Chapter 50
Modelling Examples and Modelling Projects: Overview

[AU1] **Hugh Burkhardt**

1 The Challenge

Mathematical modelling as an activity is absent from the implemented curriculum in most classrooms around the world, after the early years where counting is commonplace. Students are taught standard models and expected to apply them. To take some simple examples, proportional models are a major feature of middle-grade mathematics. Students are expected to solve problems like:

> 6 friends bought a six-pack of cola for $3.
>
> How much should each of them pay?

However, in most textbook chapters on proportion, all the situations are indeed proportional. Students are rarely asked to identify whether the situation is one for which a proportional model is appropriate. They do not meet problems like:

> If it takes 40 minutes to bake 5 potatoes in the oven.
>
> How long will it take to bake 1 potato?

where the appropriate model depends on the type of oven[1], let alone a situation like:

> King Henry VIII had 6 wives.
>
> How many wives did King Henry IV have?

– a fact with which modelling cannot help you[2].

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[1] Constant, i.e., roughly 40 min, in a regular oven; Proportional, i.e., roughly 8 min, in a microwave oven.

[2] If you are curious, try Wikipedia.

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Thus modelling presents teachers and students with a new kind of challenge. Students are accustomed to this – school is all about new challenges. However, modelling will take most teachers outside their comfort zone of professional expertise. This chapter is focused on the various pedagogical and mathematical demands of teaching modelling and applications, and how teachers may be helped to respond to them effectively.

1.1 Analysis of the Challenge

In this book there are two papers with the emphasis on analysing the challenge modelling poses on teachers and students.

Barbara Schmidt looks at obstacles from the teacher's perspective. What factors do they say are holding them back them from introducing modelling into their classrooms? She reports on a questionnaire-based study of the views of 101 teachers, half of whom take part in training in teaching modelling. There were also in-depth interviews with six teachers. The questionnaire was administered four times over a year – before, during, and after the training period, and 5 months later. It was designed to probe 14 factors. Seven are related to the kinds of obstacle that Blum has described: organizational; pupil-related; teacher-related; and materials related. (The other seven, not analysed here, have a more positive focus – on the affordances that modelling provides).

The teachers' responses identified three major factors that inhibit their teaching of modelling:

- Ninety-seven percent mentioned the time that modelling problems require as an obstacle, a view that was only slightly changed by training. This is a generic problem – the learning of mathematical concepts also requires association of reflection, building the connections that are essential for robust long-term understanding. [AU2]
- Before the training, 61% of the teachers said there was too little material available for them to use in the classroom; happily, this concern more or less disappeared during the training, for those who had it but not the control group. (This still leaves the issue of making these materials widely known to teachers, through training and other channels).
- The second obstacle that teachers mentioned was the challenge of assessing performance in modelling. Eighty-four percent of teachers mentioned it, and this view was not affected by the training. While teachers of history or first-language studies are accustomed to assessing essays and other forms of open writing, school mathematics is still dominated by "answers" that are correct or not.

The study notes the importance of teacher's beliefs about mathematics itself, which many felt does not extend to modelling. As with any study of this kind, the overt responses may conceal some deeper discomforts, of the kind that everyone feels when required to extend their professional practice.

Richard Cabassut and Anke Wagner compare French and German primary school textbooks, looking at the role that modelling plays in those curricula from an anthropological perspective. They focus on the use of the term ‘modelling’, which is rare in the French primary context. They recognize that problem-solving in the primary school includes modelling, but they do not see both of these as strategic/process skills rather than ‘knowledge to be taught’. They argue that modelling is perhaps knowledge that is preparatory for the mathematics curriculum. That modelling is not ‘knowledge to be taught’ presents challenges for teacher education.

1.2 Helping Teachers

Teachers face the biggest challenges when modelling is introduced into the curriculum. All teachers have a well-established pattern of professional practice involving a spectrum of ‘moves’ in the classroom that covers the situations they face day-by-day. For most teachers of mathematics, this spectrum does not cover all the skills that teaching modelling requires. These include the teacher:

- Moving from a view of mathematics as a large set of separate “things to learn” to seeing it as a well-connected set of concepts and skills that, used flexibly, enable you to solve problems and understand the real world better. (In the teaching of language, this would mean moving from seeing the language as a set of rules for spelling and grammar to a focus on reading and writing substantial pieces – a focus which, of course, all native language teachers have).
- [AU3] Moving from short item fragments of mathematics to tasks involving longer chains of reasoning – with a teacher’s focus on students’ reasoning – to just their answers.
- Giving students greater responsibility for their own and each others’ learning, moving them into “teacher roles” like explaining their assumptions and assessing each others’ reasoning.
- Becoming a diagnostician and adviser rather than a source of answers and summative right/wrong judgements – for modelling, it is an essential part of the students’ job to decide if their solutions and reasoning are correct (as in life).

Three of the papers in this chapter address these challenges, and the teacher [AU4] education programmes that address them. All three relate to LEMA, a Europe-wide professional development course focused on modelling.

Katja Maaß and Johannes Gurlitt describe the overall framework of the LEMA project and its evaluation. They outline the theoretical background, the design of the course and its evaluation, and the results.

The design of the course focused on the following theoretical ideas: modelling and its teaching, professional development of teachers, and their beliefs about self-efficacy and about mathematics education – procedures and formalism, processes and application. These ideas were embodied in the five modules of the course.

These addressed, respectively, modelling, tasks, lessons, assessment, and reflection. The course was piloted in six countries.

The cross-country evaluation used a pre–post control group design, investigating the following questions with the differences from the control group briefly noted here:

1. Does the professional development course influence the pedagogical content knowledge of the teachers? Improvement.
2. Does the professional development course influence teachers' beliefs about mathematics education? No change.
3. Does the professional development course influence teachers' self-efficacy regarding modelling? Improvement.
4. Does success in one dimension include success in another dimension (correlational)? Results on 1–3 above correlated at about 0.5.
5. How satisfied were teachers with the professional development course? High level of satisfaction.

The course was appreciated and had a positive effect on the teachers' pedagogical content knowledge and modelling self-efficacy, without affecting their beliefs.

Geoff Wake looks at case-study data generated by five teachers as an 'e-narrative' of their work in their classrooms at the initial stages of introducing modelling activities in the UK context, which is heavily dominated by tests that do not include modelling. He seeks to identify the issues of concern to teachers as they attempt to change their pedagogic practices to include mathematical modelling and to draw conclusions for professional learning both in general, and in relation to modelling in particular. He uses cultural historical activity theory (CHAT), which provides a tool for understanding how the work of a collective, such as teacher and pupils in mathematics classrooms, is mediated by different factors. Success in mathematics (and English) tests of a school's pupils at age 16 is crucial in the school's annual performance measure, an important factor in seeing low-risk strategies in teaching mathematics. This manifests itself in classroom discourse and behaviour that focuses heavily on 'the test', with little in the way of assessing modelling sub-competencies such as 'interpretation'. It is in this culture that the teachers need to find the motivation to adopt new pedagogies that support modelling.

We see, through the lens of CHAT, that expansive professional learning in relation to teacher knowledge cannot be left to chance. The case studies suggest that, in general, professional learning requires the intersection of three important factors:

1. The key personnel involved must have at least approximately aligned long term goals, and a professional expertise and understanding of the context that allows them to work within the rules of the system but adapt these to the benefit of the desired professional learning.
2. A climate in which new or potentially emerging rules appear to mediate an expansion in the object of activity.
3. Networks of personal relationships that facilitate ease of communication and boundary crossing.

In the particular case of teacher development in relation to modelling, the case studies point to the teachers' and students' changing roles in the classroom as being of most concern to teachers as they first use modelling activities. This has important implications for those supporting such changes through facilitating professional development.

Javier García and Luisa Ruiz-Higueras use an anthropological approach to the analysis of teachers' practices to study the changes resulting from the LEMA course in teachers' practices involving modelling and applications. They consider the problem of initial teacher training and professional development (particularly in modelling and applications) as a problem of the teaching profession more than a teacher's problem. They use the anthropological theory of didactics (ATD), which assumes that:

- In order to understand how individuals act, we need to know first how the institutions they belong to *act*.
- Mathematics as a human activity requires the integration of the *practical* with the *theoretical* aspects.

They argue that teaching modelling and applications is more than a problem that teachers face in their classroom; it should be considered a problem that the teaching profession faces due to systemic changes, in the way mathematics is being considered, in the new general aims assigned to schooling, and in the inadequate training of those responsible for developing the curriculum. The paper describes how this generates professional problems like: How can real contexts and situations be used in order to give meaning to mathematics? So, for example, teachers who like a modelling approach face didactical problems like: How can the ideas given in this textbook be restated to encourage students to explore 'variables' in real contexts?

LEMA offers teachers opportunities to develop their practice with a wide range of teaching techniques that support modelling, and to reflect and justify the why of these practices that is to develop their theoretical perspective.

1.3 Situations for Modelling

The final seven papers all present interesting and novel situations that have been used in the teaching and learning of modelling at high school or undergraduate level.

Hans Humenberger's paper is concerned with a problem of interest and concern to most of us – how does Google decide where different websites appear in the list resulting from a search. He shows how this can be tackled at undergraduate level, using a wide range of mathematical techniques including directed graphs and transition matrices that embody the Markov process structure of the problem. The general approach can be realized in more elementary form with a spreadsheet.

Matthias Brandl describes an internet portal that offers modelling problems aimed at gifted upper high school students. He describes the results of an attitude survey which suggests that mathematically gifted and highly gifted students are (very) interested in applicability and therefore in word problems with connection to real world application. He presents two examples that emphasize modelling processes:

- Using the context[3] of a champagne glass, which cone made from a circle of fixed radius, has the greatest volume
- How does the number of winners in a lottery depend on the number of participants?The first example involves straightforward algebraic modelling and the second more challenging probability theory.

Usha Kotelawala shows how basic models of reliability theory can provide motivating problems for secondary students as they develop skill and understanding in probability and algebra within secondary mathematics. The author gives and outline of reliability theory, presented as a simple application of probability, and gives simple examples such as:

- Grandma called yesterday to ask you to help her hang holiday lights for the winter season. Her lights are the old type of light strings. If one bulb fails, the whole string fails. The probability of each bulb working is 0.75. If one of the strings has only two bulbs (a short string for illuminating very small areas) what is the probability of the string working?
- FlyCheap Airlines has only one route from London to Hamburg. Recently, the company purchased a used 727 jet airplane having three engines. It can actually fly on just one engine. For each engine, the probability of it working over the course of the trip is 0.98. What is probability that the plane will be able to successfully fly? Find at least two different ways to determine your answer.

Before moving onto more complex (both "series" and "parallel").

As well as requiring reliable procedural skills (simple and complex substitution, expressing and simplifying polynomials, solving polynomial equations), the problems included opportunities for students to work on inductive reasoning for generalized models and the strategy of simplifying parts of a problem along the path to a larger solution.

Tetsushi Kawasaki and Seiji Moriya address the widespread concern in Japan at the estrangement of science from mathematics in the mathematical curriculum of Japanese senior high schools. For example, in order to connect mathematics to the fields of Newtonian physics through mathematical modelling, they provided concrete examples. The one described in some detail focused on 'Kepler's Laws'. First Kepler's "data", summarized in the first and second laws, is provided and analysed, with IT support. Then Newton's second law and that of gravitation is described. The students focus on numerical solutions using Euler's method,

[3] Nice but not directly relevant – they don't make champagne glasses that way.

confronting the error and stability issues. The evaluation suggests that this approach succeeded in increasing students' knowledge about the laws with the simulation of planetary movements and, at the same time, their making of a "mathematical development model". It also suggests that mathematics materials involving physical perspectives are effective for senior high school students.

Mette Andresen and Asbjoern Petersen describe a study of upper secondary students in Denmark using technology in modelling chemical equilibrium, a context that integrates mathematical and chemical modelling.

'Multi-disciplinarity' was prescribed in Danish Upper Secondary Schools' curriculum by governmental regulations, with requirements centred on applications of, and reflections upon each subject. The revision of mathematics teaching is intended to support the students' knowledge about 'how mathematics adds to understanding, formulating and treating problems in different subject areas' and to know about mathematical reasoning. The learning goals served as a basis for the design of multidisciplinary mathematics teaching.

This example is based on dynamic equilibrium in chemistry in which, for example, the two-way reaction comes to equilibrium when the forward reaction rate (proportional to the concentrations of A and B) and backward reaction rate (similarly for the concentrations of C and D) become equal. The students worked on a project involved both theoretical and experimental work, leading to written reports. These were analysed for their perceptions of modelling in a multi-disciplinary teaching environment. The authors found that there was little evidence of a focus on the modelling[4]. The reason for this are discussed.

Martin Bracke and Andreas Geiger describe in their paper experiences with a long-term teaching experiment, in which they included real-world modelling examples in regular lessons. They describe several challenging realistic tasks such as the design of a track for the German high-speed rail. They evaluated the tasks and showed that students were mainly interested and fascinated by this kind of modelling activities. The students reported that they have learned modelling and sometimes even changed their attitude towards mathematics.

The paper by Gabriele Kaiser, Björn Schwarz and Nils Buchholtz reports on modelling activities with even more demanding authentic modelling problems in the framework of modelling weeks with students from upper secondary level. Students' solution on one of this demanding task, namely the development of infected lady bugs, were described and show that the students reached impressive solutions from a mathematical and real world perspective. Furthermore, the results of the evaluation point out that most of the students were deeply impressed by these kinds of authentic examples and really interested in these activities.

Both approaches show that ambitious modelling examples can be dealt with in mathematics teaching, but they require long-term processes, but then lead to unexpected high results.

[4] This reflects earlier work that found that, when actual experiments are involved, making these work took most of the students' attention, leaving little for scientific reasoning.

Author Queries

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Queries	Details Required	Author's Response
AU1	Kindly check affiliation.	
AU2	Please check if edit to sentence starting "This is generic…" is okay.	
AU3	Please check if edit to sentence starting "Moving from short…" is okay.	
AU4	Please expand LEMA.	