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# EXTENDED TASKS FOR GCSE MATHEMATICS

A series of modules to support school-based  
assessment

Statistics  
and  
Probability  
Finding Out



MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

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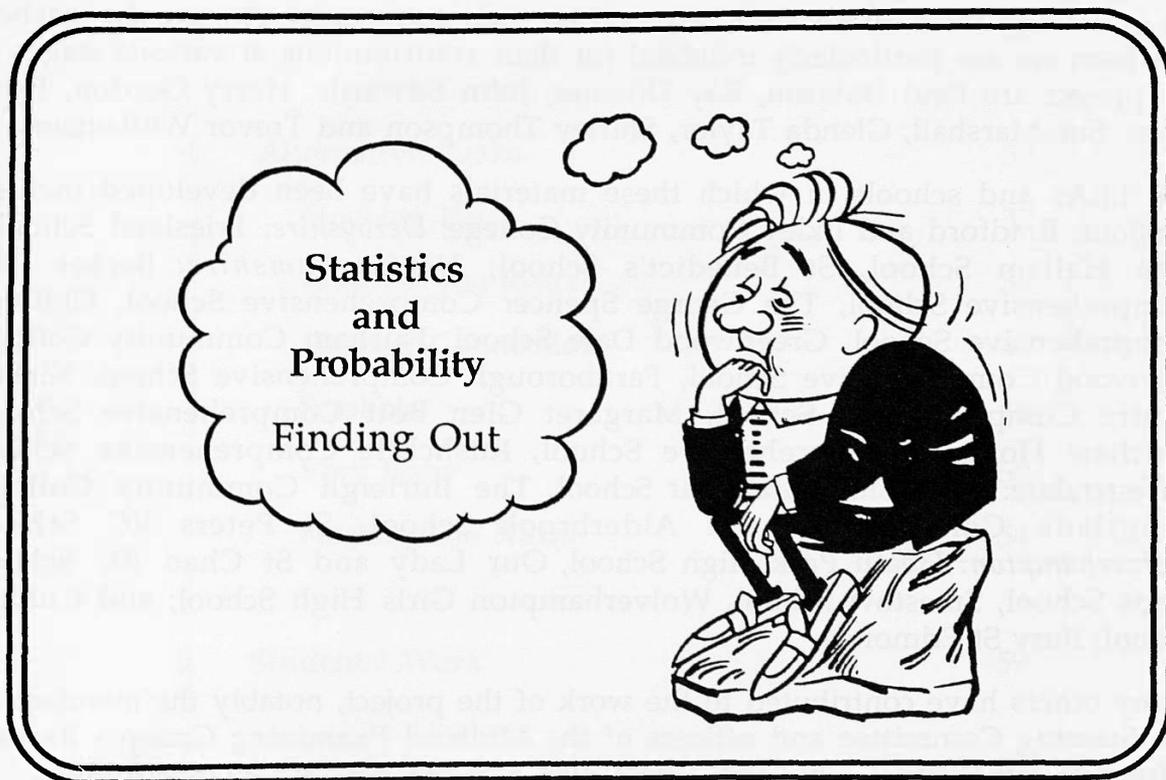


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## FOR GCSE

### MATHEMATICS

A series of modules to support school-based  
assessment



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## Authors

This book is one of a series forming a support package for GCSE coursework in mathematics. It has been developed as part of a joint project by the Shell Centre for Mathematical Education and the Midland Examining Group.

The books were written by

*Steve Maddern and Rita Crust*

working with the Shell Centre team, including Alan Bell, Barbara Binns, Hugh Burkhardt, Rosemary Fraser, John Gillespie, Richard Phillips, Malcolm Swan and Diana Wharmby.

The project was directed by Hugh Burkhardt.

A large number of teachers and their students have contributed to this work through a continuing process of trialling and observation in their classrooms. We are grateful to them all for their help and for their comments. Among the teachers to whom we are particularly indebted for their contributions at various stages of the project are Paul Davison, Ray Downes, John Edwards, Harry Gordon, Peter Jones, Sue Marshall, Glenda Taylor, Shirley Thompson and Trevor Williamson.

The LEAs and schools in which these materials have been developed include *Bradford*: Bradford and Ilkley Community College; *Derbyshire*: Friesland School, Kirk Hallam School, St Benedict's School; *Nottinghamshire*: Becket RC Comprehensive School, The George Spencer Comprehensive School, Chilwell Comprehensive School, Greenwood Dale School, Fairham Community College, Haywood Comprehensive School, Farnborough Comprehensive School, Kirkby Centre Comprehensive School, Margaret Glen Bott Comprehensive School, Matthew Holland Comprehensive School, Rushcliffe Comprehensive School; *Leicestershire*: The Ashby Grammar School, The Burleigh Community College, Longslade College; *Solihull*: Alderbrook School, St Peters RC School; *Wolverhampton*: Heath Park High School, Our Lady and St Chad RC School, Regis School, Smestow School, Wolverhampton Girls High School; and Culford School, Bury St Edmonds.

Many others have contributed to the work of the project, notably the members of the Steering Committee and officers of the Midland Examining Group - Barbara Edmonds, Ian Evans, Geoff Gibb, Paul Lloyd, Ron McLone and Elizabeth Mills.

Jenny Payne has typed the manuscript in its development stages with help from Judith Rowlands and Mark Stocks. The final version has been prepared by Susan Hatfield.

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# 1 Introduction

FINDING OUT is one of eight such 'cluster books' each offering a lead task which is fully supported by detailed teacher's notes, a student's introduction to the problem, a case study, examples of students' work which demonstrate achievement at a variety of levels, together with six alternative tasks of a similar nature. The alternative tasks simply comprise the student's introduction to the problem and some brief teacher's notes. It is intended that these alternative tasks should be used in a similar manner to the lead task and hence only the lead task has been fully supported with more detailed teacher's notes and examples of students' work.

The eight cluster books fall into four pairs, one for each of the general categories: Pure Investigations, Statistics and Probability, Practical Geometry and Applications. This series of cluster books is further supported by an overall teacher's guide and a departmental development programme, IMPACT, to enable teacher, student and departmental experience to be gained with this type of work.

The material is available in two parts

<b>Part One</b>	The Teacher's Guide
	IMPACT
Pure Investigations	I1 - Looking Deeper
	I2 - Making The Most Of It
Statistics and Probability	S1 - Take a Chance
	S2 - Finding Out
<b>Part Two</b>	G1 - Pack It In
Practical Geometry	G2 - Construct It Right
	A1 - Plan It
Applications	A2 - Where There's Life, There's Maths

This particular 'cluster book', FINDING OUT, offers a range of materials designed to support students as they pursue extended tasks relating to statistical investigations. The material has been designed and tested, as extended tasks, in a range of classrooms. A total of about twelve to fifteen hours study time, usually over a period of two to three weeks, was spent on each task. Many of the ideas have been used to stimulate work for a longer period of time than this, but any period which is significantly shorter has proved to be rather unsatisfactory.

It is important that students should experience a variety of different types of extended task work in mathematics if they are to fully understand the depth, breadth and value of the subject. Statistical tasks based upon students' everyday experiences and interests usually stimulate a great deal of enthusiasm in the classroom. The analysis of real data which is of some personal significance can be much more rewarding for students than the completion of exercises containing second hand data. Consequently, the tasks begin within real contexts drawn from everyday life.

The common element amongst all the items within this cluster is that they allow for statistical analyses of real situations, according to the individual need and ability of each student: hence the title of the cluster, FINDING OUT.

Clearly, there are many styles of classroom operation for GCSE extended task work and it is intended that this pack will support most, if not all, approaches. All the tasks outlined within the cluster books may be used with students of all abilities within the GCSE range. The lead task of *Why Are We Waiting?* may be used with a whole class of students, each naturally developing their own lines of enquiry. It is intended that all the tasks within the cluster may be used in this manner. However, an alternative classroom approach may be to use a selection, or even all, of the ideas within the cluster at one time, thus allowing students to choose their preferred context for their statistical study. There is, however, a further more general classroom approach which may be adopted. This is one that does not even restrict the task to that of a statistical nature. In this case some, or all, of the items within this cluster may be used in conjunction with those from one or more of the other cluster books, or indeed any other resource. The idea is that this support material should allow individual teacher and class style to determine the mode of operation, and should not be restrictive in any way.

Teachers who are new to this type of activity are strongly advised to use the lead tasks.

These introductory notes should be read in conjunction with the general teacher's guide for the whole pack of support material. Many of the issues implied or hinted at within the cluster books are discussed in greater detail in The Teacher's Guide.

# 2

## *Why Are We Waiting?*

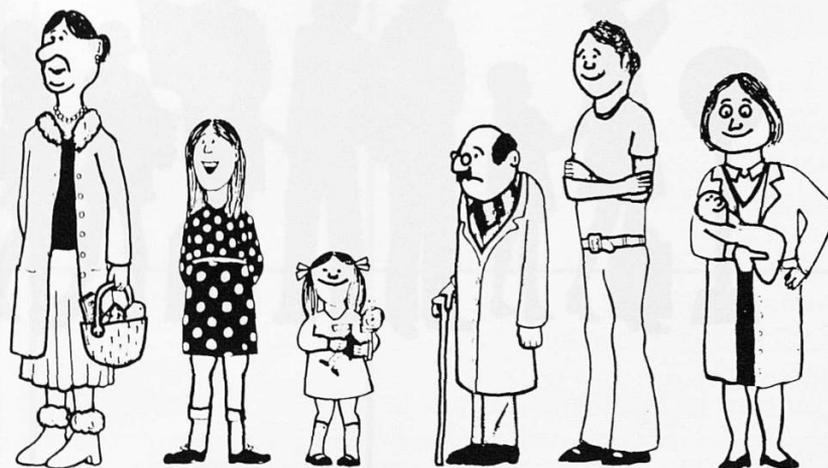
The lead task in this book is called *Why Are We Waiting?* It is based upon real life experiences, and provides a rich and tractable environment for extended coursework tasks at GCSE level.

The tasks are set out on pages 7-17 in a form that is suitable for photocopying for students.

The Teacher's Notes begin on page 18. These pages contain space for comments based on the school's own classroom experiences.



## WHY ARE WE WAITING?



This topic is based on the idea of queueing. You will have met queues many times in your life, and no doubt you will stand in many more queues in the future. During this investigation into queues, you will have the opportunity to talk about the different types of queues you have met, and you will be asked to investigate a few of them in depth.

You will be introduced to several different queueing systems and you are free to investigate them in any way you wish, although certain suggestions are offered to you. After considering the suggestions provided, you may find that you have a better idea about further investigations you could carry out.

When you have explored the queueing systems suggested on the resource sheets, you may like to look at a queue of which you are sometimes a part.

As usual with your GCSE coursework, it is important that you should ask your own questions about your work as you go along, and that you should keep an account of what you do and why you are doing it. You will need to present a final written report of the work you complete relating to this task.

## QUEUEING SYSTEMS



You should work on this task in small groups. Write down five different queueing systems you have come across at some stage in your life. You will have about twenty minutes to fill in the table on the next page.

A spokesperson from each group will then have some time to explain your agreed suggestions to the other groups in the class. You will score five points for each system which is accepted as a proper queueing system by the other groups, but one point will be knocked off for every group which also has that system, so try to think of unusual queueing systems. However, you score zero if they are not accepted.

For example

1	System A - accepted - one other group has it	= 4 points
2	System B - not accepted	= 0 points
3	System C - accepted - four other groups have it	= 1 point
4	System D - accepted - no other group has it	= 5 points
5	System E - accepted - four other groups have it	= 1 point
	Total	= 11 points

**QUEUEING SYSTEMS** : continued

No	Where have you seen this queue?	Write down some details about this queue
<b>1</b>		
<b>2</b>		
<b>3</b>		
<b>4</b>		
<b>5</b>		

## INTERVIEWS



As a part of one particular school's GCSE assessment scheme in Mathematics, each student has a discussion with one of the teachers for about ten minutes towards the end of their fifth year.

During this time students are invited to talk about the extended tasks they have carried out and they are also asked a few questions about the mathematics they have learned.

A programme of interview times is pinned on the fifth year Common Room notice-board one week in advance, so that everyone has time to think about the things they would like to discuss.

The following programme appeared for one particular afternoon.

GCSE MATHEMATICS ORAL ASSESSMENT	
DATE	FRIDAY 22 MAY AFTERNOON
TEACHER	MISS STABLES
TIME	STUDENT
2.00	Tammy Whitaker
2.10	Jason Hargreaves
2.20	Rajinder Ubhi
2.30	Melanie Nesbitt
2.40	Winston Bailey
2.50	Kung Ip
3.00	Noreen Dyson
3.10	Walter Bassett
3.20	Jenny Payne
3.30	Susie Reed
3.40	Tony Singh
3.50	Sally Nesbitt

*INTERVIEWS:* continued

One of the fourth year students decided to investigate the actual time taken for these interviews as a part of an extended task for her GCSE coursework in Mathematics.

She recorded the following data.

LENGTH OF STUDENT INTERVIEWS

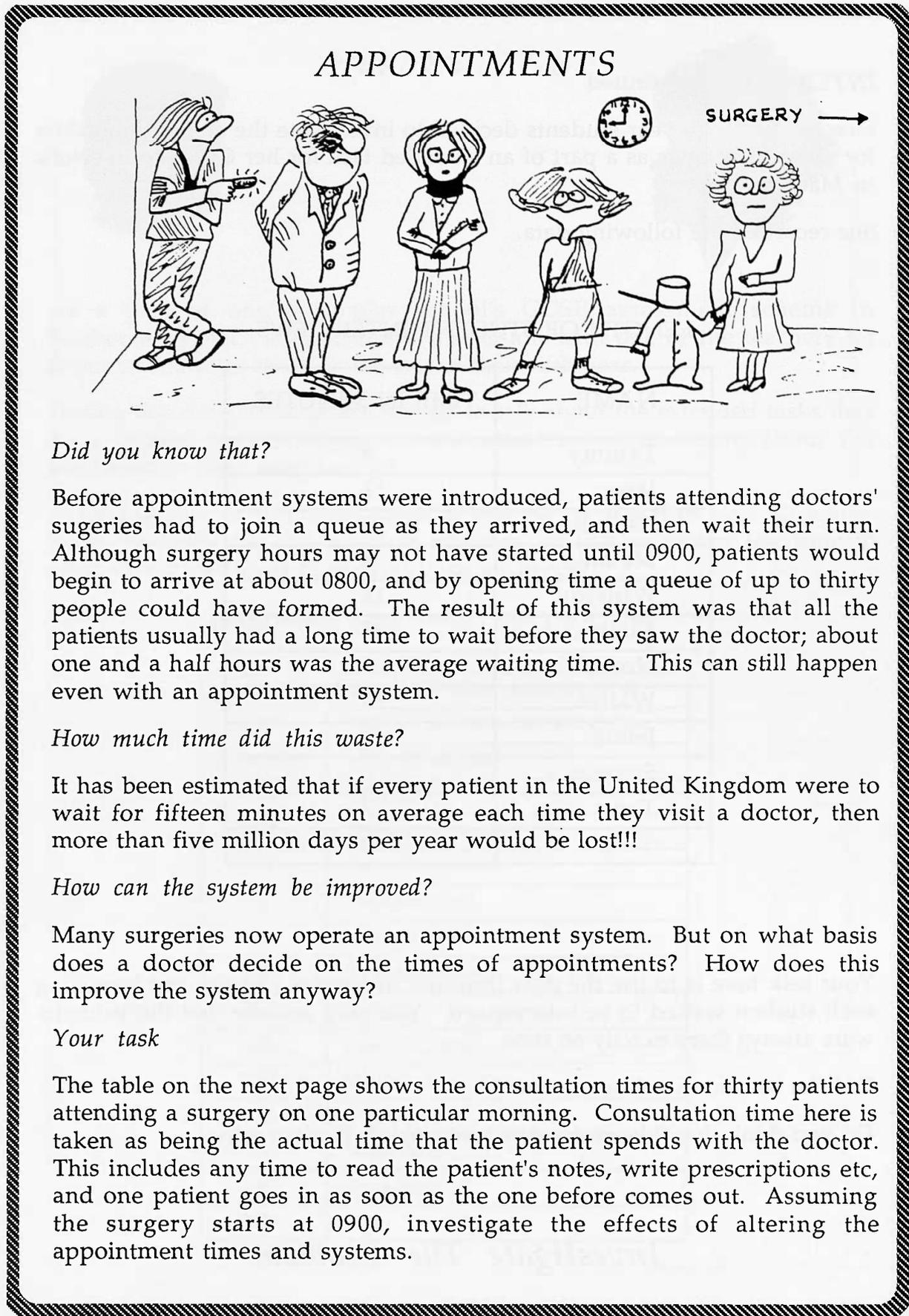
NAME	TIME IN MINUTES
Tammy	9
Jason	12
Rajinder	10
Melanie	8
Winston	12
Kung	7
Noreen	8
Walter	10
Jenny	9
Susie	7
Tony	7
Sally	10

Your task here is to use the data from the two tables to work out how long each student waited to be interviewed. You may assume that the students were always there exactly on time.

Did the teacher ever have to wait?

Do you think that this situation is reasonable? Explain why.

*Investigate The Problem*



*Did you know that?*

Before appointment systems were introduced, patients attending doctors' surgeries had to join a queue as they arrived, and then wait their turn. Although surgery hours may not have started until 0900, patients would begin to arrive at about 0800, and by opening time a queue of up to thirty people could have formed. The result of this system was that all the patients usually had a long time to wait before they saw the doctor; about one and a half hours was the average waiting time. This can still happen even with an appointment system.

*How much time did this waste?*

It has been estimated that if every patient in the United Kingdom were to wait for fifteen minutes on average each time they visit a doctor, then more than five million days per year would be lost!!!

*How can the system be improved?*

Many surgeries now operate an appointment system. But on what basis does a doctor decide on the times of appointments? How does this improve the system anyway?

*Your task*

The table on the next page shows the consultation times for thirty patients attending a surgery on one particular morning. Consultation time here is taken as being the actual time that the patient spends with the doctor. This includes any time to read the patient's notes, write prescriptions etc, and one patient goes in as soon as the one before comes out. Assuming the surgery starts at 0900, investigate the effects of altering the appointment times and systems.

*APPOINTMENTS* : continued

You should consider at least two different appointment systems and compare any aspects you consider to be important. For example

- \* the time the doctor has to wait
- \* the time the patient has to wait.

What appointment system would you advise this doctor to use? You should explain your reasons clearly.

CONSULTATION TIMES AT A GP'S SURGERY

PATIENT NUMBER	CONSULTATION
1	6
2	2
3	5
4	1
5	7
6	16
7	2
8	5
9	4
10	7
11	2
12	7
13	2
14	3
15	4
16	1
17	3
18	6
19	3
20	5
21	4
22	18
23	6
24	3
25	5
26	1
27	4
28	19
29	3
30	5

*Investigate The Problem*

## CONSULTATION TIMES I



During a typical week Dr Loveday sees a hundred patients. A frequency table showing their consultation times, correct to the nearest minute, is as shown below.

CONSULTATION TIME IN MINUTES	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
NUMBER OF PATIENTS	0	10	16	18	16	12	10	6	4	3	2	0	0	0	0	0	1	0	0	2	0

If Dr Loveday uses the appointment system you recommended after completing the task *Appointments*, what do you think is likely to happen on a typical day when he sees thirty patients. You should consider

- \* the time the doctor has to wait
- \* the time the patients have to wait

### *Investigate The Problem*

### CONSULTATION TIMES : continued

After we have collected some real data, we can use it to make a mathematical model. We can then use our model to predict what could happen in certain circumstances.

We can simulate this situation by using a hundred squares similar to the ones which are provided below. Each square represents a consultation time. Since ten patients consulted the doctor for one minute, ten squares contain the number 1. Since four patients consulted the doctor for eight minutes, four squares contain the number 8.

Cut the large square into a hundred small squares and place them in a box. Take one square at random from the box and read the number written on it. This is the consultation time for a patient. Replace the square in the box, then take out another square at random to find the consultation time for the next patient.

Why do you think we replace each square?

Use this model to simulate Dr Loveday's surgery for thirty patients on a typical day, using your recommended appointment system.

Can you improve your recommendations?

1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	3	3	3	3
3	3	3	3	3	3	3	3	3	3
3	3	3	3	4	4	4	4	4	4
4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5
5	5	6	6	6	6	6	6	6	6
6	6	7	7	7	7	7	7	8	8
8	8	9	9	9	10	10	16	19	19

## CONSULTATION TIMES II

Over a period of about a month, patients were observed in Dr Singh's surgery. The following table summarises the observations made for eight hundred patients.

PERCENTAGE FREQUENCY DISTRIBUTION OF A DOCTOR'S CONSULTING TIMES				
Consultation time interval in minutes	Consultation mean time in minutes	% frequency of patients	Cumulative % frequency	Random number range
0.5 - 1.5	1	10	10	00 - 09
1.5 - 2.5	2	16	26	10 - 25
2.5 - 3.5	3	18	44	26 - 43
3.5 - 4.5	4	16	60	44 - 59
4.5 - 5.5	5	12	72	60 - 71
5.5 - 6.5	6	10	82	72 - 81
6.5 - 7.5	7	6	88	82 - 87
7.5 - 8.5	8	4	92	87 - 91
8.5 - 9.5	9	3	95	92 - 94
9.5 - 10.5	10	2	97	95 - 96
10.5 - 11.5	11	0	97	
11.5 - 12.5	12	0	97	
12.5 - 13.5	13	0	97	
13.5 - 14.5	14	0	97	
14.5 - 15.5	15	0	97	
15.5 - 16.5	16	1	98	97
16.5 - 17.5	17	0	98	
17.5 - 18.5	18	0	98	
18.5 - 19.5	19	2	100	98 - 99
19.5 - 20.5	20	0	100	
X	X	100	X	X

We can use random numbers to simulate the distribution of these consultation times.

We can begin anywhere in the table of random digits, and read pairs of digits from that point.

For example, if we choose to begin 8th block down, 3rd block across, 2nd row, 3rd column, 4th number we get

49 29225 12200

The first pair is 49, the next pair is 29, then 22, 51, 22, 00 ... We then find 49 in the right hand column of the table. This falls in the range 44-59 and so the simulated consultation time for this patient is four minutes. The next patient has a random number 29, and this give us a time of three minutes.

**CONSULTATION TIMES** : continued

Assuming the same distribution of consultation times, use the tables to simulate your recommended queueing system for thirty patients on a typical day. Are you still happy with your recommendations?

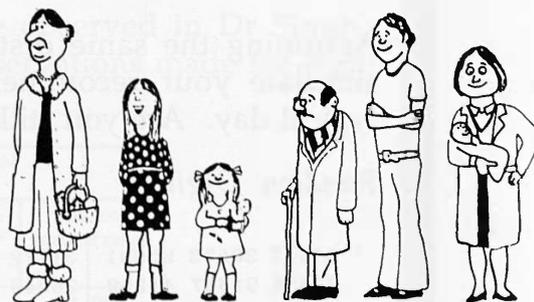
*Random Digits*

83427	30573	95991	31772	36247	74764	29667	58907	35907	70139	06428	11711
37188	91337	41259	26946	94429	08191	25547	88601	16363	68222	24702	11854
68802	72043	39060	80072	34376	57779	28872	65855	85617	05163	78336	38856
87756	46954	98326	34606	28169	63821	68438	14585	19886	49968	72255	52982
32479	61502	71229	80822	00192	32509	73582	13615	06586	55934	68238	51091
00954	58296	74188	63081	47051	10491	10533	94994	77168	53199	64161	44339
18603	55950	81881	73031	56817	16449	07624	98831	97241	47812	50023	74990
13193	30134	87624	45202	39599	88946	76166	89687	91467	94056	42948	75855
67355	33289	33603	26994	08442	16468	03601	87916	75078	01517	40364	13365
35422	27149	07464	74001	31186	86781	86444	28670	98235	05297	31868	95682
75807	84816	40285	23532	08680	73536	42985	38717	23725	40582	45238	45101
35672	03620	21061	22186	29322	85970	87642	34944	32999	26306	38562	57114
10358	33993	12445	97293	95235	34181	55570	06025	69592	65151	29370	84093
25091	67600	82364	70673	56623	49139	21713	24091	71135	68704	34304	37803
07362	62185	90928	81720	33628	27817	65168	00417	90613	14743	91644	28195
67812	63699	05909	03934	62038	76436	68917	49126	51499	81172	82120	91345
49275	65322	49226	14211	97562	12826	70288	81201	10242	55983	06569	81575
47038	18658	61204	48365	06779	29783	60191	90007	15693	54120	41242	73477
47478	10525	96481	61672	74924	69243	06223	44817	77923	46639	16546	08007
42183	61450	22726	40785	36344	68403	87973	82080	67778	93298	58912	06894
41099	61670	15324	25757	72396	97321	49578	78851	64836	00591	29791	15980
47458	02763	08098	34745	47736	12313	45516	73593	15078	73196	14595	97729
91827	67173	11236	49942	23952	66074	18112	13724	81216	33020	41698	32042
31779	77687	38474	89156	79233	86820	80648	68378	95052	26134	40517	40557
62587	04441	62927	90218	09311	18356	63640	93071	90809	41392	19993	09714
10811	39969	67359	34789	74373	15005	63614	28305	64261	95474	92745	99766
19159	81542	07824	37186	86793	67503	16780	97351	82444	70470	57871	76554
88684	53973	27498	22389	25271	83354	38577	40580	36903	22227	75806	83158
66082	10061	68475	78824	13182	25199	18286	15469	62584	67176	33742	59739
02932	05046	64735	57147	64868	23737	96782	94586	15971	47849	22998	64712
44780	29400	31838	83384	16717	25228	05071	90931	80374	02341	98801	65660
47134	54707	62427	48228	53612	99416	70965	70621	78625	11379	46763	86133
24501	40254	30090	91079	92991	31583	77839	95514	12016	82067	35612	59298
13849	56488	31359	03161	72568	62687	83020	36460	30742	41834	44608	36057
42906	95353	53183	04840	16094	90022	13115	49700	96897	42876	36655	95145
43033	89828	01285	69053	10162	88727	67113	20972	46418	35737	28977	62263
24106	91081	25229	45343	45179	51375	97690	80407	98749	29225	12200	21004
25608	35741	88043	48470	51661	50266	32983	33201	78193	09839	70489	44605
08015	46152	05996	70459	42400	59247	10947	26348	72850	95323	80554	61946
07615	54194	18878	91407	84330	64888	21925	95403	20784	71664	75714	24917

*Investigate The Problem*

## Why Are We Waiting? - Teacher's Notes

*Why are we waiting?* is designed to support students as they develop and pursue extended tasks, based upon the realistic use of a variety of statistical ideas applied to queueing situations. As with any GCSE extended task, students should be encouraged to ask their own questions and to seek answers to their questions. This situation, together with the series of broad tasks which are offered on the student's resource sheets, will involve them in



- \* extracting data from tables
- \* combining data from various sources
- \* analysing queueing data
- \* modelling queueing situations
- \* using random number tables and frequency distributions to simulate queues
- \* carrying out related calculations
- \* problem posing
- \* problem solving
- \* applying any mathematics they feel appropriate to help them with their own individual lines of enquiry
- \* making and stating assumptions, restrictions and constraints regarding their own queueing models.

Students should be provided with opportunities to reflect on their own experiences of queueing systems and perhaps use role play to simulate different systems. Their experiences and suggestions should be explored through group and class discussion. This may involve them in discussing situations such as those shown on the resource sheets, taking part in both small group and class discussions about queues, and investigating queues at bus stops, supermarkets, banks, building societies etc. This demonstrates how GCSE coursework can be completed while at the same time introducing students to, and allowing them to acquire an understanding of,

many new concepts in what may appear to be a rather difficult area of mathematics.

The early tasks may appear to be a fairly closed and directed set of worksheets. They do, however, illustrate certain concepts and contexts before leaving students to pursue their own investigations. Naturally, account is taken of this situation when applying any GCSE assessment scheme to the work. The more closed aspects of the work may well contribute to, say, accuracy but certainly not to the assessment of overall design. This would be assessed entirely in the more open aspects of the work.

It is useful to bear in mind the following

- \* It is important that students are able to handle real data and draw inferences.
- \* Handling real data can be difficult: organising lots of data can be very demanding.
- \* Students may find it helpful to use a computer to generate random numbers, compile tables and/or carry out an entire simulation if one is available.
- \* Simulations can be fun!

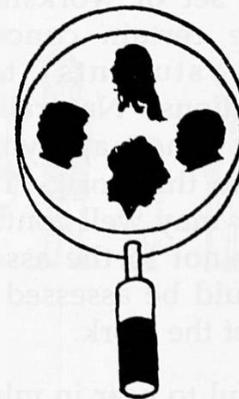
### *Understanding and Exploring the Problem*

The resource sheets *Queueing Systems*, which are on pages 8-9, should be presented to small groups of students. Allow the groups about twenty minutes to discuss the activity and complete the table on the second sheet. The amount of time needed may vary for different groups. The idea is to start from a 'zero position' and to provide students with the opportunity to discuss freely their understanding of queues and to set up their own definition. If there is no teacher introduction, or very little, as is suggested, then the discussion is likely to be broad, and more beneficial than the carrying out of the activity after a closed, tight teacher-definition of a queue.

Each group should appoint a spokesperson to report their findings to other groups. This will introduce the idea of queues in a natural way, as



well as their related differences and similarities. Students will probably begin to raise some important issues at this stage, since they will need to decide whether to accept or reject each queue suggested. Teachers ought to be prepared for some heated discussion at this stage, since many different queue classifications are likely to be presented by the groups. During classroom trials of this material, it was found beneficial to have about four groups in the classroom, and to allow the spokespersons from each group to take turns in presenting one queue for discussion. This situation prevents a particular group from dominating the discussion during the reporting back session. This activity may well take longer than expected, but this depends very much upon how lively the discussion is. It is also a nice idea to follow this initial activity by getting each group to write their own definition of a queue. Alternatively, you may wish to select key words and phrases from the reporting back session to produce a class definition.



Students will have met or heard about GCSE oral assessment in one or more subjects. This is a good opportunity to discuss the issue with them, and a nice way to develop their understanding of it. The resource sheets *Interviews*, which are on pages 10-11, introduce students to the idea of waiting times. They offer one particular, and quite simple, queueing model and they use a context which is likely to be very real to all GCSE students. It is, perhaps, best that students should initially attempt this work individually. They will encounter the questions of how to tackle the problem, and how to represent their methods and solutions. The assumption that the students are always there exactly on time may well be questioned by some. If this happens then an appropriate response may be, 'Well what do you think is likely to happen?', 'What do you suggest then?', 'How can you include that in your model?'

While working in small groups, it is then extremely useful for students to compare how they achieved their solution, how they presented it and on what grounds they made their decisions about whether or not they felt the situation was reasonable. The discussion of their approaches

will help to put over the idea that there can be many ways of tackling a problem, even if the eventual outcome is likely to be the same. It also allows each student to explain or verbalise her thoughts and approaches. This is something which is likely to help students to clarify their thinking on this issue.

An overall class summary of the variety of methods used to tackle the previous problem is then helpful. This can be achieved by highlighting some of the more common approaches as well as the more unusual ones which were adopted.

It may also be useful to have a brainstorming session on queues. Some of the questions which you may wish to introduce include

- \* Where do we meet queues?
- \* What types of queues are there?
- \* What are the merits of different types of queues?
- \* What can be the disadvantages of queues?

The aim of asking these questions is not to obtain specific answers, but to provide students and teachers with both the time and the opportunity to let their minds range over many possibilities and to provoke many suggestions. One student could be appointed as a scribe to make notes on the blackboard or on blank wall posters. The production of a poster which summarises the outcomes of the brainstorming sessions is particularly useful for later reference. The extent to which this session is useful will depend very much upon the success of the activity *Queueing Systems*. What some students achieve during this brainstorming session may already have been achieved by other students during their previous discussion.

## Devising and Planning Individual Studies

The resource sheets *Appointments*, which are on pages 12-13, may now be considered. The amount of initial support needed for this activity will depend upon individual circumstances. This is also true for the depth to which the investigation is taken. Students should be encouraged to tackle this problem in any way that they feel is appropriate. They should, however, state clearly the assumptions and constraints which they build into their model. Identification of assumptions made and constraints imposed is not an easy task for students. Sensitive individual discussion is often necessary in order to ensure that each student personally identifies these factors. These points should be discussed in their report.



The data provided in the table on page 13 was obtained from the distributions on pages 15 and 17.

It is essential that each student should reach some firm recommendations on the basis of the information provided. These recommendations will be used in the following situations.

The resource sheets *Consultation Times*, which are on pages 14-17, introduce the ideas of simulation using frequency distributions. The task *Consultation Times I*, is intended to provide a gentle introduction to statistical simulation. *Consultation Times II* may prove more appropriate for more able students. Considerable discussion may be required about how this data was collected, how random numbers are obtained and why random digit tables exist.

The discussion may well lead to a consideration of different types of distributions and their related characteristics such as skewness. The task here is to investigate further, using simulated data, the system that the student has suggested as being best for the previous situation. Their decisions, and suggested system, are based entirely on the observations made during just one isolated period.

During classroom trials, students often incorporated features from their personal experience of their own doctor's surgery.

Questions such as

- \* Do you think your system will be the best one possible over, say, a whole month?
- \* What if the patients had arrived in reverse order?
- \* Why do you think certain patients took a long time? How often do you think that they visit the doctor?

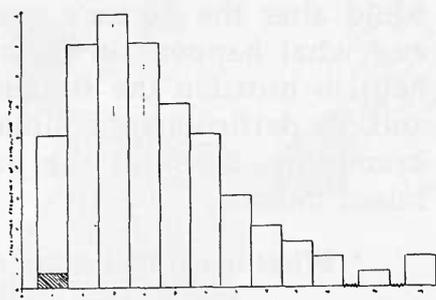
may well stimulate thoughts about how appropriate their individually suggested models may be in general.

An alternative approach to the simulation may well involve using the histogram of the given data. The histogram can be colour coded and cut up into unit pieces. These pieces can then be put into a hat, or similar container, and drawn at random. The colour on the unit piece indicates the consultation time and is, naturally, replaced in the container, and because of the way in which this is obtained it is determined by the original distribution. This approach could lead to a discussion of sampling, both with and without replacement, and the related idea of independent events.

An alternative to using random digit tables is, of course, to generate random numbers using an appropriate calculator or computer program.

Another approach, but one which is perhaps more suited to an able group, is to openly discuss how a simulation could be set up and then take it from there. Alternatively, again perhaps with a more able group, a simulation could be suggested involving the use of a twenty-sided die to produce random consultation times. The assumption here is that times will always be less than twenty minutes. It could then be left to the students to discover the weaknesses of this approach. These situations both lead on to the need for a distribution as provided on the resource sheets.

FREQUENCY DISTRIBUTION OF A DOCTOR'S  
CONSULTATION TIMES BASED ON 800  
OBSERVATIONS OVER A PERIOD OF  
ONE MONTH



### *Implementing Plans and Pursuing Ideas*

The remainder of the time available for this extended task may be used in many ways. The investigation, so far, may well have become more involved and detailed than the one outlined above, and therefore this could form the whole work. Alternatively, the investigation into the doctor's surgery may well continue from this stage onwards in any way the student may feel appropriate. A further alternative is to look at a new situation; perhaps one from the student's own experience. There are many queues which the student will have encountered both at school and in the outside world. It may be worthwhile at this stage to hold a class discussion about how we could alter the doctor's queueing system further and what happens in the real world. This will help to broaden the students' view of queueing and, in particular, the situations they have been examining. Some of the questions that may be raised include



- \* What if we had more doctors?
- \* What if we changed the time intervals?
- \* Can we minimise waiting times for the patients and the doctor?

This discussion may almost be a summary discussion, since the students will probably have raised and discussed many of these issues in their work so far. However, the chance to explain to others is again an important opportunity. During the period when the students are doing their individual work, it is probably best if they set themselves a particular problem to tackle. This gives them a definite situation to work at and report upon. Such problems include

- \* How long does it take to get through the Asda checkout?
- \* Why has the Post Office changed from multiple queues to single queues?
- \* What is the best queueing system for the school canteen?

- \* What is the best way to organise Parents' Evenings?
- \* How can we produce random numbers?
- \* Who uses what types of queue in our area?
- \* How long do cars have to wait to turn right at the end of the road?
- \* Should the Shell Service Station get another pump?
- \* How can we get cars through traffic lights more quickly?

### *Reviewing and Communicating Findings*

The assessment of this task will be based on all the work on queues including the introductory tasks from the resource sheets. Students should submit a full report of what they tried, why they tried it and what they found out.



Also included with these teacher's notes are some suggestions about where we find queues, one particular classification system and a few ideas for further investigation. However, it should be noted that lists of where we find queues and how we classify them have varied considerably in different trial classrooms. Hence, these suggestions are for teachers only, rather than for students. These ideas will need to be interpreted in the context of the local environment.

As with other extended tasks, students will benefit from discussion with their teachers and fellow students during this final stage of their work. They may also find it helpful to imagine that their reports are to be submitted to someone who has the power to implement such changes as they may choose to advocate. Indeed, it may be that a copy of a relevant section of their report will be suitable for submission to the Head of your school, to a local newspaper, or to some local committee. During the classroom trials of these materials, one school investigated the check-out system at their local supermarket. The investigation became a cross-curricular study and their final report was forwarded by the local manager to the head office.

## Where do we find queues?

Doctors  
Dentists  
Hospitals  
Petrol Stations  
Supermarkets  
Post Offices  
Banks  
Building Societies  
Hairdressers  
Crossroads  
Traffic Lights  
School:

Canteen  
Assembly  
Tuck Shop  
Dining Room  
Head Teacher's Office  
Parents' Evening  
Medical Examinations  
Corridors

and many more.

## What types of queue are there?

*Single:* e.g. bus stop

*Single into multiple:* e.g. post office, bank, building society

*Set times:* e.g. appointments at, say, hairdressers

*Numbers on arrival:* e.g. some supermarkets issue numbered tickets for their cut cheese and cooked meat counters.

etc.

## Other ideas for investigation

Mini roundabouts v Crossroads

Traffic light filters

Traffic light phasing

etc.



# 3

## *A Case Study*

### *Fourth Year*

#### *Foundation Level GCSE Group*

I have recently started to teach maths after teaching my own subject, Home Economics, for fourteen years. It was a case of redeployment or change of subject. I agreed to the change of subject and I have been given some retraining. During the trials of the GCSE material, I chose *Why Are You Waiting?* because there seemed to be a considerable amount of support for the teacher.

When I read the teacher's notes in detail, I was rather concerned by the huge amount of number work involved, but I decided to give it a go anyway. I wanted to play down the number work, really, but at the same time I wanted to let the children have a go at it. My concern was partly due to my fear of such vast quantities of data and also the ability of the group. I was very nervous about the whole thing, so I stuck to the teacher's notes but took the suggested approach of letting the children do their own data collection. I expected the initial quiz type activity to take about half of the first lesson, but it went so well that it took us well into the second one hour lesson. This gave me tremendous confidence. It was really exciting and enjoyable for all of us. After this I felt that it was full steam ahead.

The group worked through the ideas on GCSE interviews and queueing at the doctors quite quickly; at least for them. By chance, rather than by design, this was used as an introduction to some of the ideas which you could consider or do with the data, once it was collected.

Although most of the children did similar things at this stage, I was not really worried because they were all involved, and tackling the problems in their own way. Once these problems had been finished, we had a brainstorming session about queues in the local community. I was amazed at how imaginative the group were: some a bit over-imaginative! We discussed this for quite a while, with everyone chipping in whenever they wanted to. We ended up with several good ideas and comments relating to each.

The children were then given total freedom, some worked in pairs, some on their own, and others in small groups, with a maximum of four children in a group. Pairs seem to be a good number for this activity. Each group decided what they wanted to do anyway. However, they had to clear all their plans with me. This meant arranging visits out of school, contacting local shops, building societies etc. The last thing I wanted was someone to be arrested because they were acting suspiciously! Many of the children used the school telephone to arrange their visits. This does not directly relate to GCSE mathematics, but it was quite a good thing for them to do. Apart from shops and building societies, the venues included banks, post offices, the school canteen, a local restaurant, the local station before school time, bus stops and traffic junctions.

The data collected was less elaborate and not as useful as the doctor's surgery material. Many of the students just collected times and worked out averages. They did not try simulations or alternative systems, but this would not really be expected from a group like this.

I was very pleased with what they produced at the end. They wrote up their reports from the notes they kept as they went along. The write-up took a fair amount of time. Overall, I think that this was a worthwhile task for this group, and one which they enjoyed. I had no idea how to mark it though, since I had never done anything like this before.

One of my colleagues also used this project with a group of fifth year higher level students. She did it quite differently, without the individual surveys and it seemed to go just as well. You would never believe that the two sets of work came from using the same material.

# 4

## *Alternative Tasks*

Stocking Up

How Do You React?

Finding Connections

Scrabble

Very Fishy

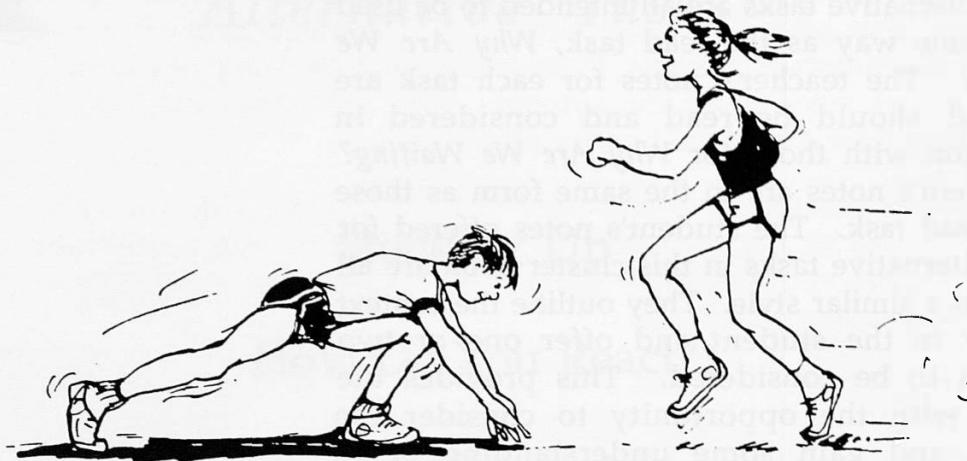
Finding The News

# Alternative Tasks

## General Notes

The six alternative tasks are all intended to be used in the same way as the lead task, *Why Are We Waiting?* The teacher's notes for each task are brief and should be read and considered in conjunction with those for *Why Are We Waiting?* The student's notes are in the same form as those for the lead task. The student's notes offered for the six alternative tasks in this cluster book are all written in a similar style. They outline the context of study to the student and offer one or two problems to be considered. This provides the student with the opportunity to consider the problem and gain some understanding of it. Students are then invited to investigate the problem in any way they wish. However, there are extension ideas which may be used if the teacher feels this is appropriate to any individual student, group or class. These suggestions provide further ideas for investigation without prescribing exactly what should happen.

## STOCKING UP



Paul and Sarah are in charge of the refreshment stall for the school sports day. They intend to sell crisps and cold drinks.

What types/flavours of cold drinks should they order?

What flavours of crisps do you think they should order?

Use the survey sheet provided to help you find out which cold drinks your class prefers. You should add any other questions you feel you need to ask.

Write your own survey sheet for crisps.

Do you think Paul and Sarah should sell anything else?

### *Investigate The Problem*

You may be able to think of other situations in which it would be useful to carry out a survey of other people's views and opinions.

After carrying out your survey you should produce a report based upon the statistical evidence you have collected.

STOCKING UP : continued

## COLD DRINK SURVEY

1. Do you prefer *still* or *fizzy* drinks?

FIZZY	
STILL	
DON'T MIND	

Tick one box

2. What flavours of drinks do you enjoy?

	LOVE IT! ☺	O.K. —	HATE IT! ☹
APPLE			
BLACKCURRANT			
LEMON			
LIME			
ORANGE			

Tick one box  
for each flavour

3. What type of orange drink do you like best?

PURE ORANGE JUICE	
ORANGE SQUASH	
FIZZY ORANGEADE	

Tick one box

4. What brand of orange drink do you enjoy most?

For example; Robinsons, Sainsburys, Schweppes, .....

Name one

## Stocking Up - Teacher's Notes

The initial task outlines a situation in which data needs to be collected. It also provides a survey sheet which can be used to collect some of the data students need as they attempt to answer the question; *What types/flavours of cold drinks should they order?*

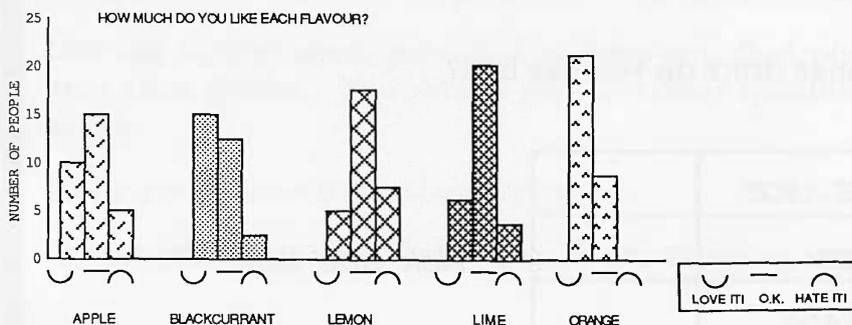
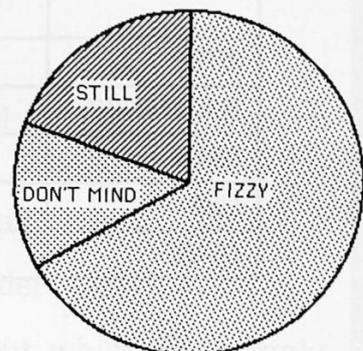
In order to answer the questions

- \* What flavours of crisps do you think they should order?
- \* Do you think they should sell anything else?

Students will need to devise their own questionnaires/survey sheets.

This task is intended to provide an opportunity for students to design and use an appropriate questionnaire, with three or more possible responses to at least some questions.

Having written their survey sheets, students will need to collect and analyse their data, using statistical techniques including graphical representation using pie-charts, pictograms or bar charts. They may also feel a need to calculate means and find medians or modes.



Students may find it interesting to arrange some tasting experiments to test whether people can detect the differences between flavours and brands of drinks/crisps. If packets of different types of crisps are emptied onto paper plates, so that people can taste, without knowing the brand or flavour, their reactions may not correspond to what they say they like. When testing drinks it may be necessary to use a blindfold because the flavour may be obvious from the colour.

It may prove quite challenging to encourage students to ask for responses on, say, a five point scale to, say, five different flavours and to attempt to analyse their findings.

If the results for the whole class are entered on a table such as the following, how could we identify the most/least popular? Which should we stock?

Name Flavour	Ali	Baljit	Carmel
A	2	1	4
B	1	4	3
C	5	2	2
D	4	5	1
E	3	3	5

Ali likes B most, then A, E, D and C least.

This situation is considered in more detail in the Shell Centre/JMB *Numeracy through Problem Solving* module : *Be a Shrewd Chooser*.

For some students, this initial situation may provide a basis for the whole of their coursework task. However, the more able students will need to move beyond the situation presented. Some students may wish to change the situation slightly and consider what types of, say, chocolate bars the school tuck shop should stock. Others may choose to collect evidence about which shoe sizes the local shop should stock. Students may wish to conduct surveys relating to, say, favourite television programmes or music. Others may choose to consider the popularity of various products which they buy regularly from their local supermarket. They may wish to consider environmental aspects such as, say, revising the speed limits along certain local roads, installing traffic lights, pelican crossings etc.

The essential features of the task are that students should

- \* Ask a question, or specify a simple hypothesis
- \* Design and use appropriate questionnaire
- \* Collect and analyse their data
- \* Reach some conclusions based upon their findings.

## HOW DO YOU REACT?



Anne and Barbara agree to take part in a simple reaction time test. They watch a screen and when they see a light flash, they have to press a buzzer. The following table shows their reaction times for a hundred light flashes.

### Who is the winner?

*Anne*

0.17	0.17	0.19	0.19	0.19	0.19	0.22	0.22	0.23	0.23
0.23	0.23	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.25
0.25	0.25	0.25	0.25	0.25	0.26	0.26	0.26	0.26	0.26
0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.28	0.28	0.28
0.28	0.28	0.28	0.28	0.28	0.29	0.29	0.29	0.29	0.29
0.29	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.31
0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.32	0.32
0.32	0.32	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.34
0.34	0.34	0.35	0.35	0.35	0.35	0.35	0.36	0.36	0.36
0.36	0.36	0.36	0.37	0.37	0.37	0.38	0.40	0.40	0.41

*Barbara*

0.20	0.20	0.21	0.21	0.21	0.22	0.22	0.23	0.23	0.23
0.23	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.25
0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.26
0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.27
0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
0.27	0.27	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
0.29	0.29	0.29	0.29	0.29	0.29	0.30	0.30	0.30	0.31
0.31	0.31	0.31	0.31	0.32	0.32	0.32	0.33	0.33	0.33
0.34	0.34	0.35	0.35	0.35	0.38	0.38	0.39	0.39	0.39
0.42	0.42	0.43	0.44	0.45	0.47	0.50	0.52	0.64	0.78

### Investigate The Problem

HOW DO YOU REACT? : continued



How quickly do you react?

You may like to work in pairs and collect your own data about your own reaction times.

The following simple test can be used to test your reaction times.

Hold out your hands, palms facing each other and slightly apart, in front of you. Ask your partner to hold a ruler upright, just above your hands and drop it without warning you. The object is to catch the ruler as quickly as possible, so that the ruler's calibrations provide a measure of your reaction time. The slower your response, the further up the ruler will your fingers grip.

You may find it interesting to investigate the following questions

- \* Do you react more quickly with your right hand or with your left hand?
- \* Do you react more quickly to something you see or to something you hear?
- \* Do you react more quickly at certain times of the day?
- \* Why not design your own simple tests of reaction times?

## *How do you React? - Teacher's Notes*

Many examples which have traditionally been used to teach statistics have failed to stimulate students' interest. Using the data provided, we pose the simple question

*Who is the winner?*

The initial stage may be to organise the data and draw graphs. Using this data students can create frequency tables and draw histograms. These provide the basis for a discussion about when it is appropriate to use the mean, median and mode.

If students decide to look at the means, they should discover that Anne is the winner, but on the other hand, Barbara is the winner if they consider the medians.

Initially, students may choose to consider measures of central tendency. However all abilities could investigate the range of values, and more able students may find it useful to calculate mean difference, variance or standard deviation.

One interesting initial activity is to ask students to work in groups and examine the data provided. Each group must decide who they think is the winner. They must then prepare a report in which they argue the case for one student, either Anne or Barbara, to be regarded as the winner. This activity can make students more aware of how statistics can be presented and manipulated. Using appropriate visual aids, a spokesperson from each group then presents their case to the whole class. Finally, the whole class can vote for

### *THE WINNER.*

Examining the data provided, is intended to sensitise students to the necessity of analysing data in a variety of ways and looking at situations from a variety of perspectives.

When students collect information for themselves in order to answer a question they have asked, they are much more likely to be interested in organising and analysing the data in order to find a solution to their own problem.

The real GCSE coursework task begins when students begin to ask themselves questions and to decide how they might collect data which could answer their questions. The completion of the initial task may have suggested ways in which they might collect, organise and analyse their own data.

Rather than competing against other students, a more fruitful activity may be for students to compete against themselves as they attempt to answer questions such as:

- \* Does the time of day affect their reaction times?
- \* How does the type of stimuli affect their reaction times?
- \* Does previous physical exercise affect reaction times to stimuli?
- \* Does the pitch of the sound affect reaction time to stimuli such as alarm clocks?
- \* Do we react more quickly to simultaneous sight and sound stimuli than we do to either separately?

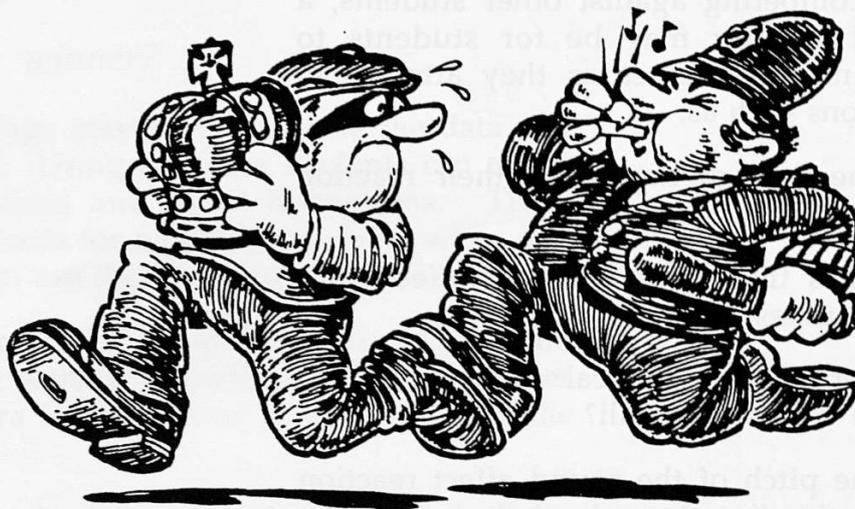
After they have asked themselves a question, students will need to design a simple experiment and collect data, which they can organise and analyse.

When working on this task, an extremely useful learning aid is the computer program TIMES, published by the Shell Centre within the pack *Teaching With a Micro MATHS 3*.

Using the computer, students can measure reaction times to a variety of stimuli in a short period of time and in a reliable way. Students who have used the program during classroom trials have enjoyed the experience, and have gained considerably from incorporating the use of modern technology in simulation situations such as

**CAR:** A simple reaction time task where you must imagine you are the driver of a car and must brake quickly when you see the brake lights of the car ahead.

## FINDING CONNECTIONS



Last night there was a robbery from the local branch of Harclays Bank. The thieves entered the bank from the car park at the back of the building. The police arrived while the thieves were still inside the building, and they had to make a hurried escape.

It had been raining heavily and the ground was quite wet. When the police examined the flower bed between the car park and the bank, they found footprints.

After examining the footprints, the police stated that they would like to interview two people. At the moment, the police are suggesting that one of them wears shoes of length 25cm and probably weighs about 50kg; the other wears shoes of length 30cm and weighs about 90kg.

Can you add to these rather sketchy police descriptions of the two intruders?

You should support your case using evidence you have collected.

### *Investigate The Problem*

*FINDING CONNECTIONS* : continued

Two variables are correlated if changes in the size of one are linked with changes in the size of the other. It is then possible to predict the size of one if we know the size of the other.

- \* You may like to collect data and investigate the relationships between  
 height and foot length  
 arm length and finger length

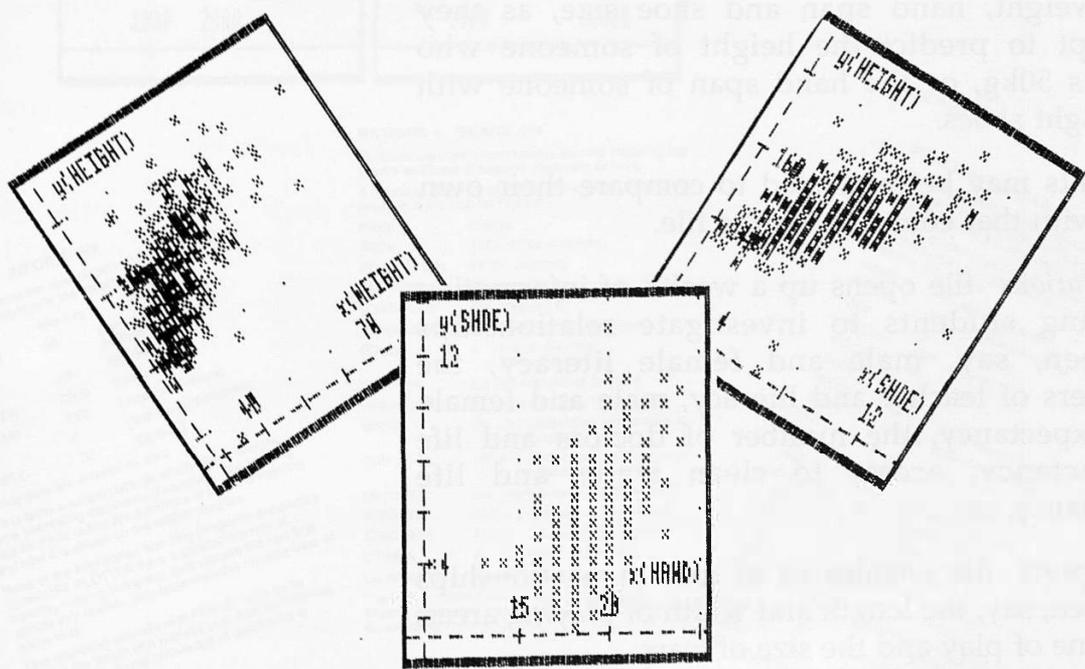
.....

- \* Are fast runners good jumpers?
- \* Does increasing the price of crisps increase the number of packets sold?

Sometimes we get connections between the sizes of two things because one is the cause of the other. For example, smoking causes lung cancer.

On other occasions we get connections between the sizes of two things because there is a common link to a third factor.

After you have collected and analysed your data, you may like to consider why you think there are connections between the things you have measured and recorded.



## *Finding Connections - Teacher's Notes*

This task provides a context within which students can begin to ask questions about the relationships between continuous variables such as foot length, height and weight. 'Are tall people heavier?' 'Do tall people have bigger feet?'. They can then collect data and create scatter graphs in order to answer their questions, as they develop a basic understanding of correlation.

Drawing a line of best fit by inspection on their scatter diagrams may enable students to answer some of the questions they have asked themselves.

More able students may wish to use more advanced methods such as the three centroid or least squares technique:

An extremely useful learning aid when working in this area of mathematics is the Shell Centre program MOUSE PLOTTER. This program contains some data files including *Pupils*, *Nations*, *Planets*, and *Sports*. MOUSE PLOTTER can read numerical information from files and plot this as a graph or a scatter diagram.

Using the file *Pupils*, students may choose to investigate the connections between, say, height and weight, hand span and shoe size, as they attempt to predict the height of someone who weighs 50kg, or the hand span of someone with size eight shoes.

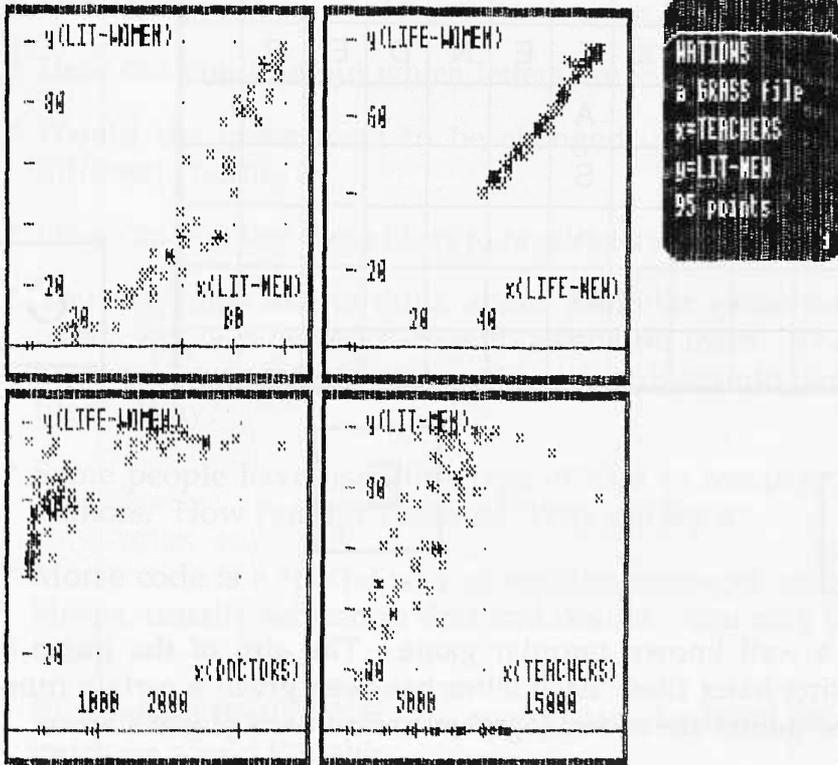
Students may be interested to compare their own data with that contained in the file.

The *Nations* file opens up a wealth of information enabling students to investigate relationships between, say, male and female literacy, the numbers of teacher and literacy, male and female life expectancy, the number of doctors and life expectancy, access to clean water and life expectancy, etc.

The *Sports* file enables us to look at relationships between, say, the length and width of playing areas, the time of play and the size of tears.

MOUSE PLOTTER also permits teachers or students to create their own files for graphing.

It is difficult to imagine a more stimulating student-directed introduction to correlation concepts. Group and class displays of the scattergraphs produced by students as they investigated the connections between a wide variety of variables aroused considerable interest in our trial schools.



MOUSE PLOTTER

**NATIONS—a GRASS file**

Statistics are drawn from various sources including the Centre for Global Education, University of York.

A database about 95 countries of the world. Here as an example is the field for France:—

NAME	Mexico
AREA	1970 (000s of sq kms)
POPULATION	67.38 (millions)
LATITUDE	19 (degrees N)
LONGITUDE	-99 (degrees E)
REVOLUTION	1910 (year of last revolution)
FOOD	2 (1=most over-fed, 5=most under-fed)
LIFE-MEN	63 (life expectancy for men)
LIFE-WOMEN	67 (life expectancy for women)
WATER	59 (% of population with access to clean water)
GNP	181.611 (Gross National Product in millions of US dollars)
MILITARY	0.4 (military spending as % GNP)
DOCTORS	563 (per million population)
TEACHERS	8934 (per million population)
LIT-MEN	86 (% adult male literacy)
LIT-WOMEN	80 (% adult female literacy)

The latitude and longitude are of the capital city. Missing data is entered as zero. See EXCURSION 3 for an activity using NATIONS.

**PUPILS—an INFORM file**

Distributed on the disc by kind permission of David Rooke and Paul Davison, PUPILS is Copyright © by Nottinghamshire County Council 1985. PUPILS contains personal data about 443 pupils in the first three years of a comprehensive school. Here as an example is one field:—

NAME	Andrew (first name)
FORM	3W (year and name)
SEX	B (G or B)
HEIGHT	168 (cms)
WEIGHT	54 (kg)
SHOE	9 (hand span in cms)
HAND	20 (Year-Month-Day)
BIRTH	701204
DAY	4
MONTH	13
AGE	08
CHILDREN	GB (in completed years)
OS	0 (order in family)
YB	1 (older brothers)
YS	0 (older sisters)
POS	0 (younger brothers)
SIZE	2 (younger sisters)
AREA	2 (position in family—2 means second eldest)
MB	MB (number of children in family)
POS	0 (area code)

See EXCURSION 2 for an activity that uses this file.

MOUSE PLOTTER

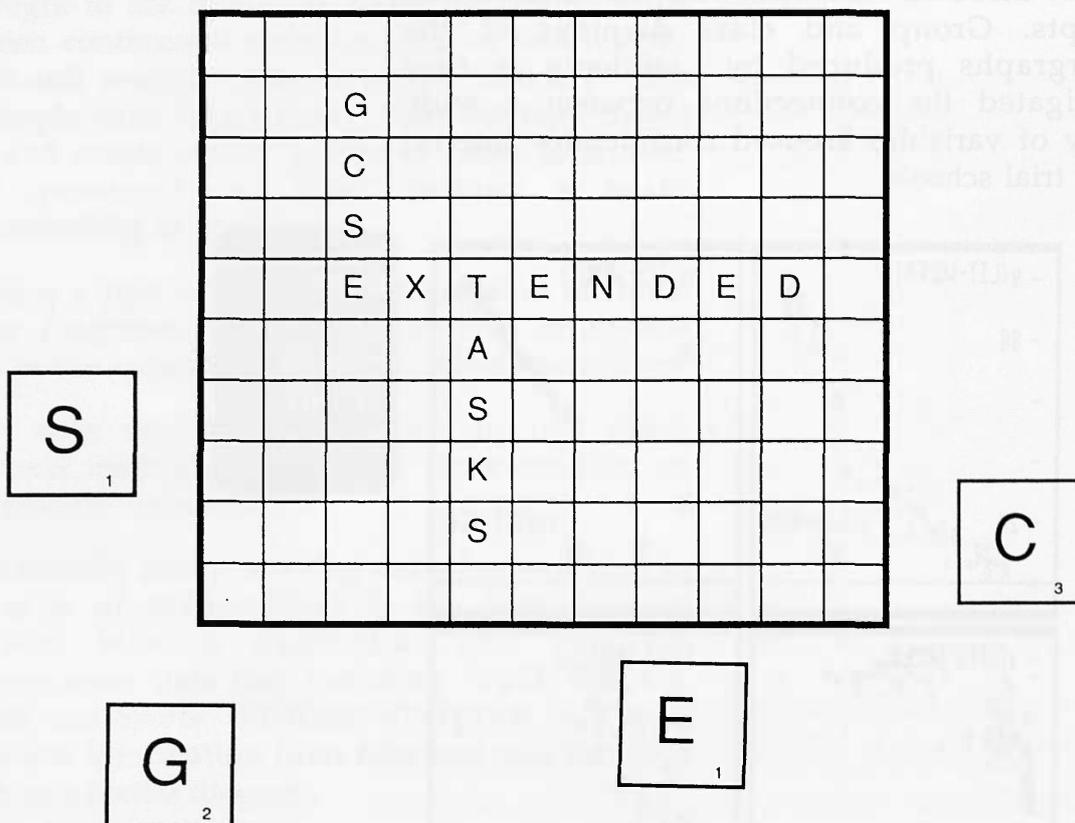
**PLANET—an INFORM file**

A database about the nine planets of the solar system. Here as an example is the record for Mercury:—

NAME	Mercury
DISTANCE	36 (from Sun—millions of miles)
YEAR	24 (Earth=1)
DIAMETER	2900 (miles)
MAXTEMP	410 (degrees Celsius)
NSAT	0 (Number of satellites)
GRAVITY	.37 (Earth=1)

Missing data are entered as zero. This is quite a simple data file which may generate some interesting classroom discussion. The missing data for Pluto raises the questions of why it is missing and how it is shown on the scatter diagram. None of the information handling programs supported by MOUSE PLOTTER have a satisfactory way of handling missing numerical data despite the fact that almost any realistic data base includes missing data of some kind. One modelling exercise with PLANET is to try to predict the length of a planet's year from its distance from the sun. Some knowledge of quadratics is necessary.

# SCRABBLE™



SCRABBLE is a well known popular game. The aim of the game is to make words using letter tiles. Each letter has been given a certain number of points. These points are added together to find each player's score.

Words must be placed on a squared board. Some squares on the board are colour coded, with labels such as *Double letter score*, *Triple word score*, etc.

There are exactly one hundred letter tiles in each pack. Every letter is included, as well as some blank tiles. Blank tiles may be used to represent any letter, but they do not count towards a player's score.

Is the game fair?

How easy is it to make words?

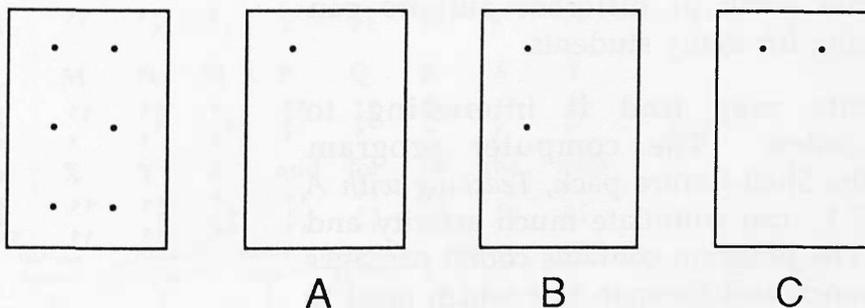
## *Investigate The Problem*

SCRABBLE is a registered trademark owned in England by J W Spear & Sons PLC

*SCRABBLE* : continued

There are many things you could investigate in depth. These include

- \* If you had invented the game, how many tiles of each letter would you have provided?
- \* How many points would you have given for each letter?
- \* How can you find out which letters are used most frequently?
- \* Would the game need to be changed if you wanted to sell it in a different country?
- \* How can you use these ideas to break secret codes eg. A=F, B=Z etc?
- \* You may also like to think about a similar game for young children using, say, a hundred tiles with words on them. The aim is to make sentences using the words. What words would you write on your tiles? Why?
- \* Some people have used this type of idea to compare the work of two authors. How can this be done? Why not try it?
- \* Morse code is a special way of sending messages using short and long bleeps, usually written as dots and dashes. You may like to investigate this.
- \* What about Braille? This is a special code for blind people. It works by touch on a grid like this.



Each dot can be flat or raised. This is another idea you may like to look into.

## Scrabble - Teacher's Notes

In order to complete this task students will need to use a commercially available version of the game SCRABBLE or a student-produced copy.

This task is intended to provide a situation within which students can specify an issue for which data is needed and is readily available inside the classroom. The initial questions: 'Is the game fair?' 'How easy is it to make words?' are intended to encourage students to examine the one-hundred tiles provided. They then need to design and use an appropriate observation sheet in order to collect data about how many of each letter is provided and how many points each letter has been given.

Having completed this first task, students need to look beyond the tiles provided. They may choose to examine pages of text and look at word frequencies or letter frequencies, as they attempt to come to terms with the reasons why E carries a different number of points than, say, C.

Topics for further discussion may include the possibility of marketing the game in France or Germany or Spain. Are letters used with the same frequency in English and Spanish?

How about the word game? Which words would you provide? How many points would each word score?

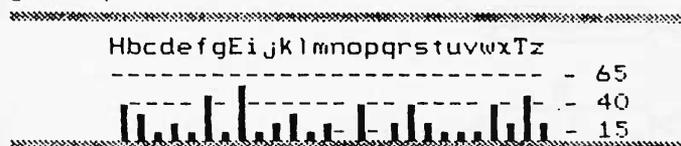
Comparing the work of different authors can prove interesting for many students.

Some students may find it interesting to investigate codes. The computer program DECODE in the Shell Centre pack, *Teaching with A Micro MATHS 1*, can stimulate much activity and speculation. The program contains coded passages of English, French and German text which need to be decoded. Moreover, the program also allows teachers, or students, to enter their own text and codes. Coding messages proved to be a very popular task in our trial classrooms, even without a computer.

# DECODE

Design and Program by Richard Phillips  
 Program Copyright © 1984 by the Shell Centre for  
 Mathematical Education

THE HwsKE zfk cEbu KToqq. jfb wJj THEbE  
 zfk f kwsrx zHoeH domHT Hfce tEER  
 tEfTorm ksbj wb efbk lwwdorm fqwrn f  
 HomHzfu, wb zorx or gorE TbEEK. oT zfk  
 THE KEf, wj ewsbKE, tbEfvorm jfb xwzr  
 tEqwz. o kfT THEbE frx qokTErEx Tw oT  
 frx THwsmHT qwrn, efbEjsq THwsmHTk. THE  
 gHwRE bfrn jwsb TodEK zoTHor THE rEiT  
 Hwsb frx f Hfqj. THE tom wrE efdE fT  
 EomHT dorstEK gfKT TER. dfbbowTT TfqvEx  
 tboEjqu, or f cEbu qwz cwoeE, ebfxqEx  
 THE orkTbsdErT zoTHwST f kwsrx frx KTwwx  
 sg zoTH f KwBT wj HskHEX dwcEdErT. Hok  
 jfeE qwwwvEx xbfzr.



An examination of Braille may arouse the enthusiasm of other students.

## BRAILLE ALPHABET

	A	B	C	D	E	F	G	H	I	J
	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠
	K	L	M	N	O	P	Q	R	S	T
	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠
	U	V	X	Y	Z	and	for	of	the	with
	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠
	W	Oblique stroke	Numeral sign	Poetry sign	Apostrophe sign	Hyphen	Dash			
	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠
Lower signs	,	;	:	.	!	( )	?	" "	" "	" "
	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠	⠠

The Morse Code can stimulate much profitable student activity.

The Morse Code was invented by an American called Samuel Morse who lived from 1791 to 1872. During a voyage across the Atlantic, the idea of transmitting messages electrically between stations which were each equipped with a transmitter and a receiver, aroused his interest. In coding the alphabet he first exhausted the possibilities of arranging the dot and the dash singly, then in groups of two or three at a time, before moving on to combinations of four signals. All numerals were allocated a five-signal combination.

If we assume that we want to transmit messages as quickly as possible, and that

- takes one unit of time (1/24 th of a second)

— takes three units of time

the gap between dots and dashes takes one unit of time

the gap between letters takes six units of time.

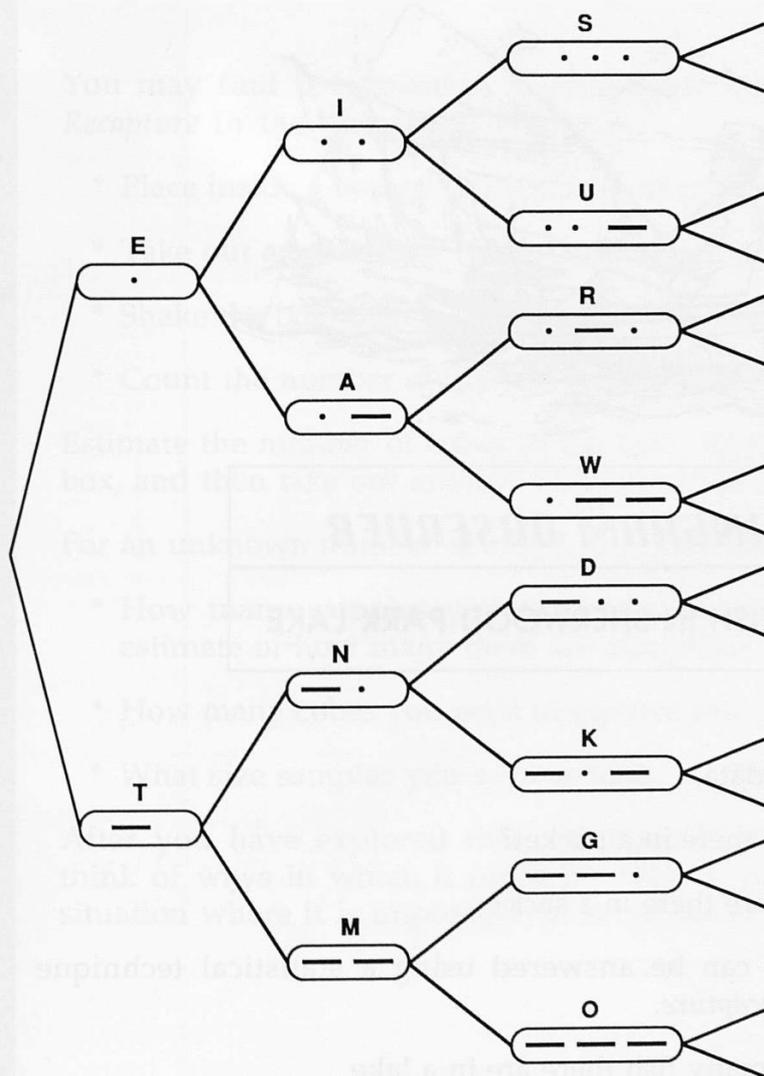
- \* Does this code seem sensible?

In order to answer this question, students will need to analyse passages of text and determine which letters are used more frequently. They will also need to determine the length of time required to transmit each letter using the Morse Code. A comparison of the results obtained, together with comments and recommendations could lead into the next question.

- \* This code depends upon the use of two symbols, a dot and a dash. Would three symbols be better? What might they be?

All of these tasks depend upon text analysis and a consideration of letter frequencies. Letter frequencies, from the most to the least frequent, are as follows

English:	ETAONRISHDLFCMUGYPWBVKXJQZ
French:	EASITNRULODCMPVQGFBJXYZKW
Spanish:	EAOSRNIDLCTUMPBGYVQHFZJXWK
German:	ENISTRADHUGMCLBOFKWVZPJQYX



2

4

8

16

MORSE CODE

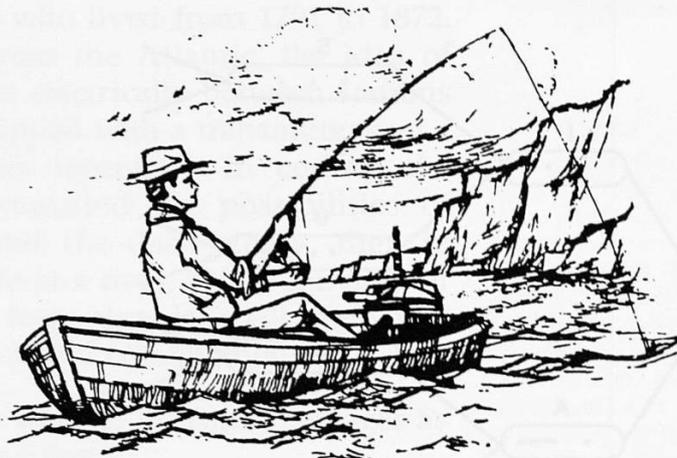
A ·—  
 B —···  
 C —·—·  
 D —··  
 E ·  
 F ····  
 G —··  
 H ····  
 I ··

J ·—  
 K —·—  
 L ····  
 M —  
 N —·  
 O —  
 P ····  
 Q —·—  
 R ···

S ...  
 T —  
 U ···  
 V ···—  
 W ·—  
 X —···  
 Y —·—  
 Z —···

1 ·—  
 2 ···—  
 3 ···—  
 4 ···—  
 5 ····  
 6 —···  
 7 —···  
 8 —···  
 9 —···  
 0 —··—

## VERY FISHY



### NOTTINGHAM OBSERVER

**ONLY 500 FISH IN SHERWOOD PARK LAKE**

How do they know?

How do they collect such data?

How many dried peas are there in a packet?

How many grains of rice are there in a sack?

Questions such as these can be answered using a statistical technique known as *Capture and Recapture*.

If we want to know how many fish there are in a lake

- \* We capture some fish and count them
- \* We mark them
- \* We put them back in the lake
- \* We take random samples of fish from the lake, and count the number of marked and unmarked fish.

*Investigate The Problem*

*VERY FISHY* : continued

You may find it interesting to investigate the technique of *Capture and Recapture* in the classroom.

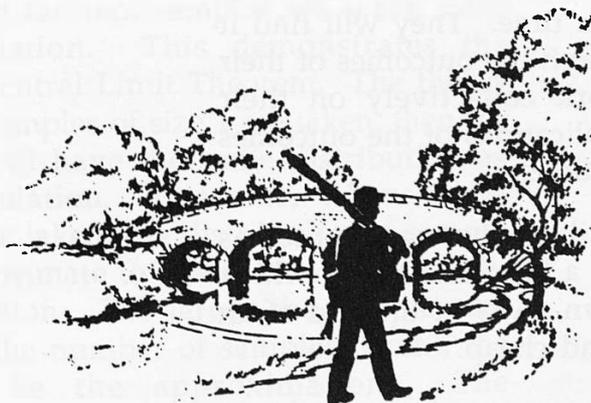
- \* Place inside a box an unknown number of identical cubes
- \* Take out a handful of cubes, mark them, and then replace them
- \* Shake the box, then take out another handful of cubes.
- \* Count the number of marked and unmarked cubes.

Estimate the number of cubes in the box. Replace your sample, shake the box, and then take out another random sample.

For an unknown number of cubes, you may like to consider

- \* How many samples you need to take, if you want to make a good estimate of how many there are altogether
- \* How many cubes you need to capture and mark
- \* What size samples you need to take.

After you have explored this technique inside the classroom, why not think of ways in which it could be used in your local environment, in a situation where it is impossible to count the entire population?



## Very Fishy - Teacher's Notes

This task provides a context within which students can estimate the size of a population by sampling, and begin to appreciate the effect of drawing different sizes of samples.

It is much more fun to estimate the size of an unknown population whose size cannot be counted. However, initially it is useful to work with populations whose size can be counted at the end of the activity to check the estimates, as students try to come to terms with the effects of

- \* taking samples of different sizes
- \* establishing what is a *random* sample
- \* drawing different numbers of samples
- \* marking different proportions of the population.

As in all statistical work, students should be encouraged to begin by asking a question or making an hypothesis. They may begin by looking at a container and estimating there are about xxx hundred cubes. The next stage is to decide how they can test their hypothesis. They need to devise a means of collecting data, then use the data collected to test their hypothesis.

Students will find it useful to work in small groups as they attempt to determine the effects of changing one variable at a time. They will find it helpful to tabulate and graph the outcomes of their experiments as they work collectively on their tasks, and discuss the implications of the outcomes of their classroom simulations.

If sampling bottles are available they can be used as an alternative to cubes in a box: dried peas which can be marked and thrown away at the end of an investigation are useful and cheap.

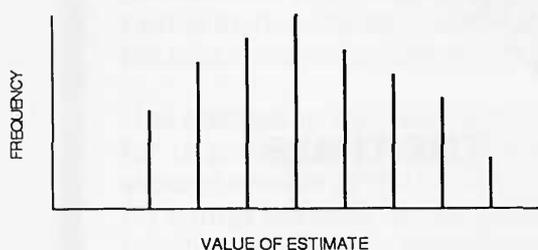
Let us assume that students are trying to estimate the size  $N$  of an unknown population of cubes. They capture, mark and replace  $C$  cubes. They then draw random samples of size  $n$ . They count the

number,  $r$ , of marked, or recaptured cubes. They may then choose to record their results as shown.

An estimate of the size of the population is

$$N = Cn/r$$

But how many estimates do we need to obtain a realistic estimate? Even if  $N$ ,  $C$ ,  $n$  are held constant,  $r$  will vary from sample to sample, giving a range of estimates of  $N$ . After repeated sampling, students may find it useful to plot a frequency graph of their estimates of the population size.



	IN CONTAINER	IN SAMPLE
CUBES ALTOGETHER	$N$	$n$
MARKED CUBES	$C$	$r$

Students can gain considerable insights into many issues relating to sampling through changing one variable at a time in this simple simulation.

A useful learning aid when considering samples and sampling are the *Probability and Statistics Programs* produced by the Microcosm Project for MEP. The program SAMPLER is particularly useful.

This program illustrates how sample means are distributed when random samples are taken from a parent population. This demonstrates the validity of the Central Limit Theorem. The theory predicts, that if samples of size 1 are taken, then the sample means will have the same distribution as the parent population. However, when larger sized samples are taken, the distribution of sample means will approximate to that of the normal (bell-shaped) distribution. The larger the sample size (and the larger the number of samples taken), the better should be the approximation. The distribution of sample means will centre on the parent population mean, whatever the sample size. The larger the sample size the less will be the spread of the means.

## FINDING THE NEWS

*The***Guardian**

**THE**  
**Sun**

**THE INDEPENDENT**

**THE TIMES**

There are many different types of daily newspapers. They differ in a variety of ways: page size, paper quality, print style, story type, target readership, style of reporting, etc.

Spend a little time thinking about some of the differences and then design one or more statistical experiments to investigate your ideas. Naturally you can explore any aspect that interests you.

The three extracts provided have been taken from one of the popular daily newspapers. These have been re-typed so that the source cannot be identified immediately. Can you find which newspaper these extracts come from?

### *Investigate The Problem*

- \* Why do different people enjoy reading different newspapers?
- \* Which newspapers are most popular?
- \* How about magazines, comics, books ....

You may like to collect some data which enables you to answer one of these questions. Alternatively, you may be able to think of some more interesting questions yourself.

FINDING THE NEWS : continued

## NEWSPAPER EXTRACTS

### EXTRACT 1

THOUGH the traditional preoccupation of British composers has been the symphony, their particular genius has been in writing for string orchestra - as in Elgar's *Introduction and Allegro*, Vaughan Williams's *Tallis Fantasia*, and Tippett's *Double Concerto*, to name only the three represented in the Manchester Camerata's ambitious but misconceived contribution to the Tippett and Debussy celebrations.

One element in the misconception was that Debussy was fobbed off with his *Syrinx* for unaccompanied flute, a work which takes no longer to play than it takes the average number of the audience to unwrap a boiled sweet. Debussy didn't write much for strings but there are the *Danse Sacree* and the *Danse Profane* for harp and strings, which could chastely have occupied the place so rudely usurped in this programme by Poulenc's *Organ Concerto*.

Anyone who is allergic to Poulenc's synthesis of Bach, Stravinsky, Handel, and sweet religious sentiment is perhaps not a competent judge of its performance. But it would surely have been more effective in church acoustics, where Gillian Weir's ghetto-blasting solo playing would at least have been at an atmospheric distance.

The concert hall of the Royal Northern College of Music is very useful but it is acoustically intractable, and not the best setting either for the *Tallis Fantasia*. John Barrow played his *Syrinx* solo from the balcony overlooking the platform, which might also have been a better place for the second orchestra in the Vaughan Williams than the neither-on-nor-off stage position to which they were actually consigned. Anyway, it failed both atmospherically and - after evidently inadequate preparation - technically.

So the major rewards of the concert were in the *Introduction and Allegro* at the beginning and the *Double Concerto* at the end. Both performances were conducted by Nicholas Braithwaite with a curiously heavy hand on the rhythms, making Elgar sound clumsy at times and Tippett unspringy. But there was some distinguished solo playing and some impressively contrasting large-scale sonorities.

*FINDING THE NEWS* : continued

## EXTRACT 2

I HAVE this theory. The amount of razzmatazz and paper generated by a given manufacturer is in inverse ratio to the importance of the product he is promoting. On this basis, if you are invited to a press conference beamed by satellite to 1,200 journalists from ten countries which starts at 8 o'clock in the morning, you could be somewhat sceptical. And you would be wrong. The product concerned, if you have not already read about it, is the Fiat Tipo - a medium sized five-door hatchback aimed at the top spot in Europe. To achieve this it will have to overtake the VW Golf and the Ford Escort, neither of which have replacements programmed for the near future, but both of which are well ensconced in the market. As far as the UK market is concerned, the Tipo should be here in June, and you can expect it to be very competitively priced - present estimates are that it will be about 10 per cent more than the Fiat Uno range, which will put it in the £6,000-£8,750 price bracket.

Nowhere is the "give the dog a bad name" syndrome more active than in the automotive world - a fact which must have given Shell cause for thought last week. Car buyers seem gifted with astonishingly long memories and can recall vividly the time when certain tyres threw chunks of tread across the road, and certain cars rusted into lacy patterns, or their engines fell out. Fiat has struggled through a similarly bad patch which saw its UK market share dwindle. But if the Tipo achieves the success planned for it, Fiat will be giving the other importers a run for their money.

First impressions of the styling are mixed. The rear of the car appears, at first glance, to have been designed by someone other than the designer of the rest of the car. But Fiat boss Vittorio Guidella counters this opinion by saying that it is a strong design - "it looks as though some giant hand is pushing the car along." In fact what pushes, or rather pulls, for this is a front wheel drive car, is a range of engines from 1100cc to 1600cc, plus a turbo diesel. There is a 16-valve 1800cc unit to come as well as an automatic gearbox and a four wheel drive car. Exactly what the specification of the car will be when they come to Britain has yet to be decided. A quick sprint around the Home Counties in a left-hand-drive car was not sufficient for a full appraisal. But it was enough to discover that the Tipo is a spacious car for its class. There are two levels of trim, the top one boasting a rather garish collection of digital instruments is instantly appealing - a noticeable improvement on previous Fiat changes.

Reliability, which has bedevilled Fiat in the past, has been attacked by a major rethink of the way in which their cars are built. The Tipo is the first to be produced by an extremely automated system in which 14 sub-assemblies, also put together by robots, are assembled almost untouched by human hand. As Andrew Cornelius pointed out in this paper on Wednesday, this is surely a system which the Rover group is agonising over. Fiat has invested more than £1bn in the Tipo. The company is obviously bent on success.

*FINDING THE NEWS* : continued

## EXTRACT 3

The British Medical Association executive is to hold an emergency debate about the crisis in the National Health Service on Wednesday, the nurses' planned day of action. The announcement supports the NHS unions' claims that consultants are sympathetic to the grievances of nurses, who will be protesting over underfunding of hospitals, low pay and overwork, caused by staff shortages.

Dr John Harvard, secretary of the BMA, said "The BMA has consistently warned that the NHS is being systematically starved of the resources it needs. Health authorities have had to contend with the 'knock'on' effect, year to year, of cutbacks. Pay awards have been underfunded by government, yet all this has occurred while our economic prospects have been improving. As a result, the health service needs an urgent injection of extra money to restore services to patients." He pointed out that the efficiency of the service depended on the goodwill and morale of those working in it - "the doctors, nurses, ambulancemen, medical secretaries, managers and all health employees." He warned "Once that is destroyed it will take years to rebuild." The BMA feels strongly that the answer to the health service problems is more money through direct taxation, not alternative finance.

Plans for the nurses' day of protest are well underway, and consultants have been making arrangements to postpone routine operations. Mr Chris Humphreys, London organiser for the National Union of Public Employees, said at least 2,500 nurses would be striking, and 15,000 to 20,000 would be joining rallies and lobbies. Nurses in 34 London hospitals have voted for action, while decisions are still expected from another four or five. Some working nurses will not wear uniform, while others operating emergency cover will display badges and armbands.

Various MPs, including Labour's health spokesman, Mr Robin Cook, will visit the hospital demonstrations. Yesterday, Mr Cook said "We understand the frustrations experienced by our nurses and believe strongly that nurses do need a new deal." Action is planned in London, Leeds, Liverpool, Manchester, Sheffield and Scotland.

Mr Rodney Bickerstaffe, Nupe general secretary, yesterday angrily rejected charges that his union agitated to provoke the protests. He said nurses and ancillary workers were "whistle blowers" willing to stand up to the Government's "kill and cure" approach to the NHS. He said his union has not organised the strike in Manchester on January 7 which sparked the protest action.

Speaking to Labour Party local government conference in Edinburgh, Mr Bickerstaffe turned his fire on the Cabinet, saying "They claim our people have been callous, unthinking and uncaring. What hypocrites. What forces nurses every four or five years to take some form of protest action? Is it because they don't like the patients? I say it's the opposite, it's because they do like them and care for them." Nupe has instructed all of its branches not to take action which harms patients in any way.

Mr Gordon Brown, Labour's shadow Chief Secretary to the Treasury, claimed yesterday that internal NHS studies showed that private sector operations for tonsils, hernias and varicose veins cost between £400 and £500 more than in the public sector, with hip replacement operations costing £1,500 more. "No-one should be in any doubt that it is the private sector that is most hit by red tape, bureaucratic mismanagement and the nightmare of never-ending charges and costs" he said.

## *Finding The News - Teacher's Notes*

This task provides a context within which students are asked a question they may be able to answer after they have collected and analysed some data. The initial question posed may seem rather broad, 'Which newspaper do these articles come from?' This broad question needs to be refined

- \* What are the articles about? Which newspapers contain this type of article?
- \* What type of language is used? Which newspapers use this sort of language?
- \* How long are the words, sentences and paragraphs?

Students need to design their own observation sheets and collect their own data from a variety of newspapers. As an aid to analysing their data they may draw graphs, calculate means, find medians and modes relating to the types of articles which can be found in different newspapers, the lengths of words used, word frequencies, the number of words in a sentence, paragraph lengths, and the shape of the related frequency distribution. The three extracts provided are taken from The Guardian. However, during classroom trials some schools chose to use extracts from alternative newspapers or real newspapers.

Students may then choose to look in depth at their favourite newspapers.

- \* What proportion of articles, advertisements etc do different newspapers contain?

Alternatively, students may prefer to consider different magazines, comics, books, etc. Some students may wish to compare texts designed for young children, teenagers and adults. They may find it interesting to consider Readability Tests and to discuss this with reference to the work of the S.E.N or Language departments.

The essential thing is that students should ask questions, or specify a simple hypothesis, which can be answered through collecting and analysing data collected from reading material.

# 5

## *Students' Work*

These six pieces of work cover a wide range of achievement. Two pieces of work are offered at each of the three levels of GCSE study; Foundation, Intermediate and Higher. These three levels are common to all GCSE schemes although the level titles differ.

The six pieces are in rank order of attainment and finish with the piece which is considered the best from the set. In Chapter 6, you will find detailed comments made on each piece by the Midland Examining Group Chief Coursework Moderator. We recommend that you should consider each piece of work in detail, make a few written comments and attempt to grade each student's work, before you read the moderator's comments.

For identification purposes, the six student's scripts are labelled S2/1 to S2/6. Because of space constraints the project team decided to reduce the size of the student's scripts, in order to include a wide range of student achievement. In addition to the loss of quality through the reduction in size, some scripts suffer from the loss of colour which originally added emphasis and clarity to the arguments presented. Nevertheless, we are hopeful that much of the strength inherent in the original scripts will become apparent as you read through the following pages.

S2/1

WHY  
ARE  
WE  
WAITING

Contents Page

1. What is a queue?
2. Different types of queues.
3. Maths oral assessment table.
4. 3rd year options Interview
5. Tally chart on options
6. Bar graph on options
7. appointments for parents evening
8. Solving the Problem
9. Comparing the appointment systems
10. Doctor Surgery
11. Conclusion
11. How can this system be improved?
12. Summary of Project

Different types of queues?

Nº.	What type of queue is it?	Details of queue
1	Queue at school	wait in single file until told to go in. Go in in certain numbers only so many at a time. a lot of pushing in.
2	Queue for the ladies toilet	Again waiting but this time you wait until the next one comes out. Not much pushing in.
3	Queue for the cinema	queue in single file, until it's time to go in. There's a lot of pushing in, in the queue.
4	Queue for Customs in an airport or Ferryport	wait in single file behind the person in front until those persons finished. Not very much pushing in.
5	Queue to see teachers at Parents evening	An appointment system is used but parents have more than 5 mins & then everybody has to wait.

During this topic I have tried to answer questions, investigate & try to improve a way in which people don't have to wait on an appointment system & I've explained & drawn up my own table.

What is a queue?

There are many types of queues, everywhere, some you don't regard as queues.

A queue is where you have to wait behind how ever many people are in front of you until you reach the front of the queue. e.g. bus queue.

An appointment system is a form of a queue because although an appointment time may have been given to you it's probably not the time you will be able to see i.e. the teacher. Appointment systems are very complicated systems especially in a doctor surgery, because there are a lot of patients all wanting to see the doctor, and not all the patients can. Because they can't be fit into the appointment system for that particular day!

Maths oral Assessment Table

As part of a schools GCSE in maths the students have a discussion with one of the teachers for about 10mins, towards the end of their 5th year. During this time students are invited to talk about extended tasks that they have carried out.

The following table appeared for one particular afternoon

GCSE Mathematics Oral Assessment	
DATE	Friday 22nd May - Afternoon
TEACHER	MISS Stables
TIME	STUDENT
2.00	Tammy Whitaker
2.10	Jason Hargreaves
2.20	Rajinder Light
2.30	Melanie Nesbitt
2.40	Eung IP
2.50	Noreen Dixon
3.00	Walker Bonsett
3.10	Terry Payne
3.20	Sunie Reed
3.30	Tony Singh
3.40	Sally Nesbitt
3.50	Winston Bailey

The above table is the GCSE oral assessment programme, for Friday 22nd May in the afternoon.

NAME	Time (in mins)
Tammy	9
Jason	12
Rajinder	10
Melanie	7
Winston	12
Eung	7
Noreen	8
Walker	10
Terry	9
Sunie	7
Tony	7
Sally	10

The above table is to show the lengths of student interviews.

Time (in mins)	Tally
7	
8	
9	
10	
12	

The tally chart is to show how many students took the same amount of time.

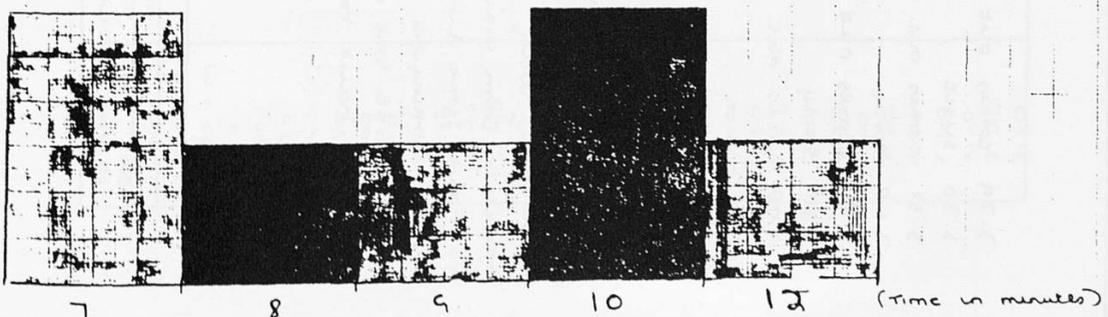
I think that this particular appointment system is quite satisfactory in that the teacher has some time for writing, notes etc. The disadvantage though is that 3 pupils each have to wait 2 minutes. Although 2 minutes is not a tremendous amount of time to wait, it's still a problem, being an oral exam the students are probably quite nervous and if they have to wait around any longer it becomes extremely nerve-wracking. Although this may not affect other students.

If each student was 2 minutes late coming out the teacher would be behind schedule & wouldn't finish on time! She'd finish at 4.24pm 34 minutes behind. She was meant to have finished at 3.50pm.

The oral assessment starts at 2.00pm

2.00	Tummy
2.09	comes out - teacher waits 1 minute
2.10	Tyson
2.22	comes out - Kaynder waits 2 minutes
2.22	Kaynder
2.32	comes out - melanie waits 2 minutes
2.32	melanie
2.40	winston - melanie comes out
2.52	comes out - kung waits 2 minutes
2.52	kung
2.59	comes out - teacher waits 1 minute
3.00	Norean
3.08	comes out - teacher waits 2 minutes
3.10	walter
3.20	comes out
3.20	jenny

A Bar Graph to show how many students took a particular time in their interview



3rd year options interview

During 3rd year all 3rd year will have to choose their subjects for the 4th year for their GCSE. They are given 10mins to talk to the year tutor. Each pupil will bring their preliminary options form to the teacher & the teacher & you will discuss your options & whether you have any ideas of what you want to do when you leave school.

There are approx 25 pupils in each form

$$\begin{array}{r} \text{forms: } 25 \times \\ \quad \quad \quad \underline{\quad 6 \quad} \\ \quad \quad \quad 150 \end{array}$$

150 pupils in 3rd year (approx)

30 pupils will be seen each day for five days

The following 30 pupils were seen on Monday 20th June.

3:29 comes out - teacher waits 1 minute

3:30 susie

3:33 comes out - teacher waits 3 minutes

3:40 Tony

3:47 comes out - teacher waits 3 minutes

3:50 Sally

4:00 comes out.

In all the teacher has to wait 11 minutes. but I think that's a good thing, so Mrs Stables had time to write notes on the student. She also had time for the next student, before he/she came in.

The only way to solve the students and the teacher from waiting would be for the teacher to have some kind of alarm/bell going off every 10 minutes so that she'd know it was time to see the next student. This way she wouldn't have to wait and neither would the students.

Tally Chart

Roller name	Tally (vertical)	Frequency
1. Lisa Wright		7
2. Rachel Powell		15
3. Louise Pike		26
4. Nicola Russell		35
5. Joanne Francis		47
6. Louise Boffey		52
7. Justine Robson		60
8. Lisa Randles		68
9. Tracy Sherratt		78
10. Wendy Hickman		88
11. Joanne Bartlett		99
12. Sally Hill		112
13. Caroline Welch		121
14. Vicky Bartlett		127
15. Alison Garnston		134
16. Massahur Lawson		143
17. Paul Tomlinson		151
18. Glyn Morgan		161
19. Marc Darnan		173
20. Paul Forster		177
21. John Cowley		179
22. Jason Guest		195
23. Massahur Highfield		207
24. Twain Hurton		220
25. Richard Frost		229
26. Carl Stanford		243
27. Stuart Brumhill		253
28. David Corbett		264
29. Paul Hule		275
30. Susan Barron		285

Time in (mins)	Tally
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	

DATE	Year	Options	Inter-view	Time	STUDENT
				9.00 a.m	Lisa Wright
				9.10	Rachel Powell
				9.20	Louise Pike
				9.30	Nicola Russell
				9.40	Joanne Francis
				9.50	Louise Boffey
				10.00	Justine Robson
				10.10	Lisa Randles
				10.20	Tracy Sherratt
				10.30	Wendy Hickman
				10.40	Joanne Bartlett
				10.50	Sally Hill
				11.00	
				11.15	Caroline Welch
				11.25	Vicky Bartlett
				11.35	Alison Garnston
				11.45	Massahur Lawson
				11.55	Paul Tomlinson
				12.05	Glyn Morgan
				12.15	Marc Darnan
				Coffee	
				1.30	Paul Forster
				1.40	John Cowley
				1.50	Jason Guest
				2.00	Massahur Highfield
				2.10	Twain Hurton
				2.20	Richard Frost
				2.30	Carl Stanford
				2.40	Stuart Brumhill
				2.50	David Corbett
				3.00	Paul Hule
				3.10	Susan Barron

Parents evening appointment system

1st year parents evening on Tuesday 17th July.  
 All parents have chosen for 7 teachers which they would like to see. Parents are allowed 5 minutes to each teacher.  
 The parents have been given their appointments time to see each particular teacher.

One parent had the following appointment times

- I Walker 5:10p.m
- P Robinson 5:15p.m
- G Ingram 5:25p.m
- P Reed 5:30p.m
- S Barton 5:45p.m
- B Morgan 6:00p.m

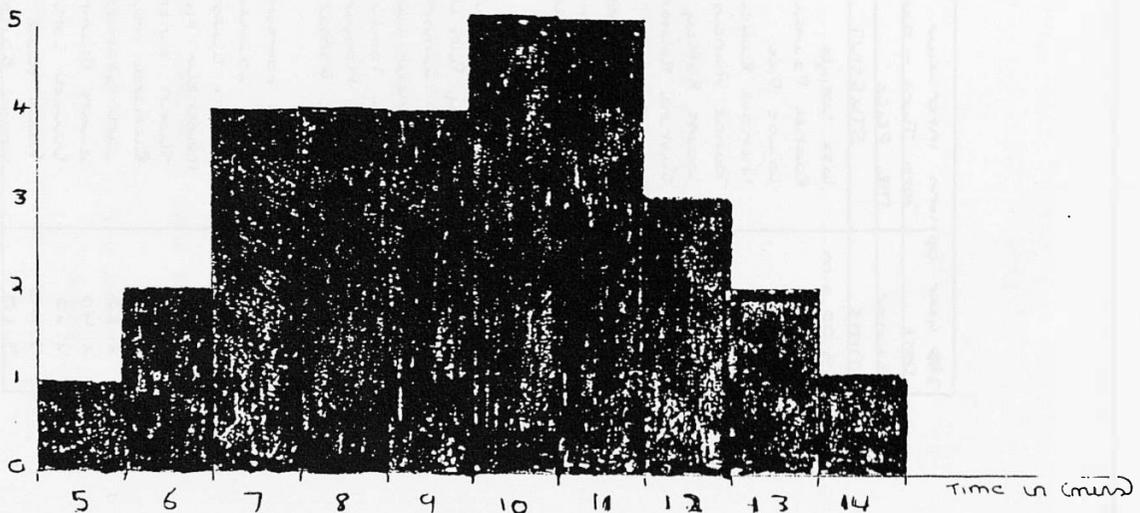
Their first appointment is at 5:10p.m & their last appointment is at 6:00p.m. The parents are given 5 mins to see & talk to each teacher. These particular parents will have to wait 25mins during the course of the evening, but this depends on whether their is a queue to see the teacher.

The problem with this kind of appointment system is that the time limit is 5mins, and if one parent is late, then the other parents are going to be late the whole evening.

Solving THE PROBLEM!

To solve this problem they should use some type of luggage which will go off every 5mins, so then the parents & teacher would know when their 5mins was up!

A Bar graph to show how many & how long the children spent with the teacher



Doctor consultation times with his patients

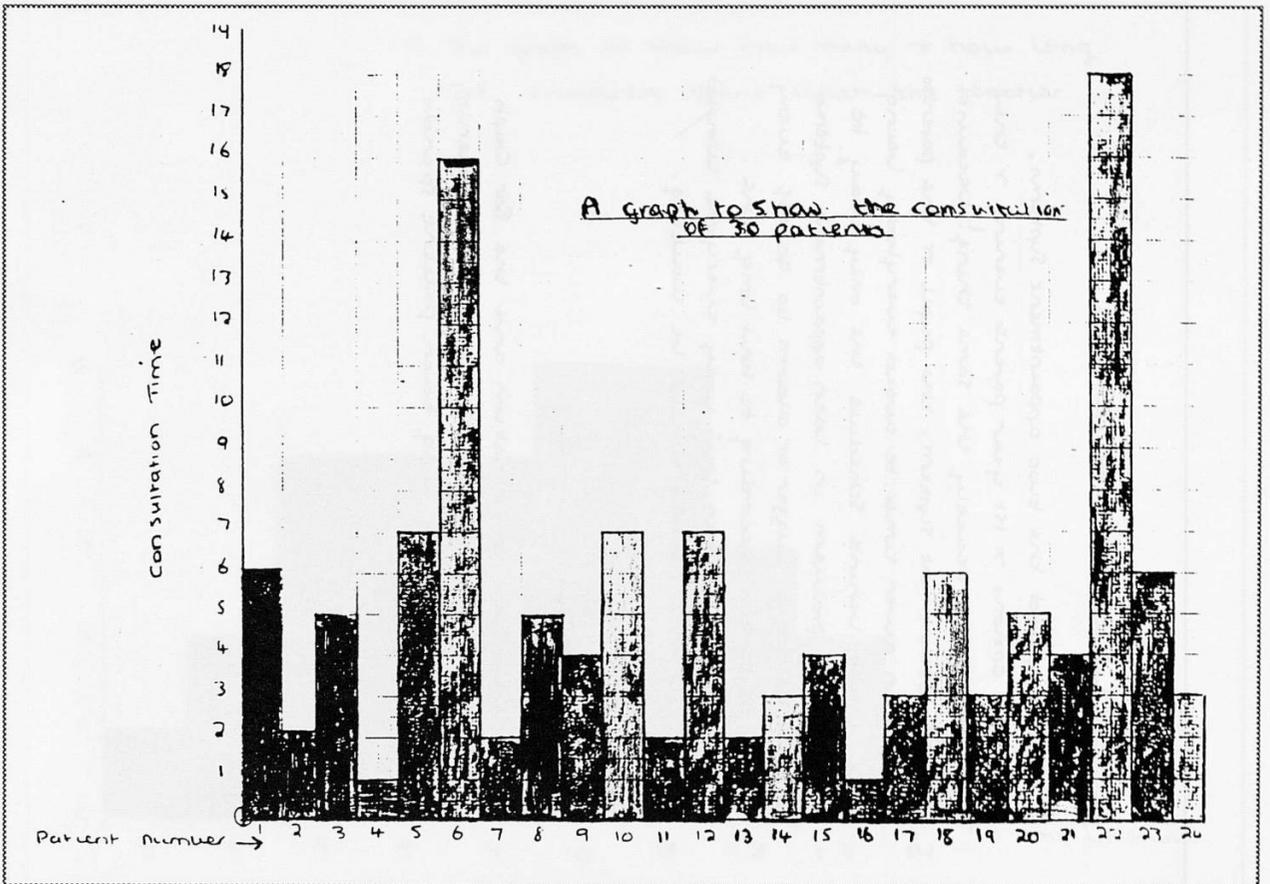
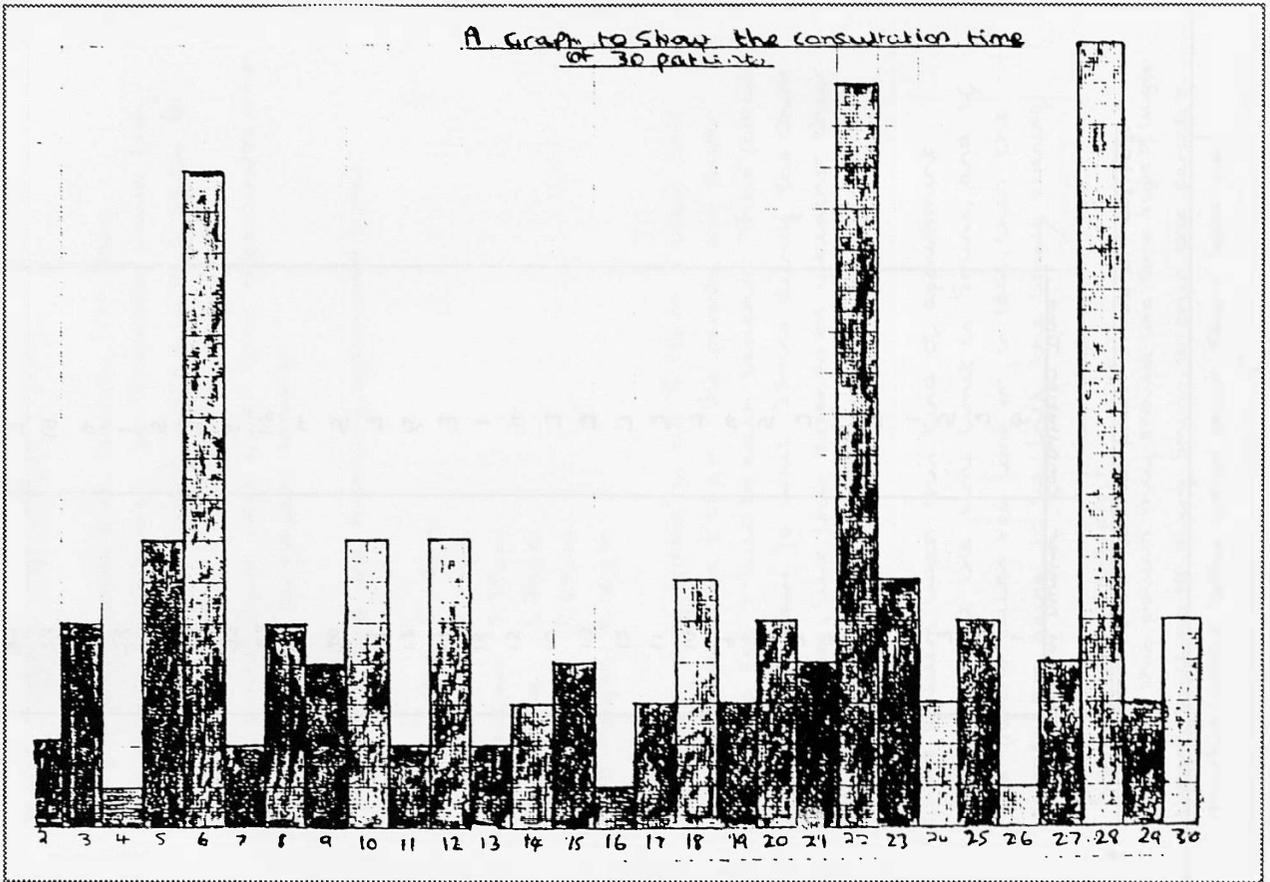
Patient number	Consultation Time
1	6
2	7
3	5
4	1
5	7
6	16
7	2
8	5
9	4
10	7
11	2
12	7
13	2
14	3
15	4
16	1
17	3
18	6
19	3
20	5
21	4
22	18
23	6
24	3
25	5
26	1
27	4
28	19
29	3
30	5

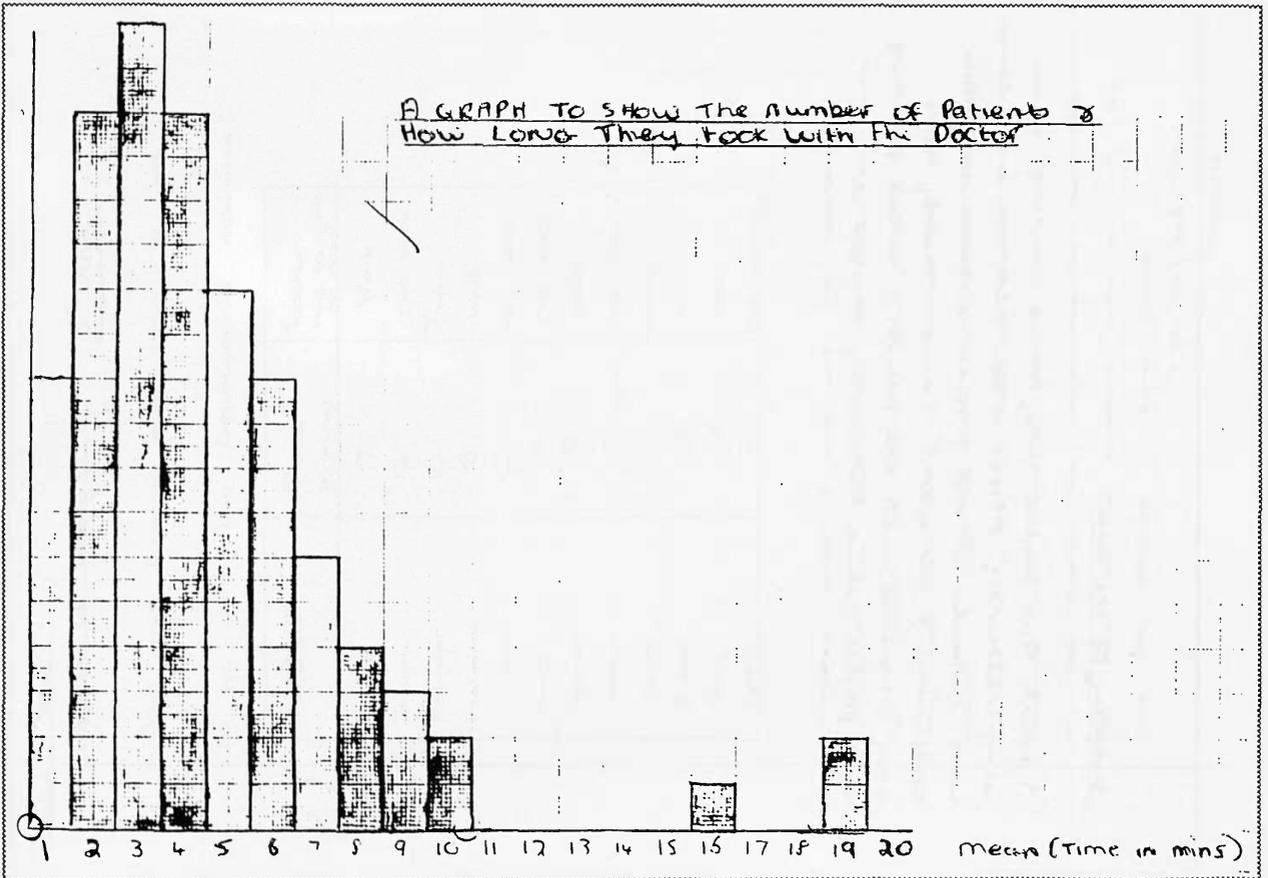
Comparing the appointment systems

I have compared the two appointment systems, 3rd year options & 1st year parents evening & they both result to exactly the same thing! Because it's an appointment system, the pupil or the parents have been given time to avoid everybody being late. Being behind schedule the only way to solve this problem in both appointment systems would be for a buzzer or alarm to go off every 5mins or 10mins according to how long the appointment is, then this way everyone would be on time & nobody would be waiting.

Doctor surgery

The tables and information and the Bar Graph are of a local practitioner who has 30 patients, it also shows how long each patient spends with the doctor.





The table below gives the assumed frequency distribution of doctors' consulting times. Observations were made over a period of a month covering some 900 consultations.

Time interval (mins)	Frequency (Consulting Time)	Cumulative Frequency	Running Total
1/2 - 1 1/2	10	10	00 - 09
1 1/2 - 2 1/2	16	26	10 - 25
2 1/2 - 3 1/2	18	44	26 - 43
3 1/2 - 4 1/2	16	60	44 - 59
4 1/2 - 5 1/2	12	72	60 - 71
5 1/2 - 6 1/2	10	82	72 - 81
6 1/2 - 7 1/2	6	88	82 - 87
7 1/2 - 8 1/2	4	92	88 - 91
8 1/2 - 9 1/2	3	95	92 - 94
9 1/2 - 10 1/2	2	97	95 - 96
10 1/2 - 11 1/2	0	97	
11 1/2 - 12 1/2	0	97	
12 1/2 - 13 1/2	0	97	
13 1/2 - 14 1/2	0	97	
14 1/2 - 15 1/2	0	97	
15 1/2 - 16 1/2	1	98	97
16 1/2 - 17 1/2	0	98	
17 1/2 - 18 1/2	0	98	
18 1/2 - 19 1/2	2	100	98 - 99
19 1/2 - 20 1/2	0	100	
	100		

Conclusion

I don't think that this particular system is very good because some patients spend a couple of minutes with the doctor, others spend more than 15mins with him/her.

I think that when the patient rings up & tells them what the problem is, one particular doctor should have all the patients they think will be over 15mins with, then the other doctors in the surgery will be able to see more patients.

How can this system be improved?

A doctor decides on what time the patient will come in and how long he thinks he will have to spend with the patient. The doctor decides this when the patient phones up or books an appointment and tells the receptionist what the problem is. The problem is that the Health Service is that you can have an appointment at 9.00a.m and not get to see the doctor until 10.00a.m, because with the health service they just give you a time regardless of how many patients are before you. The time they tell you isn't the time you'll see the doctor. So the only way for this system to be improved would be for the doctor to estimate how long he will be with <sup>the</sup> patient & to tell the receptionist so that she can give appointments to people without them having to wait long. The doctor shouldn't take any longer than 15mins & if there's more than one doctor then the patient shouldn't have to wait long.

Summary of the topic

I found this project 'why are we waiting' a very interesting project to do. There was a lot of work involved in the project i.e tables, explanation, conclusions & Bar graphs. I didn't really have any problems with the project & while producing the project I have thoroughly enjoyed it. /

Haseeba Patel

Why are we waiting?

5th Oct 2011

Does the teacher ever have to wait?

Do you think this situation is reasonable?

NAME	TIME (in mins)	Teacher has to wait or not?
Tammy	9	Wait
Jason	12	not wait
Rahner	10	not wait
Melaine	8	Wait
Winston	12	not wait
Kung	7	not wait
Norren	8	wait
Walter	10	not wait
Jerry	9	not wait
Subie	7	wait
Tony	7	wait
Sally	10	not wait

Winston took 12 mins with the teacher so I think the teacher did not have to wait for Norren to come, but when it boy's they had some many minutes with the teacher. It might be that some students may of finished talking to the teachers before their 10 mins therefore they might of called the next person in, that means they might not of been late. Not only do the teachers have to wait, but also the students in some cases.

Do you think that this is reasonable?

S2/2



Why are we worried?

During my time reading this investigation, I will be considering the following questions:  
 Whether the doctor has a dinner break or not  
 Whether the doctor was called out on calls or not  
 Amount of time per patient  
 Amount of time worked by doctors  
 How long should the doctor/patient have to wait  
 Is the time for patients long.

Because I have the knowledge to find the mean, median and mode, I decided to find them for the table of patients given.

The median is 5:30 mins per patient

The mode is 5 mins

The mean is 4:30 mins

✓ What do they mean? what are they used for?

I think each patient should receive 10 mins with the doctor. I thought this was a reasonable amount of time because the majority of patients spent more than 5 mins but less than 15 mins, so I have thought a reasonable time would be 10 mins. I used this out with the table of patient we were given, it works out that the doctor would have to work 5 hrs and 20 mins. This includes no breaks. For the patients who needed less than 10 minutes, I have given them the double amount of time. If the doctor started work at 7:00 in the morning he would finish at 2:20 in the afternoon. ✓

I think this amount of time for a doctor to work is <sup>too</sup> much. It would be better if the doctor could ~~work~~ see half the patients during the morning and the other half during the afternoon. If the doctor does spend 10 minutes with the patients, the doctor might <sup>have</sup> finished seeing the patients before the 10 minutes were up. This would mean that the doctor would have to wait for the next patient to arrive. It could be the other way around, which would mean that the patient would have to wait to see the doctor.

ADVANTAGES + DISADVANTAGES  
OF THIS SYSTEM

- 1) Doctor has some spare time to see emergency patients ~~between~~ <sup>between</sup> seeing his other patients **ADVANTAGE**
- 2) To much spare time. **DISADVANTAGE**
- 3) Less waiting time for patients **ADVANTAGE**
- 4) more waiting time for doctor **DISADVANTAGE**
- 5) No break for doctor **DISADVANTAGE**
- 6) To many working hours for doctor **DISADVANTAGE**
- 7) To many patients to see **DISADVANTAGE**. ✓ <sup>NO</sup>

Patient Number	Consultation time	Patient being made aware of waiting time (minutes)	Patient come out of waiting room (minutes)
1	6	9.00	9.06
2	2	9.06	9.08
3	5	9.08	9.13
4	1	9.13	9.14
5	7	9.14	9.21
6	16	9.21	9.37
7	2	9.37	9.39
8	5	9.39	9.44
9	4	9.44	9.48
10	7	9.48	9.51
11	7	9.51	9.53
12	7	9.53	10.00
13	2	10.00	10.02
14	3	10.02	10.05
15	4	10.05	10.09
16	1	10.09	10.10
17	3	10.10	10.13
18	6	10.13	10.17
19	3	10.17	10.22
20	5	10.22	10.27
21	4	10.27	10.31
22	18	10.31	10.49
23	6	10.49	10.55
24	3	10.55	10.58
25	5	10.58	11.03
26	1	11.03	11.04
27	4	11.04	11.08
28	19	11.08	11.27
29	3	11.27	11.30
30	5	11.30	11.35

Patient Number	Consultation time
1	6
2	2
3	5
4	1
5	7
6	16
7	2
8	5
9	4
10	7
11	2
12	7
13	2
14	3
15	4
16	1
17	3
18	6
19	3
20	5
21	4
22	18
23	6
24	3
25	5
26	1
27	4
28	19
29	3
30	5

Table 3 Consultation times of a GP's surgery

(GP means General Practitioner which is a local doctor)

WHY ARE WE WAITING?

SYSTEM 2 This is the system where the patient sees

Patient Number	consultation time	return of patient	Time to go in to see doctor	Time to go in to see doctor (Time)
1	6 mins	"	7.00	7.06
2	5	"	7.06	7.11
3	6	"	7.11	7.17
4	5	"	7.17	7.22
5	6	"	7.22	7.28
6	3	"	7.28	7.31
7	5	"	7.31	7.36
8	4	"	7.36	7.40
9	3	"	7.40	7.43
10	4 mins	"	7.43	7.44
11	6	"	7.44	7.50
12	2	"	7.50	7.52
13	6	"	7.52	7.58
14	1	"	7.58	7.57
15	1	"	7.57	10.00
16	2	"	10.00	10.02
17	4	"	10.02	10.04
18	17	"	10.04	10.23
19	16	"	10.23	10.37
20	3 mins	"	10.37	10.42
21	4	"	10.42	10.46
22	7	"	10.46	10.53
23	17	"	10.53	10.07
24	3	"	10.07	10.10
25	4	"	10.10	10.14
26	3	"	10.14	10.17
27	5	"	10.17	10.22
28	2	"	10.22	10.24
29	1	"	10.24	10.25
30	1	"	10.25	10.26

12<sup>th</sup> October 87

Why Are We Waiting?

SYSTEM 1

We were asked to use our system (6) on another table. We used random digits to get times of consultation per patient.

- 7 blocks down
- 1 block across
- 3<sup>rd</sup> row down
- 2<sup>nd</sup> column
- 3<sup>rd</sup> number
- 5 blocks down
- 2<sup>nd</sup> block across
- 2<sup>nd</sup> row down
- 3<sup>rd</sup> column
- 1<sup>st</sup> number

796 37266 70276 84728 44742 18010 11577 74335  
 58277 23462 76911 20440

SYSTEM 1

Patient Number	CONSUATION Time	APPOINTMENT Time	TIME TO WAIT FOR NEXT PATIENT TO COME
1	6 mins	7.00	4 mins
2	5 mins	7.10	5 mins
3	6 mins	7.20	4 mins
4	5 mins	7.30	5 mins
5	6 mins	7.40	4 mins
6	3 mins	7.50	7 mins
7	5 mins	10.00	5 mins
8	4 mins	10.10	6 mins
9	3 mins	10.20	7 mins
10	4 mins	10.30	6 mins
11	6 mins	10.40	4 mins
12	2 mins	10.50	8 mins
13	6 mins	11.00	9 mins
14	1 mins	11.10	7 mins
15	1 mins	11.20	8 mins
16	2 mins	11.30	8 mins
17	4 mins	11.40	6 mins
18	17 mins	12.00	1 min
19	16 mins	12.20	4 mins
20			
21			
22			

M9C Donald's  
TABLE TO SHOW HOW LONG IT  
TOOK TO BE SERVED

CUSTOMER	WAITING TIME, TO ORDER	HOW LONG IT TOOK TO ORDER	HOW LONG THEY WAITED TO RECEIVE THEIR FOOD	
1	0 SECS	1 1/2 mins	2 mins	Fries, Burger, chips coca, tea.
2	3 mins	1/2 min	1 min	Fries & Drink
3	4 mins	2 mins	3 mins	Had to wait for chips to be done
4	9 mins	1 min	30 SECS	Just bought Fries
5	10.30 mins	1 1/2 mins	1 min	Cheese burger & Drink
6	13.30 mins	1 min	5 mins	Had to wait for chicken nuggets

DATE I WENT: THURSDAY  
 TIME I ARRIVED AT MCD'S:  
 3.45  
 NO. OF TABLES SERVING:  
 3  
 BUSY OR NOT?  
 BUSY

# MACDONALD'S

**A**s a part of this investigation I decided I would take a trip to Mac Donald's, to investigate their queuing system, and to make a list of improvements which could be made. If it is necessary I choose to take a trip to Mac Donald's because I thought it was a good place for this part of the investigation. ✓

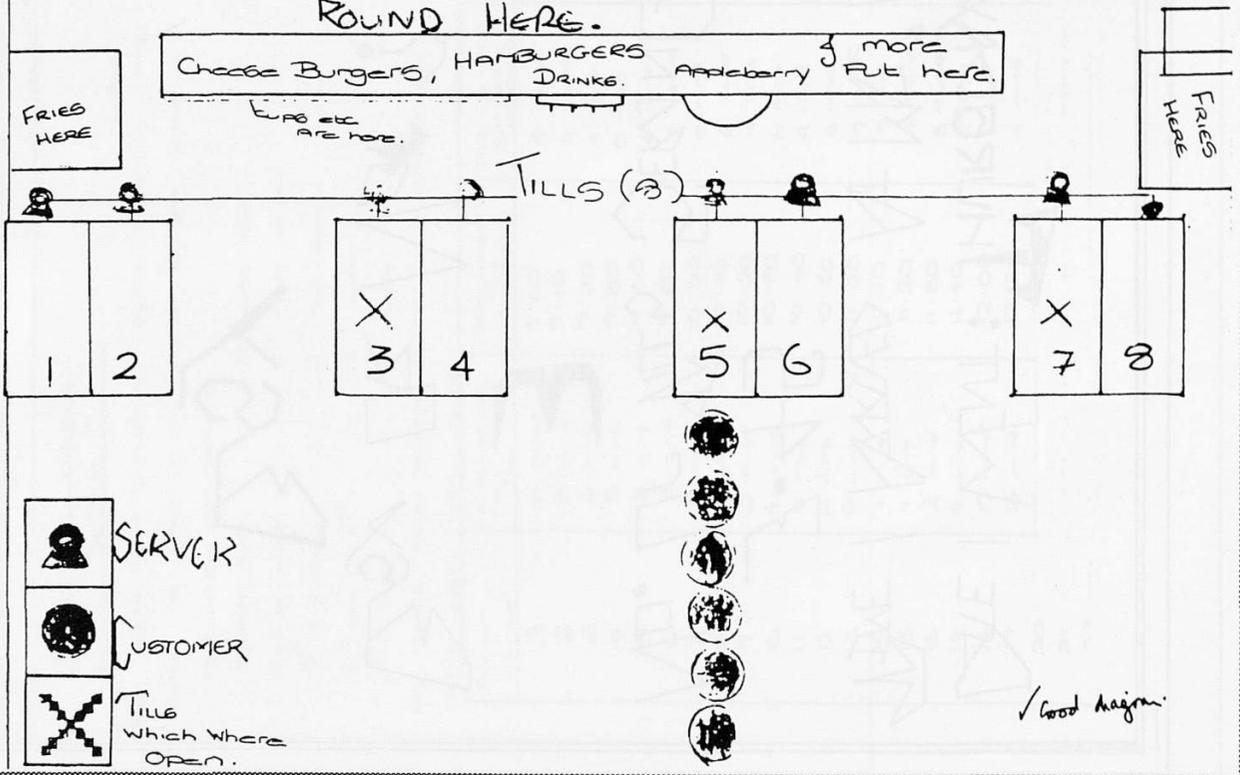
**I** went on Thursday October 15<sup>th</sup> after school, I arrived there just after quarter to four. To my surprise the que's were very long. There was only 4 tills serving. I decided I would time how long it took for each customer to be served. I went to one certain till & stood there, taking the times. This particular line was a line of 6 people. ✓

# RESULTS

I found that the waiting time was too long. People who wanted just one thing had to wait a considerable long amount of time. As you can see from the table of Results. ✓

## MACDONALD'S

ALL FOOD ETC. IS MADE ROUND HERE.



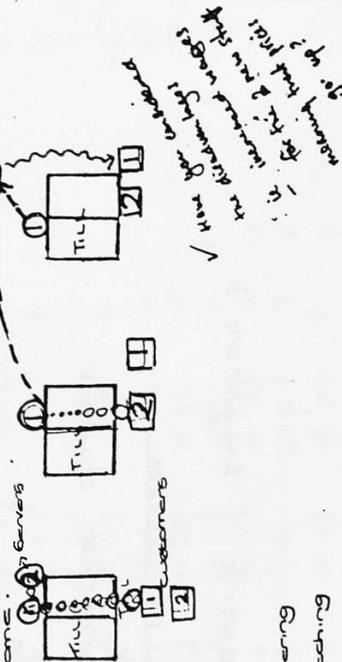
# CONCLUSION

## IMPROVE THE QUEUING SYSTEM :-

### Now?

Will be improvement. Could be more of the bills could be open, which would mean less a decrease of people in each queue ✓

Another, 2 servers per bill, this would decrease the amount of time waited by customers. So, one of them could take the order the other person would go and fetch it, so the order taker could take another order whilst he/she was waiting for the order before to come.



### KEY:

- ordering
- fetching
- order
- customer
- receives food.

Could

Another improvement which could be made is to have more of the food cooked before hand, e.g. Chicken nuggets. If you order them you have to wait a considerable amount of time. And they should be pre-ordered, if they now that the fries are going to run out, they should start cooking them before hand. They should employ more people, to help around & cook.

### Queuing:

They could change the queuing system by having different lines for different amounts of food. Say if you wanted just 1 item then you'd go to a que which serves 1 item only, if you ordering a large amount then you would go to another till. Depending on the amount of food you want, you go to the appropriate till. This saves time for people who want to order a small amount of food. And it also spreads all the people out on the tills (if all tills were serving) ✓

I think they should show a light on or something to show that the tills are serving, because I found that there would be a long line of people behind a till, whilst there is not any on another, this is caused because the public do not know that till is serving. ✓ Good idea.

## S2/3

Coursework : Assignment 3

### Types of Queues

a  
 bank  
 supermarket checkout  
 bus  
 train  
 doctors  
 opticians  
 hospitals

b  
 single queue  
 individual appointments  
 first come, first served.

c  
 bank  
 supermarket  
 bus  
 train  
 doctors  
 opticians  
 hospital  
 public house  
 queue  
 single  
 single  
 single  
 individual appointments  
 individual appointments  
 individual appointments  
 first come, first served.

Table 1

Time Appt	Duration	student waiting	teacher waiting
2.00	2.09		+1
2.10	2.22		
2.20	2.32	+2	
2.30	2.40	+2	
2.40	2.52		+1
2.50	2.59		+2
3.00	3.08		
3.10	3.20		+1
3.20	3.29		+3
3.30	3.37		+3
3.40	3.47		
3.50	4.00		
		6	13

The Tutor has had to wait 13 minutes in between interviews the students wait 6 minutes.

If the tutor wanted a short break in between interviews this system would be fine. Assuming all the students were in reasonable time for their interviews a quicker method would be one student finishes his interview and the next one goes in. Or a first come first served system by which tutor sees the students as they arrive. In this system he would never have to wait.

TABLE 1

<u>Appt time</u>	<u>consultation time</u>	<u>7 minute</u>	<u>Appt system</u>
<u>patient</u>	<u>Duration</u>	<u>patients waiting</u>	<u>after waiting</u>
9-00	9-00	6	1
9-07	9-07	2	1
9-14	9-14	5	
9-21	9-21	1	3
9-28	9-35	7	6
9-35	9-51	16	
9-42	9-53	2	9
9-49	9-58	5	4
9-56	10-02	4	2
10-03	10-10	7	
10-10	10-10	2	5
10-17	10-17	7	
10-24	10-24	2	5
10-31	10-31	3	4
10-38	10-42	4	3
10-45	10-45	1	3
10-52	10-52	3	6
10-59	10-59	6	4
11-06	11-06	3	1
11-13	11-13	5	4
11-20	11-20	4	2
11-27	11-27	18	3
11-34	11-45	6	11
11-41	11-51	3	10
11-48	11-54	5	6
11-55	11-59	1	4
12-02	12-02	4	1
12-09	12-09	19	
12-16	12-28	3	12
12-23	12-31	5	8
		2hrs 39min	53 minutes

The 7 minute interval system didn't really work for anyone the doctor had to wait a total 53 minutes and the patients waited 56 minutes in total. Which means an average of less than two minutes a patient. On the other hand the doctor also had to wait approximately two minutes per patient. The consultation time for the whole of his surgery was 2 hours and 39 minutes yet it actually took 3 hours and 36 minutes. The doctor spent on average 5.3 minutes consultation time with his patients.

On looking at the mean consultation time at the doctors it would be  $159 \div 30 = 5.3$  minutes consultation time. The mode time would be 3 minutes and 5 minutes.

The system which would suit the doctor best would be the 5 minute appointment system. Though, the whole surgery the doctor would only have to wait for 10 minutes. Which is an average of  $\frac{1}{3}$  of a minute for each patient. Also he would finish his surgery in only 2 hours 48 minutes which saves him 47 minutes compared to the 7 minute system. Unfortunately his patients would not fare as well they would have to wait for a total of 3 hrs 14 minutes. Which average out at 6 minutes per person waiting time.

Tally chart - consultation time in doctors surgery

Time	Tally	Frequency
1		3
2		4
3		5
4		4
5		5
6		3
7		3
16		1
18		1
19		1
<b>Total frequency</b> - 36		

TABLE 2

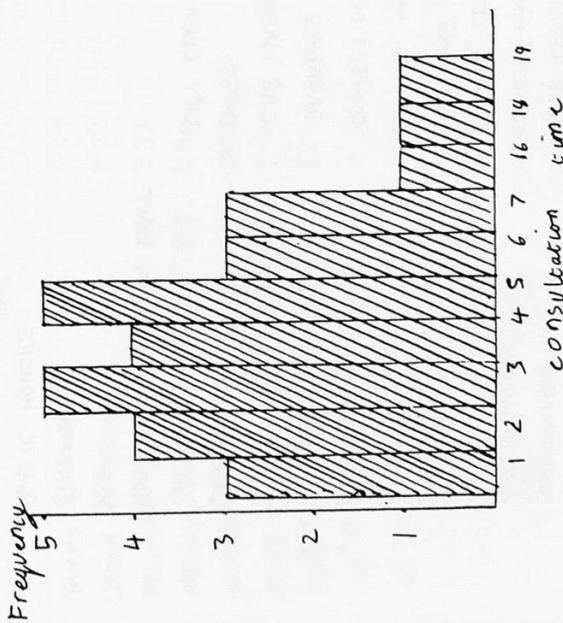
App time	Consultation time	Duration	5 minutes	Appet	Systec
patient	patient		waiting	doctor	waiting
9:00	9:00	6	1		
9:05	9:06	2			
9:10	9:15	5		2	
9:15	9:16	1			
9:20	9:27	7			4
9:25	9:43	16	2		
9:30	9:45	2	13		
9:35	9:50	5	10		
9:40	9:54	4	10		
9:45	10:01	7	9		
9:50	10:03	2	11		
9:55	10:10	7	8		
10:00	10:10	2	10		
10:05	10:12	3	7		
10:10	10:15	4	5		
10:15	10:19	1	1		
10:20	10:20	3		2	
10:25	10:25	6	1		
10:30	10:31	3			
10:35	10:35	5			1
10:40	10:40	4			1
10:45	10:45	18			
10:50	11:03	6	13		
10:55	11:09	3	14		
11:00	11:12	5	12		
11:05	11:17	1	12		
11:10	11:18	4	8		
11:15	11:22	19	7		
11:20	11:41	3	21		
11:25	11:41	3	19		
11:44	11:49	5	3	14	16
<b>2hrs 39min</b>			<b>3</b>	<b>14</b>	<b>16</b>

Appic system using a frequency distribution

patient	Appr time	
1	2	16
2	5	17
3	4	18
4	7	19
5	1	20
6	7	21
7	2	22
8	8	23
9	2	24
10	1	25
11	3	26
12	4	27
13	5	28
14	1	29
15	2	30

Using table 4 I generated some numbers and then used the numbers on a frequency distribution chart. Using this I put the appointment time down for 30 patients. What I do find surprising is that the system above is more efficient than the 5 minute appointment system I devised. The doctor would be best advised to use this system as this surgery only takes 2 hrs and 9 minutes which would be a saving of 40 minutes over the 5 minute appointment system. In a total of 129 minutes for 30 patients this would give the doctor 4.3 minutes with each patient.

Histogram to show frequency of consultation time



Using table 4  
 6th Block down  
 2nd Block across  
 4th row  
 5th column  
 3rd Number

18 61 47 85 04 82 11 91 19 08 32 52 61 05  
 24 56 00 88 33 55 67 57 64 53 68 75 92 83  
 96 63.

## S2/4

### QUEUES

PLACES WHERE SOME FORM OF QUEUEING EXISTS

#### SINGLE QUEUES NON APPOINTMENT

POST OFFICES	RAILWAY TICKET OFFICE
NEWSAGENTS	BUS
** BANKS	THEATRE
FISH + CHIP SHOPS	TRAIN STATION
TAKEAWAYS	BUTCHERS
SUPERMARKETS	SHOE REPAIRERS
GREENGROCCERS	SPORTS GROUNDS
CAFES	SPORTS CENTRES
BUS STOPS	SWIMMING BATHS
RENT OFFICES	SCHOOL LUNCH
** GAS BOARD	CONFECTIONERS
** ELECTRICITY BOARD	GARAGES
PUBLIC CONVENIENCES	CAR WASH
LAUNDERETTES	* DHSS
CHEMIST	DRY CLEANERS
HOSPITAL CASUALTY DEPT.	
ZOO	
CINEMA	
PUBLIC HOUSES	
CLUBS	

### APPOINTMENTS

HOSPITAL	JOB CENTRE
DOCTOR	EMPLOYMENT OFFICES
DENTIST	BANK MANAGER
OPTICIAN	SOLICITOR
* DHSS	RESTAURANTS
JOB INTERVIEW	COLLEGE INTERVIEWS
	INTERVIEWS AT WORK

\* DHSS On an initial visit to the DHSS a person takes a ticket from a machine and takes a seat, when the number on the ticket is called out the person then goes to the counter for interview. I believe this system is to stop queue jumping and also to help people who cannot stand about for a long time in queues eg old people, pregnant women etc.

\*\* These establishments would usually have more than one queue operating at any one time

DOCTORS CONSULTATION TIMES

In table 3 the total time the doctor spends with his patients is 159 minutes. If we divide this time between the 30 patients, this gives us an average interview time with each patient of 5.3 minutes each. From the table we can see that 21 out of the 30 patients spend less than this time actually in the doctors surgery. Six patients are only marginally over the 5.3 minutes average and three patients are taking up 5.3 minutes of the doctors time which is in fact one third of the total surgery time.

There is never going to be a system which allows for 100% efficiency in an appointment system, so we must try and come up with one that saves the doctor the greatest amount of his valuable time. From the data given in table 3 we must assume from the average of 5.3 minutes that this would be a typical day in the surgery and build an appointment system based on or near that average.

One knows there will be patients who require more of the doctors attention than others, and these patients must be accommodated with as little inconvenience as the doctor and fellow patients as possible. I would conclude that an appointment system based on five minute intervals would seem to be the best for this particular surgery. In my experience of going to the doctors I would say that nearly 100% of patients with an appointment would turn up at least five minutes before the appointment was due. This would also serve to save the doctor valuable time.

STUDENT	INTERVIEW TIME	INTERVIEW ENDS	TEACHER WAITING TIME	STUDENT WAITING TIME
TAMMY WHITAKER	2:00	2:09	1 MINUTE	
JASON HARGREAVES	2:10	2:22		* 2 MINUTES
RAJINDER UBHI	2:20	2:32		* 2 MINUTES
MELANIE NESBITT	2:30	2:40		
WINSTON BAILEY	2:40	2:52		* 2 MINUTES
KUNG IP	2:50	2:59	1 MINUTE	
NOREEN DYSON	3:00	3:08	2 MINUTES	
WALTER BASSETT	3:10	3:20		
JENNY PAYNE	3:20	3:29	1 MINUTE	
SUSIE REED	3:30	3:37	3 MINUTES	
TONY SINGH	3:40	3:47	3 MINUTES	
SALLY NESBITT	3:50	4:00		

\* ACTUAL TIME OF INTERVIEWS

The tutor has to wait on six occasions for a total of 11 minutes.

Students have to wait on three occasions for a total of 6 minutes.

I would say that this system is quite reasonable but could be perhaps improved if the appointments were made at 5 minute intervals. This would have the effect of reducing the amount of time the tutor has to wait for students as the tutors time is arguably more valuable than the students.

FOUR MINUTE APPOINTMENT SYSTEM DISADVANTAGES

- \*\* NO BREAK FOR DOCTOR
- \* LATER PATIENTS HAVE TOO LONG TO WAIT

FIVE MINUTE APPOINTMENT SYSTEM ADVANTAGES

- 1/ DOCTOR GETS CHANCE TO WRITE UP NOTES
- 2/ PATIENTS MAXIMUM WAIT REDUCED
- 3/ POSSIBILITY FOR DOCTOR TO TAKE A BREAK IF NEEDED

As can be seen from the figures on the last page the four minute system is impractical especially for the patients. Whereas the five minute system is much more fair to patients and allows the doctor a brief respite from the pressures of his job.

PATIENT No	4 MINUTE APPOINTMENTS				5 MINUTE APPOINTMENTS				CONSULTATION TIME
	APPOINTMENT TIMES	ACTUAL TIME SAW DOCTOR	DOCTOR WAITING	PATIENT WAITING	APPOINTMENT TIME	ACTIVE TIME SAW DOCTOR	DOCTOR WAITING	PATIENT WAITING	
1	9:0	9:0	0	0	9:0	9:0	0	0	6
2	9:04	9:06	0	2	9:05	9:06	0	1	2
3	9:08	9:08	0	0	9:10	9:08	0	0	5
4	9:12	9:13	0	1	9:15	9:13	2	0	1
5	9:16	9:16	3	0	9:20	9:14	6	0	7
6	9:20	9:21	0	1	9:25	9:21	4	0	16
7	9:24	9:37	0	13	9:30	9:37	0	7	2
8	9:28	9:39	0	11	9:35	9:39	0	4	5
9	9:32	9:44	0	12	9:40	9:44	0	4	4
10	9:36	9:48	0	12	9:45	9:48	0	3	7
11	9:40	9:55	0	15	9:50	9:55	0	5	2
12	9:44	9:57	0	13	9:55	9:57	0	2	7
13	9:48	10:04	0	16	10:00	10:04	0	4	2
14	9:52	10:06	0	14	10:05	10:06	0	3	3
15	9:56	10:09	0	13	10:10	10:09	1	0	4
16	10:0	10:13	0	13	10:15	10:13	2	0	1
17	10:04	10:14	0	10	10:20	10:14	6	0	3
18	10:08	10:17	0	9	10:25	10:17	8	0	6
19	10:12	10:23	0	11	10:30	10:23	7	0	3
20	10:16	10:26	0	10	10:35	10:26	9	0	5
21	10:20	10:31	0	11	10:40	10:31	9	0	4
22	10:24	10:35	0	11	10:45	10:35	10	0	18
23	10:28	10:53	0	25	10:50	10:53	0	3	6
24	10:32	10:59	0	27	10:55	10:59	0	4	3
25	10:36	11:02	0	26	11:00	11:02	0	2	5
26	10:40	11:07	0	27	11:05	11:07	0	2	1
27	10:44	11:08	0	24	11:10	11:08	2	0	4
28	10:48	11:12	0	24	11:15	11:12	3	0	19
29	10:52	11:31	0	39	11:20	11:31	0	11	3
30	10:56	11:34	0	38	11:25	11:34	0	9	5

\*\* \*

INVESTIGATION INTO 4.5.6 MINUTE APPOINTMENT SYSTEM BASED ON RANDOM DIGIT INFORMATION! AND FREQUENCY TABLE COVERING 20 PATIENTS.

PATIENT NO.	1	2	3	4	5	6	7
1	5	3	16	3	3	4	4
2	19	3	5	1	2	3	19
3	3	4	5	7	2	3	8
4	4	1	3	8	7	1	1
5	4	19	7	4	4	9	2
6	5	3	16	4	19	7	19
7	6	4	4	6	4	2	3
8	6	3	2	2	6	2	10
9	2	1	5	9	3	4	3
10	1	5	16	3	5	5	5
11	2	5	3	3	7	6	4
12	9	2	2	4	2	3	2
13	1	19	3	4	5	4	3
14	9	3	5	4	19	4	5
15	4	6	4	19	5	2	3
16	4	7	4	7	5	4	5
17	4	5	3	4	5	4	6
18	8	5	1	1	3	6	4
19	6	10	5	4	9	4	3
20	2	5	8	2	1	7	2

TOTAL MINUTES	104	113	120	99	119	90	7
AVERAGE TIME	$\frac{104}{20} = 5.2m$	$\frac{113}{20} = 5.65m$	$\frac{120}{20} = 6m$	$\frac{99}{20} = 4.95m$	$\frac{119}{20} = 5.95m$	$\frac{90}{20} = 4.5m$	$\frac{11}{20} = 5.5m$
4 MINUTES COMMENTS ON PRACTICALITY	4 MINUTES APPOINTMENT SYSTEM NOT FEASIBLE HERE	NOT FEASIBLE	NOT FEASIBLE	NOT FEASIBLE	NOT FEASIBLE	POSSIBLE BUT 9 PATIENTS ABOVE AVERAGE	NOT FEASIBLE
5 MINUTES	5 MINUTE APPOINTMENT BEST FOR THIS CASE	POSSIBLY BEST FOR DOCTOR	POSSIBLY BEST FOR DOCTOR	IDEAL	POSSIBLY BEST FOR DOCTOR	POSSIBLY BEST FOR DOCTOR	POSSIBLY BEST FOR DOCTOR
6 MINUTES	6 MINUTE SYSTEM WOULD WASTE DOCTORS TIME	DOCTOR PROBABLY WASTES TOO LONG	POSSIBLE BUT BELONG IN THE 50 DOCTOR WAITING	NOT PRACTICAL	POSSIBLE FENER BELOW AVERAGE	NOT PRACTICAL TOO MUCH WAITING FOR DOCTOR	NOT PRACTICAL

From the information collected from the frequency chart and the Random digit table I would without doubt recommend to this doctor that he adopt a five minute appointment system. From the figures above it would appear that this would be the system which would be most convenient to him and his patients. As I have said previously no system can be totally efficient but I believe from the figures produced that the five minute system would work best for him.

S2/5

# WHY ARE WE WAITING

WHY ARE WE WAITING?

Q1

In your group write down five different queuing systems that you have come across at some stage in your life. Your group has about 20 minutes to fill in the table below.

You will then have some time to explain your results to the other groups in the class. You will score five points for each one accepted as a proper queuing system by the other groups but one point will be knocked off for every group who also has that system, so try to think of unusual ones, however, you score zero if they are not accepted.

Eg,

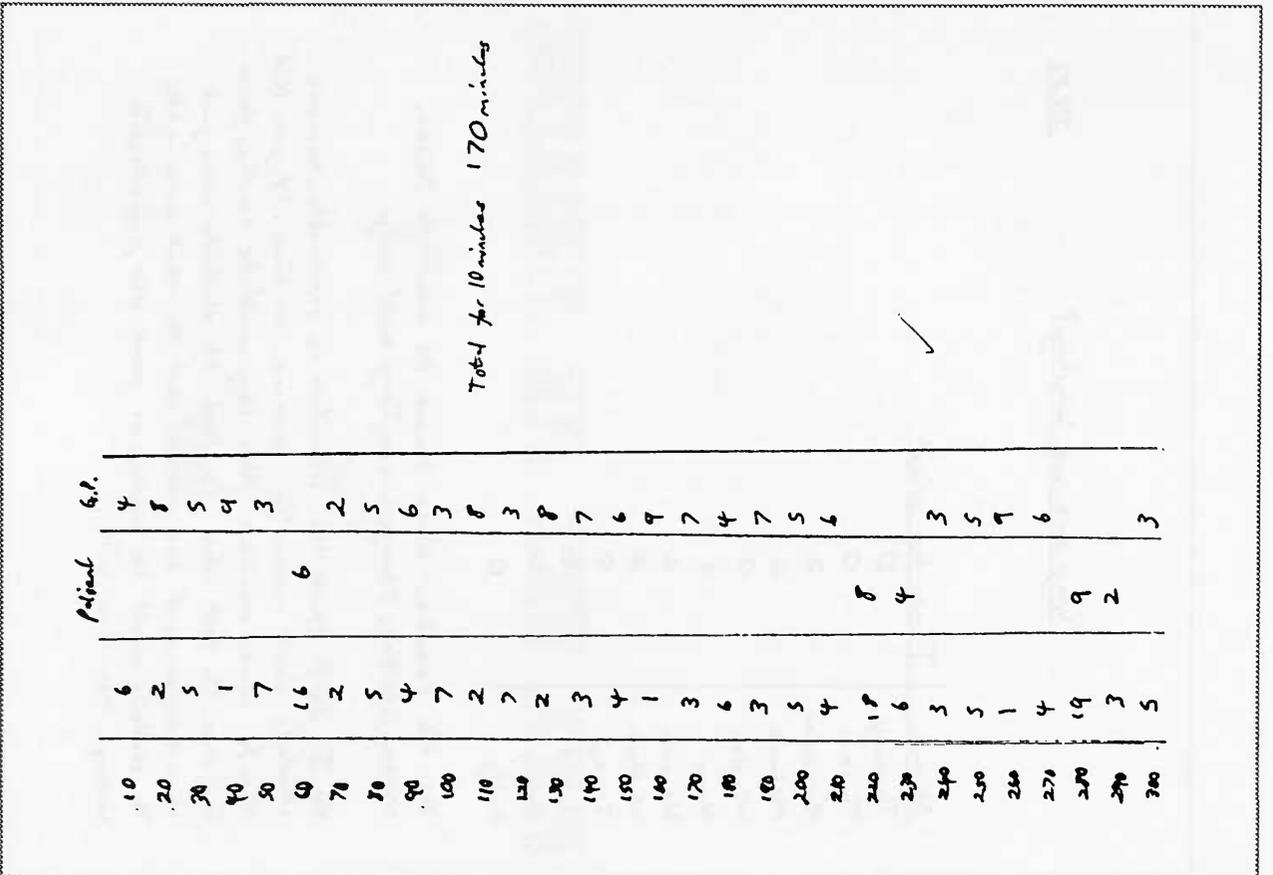
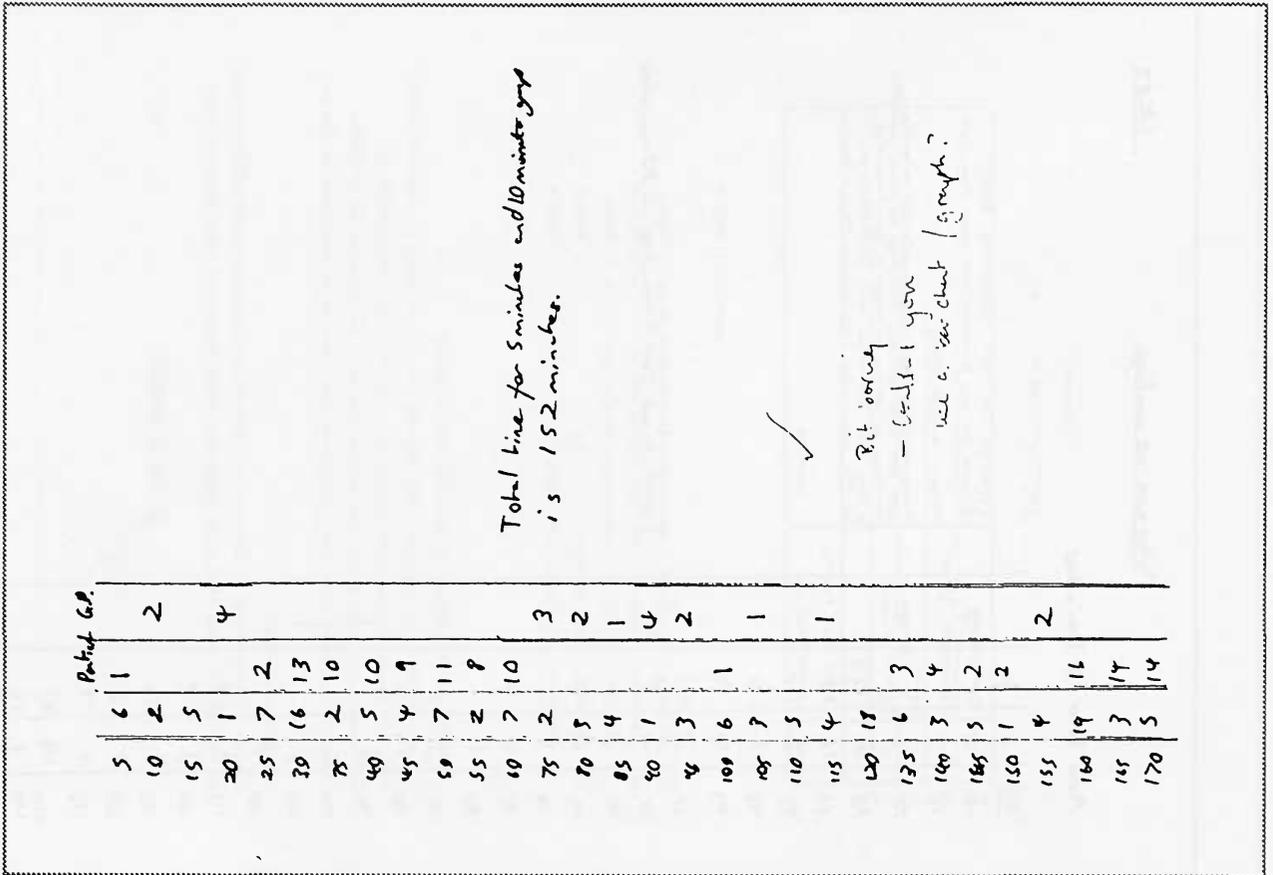
- 1 System A - accepted - one other group has it - 4 points
- 2 System B - not accepted - 0 points
- 3 System C - accepted - four other groups have it - 1 point
- 4 System D - accepted - no other group has it - 5 points
- 5 System E - accepted - four other groups have it - 1 point

Total = 11 points

No	Where have you seen this queue?	Details
1	T.S.B.	We get the same line every time we go to the bank and they go to the bank to get money and the bank is full in the morning and the queue is 150 people waiting for their money
2	Dinner Queue	Queue in line for
3	At the bank	Queue in line for
4	Motorways Jam	Queue in an 2-lane line then split into 4-lane to take 4 lanes each
5	Alberta	Definition of a queue. ✓

(21)





OUR MAIN SURVEY  
OF A QUEUING SYSTEM

We decided that to do our survey we would go down to the Halifax Building Society and study their queuing system.

We contacted them and made an appointment to go down at 8.45 and then study the system of queuing that the Halifax uses. The system is for people coming in to form one queue. The person at the head of the queue, ie the one who has waited the longest, moves to an empty till whenever one becomes available. This avoids several small queues forming at each till with the consequence of people jockeying for a position at the shortest quickest till.

The Manager thought that the system was very good and worked well and the customers liked it. The advantages about the system are that it is completely fair and when you are at the till it is very confidential because you don't have a person looking over your shoulder. The only one disadvantage that I could find is that it needs more space than the normal way of queuing. A point to note is that the Halifax was the first place in Loughborough to use this system, but it has been used for a long time in other countries.

At nine o'clock there were already 5 people waiting to go in and when the doors were opened the five came in and I started to record the times. The times that I was recording were the time the customer came in to the queue, the time the customer went out of the queue to the till and the time the customer left the till. I did the survey for the first hour and then we had a break when we had some coffee. After this half an hour break we did some more surveying for half an hour. We did some specific surveys of certain people, 4 males and 4 females. This specific survey was to find out the following: how long a person stayed in the main queue and how long they spent at the till. We did this for the eight people and then we had a talk to the manager about the system.

CONCLUSIONS DRAWN FROM THE HALIFAX DATA

During the 1 hour, and 30 minutes surveyed, 135 people came into the Halifax to do transactions at the

WHY ARE WE WAITING  
DOCTOR'S APPOINTMENT TIMES

Working on the results of the list we were given I discovered that the mean consultation time was 5.7 minutes.

If we round off the time between appointments to 5 minutes, this would work out quite well, except that if you got someone like number 22 on the list with an actual consultation time of 18 minutes then all the people after this would be waiting for a long time. If we had the scheduled consultation time as 10 minutes then the waiting time would be 170 minutes, which would be a reduction on the 5 minute waiting time. The latter would mean a total waiting time of 230 minutes.

An alternative to both these consultation times is some way in which we can allow the doctor to catch up on his consultation time. I thought of the idea of having a gap of 10 minutes every one hour in the appointments to allow for this delay to be made (keeping the 5 minute consultation time). I tried this and it cut the total patient waiting time to some extent, ie to 152 minutes.

PERSON	* APPOINTMENT *	* LENGTH OF	* TIME OF	* TIME DUE	* PATIENT	* G.P.
	* APP.	* TIME OF	* TO LEAVE	* WAITING	* WAITING	* TIME
1						
2	6	0	5			
3	5	5	10	1		
4	6	10	15	1		2
5	2	15	20			
6	2	20	25	1		2
7	1	25	30			3
8	3	30	35			4
9	5	35	40			2
10	2	40	45			4
11	3	45	50			3
12	1	50	55			12
13	2	55	60			4
14	1	70	75			3
15	4	75	80			4
16	16	80	85			4
17	2	85	90			1
18	10	90	95	1		
19	7	95	100	5		
20	9	100	105	1		
21	7	110	110	15		
22	7	115	115	12		
23	3	120	120	1		
24	5	130	140			
25	5	135	140			
26	8	140	145			
27	9	145	150	1		
28	4	150	155	7		
29	4	155	160	7		
30	1	160	165	0		
		165	170			
TOTAL				111	40	

Total time in waiting Minutes is 151 minutes  
which is 2 Hours 31 minutes.

tills. The mean length of time a person spent at the till was 2 minutes 7.14 seconds, but the modal length was usually about 1 minute or less. During the 1 hour 30 minutes there was only a staggeringly small amount of man-waiting time of 2 hours 18 minutes, which means the mean waiting time per person was 53.4 seconds. (An additional fact discovered was that the average number of tills open over the time was 3.5 approx).

These figures show that the system is very efficient, because at one time when a person spent 16 minutes at the till there were 31 other people doing their transactions, which shows that if one person is a long time it does not delay the whole system.

Out of the specific survey which was taken in the last hour when business was getting much busier the average time of waiting was 1 minute 13.6 seconds.

IMPROVEMENTS WHICH COULD BE MADE

The overall system of queuing is very efficient and I do not think it could be improved, but one improvement that could be made is to have one cash desk purely for withdrawing cash. This, I think, would take out the short quick visits from the general queue and make the overall wait of those with bigger transactions shorter, i.e. one 3 minute transaction is quicker than 3 1 minute transactions, because of time needed moving to and from the till. Presumably a 'one job' till would move faster than a multi-job till.

*Very good piece of work.*



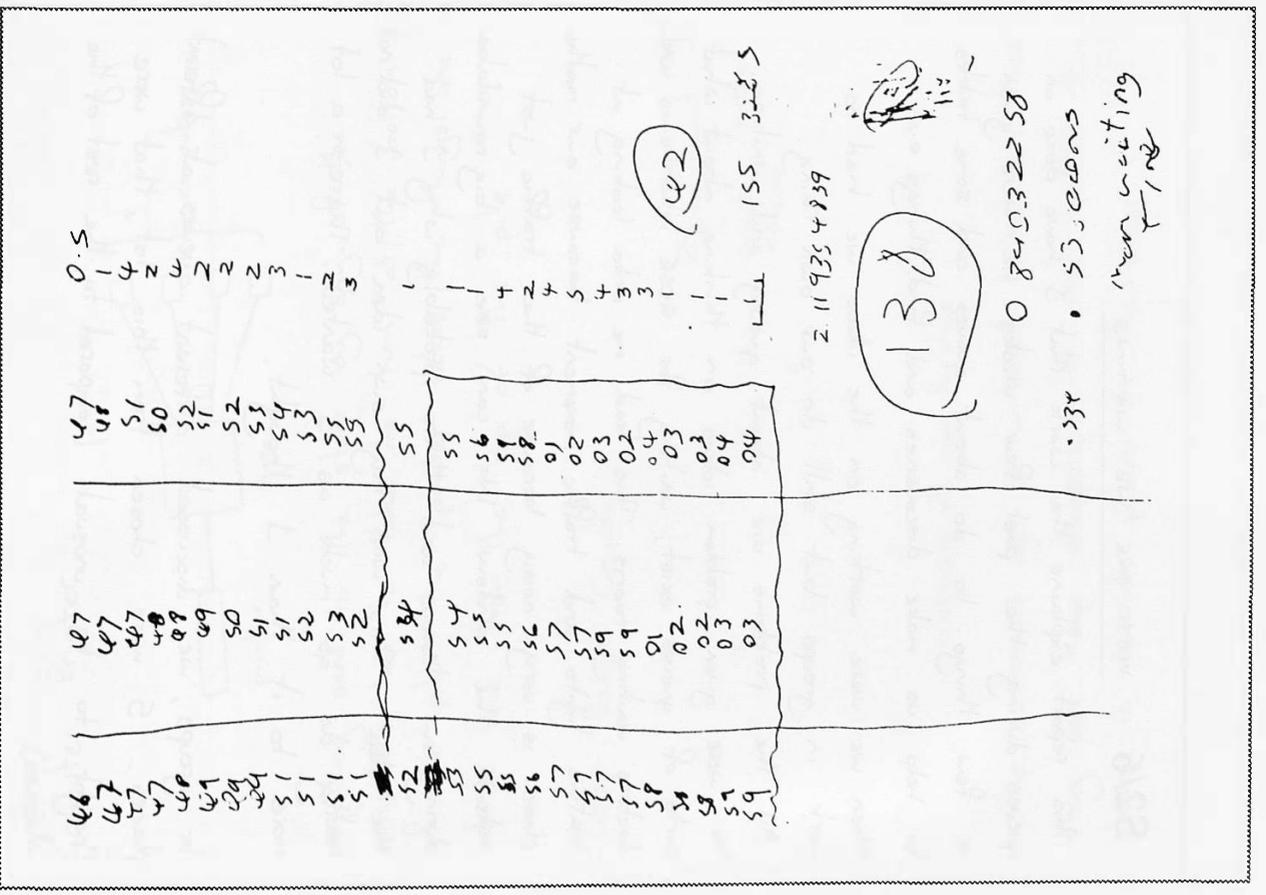


Age	Time	Time taken	Number of people	Time in	Time out
M B	3	1:42	49	2:53	4:14
M G	5	3:20	1:30	2:53	4:14
M G	4	4:19	1:58		
M	6				

Age	Pos	At the	At the	At the
G	5	2:53	3:20	3:10
M G	4	4:14	4:21	4:45
M G	6	2:15	2:20	5:2
O M	2	2:11	3:34	1:18
O M	5	3:21	2:41	1:23
M M	4	2:21	6:11	1:21
M M	11	6:04	6:53	1:07
O M	13	6:37	2:22	5:16
M	11	7:12		14:97

Total cosines M=30 in 14:60  
F=25  
Average time of entry



**S2/6**

WHY ARE WE WAITING ?

This report explains the work that I have done on queues during the past few weeks. We were given a few things to do about queues and some tables to help us make discoveries and find things out.

When we were working on the ideas we had to work in groups but still do our own thing.

All the problems are about queuing and waiting. We were given problem ideas on thinking about what sorts of queues exist, waiting for GCSE interviews and doctors waiting rooms. This led me onto looking at traffic lights and traffic movement because our maths class is very noisy because of the traffic just outside the window. We can see a big roundabout during our lessons and that's probably why I had the idea. This was my own idea but I did not really do as much as I wanted. There's a lot more to it than I thought.

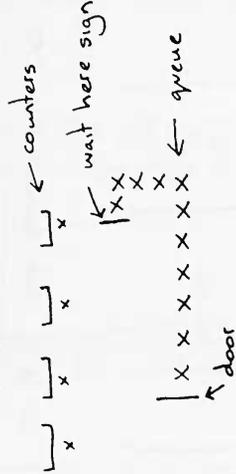
In groups, we discussed different queues at different places. 5 were chosen from this list, that were thought to be unusual (compared to the rest of the classes).

There was some disagreement about what a queue was. My definition:

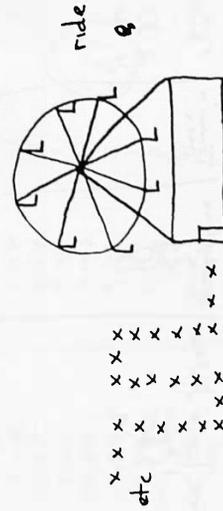
A sequence/order of people +/or objects waiting for something. A queue progresses according to ~~the~~ time passed.

Our 5

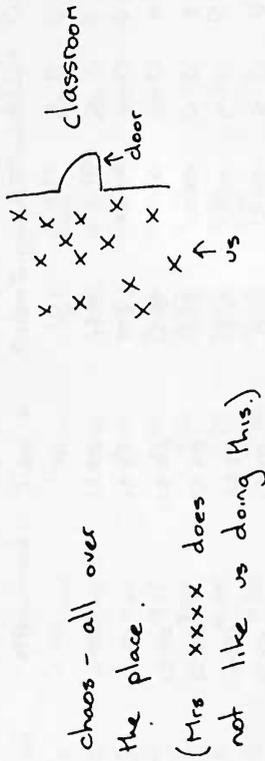
Abbey National



Allton Towers



### Waiting for maths

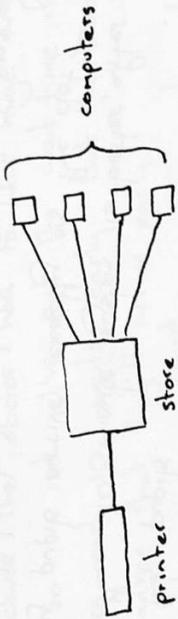


We were then given some tasks to complete concerning 'interview' times.  
We had to work out whether it was an effective appointment system or not and whether the teacher + pupils had to wait.

Different appointment times were tried.

	TIME GO IN	TIME SUPPOSED TO GO IN	TIME GOES OUT	TIME SUPPOSED	TEACHERS WAIT	PUPILS WAIT
1.	2.00	2.00	2.09	2.10	0	0
2.	2.10	2.10	2.21	2.20	1	0
3.	2.22	2.20	2.31	2.30	0	2
4.	2.32	2.30	2.39	2.40	0	2
5.	2.40	2.40	2.52	2.50	0	0
6.	2.52	2.50	2.59	3.00	0	2
7.	3.00	3.00	3.08	3.10	1	0
8.	3.10	3.10	3.20	3.20	2	0
9.	3.20	3.20	3.29	3.30	0	0
10.	3.30	3.30	3.37	3.40	1	0
11.	3.40	3.40	3.47	3.50	3	0
12.	3.50	3.50	4.00	4.00	3	0

### Computer Printer



### Coop Bacon Counter



The customers get a numbered ticket and then they can keep shopping. The next number is shown in several places in the shop

No.	APPOINTMENT TIME	ACTUAL TIME GO IN	TIME OUT	SUPPOSED	DOCTOR'S WAIT	PATIENT'S WAIT
1	9:00	9:00	9:06	9:10	4	0
2	9:10	9:10	9:12	9:20	8	0
3	9:20	9:20	9:25	9:30	5	0
4	9:30	9:30	9:31	9:40	9	0
5	9:40	9:40	9:47	9:50	3	0
6	9:50	9:50	10:06	10:00	4	6
7	10:00	10:06	10:08	10:10	2	0
8	10:10	10:10	10:15	10:20	5	0
9	10:20	10:20	10:24	10:30	6	0
10	10:30	10:30	10:37	10:40	3	0
11	10:40	10:40	10:42	10:50	8	0
12	10:50	10:50	10:57	11:00	3	0
13	11:00	11:00	11:02	11:10	8	0
14	11:10	11:10	11:13	11:20	7	0
15	11:20	11:20	11:24	11:30	6	0
16	11:30	11:30	11:31	11:40	9	0
17	11:40	11:40	11:43	11:50	7	0
18	11:50	11:50	11:56	12:00	4	0
19	12:00	12:00	12:03	12:10	7	0
20	12:10	12:10	12:15	12:20	5	0
21	12:20	12:20	12:24	12:30	6	0
22	12:30	12:30	12:48	12:40	0	8
23	12:40	12:48	12:54	12:50	0	4
24	12:50	12:54	12:57	13:00	3	0
25	13:00	13:00	13:05	1:10	5	0
26	1:10	1:10	1:11	1:20	9	0
27	1:20	1:20	1:24	1:30	6	0
28	1:30	1:30	1:49	1:40	0	9
29	1:40	1:49	1:52	1:50	0	2
30	1:50	1:52	1:57	2:00	0	0

	APPOINTMENT	ACTUALLY IN	PUPILS WAIT	TEACHER'S WAIT
1.	2:00	2:00	0	0
2.	2:09	2:09	0	0
3.	2:18	2:21	3	0
4.	2:27	2:31	4	0
5.	2:36	2:39	3	0
6.	2:45	2:51	6	0
7.	2:54	2:58	4	0
8.	3:03	3:06	3	0
9.	3:12	3:16	4	0
10.	3:21	3:25	4	0
11.	3:30	3:32	2	0
12.	3:39	3:39	0	0

The average interview time was 9 mins. The 9 minute system means the pupils have to wait, and the teacher doesn't.  
 The 10 minute system means it is fairer as the teacher waits only a small amount of time as do some of the pupils.  
 I think the 10 minute system is better, after all how many pupils arrive exactly on the dot?  
 I then looked at the doctor problem. I was given the consultation times of 30 patients. This is what I did with the data.

I thought that I would try a 5 min appointment system.

No	Appointed Time in	Out	Supposed	Doctors	Patients
1	9:00	9:06	9:05	0	0
2	9:05	9:08	9:10	2	1
3	9:10	9:15	9:15	0	0
4	9:15	9:16	9:20	4	0
5	9:20	9:27	9:25	0	0
6	9:25	9:43	9:30	0	2
7	9:30	9:45	9:35	0	13
8	9:35	9:50	9:40	0	10
9	9:40	9:54	9:45	0	10
10	9:45	10:01	9:50	0	9
11	9:50	10:01	9:55	0	11
12	9:55	10:03	10:00	0	8
13	10:00	10:10	10:05	0	10
14	10:05	10:12	10:10	0	7
15	10:10	10:15	10:15	0	5
16	10:15	10:19	10:20	0	4
17	10:20	10:23	10:25	2	0
18	10:25	10:31	10:30	0	0
19	10:30	10:34	10:35	1	1
20	10:35	10:40	10:40	0	0
21	10:40	10:44	10:45	1	0
22	10:45	11:03	10:50	0	0
23	10:50	11:09	10:55	0	13
24	10:55	11:12	11:00	0	14
25	11:00	11:17	11:05	0	12
26	11:05	11:18	11:10	0	12
27	11:10	11:22	11:15	0	8
28	11:15	11:41	11:20	0	7
29	11:20	11:44	11:25	0	21
30	11:25	11:49	11:30	0	19

The 5 min appointment system mean that some patients have a very long wait. Assuming that they don't mind, this is better for the doctor who isn't working time.

First I tried a 10 minute system, this didn't work too well because the doctor had to wait huge amounts of time. So in red, I've put the wait time if the patients were each 10 minutes early. But the Doctor can't guarantee that they'll arrive early.  
 All the systems + appointment times were work out assuming that patients arrived exactly for their appointment time, unless I assume they arrive early.

No	Time taken	Appointment	Actual	Doctor	Patient
1	6	9:06	9:00	0	0
2	1	9:05	9:06	0	1
3	5	9:10	9:10	3	0
4	4	9:15	9:15	0	0
5	5	9:20	9:20	1	0
6	2	9:25	9:25	0	0
7	1	9:30	9:30	3	0
8	6	9:35	9:35	4	0
9	5	9:40	9:41	0	1
10	3	9:45	9:46	0	1
11	3	9:50	9:50	1	0
12	2	9:55	9:55	2	0
13	7	10:00	10:00	3	0
14	7	10:05	10:07	0	2
15	19	10:10	10:14	0	4
16	3	10:15	10:33	0	18
17	2	10:20	10:36	0	16
18	1	10:25	10:38	0	13
19	4	10:30	10:39	0	9
20	3	10:35	10:43	0	8
21	4	10:40	10:46	0	6
22	5	10:45	10:50	0	5
23	6	10:50	10:55	0	5
24	5	10:55	11:01	0	6
25	3	11:00	11:06	0	6
26	4	11:05	11:09	0	4
27	7	11:10	11:13	0	3
28	16	11:15	11:20	0	5
29	18	11:20	11:36	0	16
30	2	11:25	11:54	0	29

It then struck me that the same people (patients) could have gone to the doctors on this particular day but been seen in a different order. They would have just been in a different order when they made their appointments.

I think that the waiting will be about the same but I'm not sure so I tried it.

I wrote the consultation times on a piece of paper.

I then folded them up and jumbled them up.

I then got the times in a new order.

Both sets of figures show a similar pattern. It takes a long time to get rid of patient long waits after one long consultation time. The second set had 2 out of the 3 long times near the end and so this had less effect.

A summary:

	Original	Jumbled
No. of patients waiting	21	20
Total patients waiting time	197	158
Average	6:57	5:27
Total Doctors waiting time	10	17
Average	0:33	0:57

So this can make a difference, but not a lot.

So, probably the 5 minute system is better. It was OK for the original and jumbled patients. We were introduced to Random no. systems and asked to test our decisions. I suppose this in a way was the same as I did with my jumbled list. I decided to do this anyway.

I followed the instructions to give 3 different sets of random nos. Each of these were different of course and I decided to test just one because I had already completed two tests.

Again a similar pattern showed to get rid of the backlog caused by the single long consultation time

Summary

No of patients waiting 17

Total patient waiting time 119

Average 3.97

Total doctor waiting time 25

Average 0.83

These figures are a lot better because there is only one long time.

This still seems satisfactory and easy to operate (i.e. 5 mins rather than anything awkward). I'll stick with this.

Consultation times won't always take the same time, some days all the patients may take 1-2 mins, other days maybe they'll all have 10-18 minute appointments. There isn't one perfect answer, consultation times vary from day to day.

Different times for appointments may work eg the patient goes to the doctor and is asked how long they think the problem will take. (i.e. 5-10-15 mins) but there will be some who say 5 mins and they need 15, there'll be some who say 15 and only need 5! So that would be useless.

Patients could be asked to come 10 minutes earlier than their real time but that would involve a long wait for the patients, who'd get quite annoyed!

A break in the appointment times (9.00-9.10 → 9.30 etc) would mean catching up time but then as appointment times vary, this wouldn't always work too well either, though probably as appointment times run late, they could go on into the break.

Different systems eg tickets and allocated time for each ticket may work better but as all systems, it has its good points and bad

In theory, the best would be to ask the patients to come early, in practice it wouldn't be very good as patients wouldn't like arriving to be kept, possibly, waiting.

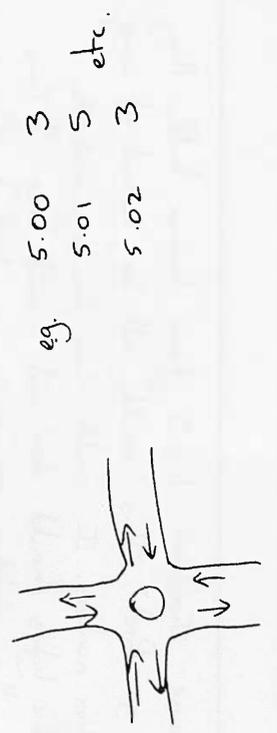
No	Time	Appointment	Actual	D	P
1	3	9.00	9.00	0	0
2	2	9.05	9.05	2	0
4	19	9.10	9.10	3	0
4	7	9.15	9.29	0	14
6	2	9.20	9.36	0	16
6	4	9.25	9.38	0	13
8	3	9.30	9.42	0	12
8	8	9.35	9.45	0	10
10	2	9.40	9.53	0	13
10	3	9.45	9.55	0	10
12	2	9.50	9.58	0	8
12	1	9.55	10.00	0	5
14	3	10.00	10.01	0	1
14	8	10.05	10.05	1	0
16	4	10.10	10.13	0	3
16	4	10.15	10.17	0	4
18	4	10.20	10.21	0	1
18	1	10.25	10.25	0	0
20	4	10.30	10.30	4	0
20	4	10.35	10.35	1	0
22	2	10.40	10.40	1	0
22	8	10.45	10.45	3	0
24	1	10.50	10.53	0	3
24	1	10.55	10.55	1	0
26	7	11.00	11.00	4	0
26	6	11.05	11.07	0	2
28	3	11.10	11.13	0	3
28	2	11.15	11.16	0	1
30	3	11.20	11.20	2	0
30	3	11.25	11.25	2	0

Random no.	Mean time	Appointment	Actual	D	P
33	24	55	→ 3	2	4
18	58	35	→ 2	4	3
98	16	86	→ 19	2	7
86	59	16	→ 7	4	2
17	70	50	→ 2	5	4
51	41	30	→ 4	3	3
31	63	62	→ 3	5	5
88	08	06	→ 8	1	1
21	11	16	→ 2	2	2
30	17	49	→ 3	2	4
25	67	13	→ 2	5	2
08	53	91	→ 1	4	8
43	12	97	→ 3	2	16
91	06	61	→ 8	1	5
49	79	26	→ 4	6	3
51	75	21	→ 4	6	2
57	95	07	→ 4	10	1
8	35	24	→ 1	3	2
58	24	63	→ 4	2	5
56	65	45	→ 4	5	4
23	39	79	→ 4	3	6
91	56	57	→ 2	4	4
7	69	11	→ 8	4	6
2	83	83	→ 1	5	4
83	73	14	→ 1	7	2
74	53	51	→ 7	6	7
43	02	65	→ 6	4	2
22	10	95	→ 3	1	4
31	09	86	→ 2	2	5
43	9	99	→ 3	1	7
			→ 3	1	19

This random no. system could be used in connection with Traffic lights or the roundabout outside our classroom as I mentioned before.  
 i.e. Traffic lights don't stay green for 35 seconds, red for 35 seconds + amber for 35 seconds, it all depends on the traffic.

At certain rush hours (5pm-6pm on our roundabout) traffic will be heavier than at other times. Our roundabout hasn't got traffic lights and it gets blocked up twice a day (i.e. 8.30am and 5.30 pm roughly). I wonder if traffic lights which were carefully worked out would keep the traffic moving better?

The length each colour has to stay has to be worked out  $\therefore$  the random nos could be used. I could survey how many cars arrive at different times and at different junctions.

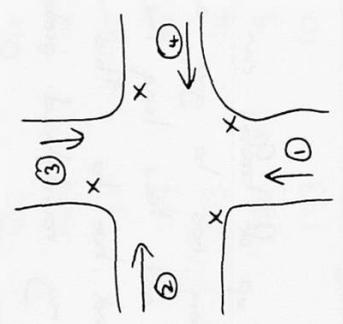


eg. 5.00 3  
 5.01 5 etc.  
 5.02 3

The rate of arrival could be measured every hour. (3.00 - 3.59 - 4.00 - 4.59 etc). From these figures, I could use the random nos to roughly fit each of these numbers. The no of cars that arrive differ every day.

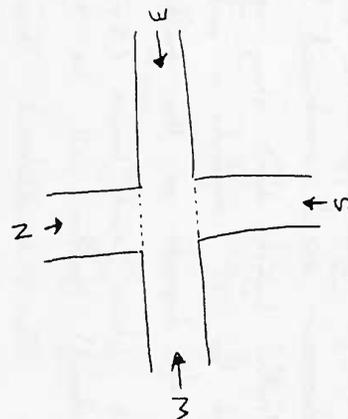
(I could also measure the time each car wants.) Depending on how often cars arrive the traffic lights have to be ready to 'dismiss' the cars at the right time and to allow other cars to pass.

Junction	Random Nos	No arrived	$\frac{1}{2}$ min later	Time	Remains	Lights
1	21	13	4.00	1 min	R	4 $\leftarrow$
2	11	19	4.00 $\frac{1}{2}$	$\frac{1}{2}$ min	G	3 $\downarrow$
3	19	14	4.01	1 min	R	2 $\rightarrow$
4	15	18	4.01 $\frac{1}{2}$	$\frac{1}{2}$ min	G	1 $\uparrow$



Because Junctions 1 & 3 have heavier traffic they can go first. This could all be looked at using random numbers. I could then decide what the traffic lights should do and could see if this helps the problem on our roundabout. There is a system a bit like this as you travel towards Derby from our school.

This is all rather complicated and I am not sure whether I could do this but it just interested me. Then I found, by luck, a similar idea in a book. It was not about our roundabout but I was still interested. I am not sure of some of the things in the book but I just used the ideas.



This is a crossroads and E-W traffic has priority. All traffic has to go straight on and no turning is allowed.

The build up of traffic can be looked at by using random nos to give arrival times for both directions and how long the N/S traffic is willing to wait and has to. This is almost exactly the same as my roundabout problem.

Some times were given for gaps between arrivals. These were generated to follow an Exponential distribution with mean rate of 7.5 per minute (i.e. one every 8 seconds). (I said I did not understand everything in the book!).

E-W	
1	5
7	11
2	23
1	4
19	1
14	6
1	4
8	33

What happens?

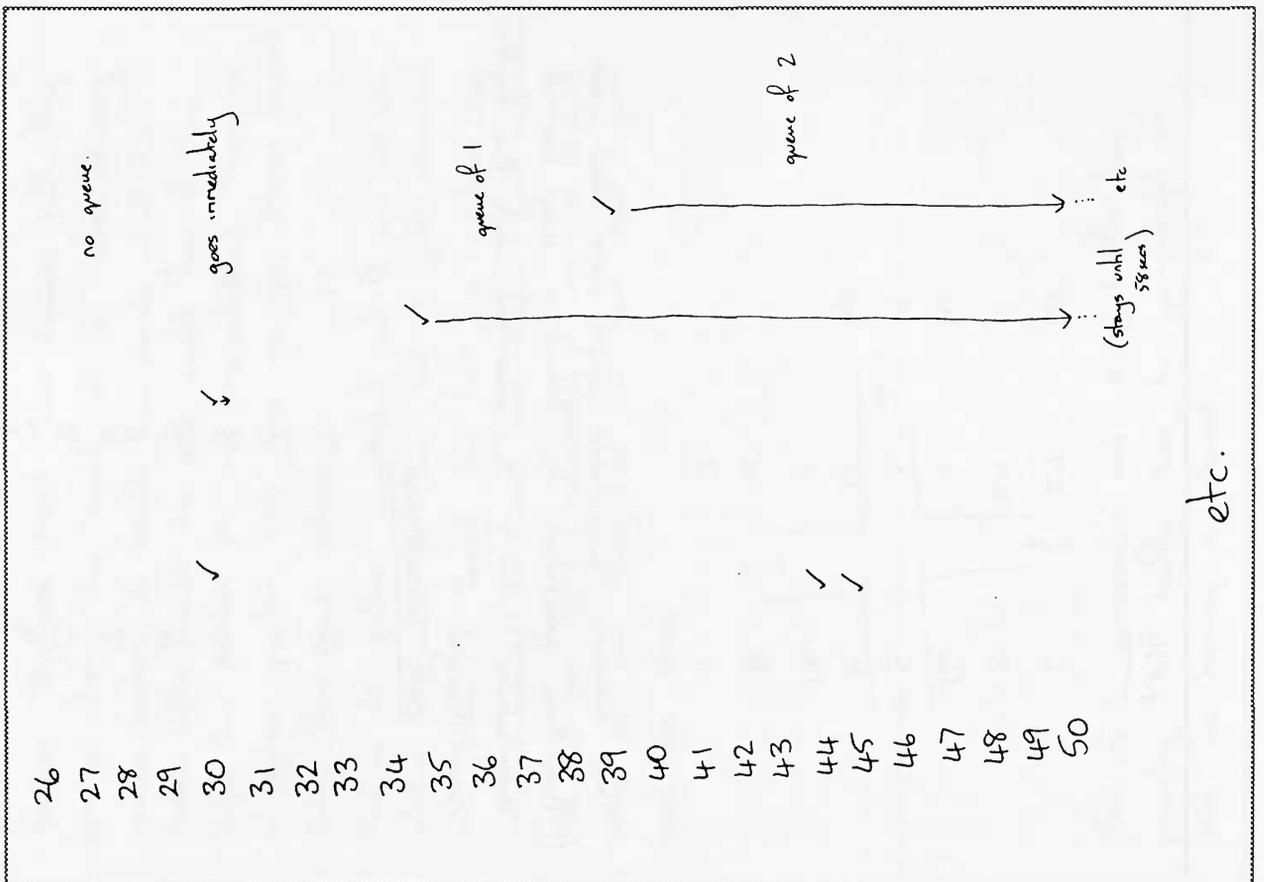
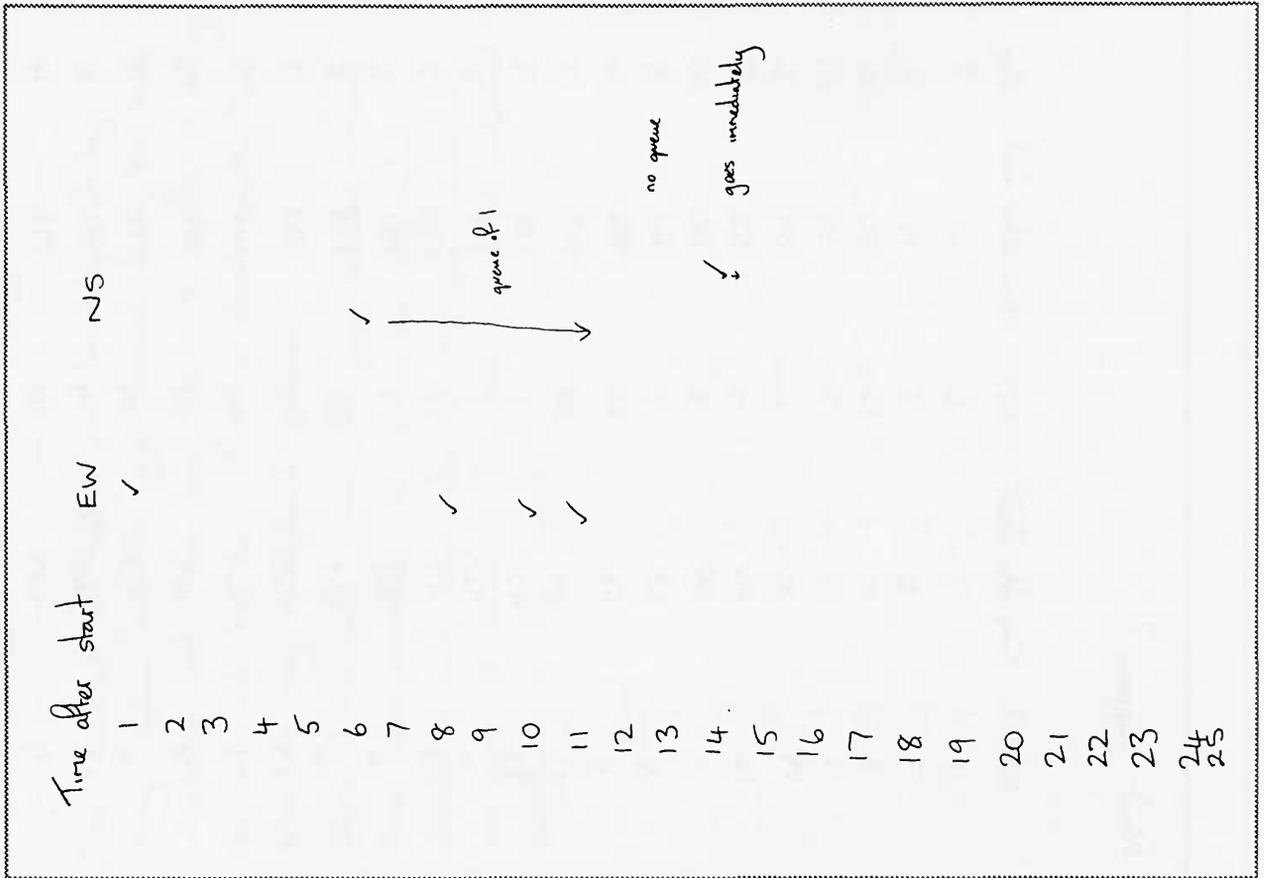
EW	Time after start	NS	Time after start	Gap
1	1	6	6	8
7	8	8	14	6
2	10	16	30	14
1	11	5	35	10
19	30	4	39	6
14	44	13	52	7
1	45	8	60	10
8	53	1	61	9
5	58	27	88	4
11	69	29	117	7
23	92	1	118	7
4	96	1	119	8
1	97	11	130	7
6	103	1	131	6
4	107	4	135	8
33	140	2	137	7
1	141	10	147	5
8	149	11	158	11
4	153	14	172	4
11	164	9	181	4
9	173	10	191	4
<u>173</u>		<u>191</u>		

N-5

6	27	10
8	29	11
16	1	14
5	1	9
4	11	10
13	1	
8	4	
1	2	

The minimum acceptable gaps were also given.  
 (This time generated to follow a Log Normal distribution !!!???) I understand what the figures mean though.

<u>GAPS ACCEPTABLE</u>		
8	4	5
6	7	11
14	7	4
10	8	4
6	7	4
7	6	
10	8	
9	7	



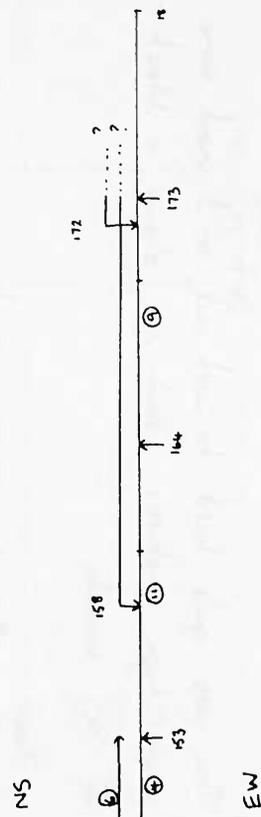
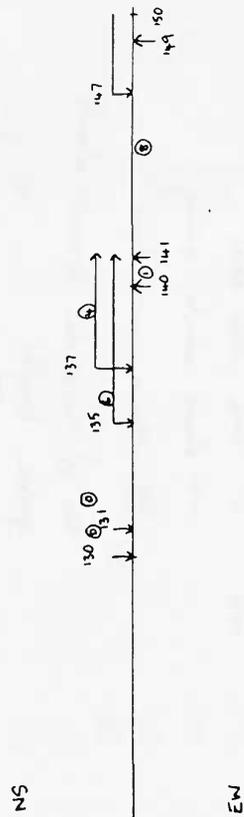
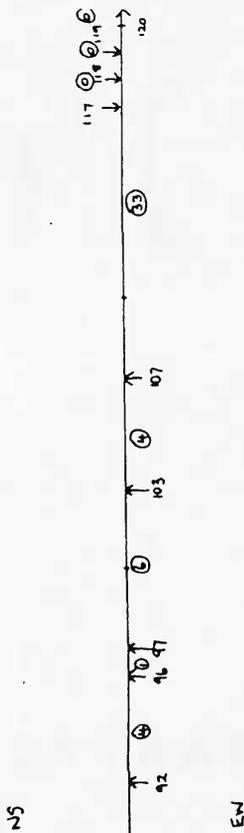


This is a very unrealistic situation and it only covers a three minute period. This makes me realise even more what a difficult problem I set myself but at least I can see how this sort of thing can be worked out. Obviously this could be done much better by a computer.

The situation at the moment seems OK. There is never a queue longer than 3 cars and the average waiting time for N-S cars is just about 5 secs.

Obviously this would have to be looked at over a long period of time with different rates of arrival for different times of day and for each direction. This would have to be based on real figures collected over a long time

The random nos are useful in this sense i.e. for queuing but it can be used in many other ways (eg computer games etc). There is much more to a queue, as I discovered than just people/things lining up and waiting.



# 6

## Moderator's Comments

### *Why Are We Waiting? S2/1*

#### Foundation Level

#### Grade E

This project represents a very thorough study by a Foundation Level candidate and she achieves above the target grade at this level.

The positive points of this project are

- \* There is a strong personal involvement in the writing. The initial work is all based upon a situation she clearly experiences.
- \* The scene is set well at the start of the project with a nicely explained introduction.
- \* Although repetitive, the latter part of the project is accurate and explains the tasks undertaken to study the queueing systems encountered in the school and at a doctors.
- \* All data is well represented in tables and there are adequate techniques applied to resolve the difficulties suggested by the candidate.
- \* There is insufficient depth to the project to justify inclusion at a higher level yet, at this level, the project is a splendid, practical study undertaken with real enthusiasm by this candidate. In fact, a frequent feature of projects submitted by Foundation Level candidates is the enthusiasm with which they undertake the work; enthusiasm which is sometimes not evident in higher level projects.
- \* The argument of the case is well stated, if a little wordy in places, and easy to follow through the write-up.

## *Why Are We Waiting?* S2/2

### **Intermediate Level**

#### **Grade E**

The cover of this piece sets the tone for the rest of the writing - there is an emphasis on presentation, description and tidyness and not enough focus on the mathematics of the study.

The impression given by the author is of an Intermediate candidate taking great pains to present the work in as precise a fashion as possible but avoiding the real 'business' of tackling the problems of queues.

The structure of the project is not as well thought out as might have been the case. The initial comments are made before any evidence has been presented to support them. Although mean, mode and median are stated there is no working to suggest how these have been obtained nor their significance. She has some nice ideas but does not seem to appreciate all the problems of an appointment system. No comment has been made as to the assumptions made when comparing appointment systems. There is no evidence of the 'waiting times' for the patients at the surgery.

The visit to MacDonalds was a nice touch and showed some personal involvement in the study, but how did she record the data? Did the people all arrive at the same time? How long did they wait in total? Was this an 'ongoing' queue? Why did she study only six people? All these points could be considered to improve the study.

My overriding impression is of a beautifully presented study with some novel items to improve the operating system but of very limited mathematical process and technique. If submitted at Foundation Level the grade achieved would be above target grade, which is below that expected of a target grade student at this level.

## Why Are We Waiting? S2/3

### Intermediate Level

#### Grade D

Queues are a part of our everyday life and the idea of studying the delays we all suffer is a challenging one. This piece of Intermediate work looks at some aspects of the problem in some detail and is commensurate with what might be expected to just achieve the target grade in this level.

The task has been undertaken by a mature student and demonstrates some of the shortcomings experienced in the work of those unaccustomed to the investigative method. There are tables of results, analysis, comments but little introduction to the sections in the study and little reflection upon the problems she is trying to resolve.

The calculations she makes all seem accurate and support the deductions based upon them, though I am 'dubious' about the 'histogram'. At the conclusion of the task she uses random number generation to simulate another queueing system but fails to explain the system simulated!

The task concentrates largely upon one place where queueing is found - the doctors. This is acceptable but makes a lot of assumptions that we 'know' what is going on in the study. This piece of coursework would have received a higher grade if she had explained

- \* How does she collect the data?
- \* Is this data real?
- \* Did she persuade the doctor to change from five minute to seven minute appointments?

There is adequate use of mathematical technique but the task is not really extended in any way. I feel that there is little 'personal' involvement in the study and it is rather narrow.

## *Why Are We Waiting? S2/4*

### **Intermediate Level**

#### **Grade C**

This is a nice, if brief, piece of work. The student is clearly mature, for the references are those of an adult not a school pupil.

The justification for my grading is that he has handled the set tasks competently, without elaboration but with clear data recording. Having completed the task he has extended it by a limited amount using the simulations of random digits. He has not extended the work to any great degree but has recorded his findings clearly and without unnecessary embellishment.

I would not feel justified in giving a grade higher than C since he has shown no greater depth of understanding or more wide ranging enquiry into the problem. To go beyond the C, I feel I should like to see comments upon the shortcomings of the system used to simulate the patients' consultations and great personal involvement of the student in the project. I should also like to see evidence that the student is prepared to extend the scope of the work beyond the one situation considered in the write-up and to actually do this. Finally, I should consider the inclusion of a coherently explained plan of development, which follows as a thread through the project, with realisation of the shortcomings and strengths of the plan to be a necessity to achieve higher grades.

Despite this, the candidate has achieved a competent project with a good mastery of Intermediate skills.

## *Why Are We Waiting? S2/5*

### Higher Level

#### Grade B

The apparent weight of work here is due to the inclusion of so much of the source material. The actual write-up is relatively brief but is well presented.

The initial problem is well answered and to the point. He has done the job without great elaboration and with very little comment - which is a shame as I feel he should be demonstrating the ability to interpret his findings. Were the topic to end here it would barely be a C. However, the extra weight is added when he gets real personal involvement into what he is doing.

I am pleased to see that he has broken from the confines of the 'Waiting Room' and gone into the community. From his writing of the study in the Halifax, he does know what he is doing and is able to interpret his findings. I should like to have seen more mention of problems in recording data when in the Halifax, and a more precise write-up of the data in neat form. It might have been nice to see a comparison with a bank or similar institution not using the queueing system he studied. This would have raised the grade expectation.

Despite these comments, the strength of this work is that he has been involved in a real problem and not a contrived situation in the classroom. This has obviously heightened his involvement in the entire process. I feel this is just a B.

*[The 'source material' referred to in paragraph one has not been included as part of this student's work in Chapter 5.]*

## Why Are We Waiting? S2/6

### Higher Level

#### Grade A

Well, if I received this piece of work to moderate I'm sure I would be happy to pass it as a grade A but I might find it difficult to justify.

The overall impression is one of a mature approach and understanding of the problem. This is shown in mature references but less clear writing about the task. There is clearly ability and breadth of knowledge within the mind undertaking the study, but technical difficulties with the finished article.

In support of this statement I cite

- \* The introduction was wide ranging, lucid but plunged straight into the first table of results without clearly identifying its purpose.
- \* The origin of the data was not made clear.
- \* The 'random number system' has been used but not clearly explained, neither do I feel this section to have been concluded satisfactorily.

Finally, she has set herself a very difficult task with the 'roundabout' problem. Nevertheless she has gone into this in some detail, not all of which she acknowledges is understood. Where I feel she has fallen down is that she has not spent adequate time in clarifying this extension work she has undertaken.

Although using reference material to expand a situation is very laudable, the project *must* make it clear that the references have been properly understood and utilised.

In the end she had analysed the problem and made useful comments, but the style and 'feel' of the project and the mathematics within it do not seem to be personal, nor fully explained.



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