TABLE 5	6	11

So the pattern for posts in each table goes up in 1's, and the pattern for fences in each table goes up in 2's, and they are all odd numbers.

Also the pattern for posts, and for fences in each table is that the pattern between posts in table 1 is 2's, and the pattern between fences is 3's. The pattern between posts in table 2 is 3's, and the pattern between fences is 5's. The pattern between posts in table 3 is 4's, and the pattern between fences is 7's. The pattern between posts in table 4 is 5's, and the pattern between fences is 9's. The pattern between posts in table 5 is 6's, and the pattern between fences is 11's.

I will now show a kind of table to make it more clearer for you.

	DIFFERENCES
TABLE 1	1
TABLE 2	2
TABLE 3	3
TABLE 4	4
TABLE 5	5

So, this means the differences between each table is 1.

To finish it all off. I found out a formular to find out the number of posts.

If someone gave me a larger number, e.g. 1 x 20, I would need to know the answer straight away, without having to work it all out. I investigated this, and I found out how to solve it. If there was a 1 x 5 grid, I would add 1 onto the number of rows which would be the number 1, and also add 1 to the number of columns which would be 5. The formular which I would use for this would be:-

$$R + 1 \times C + 1 = P \text{ (posts)}$$

The 'R' stands for rows, and the 'C' stands for columns.

If someone asked me how many posts there were in a 5 x 20 grid, the answer would be:-

$5 + 1 = 6$ $\quad$ $20 + 1 = 21$

$6 \times 21 = 126$ posts.

To find out the number of fences for a large grid, here is the formular I would use

$$R \times C - 1 + P = F$$

BARRIERS cont...

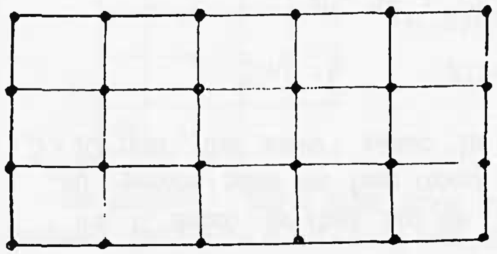
The 'R' again stands for rows and the 'C' again stands for columns.

You times the number of rows by the number of columns. You then take one away. Then add the number of posts to that, and that gives you the number of fences.

Overall, I enjoyed doing this investigation, and I found it quite interesting.

12/6 Barriers

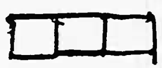
The diagram below shows a barrier system made up of posts • and fences ▬

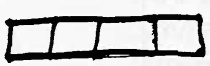


The barriers are used to section off areas of land for many reasons; for example, to keep sheep in.
The object of the investigation is to find out how many barriers and posts are needed for whatever reason.
Simple cases were tested first, to see if any form of pattern could be obtained.
Working horizontally; simple cases were tested as shown below.

 4 Posts 4 Fences

 6 Posts 7 Fences

 8 Posts 10 Fences

 10 Posts 13 Fences

 12 Posts 16 Fences

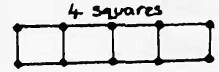
From the simple cases above, already a pattern has been formed.

The results have been put into a table so that it is easier to see any pattern(s) which may occur.

Number of Squares	1	2	3	4	5
Number of Posts	4	6	8	10	12
Number of Fences	4	7	10	13	16

On the table above, it can be seen that as the squares increase, the number of posts increase by 2 and the number of fences increase by 3.
For example,

4 squares

 = **10 Posts** and **13 fences**

5 squares

 = **12 Posts** and **16 fences**

The difference between there being 4 squares and 5 squares is that there are 2 more posts and 3 extra fences.
From knowing this, predictions can be made for instance, I can predict that if there are 10 squares; there would be 22 posts and 31 fences. To test my prediction, I have drawn it below

 22 posts 31 fences.

However, I would not be able to predict a high number of squares, without using some kind of rule or formula as mistakes are liable to be made

One possible rule which was found was one which found the number of fences. Another table was drawn but just showing the number of squares and fences so that the rule could be tested

SQUARES	x 3 + 1	=	FENCES
1	x 3 + 1	=	4
2	x 3 + 1	=	7
3	x 3 + 1	=	10
4	x 3 + 1	=	13
5	x 3 + 1	=	16
6	x 3 + 1	=	19

The rule is, that the number of squares multiplied by 3 and added by 1, equals the number of fences

$S \times 3 + 1 = F$

As can be seen on the table, this rule works.

Also, another rule for working out the number of posts could be used. The rule being that the number of squares multiplied by 2 and added by 2 is equal to the number of posts.

$S \times 2 + 2 = P$

This rule was tested on the table below.

SQUARES	x 2 + 2	=	POSTS
1	x 2 + 2	=	4
2	x 2 + 2	=	6
3	x 2 + 2	=	8
4	x 2 + 2	=	10
5	x 2 + 2	=	12
6	x 2 + 2	=	14

As seen on the table, this rule also WORKS

Also, if the number of squares was wished to be obtained, another simple rule was found. That was, if the number of Posts were divided by 2 and minused by 1, then this would be equal to the number of squares. This too, was tested below on a table, and proved to work.

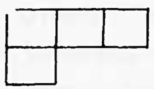
POSTS	÷ 2 − 1	=	SQUARES
4	÷ 2 − 1	=	1
6	÷ 2 − 1	=	2
8	÷ 2 − 1	=	3
10	÷ 2 − 1	=	4
12	÷ 2 − 1	=	5
14	÷ 2 − 1	=	6

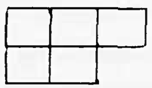
However, everything which has been tested so far has only been proven to work for the barriers which go across in rows. As shown below for example.

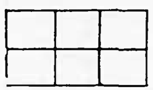
This is obviousley the same for columns. When they were tested on barriers which didn't just go straight across; the rules did not work and the patterns were different. So more investigations were to be done in seeing if a rule, pattern or formula could be found which worked for both the barriers which formed straight lines and columns or barriers which were in blocks.

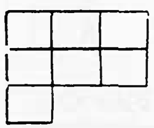
As the previous 3 rules did not apply to barriers which were not formed in horizontal or vertical lines, more simple cases were drawn to try and determin as to why different results were obtained from the different barriers.
Starting with 3 squares in a row, they were gradually built up into a barrier system of 9.

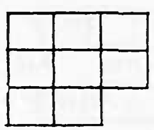
 3 squares 8 Posts 10 fences

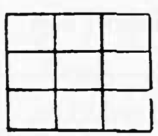
 4 squares 10 Posts 13 fences

 5 squares 11 Posts 15 fences

 6 squares 12 Posts 17 fences

 7 squares 14 Posts 20 fences

 8 squares 15 Posts 22 fences

 9 squares 16 Posts 24 fences.

From looking at the number of Posts and fences overleaf it does not **appear** as if a possible rule or pattern could be gained from these results. Below a table has been drawn to show the form of intervals between the number of posts and fences to try and find a type of pattern.

Intervals between.		2			2		
Posts	8	10	11	12	14	15	16
Fences	10	13	15	17	20	22	24
Intervals between.		3	2	2	3	2	2

The intervals above do follow a pattern but none of which a rule can be formed from.
Below a table was drawn showing the number of squares, posts and fences to see if this would help the investigation.

SQUARES	+	POSTS	-1 =	Fences
3	+	8	-1 =	10
4	+	10	-1 =	13
5	+	11	-1 =	15
6	+	12	-1 =	17
7	+	14	-1 =	20
8	+	15	-1 =	22
9	+	16	-1 =	24

Fortunately from the table above, I was able to find a pattern. This is shown in red on the table.
If the number of squares are added to the number of posts and then minused by 1, then this will equal the number of fences.
This can be written. $S_{(squares)} + P_{(posts)} - 1 = F_{(fences)}$. $S+P-1=F$.

This rule works for barriers which are formed horizontally or vertically and also in blocks.

If a rule, such as S+P-1=F, can be made to work out the number of fences then it should also be possible for a rule to be made which could find out the number of squares and one which could find out the number of posts. This was previously done for the rules which only worked for horizontal and vertical barriers.

So, I then rotated the rule which equals the number of fences and formed two tables to show them.

The table below is to show how the number of posts can be found.

FENCES	-	SQUARES	+1 =	POSTS
10	-	3	+1 =	8
13	-	4	+1 =	10
15	-	5	+1 =	11
17	-	6	+1 =	12
20	-	7	+1 =	14
22	-	8	+1 =	15
24	-	9	+1 =	16

The rule F(fences) − S(squares) + 1 = P(posts). F−S+1=P, works for all types of barriers, along with the rule which calculates the number of squares on the table overleaf.

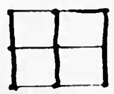
FENCES	-	POSTS	+1 =	SQUARES
10	-	8	+1 =	3
13	-	10	+1 =	4
15	-	11	+1 =	5
17	-	12	+1 =	6
20	-	14	+1 =	7
22	-	15	+1 =	8
24	-	16	+1 =	9

The rule for finding the number of squares can be written:-

f−P+1=S.

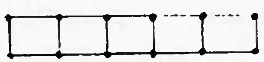
It was noticeable that different rules were used for the different forms of barrier systems. For example; previously it showed that the rule '$S \times 3 + 1 = F$'. worked for horizontal or vertical lines but not for a block. So, why didn't the rules which were previousl found work for all forms of barriers? The obvious test which can be done is to compare some cases of horizontal barriers to those which are formed in a block. system.
For example; below shows a horizontal barrier system and one which has been formed in a block; both of which have an area of 4 squares.

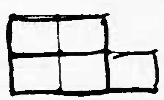
(a)  10 Posts 13 Fences

(b)  9 Posts 12 Fences

Barrier system (a) uses 10 posts and 13 fences whilst (b) uses only 9 posts and 12 fences.
Below the same has been tested on other barrier systems to see if a similar pattern can be obtained

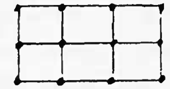
For 5 squares

 12 Posts 16 Fences

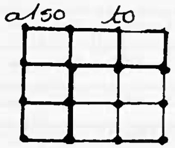
 11 Posts 15 fences

For 6 squares

 14 Posts 19 Fences

 12 Posts 17 fences

From the cases which have been drawn it appears that when the barrier systems are formed in 'block shapes', rather than rows or columns, then this reduces the number of posts and fences.
This is because that when a barrier system is formed in rows or columns, with each square, 2 posts and 3 fences are added on every time. But when forming barriers in 'blocks' like the one above using 6 squares; only 1 post and 2 fences are added with each square. However, the beginning of each row in the block will always have an extra post and fence on it as it is the start of a new row, this applies also to the first row of every block.

 16 Posts 24 fences.

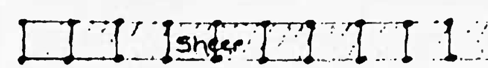
As can be seen on the barrier-system of 9 squares above; the posts and fences in red show where posts and 3 fences have been added. In black, there has been only 1 post and 2 fences have been added. If this barrier system of 9 squares had been formed in a horizontal vertical line then the outcome would have been 20 posts and 28 fences.

So, what use has a barrier system and who would use it?
A barrier-system could be used on a farm. It is probable that it would be used to keep livestock or crops in it.
A farmer has to be as economical as possible; so he would want a barrier system that is the cheapest which could be obtained and one which is the mo economical. Therefore, the farmer would not want to use more posts and fences than what he actually needed.
It is noticeable that there are a reduced number of posts and fences when the barriers are formed in a block. So it is probable that the farmer would opt for a block barrier system rather than one which forms rows or columns.
Below is a table which shows the most economical forms of block barriers systems for a certain number of square

SQUARES	POSTS	FENCES
1	4	4
2	6	7
3	8	10
4	9	12
5	11	15
(Cont.) 6	12	17
7	14	20
8	15	22
9	16	24
10	18	27

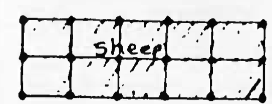
Although there is no formula to this table, it does tell us which would be the most economical way in which to arrange the squares.
For instance, a farmer needed a barrier-system from which he could keep sheep in. The following two barrier-systems are possiblities of which he could choose from.

BARRIER-SYSTEM A



This has an area of 10 squares. The farmer is considering keeping sheep in it. It uses 22 posts and 31 fences.

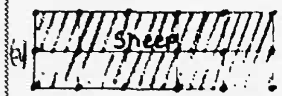
BARRIER SYSTEM B.



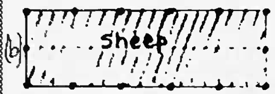
This also has an area of 10 squares. But this uses only 18 posts and 27 fences. Therefore, It is most probable that the farmer would opt for 'Barrier-system B'.

However; having established a more economical form of barrier-system, it seems unlikely that the farmer would either

Want or need the fences that are dividing up the area in the middle.
It seems very unlikely that the farmer would use extra posts and fences and pay the extra cost of, dividing the sheep into sections of the field, unless there is a specific reason for doing so

(a)


It seems unlikely that the sheep would be kept like this.

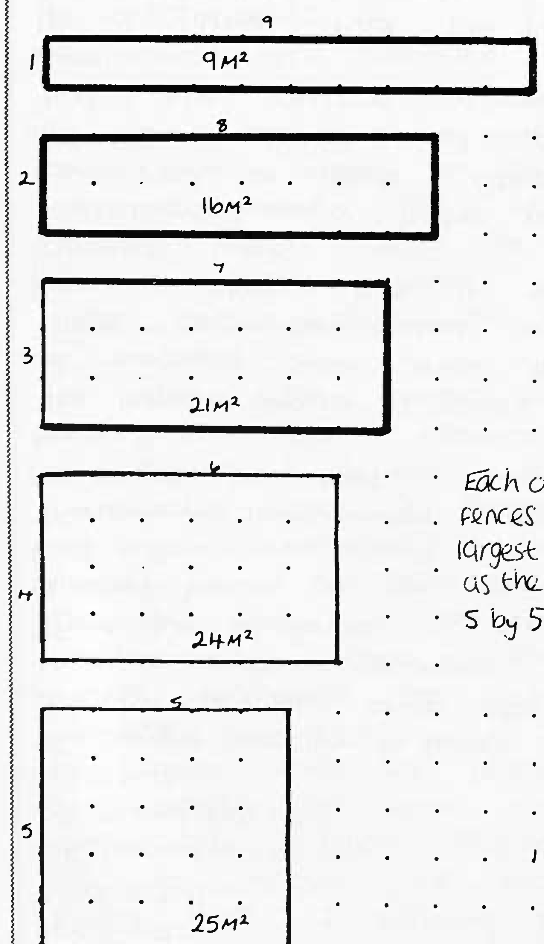
(b)


It is much more probable that 'barrier system b' would be used.
Thus would reduce the number of posts and fences and also the cost.
Barrier system a uses 27 fences and 18 posts whilst (b) uses only 14 fences and 14 posts. Thus would prove to be more economical to the farmer.

However, apart from wanting more economical fencing, the area is also very important. Obviousley, the positioning of the fencing will effect the area. It is probable that the farmer would want the largest area obtainable from the fences and posts.
So if a farmer had 20 fences and 20 posts, What would be the largest are obtainable from these?

On the 'square spotty paper' overleaf, all the areas which are obtainable from 20 fences have been drawn.
Given that each fence is only 1 metre long, it can be seen that the largest area is $25m^2$, whilst the smallest area is only $9m^2$.
There is quite a difference between the largest and smallest area, so the positioning of the fencing really does effect the outcome of the area, which would be important to a farmer.

The different areas obtainable from 20 fences.



Each of these areas all use 20 fences As can be seen, the largest area obtainable is 25m², as the fences are arranged 5 by 5.

1 fencing panel = 1 metre long

The next question which can be posed is whether there is an easier method of finding the largest or the smallest area from the number of fences given from previous work, it is noticeable that whenever the barrier-system has 4 sides, the number of posts and fences are always the same.

Overleaf, different examples of fences have been drawn to demonstrate how the positioning of the fences effects the area.

For finding the smallest area for any number of fences, the fences are divided by 2 and minused by 1. $F \div 2 - 1 = \text{AREA}$

This has been previousley tested and proves to work. For example, there are 16 fences.

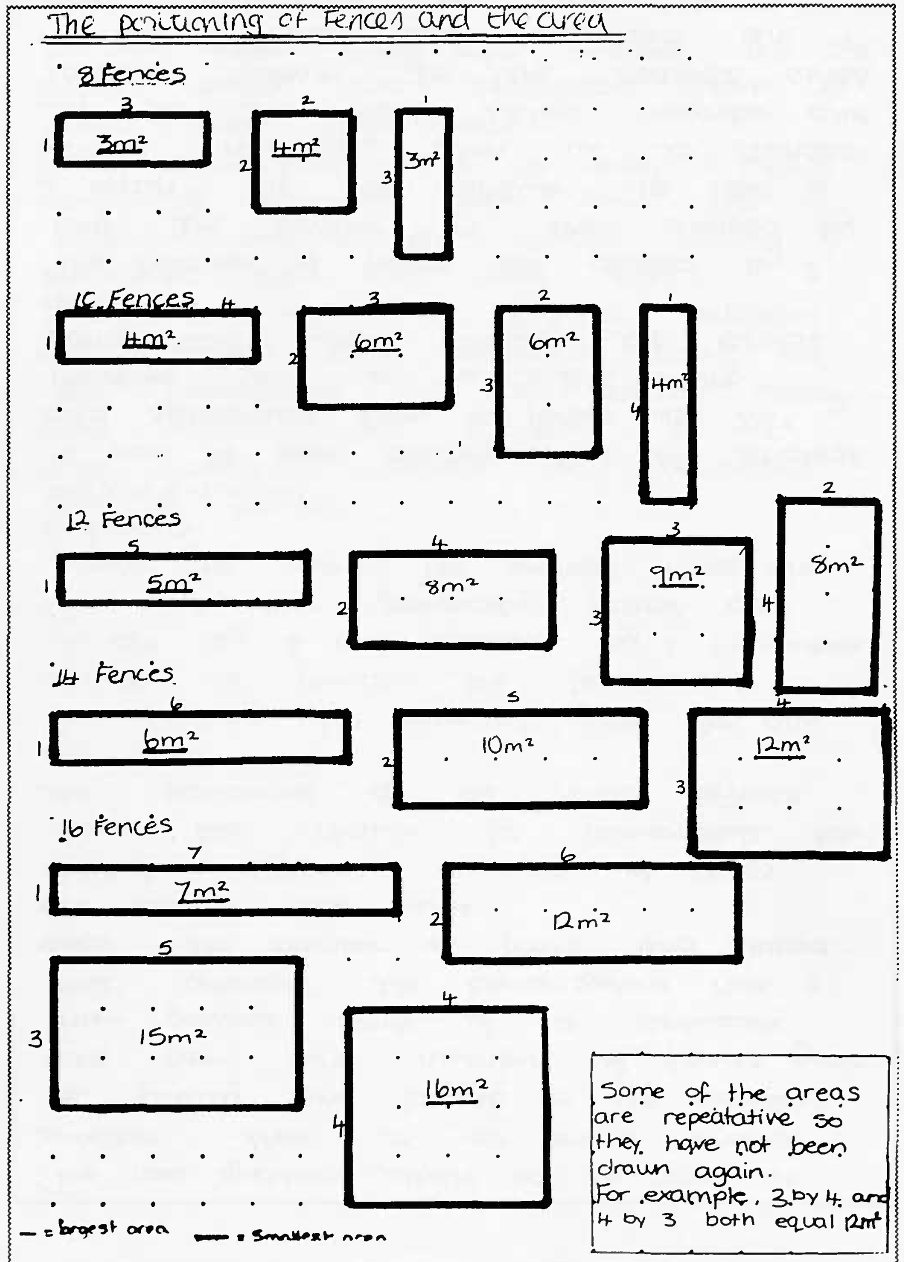
$16 \div 2 = 8 - 1 = 7m^2$.

It can be seen overleaf that the smallest area obtainable from 16 fences is $7m^2$.

However, it is a little more complicated when finding the largest area.

The number of fences are divided by 2, and the answer is then divided by 2 again. If the answer is not a whole number, then it is rounded up to the next whole number and rounded down to the nearest whole number. These two numbers are then multipled to find the largest area.

For example,
There are 14 fences.
$14 \div 2 = 7$.
$7 \div 2 = 3.5$.
3.5 rounded up = 4
3.5 rounded down = 3.
$4 \times 3 = 12$.
The largest area for 14 fences is $12m^2$. Overleaf (dotty paper) it can be seen that this is correct.
Here is another example;
There are 12 fences.
$12 \div 2 = 6$.
$6 \div 2 = 3$
This does not need rounding up or down, so it is just multiplyed by itself.
$3 \times 3 = 9$.
The largest area for 12 fences is $9m^2$. Overleaf it can be sean that this is correct.
This can be tested on any number of fences which has 4 sides and will always work.
Predictions using higher numbers of fences can now be made. Obviousley these have not been drawn out to check them but the method above will always work.
If a farmer had 198 fences, what woul be the largest area obtainable?
$198 \div 2 = 99$
$99 \div 2 = 49.5$
$49 \times 50 = 2450m^2$.
The largest area obtainable for 198 fences would be $2450m^2$.



What would be the largest area obtainable for 546 fences?

$546 \div 2 = 273$

$273 \div 2 = 136 \cdot 5$

$136 \times 137 = 18632$.

The largest area obtainable would be $18632\,m^2$.

So now when given a certain number of fences, the largest or the smallest area can be found. Also, it is known that the posts will be the same numbe as the fences when using a 4 sided barrier system. So now, cost can be included.

For all of the previous work, the fencing panels have been assumed to be 1 metre in length. However, this is rather unrealistic as 2 metre fencing panels are more commonly used, unless a 1 metre panel had been specially made.

So if cost is to be used realistically, then the length of the panels should be worked out in 2 metre lengths.

A farmer has 24 fencing panels (and 24 Pos He wants to use the largest area obtainable from these. Also he woul like to know the total cost.

Given that 1 fencing panel which is 2 by 2 costs £9 including V.A.T and 1 post, 3 metres high costs £3 including V.A.T. So What would be the largest area obtainable for the farme and how much would this cost him?

The largest area:-

24 Fencing panels (each 2 metres in length)

$24 \div 2 = 12$

$12 \div 2 = 6$

(1 metre) = $6 \times 6 = 36$

(2 metres) = $12 \times 12 = 144\,m^2$

The largest area equals $144\,m^2$.

1 fencing panel = £9.

$24 \times 9 =$ £216.

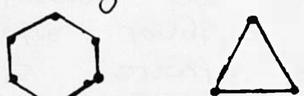
1 post = £3.

$24 \times 3 =$ £72

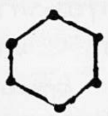
£216 + £72 = £288.

From 24 fencing panels the farmer would obtain an area of $144\,m^2$ and the total cost for this would be £288

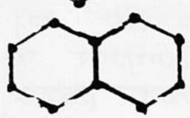
The previous work included barrier-systems which used 4 sides. But it would be interesting to see if there is a different outcome when investigating barrier-systems in other shapes; such as Hexagons or triangles.



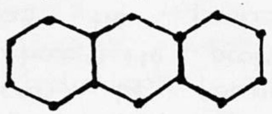
I decided to investigate hexagons.
Below are some simple cases of barrier-systems using hexagons.



6 Posts
6 fences



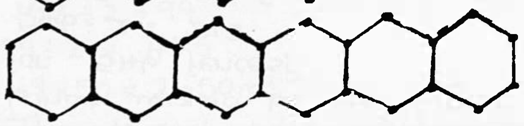
10 Posts
11 Fences



14 Posts
16 fences

18 Posts
21 fences

22 Posts
26 fences



26 Posts
31 fences

A table was then formed showing the results from the simple cases on the previous page.

No. of Hexagons	1	2	3	4	5	6
No. of Posts	6	10	14	18	22	26
No. of fences	6	11	16	21	26	31
	5	5	5	5	5	

In the table above it can be seen that the number of posts increase by 4 and the number of fences have intervals which increase by 5 each time as the hexagon increase. These have been shown in red on the table.
From this I found some rules which could be used to find the number of fences, posts or hexagons. Three tables have been formed to demonstrate the rules.

Hexagons	+	Posts	-1	= fences
1	+	6	-1	= 6
2	+	10	-1	= 11
3	+	14	-1	= 16
4	+	18	-1	= 21
5	+	22	-1	= 26
6	+	26	-1	= 31

As can be seen from the table; if the number of hexagons and posts ar added together and then 1 is minused from the answer, then this will equal the number of fences. So it can be said that the rule for working out the number of fences is :- $H + P - 1 = f$.
Similarly the rules which are used for finding the number of hexagons and

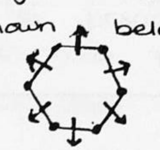
posts can be found by rotating the first rule round a little.
Again the tables below demonstrate the two rules.

Fences	- Hexagons	+1 = Posts
6	- 1	+1 = 6
11	- 2	+1 = 10
16	- 3	+1 = 14
21	- 4	+1 = 18
26	- 5	+1 = 22
31	- 6	+1 = 26

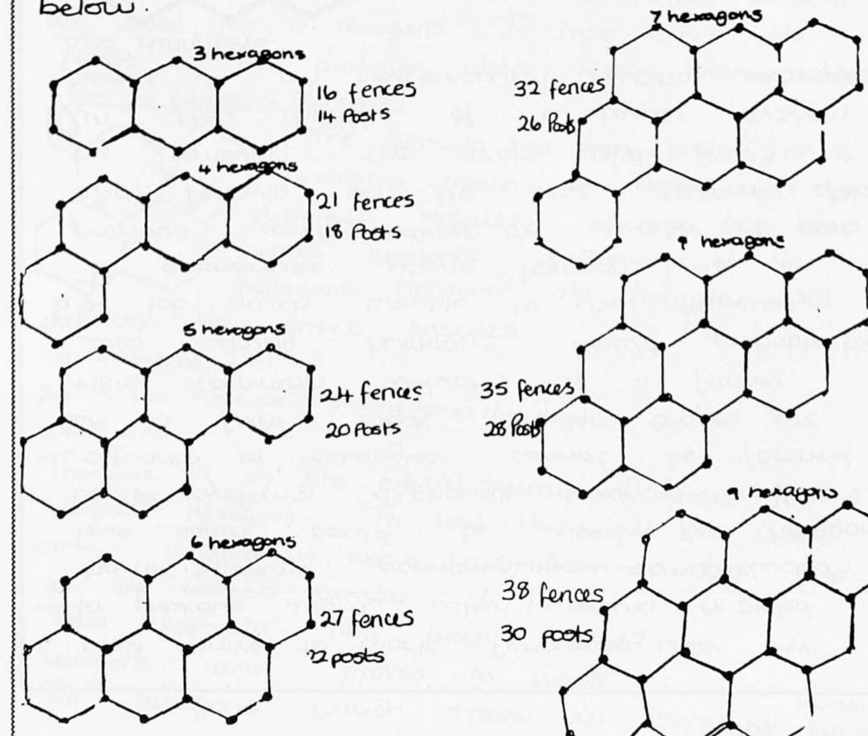
The table shows that by minusing the number of hexagons from the fences and then by adding 1, this equals the number of posts. So by wanting to know the number of posts the rule, F-H+1=P can be used.

Fences	- Posts	+1	= Hexagons
6	- 6	+1	= 1
11	- 10	+1	= 2
16	- 14	+1	= 3
21	- 18	+1	= 4
26	- 22	+1	= 5
31	- 26	+1	= 6

As can be seen on this table, when the number of posts are minused from the fences and then 1 is added this equals the number of hexagons. So if wanting to obtain the number of hexagons from the fences and posts the rule:- F-P+1=H can be used.
These rules apply to hexagons 'travelling' in all of its angles, as shown below



Just like the barrier systems using 4 sides, it would be interesting to see how the most economical way of forming the hexagons would be. To do this the rules should be tested on hexagons other than those which form single lines of them; as on the 4 sided barrier systems, the rules which were found for horizontal or vertical lines did not work when they were formed in blocks.
The rules have been tested on simple cases of different hexagons below.



A table was then formed to show the number of hexagons, fences and posts, so that the rules which were found previously could be tested.

No. of Hexagons	3	4	5	6	7	8	9
No. of Posts	14	18	20	22	26	28	30
No. of fences	16	21	24	27	32	35	38

Of coarse it is expected that the results obtained will be different from those which were found for the other simple cases as the hexagons are in a different positioning.

The three rules:- $H+P-1=F$, $F-H+1=P$ and $F-P+1=H$ can all be tested on the table above, and in doing so, proves that they work. However, to show that they work, a simple case has been tested below using the three rules

For 5 hexagons

$H+P-1=F$.

$5+20-1=24$

24 being the number of fences which is correct

$F-H+1=P$.

$24-5+1=20$.

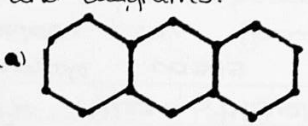
20 being the number of fences which is correct

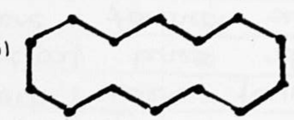
$F-P+1=H$

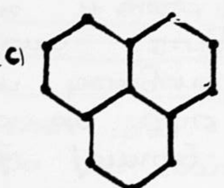
$24-20+1=5$

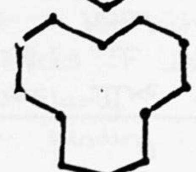
5 being the number of hexagons which is also correct. These three rules can be tested on all the results from the table above and prove to work for every one.

In previous work, when using 4 sided barrier-systems, economising was included. This again could be used for hexagon barrier-systems although hexagons unlike squares or rectangles cannot be formed in a fixed block. Irregular shapes are more common which, if a farmer was using hexagons would probably them to be too much trouble to use. However, To economise using hexagons as in previous work; reducing the number of posts and fencing is the best method often by removing the posts and fences in the middle of the barrier system. Below is an explanation using examples and diagrams.

(a)  This uses 16 fences and 14 posts

(b)  This uses 14 fences and 14 posts

(c)  This uses 15 fences and 13 posts

(d)  This uses 12 fences and 12 posts

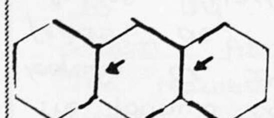
As can be seen on the previous page, barrier-system (d) uses the least posts and fences. Not only has it no fences and posts in the middle, the 3 hexagons are as compact as possible. Therefore, it can be said that this is a more economical barrier-system.

So far, the rules which have been formed have only been able to be used when knowing either the number of hexagons and posts, the hexagons and fences or the posts and fences. It would be much simplier and less time consuming if when knowing a certain number of hexagons, could calculate the number of fences without having to count each fence.

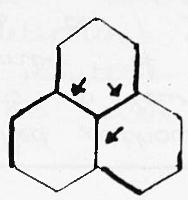
By studying different numbers of hexagons and their fences, I have formed a simple formula which can be used to calculate the number of fences just by knowing the number of hexagons.

By multiplying the number of hexagons by 6. (which is the number of sides of a hexagon) and then minusing from the answer the number of 'dividing fences' in the barrier-system this will obtain the overall number of fences.

The 'dividing fences' are shown below. They have been shown in red.



2 dividing fences



3 dividing fences.

Below demonstrates how the formula works

For 4 hexagons (travelling across)

(a)

(The arrows are pointing to the dividing fences.)

I wish to know the number of overall fences for 4 hexagons without having to count them.

The number of hexagons are multiplied by 6.

6×4 hexagons $= 24$

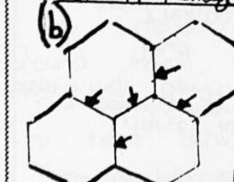
There are 3 dividing fences which are shown above in red. So 3 is then minused from the 24.

$24 - 3 = 21$.

Therefore, 21 is the overall number of fences which are used for 4 hexagons. To test this the fences on the diagram above have been counted

For 4 hexagons. (compact)

(b)



The formula has been tested on 4 hexagons again, as there will be different results caused by the more compact positioning of the hexagons. However the formula still produces the correct results.

6×4 hexagons $= 24$

$24 - 5$ (as there are 5 dividing fences this time)

$24 - 5 = 19$.

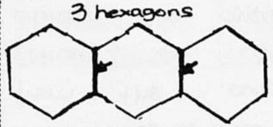
Therefore 19 is the overall number of fences for 4 'Compact' hexagons. To test this, all the fences on diagram (b) above have been counted and is the correct number of fences.

This formula has been tested on other hexagons and proves to work.

For hexagons which travel in straight (across) lines

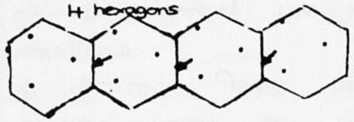
The formula could be used without actually looking at the hexagons to count the dividing fences, as the number of dividing fences will always be 1 less than the number of hexagons. Below these simple cases Prove this.

3 hexagons



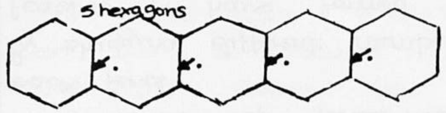
There are 3 hexagons and 2 dividing fences'.

4 hexagons



There are 4 hexagons and 3 'dividing fences

5 hexagons

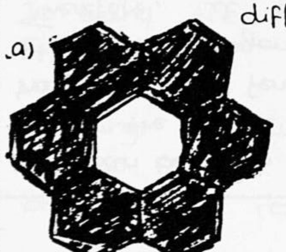


There are 5 hexagons and 4 dividing fences.

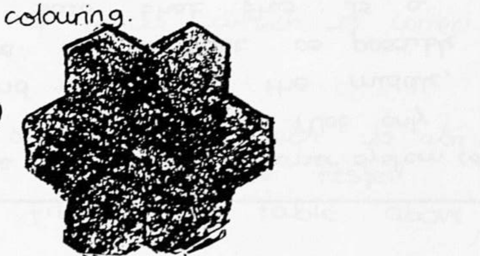
Unfortunately, the same cannot be said about hexagons which don't travel 'acrosswards'. There are so many different ways in which the hexagons could be 'compacted' together that there is no reliable, fixed pattern like the one above for hexagons which travel straight across.

An example of this is below. The two barrier-systems both look as if they consist of 7 hexagons apart from the different colouring.

a)



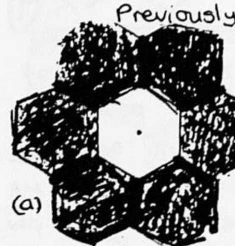
(b)



Barrier-system (a) consists of only 6 hexagons (indicated by the orange colour) but barrier-system (b) consists of 7 hexagons.

Barrier systems consisting of 6 and 7 hexagons can be shown in many different ways but on the diagrams overleaf they show how a barrier-system which appears to consist of 7 hexagons could also consist of 6. Barrier-System (a), containing 6 hexagons is formed so that it is in a 'circluar' shape with 3 hexagons on the left and 3 on the right leaving a hollow in the middle. But on barrier-system b, the hollow is an extra hexagon. Both barrier-system (a) and (b) use 30 fences.

This is demonstrated below, using the formula Previously found.

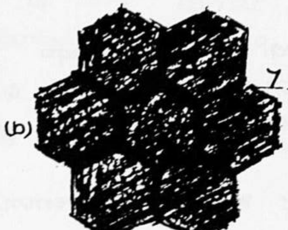


6 hexagons

6 dividing fences

(a)

(b)



7 hexagons

12 dividing fences

Barrier-system (a) = 6 hexagons.

$6 \times 6 = 36$.

$36 - 6 = 30$. (6 being the number of dividing fences)

30 being the overall number of fences.

The 'dividing fences' are indicated by the colour red

Barrier system (b) = 7 hexagons.

$6 \times 7 = 42$

$42 - 12 = 30$. (12 being the number of dividing fences)

30 being the overall number of fences.

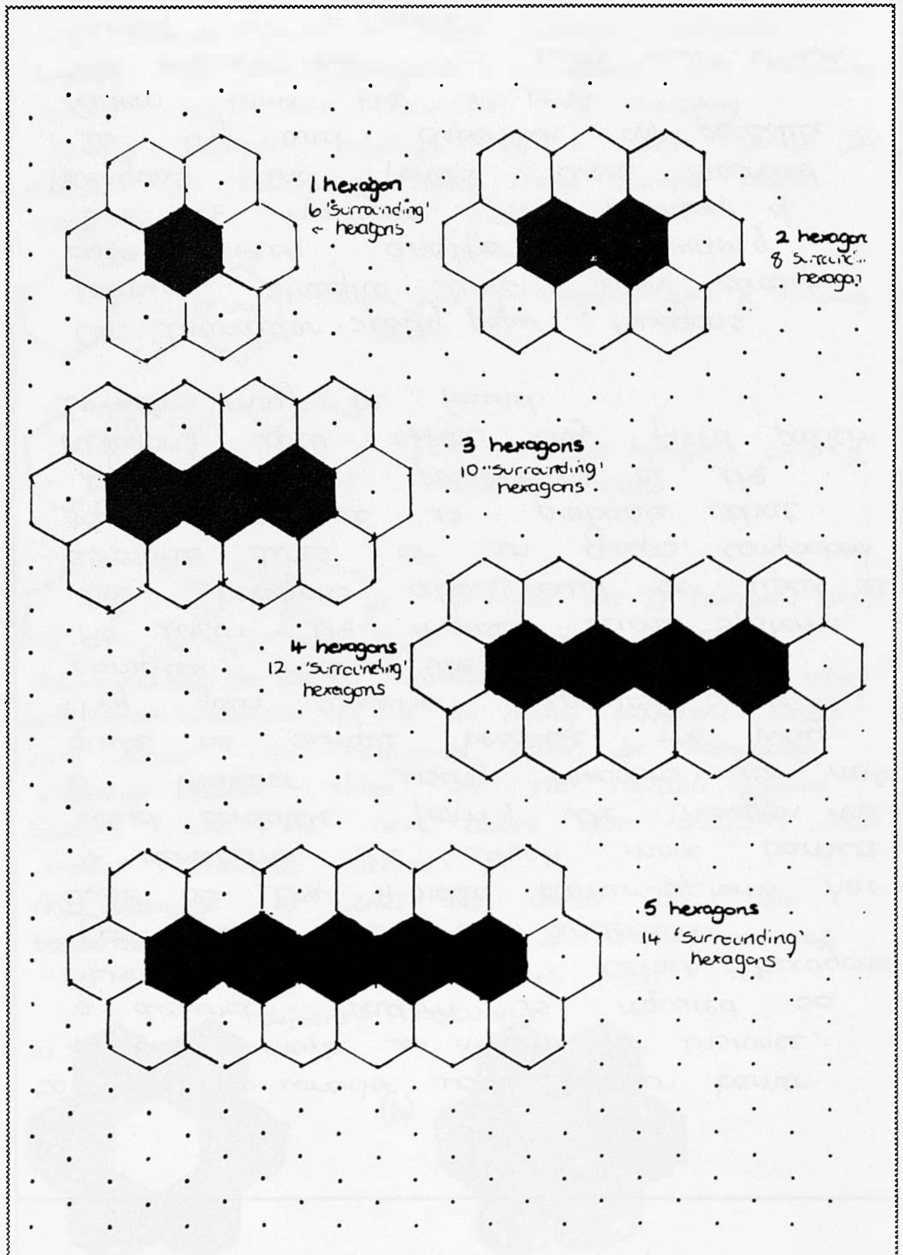
So rules and patterns can only be formed depending upon the positioning of the hexagons.

so In order to consider using hexagon barrier-systems, maybe in a farm for instance, a distinct pattern is required so that the way in which hexagons fit together can be understood.
Just as the 4-sided barrier-systems had 4 directions in which more barriers could continue from, the hexagon has 6. However, using hexagons is not quite as simple because the way they join together appears quite complex, rather like a honeycombe.
As with the 4-sided barrier systems, the hexagons also could be used in straight lines or in groups, 'compacted' together so it is probable that the different positioning of the hexagons will effect any fixed pattern which may be found.

On 'triangular spotty paper', hexagons forming straight lines were drawn also with another surrounding 'line' of hexagons. The number of hexagons and fences were recorded to try and determin any possible pattern from the results.
The results from this were then shown overleaf on a table.

No. of hexagons		No of surrounding hexagons		No. of fences		No of Surrounding fences.
1	5	6	0	6	18	24
2	6	8	3	11	19	30
3	7	10	6	16	20	36
4	8	12	9	21	21	42
5	9	14	12	26	22	48

The intervals between the number of fences and hexagons were indicated in green as they form some varied patterns.
For instance, the intervals between the number of hexagons and the number of surrounding hexagons begin at 5 and increases by 1 as each hexagon increases by 1.
A rule which could link the number of hexagons with the surrounding hexagons is by multiplying a given number of hexagons by 2 and then adding 4 to equal the number of surrounding hexagons. Some examp to test this are shown below.
For 2 hexagons.
$2 \times 2 = 4.$ $4 + 4 = 8.$
8 being the number of surrounding hexagons which can be seen on the table above.
for 3 hexagons.
$3 \times 2 = 6.$ $6 + 4 = 10$
10 being the number of surrounding hexagons which also can be seen above
for 5 hexagons.
$5 \times 2 = 10.$ $10 + 4 = 14.$
14 being the number of surrounding hexagon which is also correct
$H \times 2 + 4$ = No. of Surrounding Hexagons.



Using more 'triangular spotty paper' the next surrounding 'line' of hexagons were drawn.
A table was then formed to show the number of hexagons and the number of fences for all three lines.

No. of hexagons	2nd surrounding line of hexagons	3rd surrounding line of hexagons	No. of fences	No of fences on 2nd line	No. of fences on 3rd line.
1	6	12	6	24	42
2	8	14	11	30	48
3	10	16	16	36	54
4	12	18	21	42	60
5	14	20	26	48	66

It is noticeable on the table that apart from the actual number of hexagons at the start (colour coded red on the table) the number of hexagons which are formed on the 2nd and 3rd surrounding 'lines' are all multiples of 2.
Also the number of fences for the hexagons in the 2nd and 3rd surrounding 'lines' are all multiples of 6.
The figures which are marked in black on the table show the intervals which occur between the hexagons and the fences.
From this, then it could be said that if two hexagons were used, and more hexagons were needed to be placed around them as compact as possible, the pattern would be that there would be 2 hexagons in the middle, 8 hexagons surrounding that and then 14 hexagons

surrounding that. This sounds complecated to understand but below is a diagram and a simple explanation of how when given a number of hexagons; the surrounding line of hexagons, and the line surrounding that can be calculated.

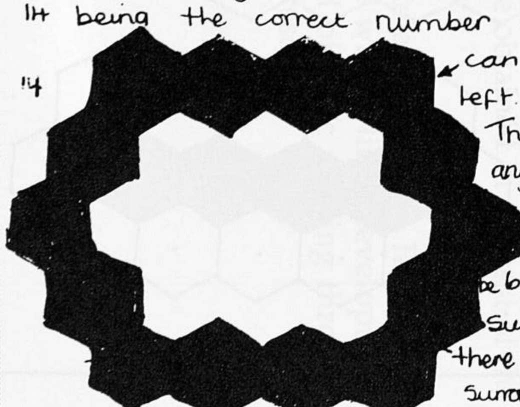
2

If you have 2 hexagons which uses 11 fences and you wish to know the number of surrounding hexagons. The rule which was previasley found: H×2+4 (but which can only be used for finding the 2nd line of surrounding hexagons) can be used 2×2+4 = 8. 8 being the number of surrounding hexagons which can be seen below.

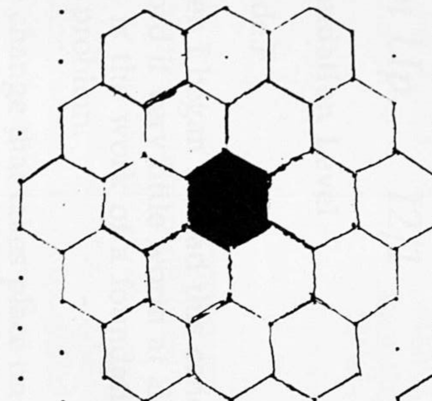
8

6 is then added to this, as on the table overleaf it shows that when 6 is added it equals the number of hexagons needed for the 3rd surrounding line. This was then done. +6=14. 14 being the correct number of hexagons which can be seen on the left.

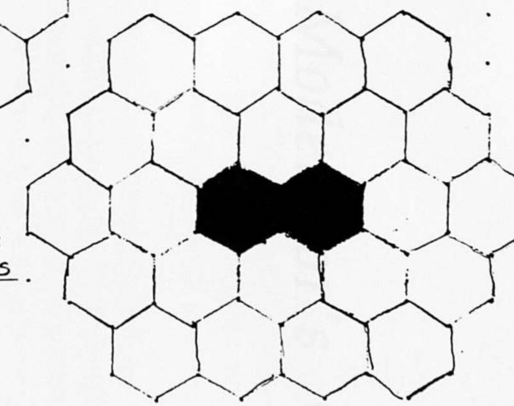
14



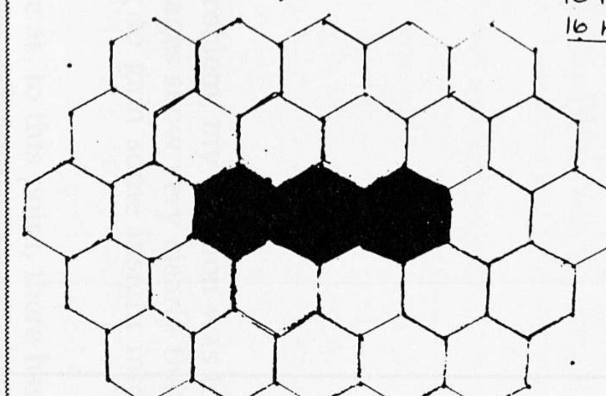
This can be continued for any following 'lines' of hexagons which may occur. There will always be 6 more hexagons on each surrounding line than there was on the previous surrounding line of hexagons.



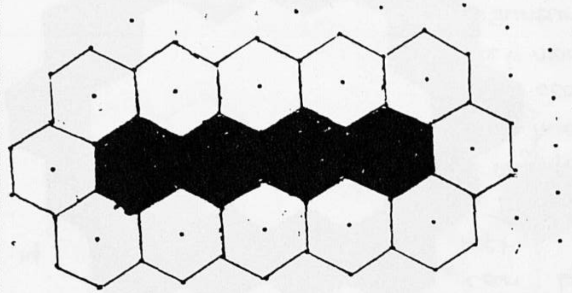
1 hexagon = 6 fences
6 hexagons = 24. fences
12. hexagons = 42 fences



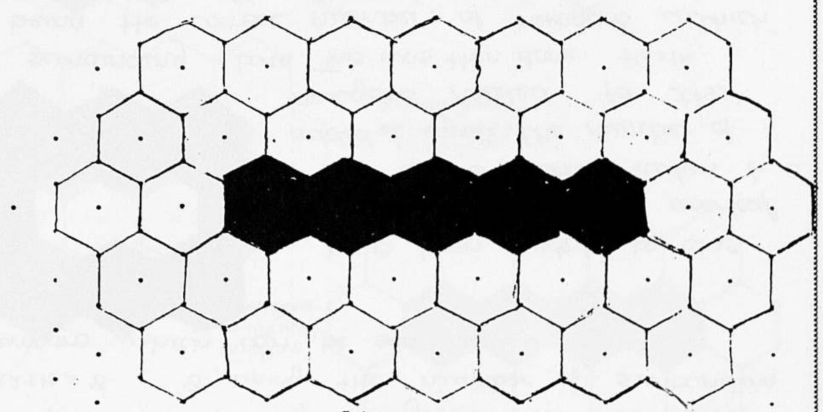
2 hexagons = 11 fences.
8 hexagons = 30 fences
14 hexagons = 48 fences



3 hexagons = 16 fences
10 hexagons = 36 fence's
16 hexagons = 54 fences



4 hexagons = 21 fences
12 hexagons = 42 fences
18 hexagons = 60 fence



5 hexagons = 26 fences
14 hexagons = 48 fences
20 hexagons = 66 fences.

Every surrounding line increases by 6 hexagons, more than the previous line. Apart from the 2nd line of hexagons which can be found by multiplieing the 1st line of hexagons by 2, and then adding 4. This can be tested on the case above for 4 hexagons.
4x2=8. 8+4=12. 12 being the number of hexagons in the next surrounding line, which is correct. 6 can then be added to produce the number of hexagons for the next surrounding line. 12+6=18. 18 is also correct. By adding 6 to this, the number of hexagons for the next surrounding line is produced; and so on.

6 *Moderator's Comments*

Pin Up I2/1

Foundation Level

Grade F

When I began to read this candidate's response to the problem, my reaction was to accord if very little worth at all. In fact, the first eight pages show very clearly that this is the work of a foundation level candidate trying to gain some insight into the problem.

The change that takes place on page 9 is quite remarkable as, to this point, there has been little structure to the task but there is suddenly a logical development in the remainder of the problem. The suspicion is that she has been given a push in a direction here with the use of lines of posters using four pins to maximum effect. Of course, helpful nudges are entirely justified, provided the marking recognises this. It is better to get a pupil working than to see them stuck and helpless, hence gaining few eventual marks.

Having drawn diagrams, she tabulates results clearly (has she been helpful with the 2 x 2, 2 x 3 etc arrangements?) and then proceeds to tackle the longitudinal arrangements using three pins. The lack of development into 'square' arrangements etc is to be expected at this level. In the 'conclusion' she gives some examples, but this is really to make use of the knowledge obtained in the rest of the task.

This is a nice piece of work by a target grade candidate at this level. It shows a satisfactory grasp of essentially a single stage problem with a little development and some comments upon the work. There is a strand of logic running through the latter part of the project and the write up is clearly presented.

Barriers *I2/2*

Foundation Level

Grade F

The candidate has given a nice, clear introduction to the task. He has defined his terms of reference but has failed to indicate the method by which he will investigate the problem. In fact, he seems not to have really planned the general approach to the task and the coursework is rather 'bitty'.

I like the way he has headed his sections of work with questions which could, if developed properly, have given rise to a good project. However, he seems to have felt it necessary to move from each, possibly good, 'jumping off' point to a new question. In consequence he never really gets any depth to the study and he fails to explain why any of the patterns he observes actually occur. (This would be unlikely in a candidate at this level.)

When I read the first two pages, I felt the candidate was going to produce a good, extended piece of work as he set up the developing situation of one square, two squares, three squares etc, but then he failed to capitalise on this good beginning. He seems to miss the point completely with the next piece of work, then notices a pattern on the 'linear' arrangement, says he can predict results but then doesn't show how to do so. The remainder of the project follows a similar pattern of some logical progression, filled in with 'flights of fancy'.

Although I would prefer to see more tenacity in sticking to one aspect of a problem and developing it, I would be prepared to reward the positive achievement of this candidate in the more rational sections of the project. He clearly has the ability to detect and develop pattern, though he didn't have the insight to spot the Triangle Numbers hiding behind the factors of four at the end of the work and he never indulges in generalisations. Perhaps, if he had not dissipated his energies on some of the verbiage in the work, and concentrated on the mathematically more worthy sections, his mark might have been higher. It is a justifiable intrusion to point out to candidates that some aspects of their work might not gain much recognition.

Barriers I2/3

Intermediate Level

Grade D/E

This offering is in complete contrast to other pieces of work. It is brief, contains almost no commentary and has few examples of barrier systems used to gather the data to generalise upon. Its conclusions are in the form of formulae and has, I feel, been regarded as a problem rather than an investigation.

The work should achieve a grade in excess of the target at Foundation Level but is not really a wholly satisfactory piece at Intermediate. I should be concerned if all the items in a candidate's folio were in the same vein as this.

However, it is precise, algebraic techniques are used carefully and results are clear, if not in recognised tabular fashion. The lack of comment is a flaw as it would lead one to consider the first section to be entirely wrong. What has been done, is to create 'hollow' barrier systems here, but no mention has been made of this fact! In the second portion only rectangular barrier systems are made and yet these are quite satisfactorily generalised. A pity evidence is not presented to demonstrate the validity of the generalisations. Such methods as have been used have to be deduced from the style of the writing - a process I do not wish to indulge in as a moderator. I am not in a suitable position to infer such things away from the classroom. Candidates must be aware of the importance of clear communication of the process of mathematics through their writings.

Barriers I2/4

Intermediate Level

Grade C/D

This is quite a long piece of work showing dedication to the task by an Intermediate pupil. I should expect this to gain a high grade at this level, though it does show some flaws which might exclude it from achieving the fullest marks.

There seems to exist some confusion in the writer's mind between the terms 'barriers' and 'enclosures'. This confusion never seems to be satisfactorily resolved as the term 'fences' is introduced unnecessarily.

He starts nicely, building up a sequence of longitudinal enclosures and seeks to generalise these results. This he satifactorily does though not without error - note use of brackets and the term 'extra' enclosures which is incorrect in the 'fences' generalisation. He, nevertheless, produces plenty of evidence of testing for his formulae with the inclusion of numerical examples.

Having recognised the weakness of the generalisation, in applying only to the longitudinal arrangement, he then sets about explaining the difference in 'block' arrangements. He begins with a nice 'spiral' build up of enclosures though he lapses this on the 13th diagram. He is not averse to making observations on his work but does not always satifactorily follow these up.

In conclusion he produces graphs which are not altogether appropriate but I can understand the justification for these. In fact the block arrangement defeats him and he is not able to properly explain the sequences of results. However, I like the way he concludes by setting himself explanatory examples and working through them, even though he is still applying brackets incorrectly to the situation, and omits the possible working of the type

$$2(n-1) + 4 = 24$$

$$2(n-1) = 20$$

$$n-1 = 10$$

$$\text{therefore } n = 11$$

to demonstrate his grasp of the situation.

The second part of the working in 'block arrangements' seems not to relate to the original problem and he gives the impression of 'trailing off' and not achieving a convincing end to the project.

Barriers I2/5

Higher Level

Grade C

What a nicely crafted investigation! Real attention to detail has been paid and presentation has been carefully attended to. This is certainly above the level of work expected at target grade in the Intermediate level candidate.

There is a nice thread of logical development following through the work, slightly pedestrian at times, but a useful approach leading the candidate, and the investigation on. Of course, there are obvious omissions. Once the scene has been set in the introduction, the use of non-rectangular arrangements never reappears. The development of 1 by n, 2 by n, 3 by n, 4 by n ... always repeat already drawn examples, so 2 x 5 reappears as a 5 x 2. No comment is made to indicate that these have been recognised as congruent.

Features of the work are then, that a system of development has been set up and adhered to through the work. This is largely single stage and is not modified to cope with non-rectangular cases. The work is nicely commented upon but contains minimal explanation as to why results occur. There is some generalisation in the conclusion, but of a limited nature, and technically inaccurate. The work, as a whole, appears accurate despite the algebraic error mentioned and a transcription error on the same page.

Despite these comments a great deal has been achieved by this candidate, though marks may have been improved further with more examples of usage of the findings and, perhaps, some reduction of the time spent on all those early cases to give attention to non-rectangular cases or areas in barriers.

Barriers I2/6

Higher Level

Grade B

What a lot of work has gone into this solution! Of course this should not be allowed to cloud judgement about the worth of what has been produced. A great deal of the solution consists of writing which is more appropriate to an English essay, and part of what is hoped for in Higher Level candidates is an efficient use of mathematical notation. This aside, the commentary does provide valuable insight into the processes undertaken by the candidate. I feel the finished result is what I should expect of a candidate of target grade at Higher level. The general impression is of a 'skimming' at a wide range of components to the task but perhaps lacking the depth needed for the achievement of the highest grade available.

The work spans

* Barriers in lines and blocks
* Areas included in barriers
* Barriers in non-rectangular arrangements.

This, clearly, is more than a single stage problem and is extended well beyond the scope of the original task. She achieves a good explanation of the relationships obtained in longitudinal arrangements of squares and in blocks, with Euler's rule. The problem with the latter 'rule' being the proliferation of variables. She only sets herself the task of resolving this problem when she deals with hexagons, which is a far more complex problem.

From here she passes quickly over the very sensible approach to areas within barriers and on to the hexagons. At this point she constructs her argument nicely but rather labours the findings, perhaps disguising an incomplete explanation of the situation.

The argument and accuracy of the piece is good and the presentation superb, but it leaves just a slight feeling of dissatisfaction with the final result.

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