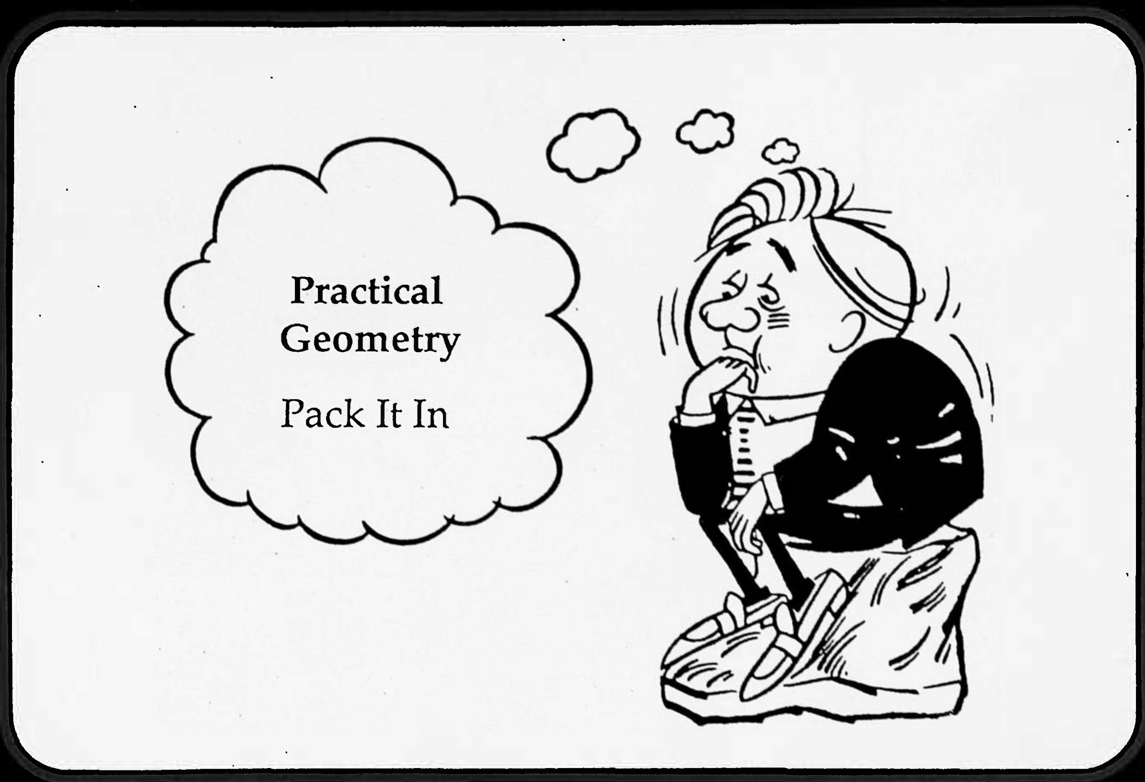
EXTENDED TASKS FOR GCSE MATHEMATICS

A series of modules to support school-based assessment



MIDLAND EXAMINING GROUP

SHELL CENTRE FOR MATHEMATICAL EDUCATION

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EXTENDED TASKS

FOR GCSE

MATHEMATICS

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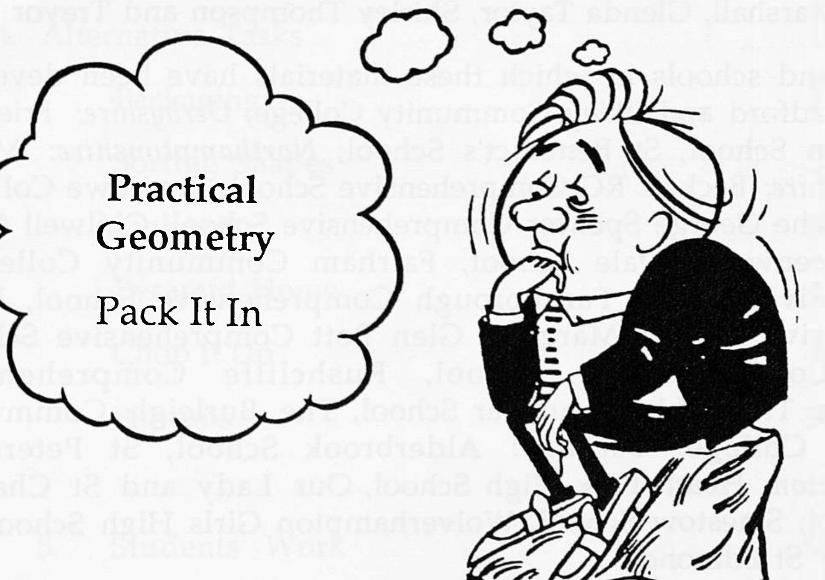

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SHELL CENTRE FOR MATHEMATICAL EDUCATION

National STEM Centre
N23923

Authors

This book is one of a series forming a support package for GCSE coursework in mathematics. It has been developed as part of a joint project by the Shell Centre for Mathematical Education and the Midland Examining Group.

The books were written by

Steve Maddern and Rita Crust

working with the Shell Centre team, including Alan Bell, Barbara Binns, Hugh Burkhardt, Rosemary Fraser, John Gillespie, Richard Phillips, Malcolm Swan and Diana Wharmby.

The project was directed by Hugh Burkhardt.

A large number of teachers and their students have contributed to this work through a continuing process of trialling and observation in their classrooms. We are grateful to them all for their help and for their comments. Among the teachers to whom we are particularly indebted for their contributions at various stages of the project are Paul Davison, Ray Downes, John Edwards, Harry Gordon, Peter Jones, Sue Marshall, Glenda Taylor, Shirley Thompson and Trevor Williamson.

The LEAs and schools in which these materials have been developed include *Bradford:* Bradford and Ilkley Community College; *Derbyshire:* Friesland School, Kirk Hallam School, St Benedict's School; *Northamptonshire:* Manor School; *Nottinghamshire:* Becket RC Comprehensive School, Broxtowe College of Further Education, The George Spencer Comprehensive School, Chilwell Comprehensive School, Greenwood Dale School, Fairham Community College, Haywood Comprehensive School, Farnborough Comprehensive School, Kirkby Centre Comprehensive School, Margaret Glen Bott Comprehensive School, Matthew Holland Comprehensive School, Rushcliffe Comprehensive School; *Leicestershire:* The Ashby Grammar School, The Burleigh Community College, Longslade College; *Solihull:* Alderbrook School, St Peters RC School; *Wolverhampton:* Heath Park High School, Our Lady and St Chad RC School, Regis School, Smestow School, Wolverhampton Girls High School; and Culford School, Bury St Edmonds.

Many others have contributed to the work of the project, notably the members of the Steering Committee and officers of the Midland Examining Group - Barbara Edmonds, Ian Evans, Geoff Gibb, Paul Lloyd, Ron McLone and Elizabeth Mills.

Jenny Payne has typed the manuscript in its development stages with help from Judith Rowlands and Mark Stocks. The final version has been prepared by Susan Hatfield.

Contents

1 *Introduction*

PACK IT IN is one of eight such 'cluster books', each offering a lead task which is fully supported by detailed teacher's notes, a student's introduction to the problem, a case study, examples of students' work which demonstrate achievement at a variety of levels, together with six alternative tasks of a similar nature. The alternative tasks simply comprise the student's introduction to the problem and some brief teacher's notes. It is intended that these alternative tasks should be used in a similar manner to the lead task and hence only the lead task has been fully supported with more detailed teacher's notes and examples of students' work.

The eight cluster books fall into four pairs, one for each of the general categories: Pure Investigations, Statistics and Probability, Practical Geometry and Applications. This series of cluster books is further supported by an overall teacher's guide and a departmental development programme, IMPACT, to enable teacher, student and departmental experience to be gained with this type of work.

The material is available in two parts

Part One		The Teacher's Guide
		IMPACT
	Pure Investigations	I1 - Looking Deeper
		I2 - Making The Most Of It
	Statistics and Probability	S1 - Take a Chance
		S2 - Finding Out
Part Two	Practical Geometry	G1 - Pack It In
		G2 - Construct It Right
	Applications	A1 - Plan It
		A2 - Where There's Life, There's Maths

This particular 'cluster book', PACK IT IN, offers a range of support material to students as they pursue practical geometry tasks within any GCSE mathematics scheme. The material has been designed and tested, as extended tasks, in a range of classrooms. A total of about twelve to fifteen hours study time, usually over a period of two to three weeks, was spent on each task. Many of the ideas have been used to stimulate work for a longer period of time than this, but any period which is significantly shorter has proved to be rather unsatisfactory. The practical geometry tasks are intended to stimulate students' interest in, and understanding of, the three-dimensional world in which they live. Many geometrical discoveries are made experimentally. However, this experimental approach can be followed up and reinforced, using reasoning and proof. Geometry also provides excellent opportunities for making and testing hypotheses.

It is important that students should experience a variety of different types of extended task work in mathematics if they are to fully understand the depth, breadth and value of the subject. The tasks within this cluster begin within real life situations, and they are intended to be tackled practically. However, it is important that this practical approach should be followed up using reasoning, calculation and proof, according to the individual need and ability of each student. The common element amongst all the items within this cluster is the idea that they are designed to develop spatial awareness.

Clearly, there are many styles of classroom operation for GCSE extended task work and it is intended that this pack will support most, if not all, approaches. All the tasks outlined within the cluster books may be used with students of all abilities within the GCSE range. The lead task of Anyone For Tennis may be used with a whole class of students, each naturally developing their own lines of enquiry. It is intended that all the tasks within the cluster may be used in this manner. However, an alternative classroom approach may be to use a selection, or even all, of the ideas within the cluster at one time, thus allowing students to choose their preferred context for their practical geometry task. There is, however, a further more general classroom approach which may be adopted. This is one that does not even restrict the task to that of a practical geometry nature. In this case some, or all, of the items within this cluster may be used in conjunction with those from one or more of the other cluster books, or indeed any other resource. The idea is that this support material should allow individual teacher and class style to determine the mode of operation, and should not be restrictive in any way.

Teachers who are new to this type of activity are strongly advised to use the lead tasks.

These introductory notes should be read in conjunction with the general teacher's guide for the whole pack of support material. Many of the issues implied or hinted at within the cluster books are discussed in greater detail in The Teacher's Guide.

2 *Anyone For Tennis*

The lead task in this book is called *Anyone For Tennis*. It is based on a real life situation and provides a rich and tractable environment for practical geometry coursework tasks at GCSE level.

The task is set out on pages 7-9 in a form that is suitable for photocopying for students.

The Teacher's Notes begin on page 10. These pages contain space for comments based on the school's own experiences.

ANYONE FOR TENNIS

During this task you are asked to assume the role of the company designer. The design tasks involve you in using your mathematics to help you to decide upon packaging for tennis balls. There are many things to consider and these will become more obvious as you work through the project.

As you work on this task you are entirely free to ask your own questions and to use your own ideas. It is a good idea for you to keep a record of the work you do, together with sketches of any packages you make as well as the questions and answers you come up with. Your notes and sketches will help you to write your report at the end of your study. You should also keep any models that you make.

The first part of the work involves you in looking at two boxes which have been suggested as being suitable containers for packing three tennis balls. Your task is to look at various aspects of these boxes, and to decide which is the better of the two.

The second, and the major part of the task, gives you freedom to design and investigate your own ideas for packaging any number of tennis balls. Here, you are free to experiment as much as you like. You must, however, produce a Marketing Report for the packages you design.

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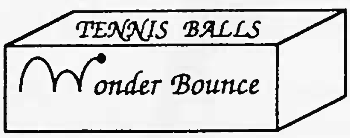
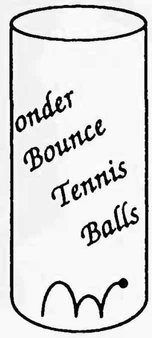
WONDERLAND SPORTS



THE CUBOID BOX



THE CYLINDRICAL BOX

The WONDERLAND SPORTS Company have decided to produce a package for three of their 'WONDER BOUNCE' tennis balls. Two designs have been suggested by a design team. Imagine that you are the Marketing Director and that you have to choose the most suitable design of the two.

You may find it useful to consider several aspects of these boxes, for example

* What are the internal dimensions of the boxes?
* What is the minimum volume required for three tennis balls?
* What is the volume of each box?
* How much space is wasted in each box?
* How much card is required for each box?
* How can you cut out these boxes from standard sheets of card measuring 569mm by 841mm?
* Anything else that you think is important.

Investigate The Problem

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WONDERLAND SPORTS COMPANY

MARKETING REPORT

PROJECT DESCRIPTION

Design of packaging for three Wonder Bounce tennis balls

SPECIFICATION	CUBOID BOX	CYLINDRICAL BOX
Dimensions		
Volume		
Excess volume		
Card required		
Number from standard sheet		

OTHER CONSIDERATIONS:

RECOMMENDATIONS:

Signed: _______________

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Anyone For Tennis - Teacher's Notes

This coursework task focusses upon practical geometry skills. It demonstrates very effectively how coursework tasks can form an integral part of the learning process for all students. This task provides an opportunity for students to complete a coursework task, as they acquire and consolidate geometrical concepts and skills relating to area and volume, as well as to drawing nets and making three-dimensional shapes.

The amount of mathematics used, and the syllabus content covered, will depend upon the individual ability of students, since they will only choose to use mathematics when they perceive a need to do so. Consequently, individual needs and ability will determine both the scope and depth of the mathematics applied to this task.

The task may also be used by students who have previously considered some aspects of these mathematical ideas. In this case, the task can be used to extend and enrich their previous experiences.

Teachers who have used this topic for GCSE coursework, have found it to be an excellent motivator, and one which interests a wide range of students. Initially, many teachers were rather concerned about the problems that can arise when using this type of topic, or that the ideas in the work might not take off and lead to high quality coursework. However, these concerns did not develop into real classroom problems, and teachers experienced no problems in keeping the work going.

From classroom trials, the following suggestions and comments have been offered by the teachers involved

* Have the confidence to keep going
* Use real tennis balls in the classroom, even though this may initially cause some chaos
* Use a reporting back session at the end of the first stage to allow all students to talk about the problem

* A lot of glue, sellotape and card is needed. Card can be collected from boxes at home, in school and at the supermarket. Anything will do. Paper can be used for early trial designs

* Encourage both group work and individual work

* Encourage students, at all times, to keep a record of their ideas, problems etc, to help with the writing of their report.

Understanding and Exploring the Problem

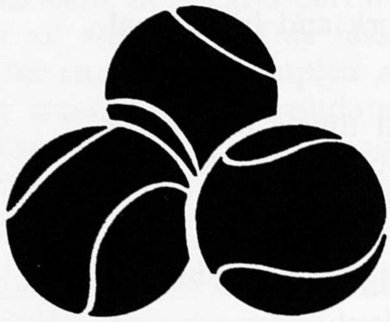
This task is explored from a single starting point, which is designed to provide an entry for students of all abilities. This starting point is presented on the worksheet *Anyone For Tennis*. In essence, the task involves students in considering and comparing two packages, each of which holds three tennis balls. Students are then free to design and make their own packages.

The worksheets *Wonderland Sports* show the two suggested containers, one of which is cylindrical and the other a cuboid. A variety of questions which could be considered, is listed on page 8. You may, or may not, choose to use this list of questions.

The worksheet on page 12 was produced by a teacher in one of our trial schools. This worksheet was produced after a group brainstorming session, and was simply a summary of students' ideas. However, even in this classroom, it was used by some students as a list of questions to be answered, and hence was rather too directive.

What is essential, is that students should work in small groups as they tackle this problem practically, and that they should discuss issues such as the ones listed. Moreover, students need to record their responses to this initial problem through sketching and making these simple packages, as well as through carrying out calculations and discussion. The first stage ends as each student completes a Marketing Report.

PACKAGING TENNIS BALLS



Draw the NET for your chosen package accurately or to scale.

Make the package out of card.

Work out the area of the card used.

Work out the volume of the space enclosed inside your package.

Work out the volume of the 'wasted' space.

Work out $\dfrac{\text{volume of wasted space}}{\text{volume of space enclosed}}$ as a percentage

Repeat for the same shape where all the dimensions are multiplied by 2.

How many tennis balls will this shape hold?

Repeat for the same shape where the *base* dimensions are multiplied by 2, but the height remains the same.

Compare your answers with the answers obtained by at least two other groups.

Make a note of your observations and conclusions.

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Devising and Planning Individual Studies

As students work on the first stage of this task, and more particularly, as they fill in the sections of their Marketing Reports dealing with OTHER CONSIDERATIONS and RECOMMENDATIONS, they should be looking beyond the rather closed starting point provided. Students should be moving forward to the second stage, during which they devise and plan their own package for any number of balls. Some teachers have preferred to continue to limit the number of balls to three, but other teachers have found it more interesting to move on to, say, four or five balls.

The following aspects need to be discussed

* the cost of production
* persuasive marketing techniques

As do issues such as

* fancy shaped packages
* attractive decoration of plain, cheap packages.

Discussions relating to

* tessellations of nets
* strength of packages
* packaging of packages
* future use of packages

can all arouse tremendous student response. However, such issues should not dominate the task; mathematical activity must prevail.

The amount of time required for these first two stages will vary, according to the ability and previous experience of the group. Some Foundation Level students may use the whole of their twelve to fifteen hours responding to the initial task and completing their Marketing Report: this is quite acceptable.

Implementing Plans and Pursuing Ideas

Most students will find it more exciting to move through the initial task sufficiently rapidly to be able to design and make their own package within the time-scale allowed. All students should have an opportunity to make a personal contribution to their enquiry. This may simply be producing their own design and report about a three ball package, or it could be that students work independently, using their own ideas on a related packaging topic.

As students move through this third stage, it is important that they should continue to feel able to draw upon the support of their teacher, and of their fellow students. Even after careful thought, and accurate drawing, cutting and sticking, unexpected things will happen. Students will need to discuss what has happened, in order to determine why it has happened, before they get down to revising and recalculating the dimensions of their nets.

Students may wish to discuss questions such as 'Which is the best?' but they may not be sufficiently clear about what they mean by *best*. They may find it useful to refer back to the list of questions on the early worksheets, or they may prefer to ask questions of their own.

During classroom trials, one fruitful teacher input at this stage, was to suggest that students should make a box which was twice as big. This leads on to many new questions, and unexpected results for students. It also provides a stimulating entry into more advanced areas of mathematics in a very practical way. This approach can also produce some surprising results for teachers, and why not?

The teachers in our trial schools produced the following list of topics for related work.

* Looking at ratios of areas and volumes
* Packaging for economy
* What if the card comes on a roll?
* Links with CDT

* Using other types of balls
* Varying single lengths etc
* Mass production
* Stacking the boxes
* Making a box for the boxes
* Investigating commercial boxes
* How many different nets are there?
  Which is the best?

Reviewing and Communicating Findings

With any practical task, there is a particular need to ensure that sufficient time is given to this final stage. Students frequently wish to carry on, making and doing. They forget that time needs to be allocated to organising their findings, recording more formally, and reflecting upon the outcomes of their activities.

If students have been conscientious about keeping a record of their progress throughout the two to three weeks, this final stage can be completed much more easily. Their records should contain sketches showing the dimensions of the packages considered and actually made, together with comments about what they tried, and changes made. The final report *must* contain a discussion of mathematical ideas used and calculations carried out. At this stage, it often proves useful to organise an exhibition of students' final packages, and to ask each student to explain in simple terms to their fellow students, why they chose their particular package.

The assessment will be based upon the final report and the model(s) presented, as well as on students' ability to communicate orally and informally, as they progress through the task. Practical tasks provide excellent opportunities for oral assessment of an informal nature: students who experience difficulty in communicating mathematically often find it much easier to discuss what they have done and why they did it, when they have something to handle, as they explain to teachers and fellow students.

3 A Case Study

I decided to set my students a task involving making a tennis ball container to hold either three, four or five balls. I felt this offered the students more scope than a box to hold exactly three. The project, which was to last three weeks, became

* Make a container for three, four or five tennis balls.
* Double the sides of the container and make a larger one.

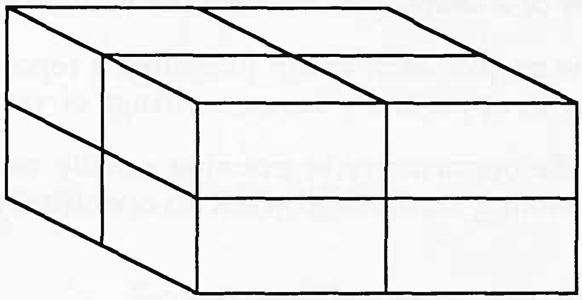
Tennis balls were borrowed from the PE Department and students, working in groups of two, three or four, were provided with plenty of card, scissors, sellotape and rough paper. The project was undertaken by both of my fourth year classes - one middle and one more able set.

Students considered various shapes and I provided opportunity for discussion about their mathematical names and features. Many complex shapes were explored, even truncated tetrahedra, but mainly prisms and pyramids.

Once students had made their two containers they were encouraged to ask themselves relevant questions relating to the task, for example

* How much card did the first/second container use?
* Is there any connection between the two?
* What is the volume of the first/second container?
* Is there any connection between the two?
* How many balls can be accommodated in the larger container and how does this compare with the smaller?
* What is the percentage of wasted space in each of the containers?
* How efficient is the container? - This could be relevant to volume or area, or even how well they stacked.
* Did doubling the size of the box have the effect anticipated?
* How did the balls pack into the larger shape?

Almost all students were surprised that doubling the sides produced such large containers. Valuable discussions ensued on the relative size of the containers, opinions fluctuating widely. I was pleased to observe one student explaining to his group that their big tetrahedron was eight times bigger than the small one. He justified this using the original rectangular boxes the balls came in; explaining that doubling the length would require one extra box; doubling the width would need a total of four boxes; and finally doubling the height would require eight boxes in all.

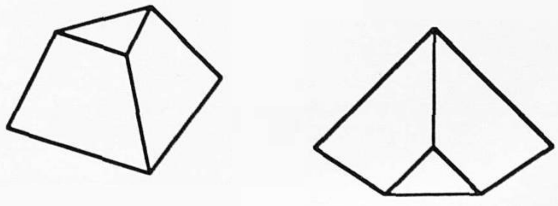


Two of my less well motivated students really warmed to the task. They chose to make an hexagonal prism for their basic container. The larger container unfortunately lacked rigidity, and they decided to use pieces of dowelling obtained from the Design Area to support the shape.

They found the volume of both of their containers by considering the hexagon as six equal triangles. It became clear to them that the volume was eight times bigger, but they could not accommodate eight times the number of balls using any arrangement in their container. They concluded that the balls 'didn't fit together well'. This gave an opportunity to discuss 3D tessellations with them. They produced by far their best piece of coursework to date as a result of using this topic.

Another couple of students began by making a tetrahedron, but realised that there was a lot of wasted space in it, so they 'cut the top off'. Finding the volume of such a shape was very difficult. One particular student went away and 'discovered from a book' the basics of trigonometry, and thereby worked out the height of the original shape; i.e without the top cut off, and went on to work out the volume of their container.

Another student very cleverly spotted that if you turn the shape on its side you could measure the former height (see diagram). She tried to use a scale factor to find the volume of the small section which had been removed, but she used the length scale factor rather than the volume scale factor. After some discussion, she realised that she should have cubed the length factor to get the volume scale factor.



Some students looked at efficiency, comparing % waste for several shapes. For instance, the small cylinder was the most efficient shape overall, but when it was doubled it became one of the least efficient. The original boxes were also included in the survey.

This activity created a real 'buzz' in the classroom. The work produced was of a very high standard, and I feel that the students learnt a great deal whilst enjoying themselves. I think it unlikely, for instance, that any of them will now forget that by doubling the lengths of a shape, you increase the volume eight times.

At the conclusion of the project, each group presented a report to the class on their findings, and there was an opportunity for an exchange of views and findings.

The project provided the opportunity to examine a wide variety of mathematical concepts, as well as enabling students to work co-operatively. Some of the areas covered include

* Volume of prisms, pyramids and spheres
* Length scale factor/Area scale factor/Volume scale factor
* Appropriate accuracy
* Use of appropriate units for length, area and volume
* Considering 3D shapes
* Tessellation
* Trigonometry

4 *Alternative Tasks*

Packaging

Sorting Shapes

Linkages

Pyramid Home

Chop It Up

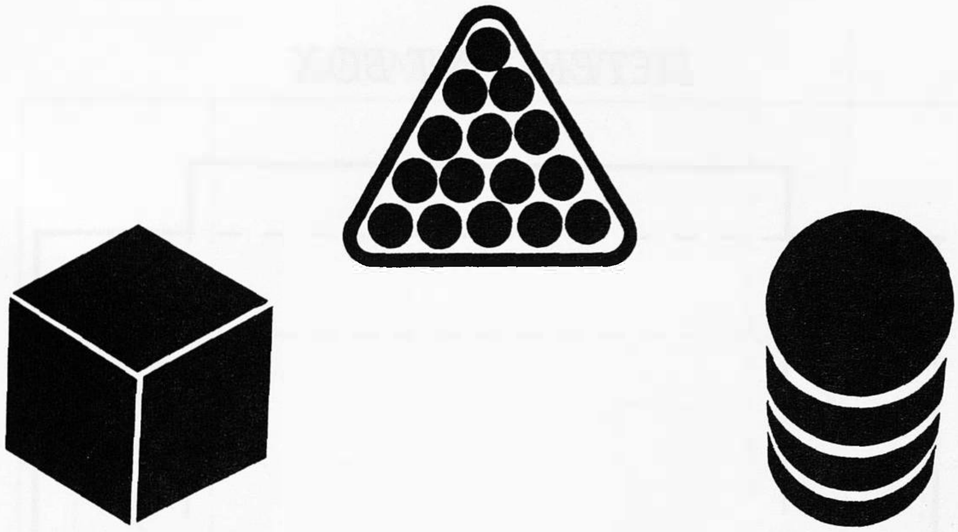
Pop-ups

Alternative Tasks

General Notes

The six alternative tasks are all intended to be used in the same way as the lead task, *Anyone For Tennis*. The teacher's notes for each task are brief and should be read and considered in conjunction with those for *Anyone For Tennis*. The student's notes are in the same form as those for the lead task. The student's notes offered for the six alternative tasks in this cluster book are all written in a similar style. They outline the context of study to the student and offer one or two problems to be considered. This provides the student with an opportunity to consider the problem and gain some understanding of it. Students are then encouraged to investigate the problem in any way they wish. Some further suggestions are offered which may be used if the teacher feels this is appropriate for any individual student, group or class. These suggestions provide further ideas for investigation without prescribing exactly what should happen.

PACKAGING



During this project you will be looking at a variety of different types of packaging. Using the ideas you see, together with your own, you are going to design and make a container to package an item of your own choice.

First, you are provided with the nets of boxes which are used to package some well known commodities. Study the nets carefully, then cut them out and assemble them. For each example, comment on the features which are of particular importance for the item packaged. Initially, you may like to work in small groups of about two to four, each making a different selection from the given nets, and then comparing what you have made. There are many points which you may choose to talk about in your group. Make sure that you keep an individual note of all of these points, and jot down your own ideas on each of these.

Later on in your study, you will be asked to choose an item which is packaged in two or more ways. You will need to draw accurate nets for the examples you choose, and discuss the similarities and differences between them. You are then asked to use the ideas you have considered and learnt about, to design your own packs.

Throughout this work you, and the rest of your group, will find it extremely helpful to consider real life situations by looking at everyday packaging already in use. Your group, together with your teacher, may like to collect as many boxes and packs as you can, during the time you spend on this work. These may well give you some good ideas for your own work.

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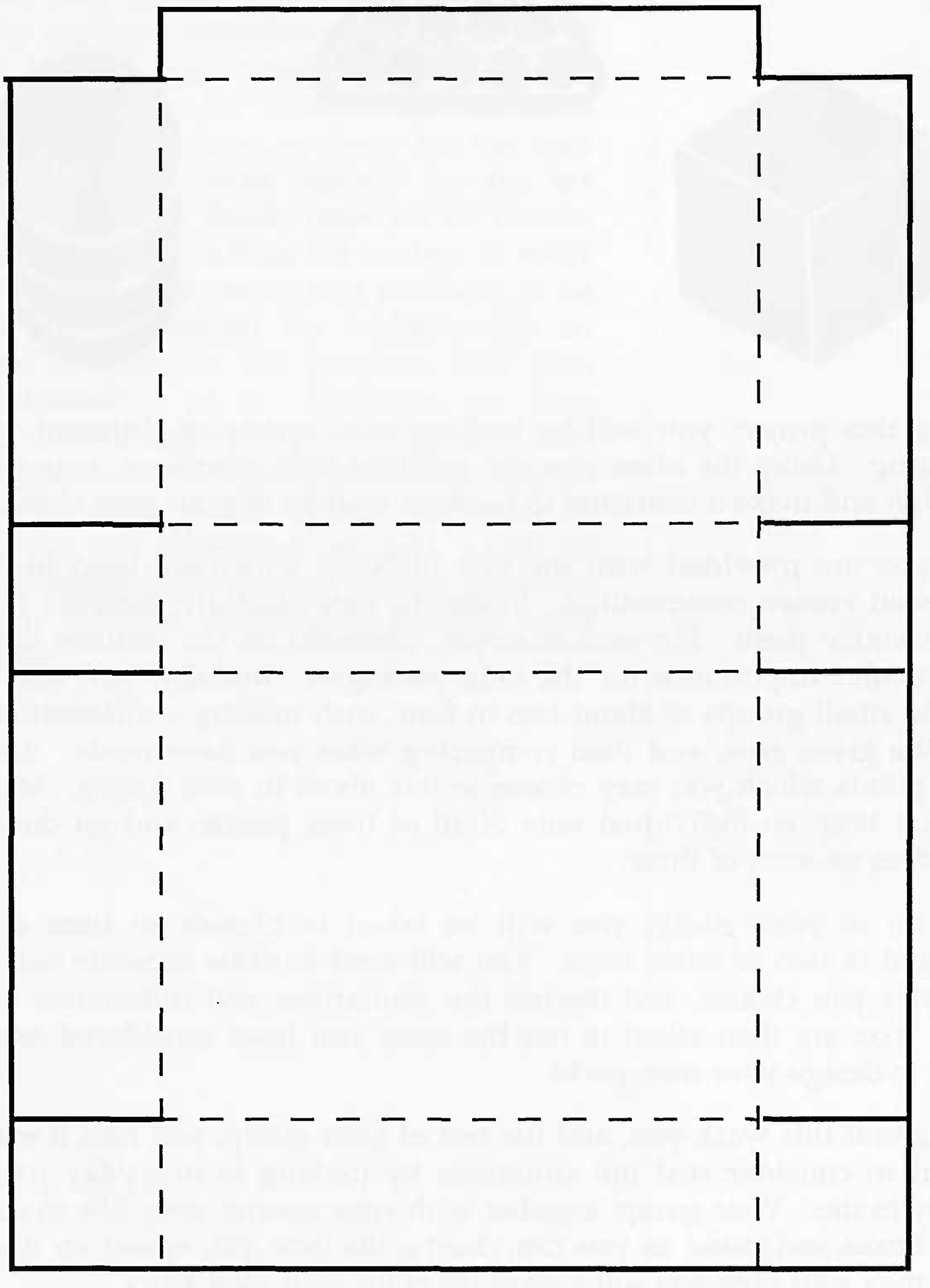
PACKAGING : continued

DETERGENT BOX



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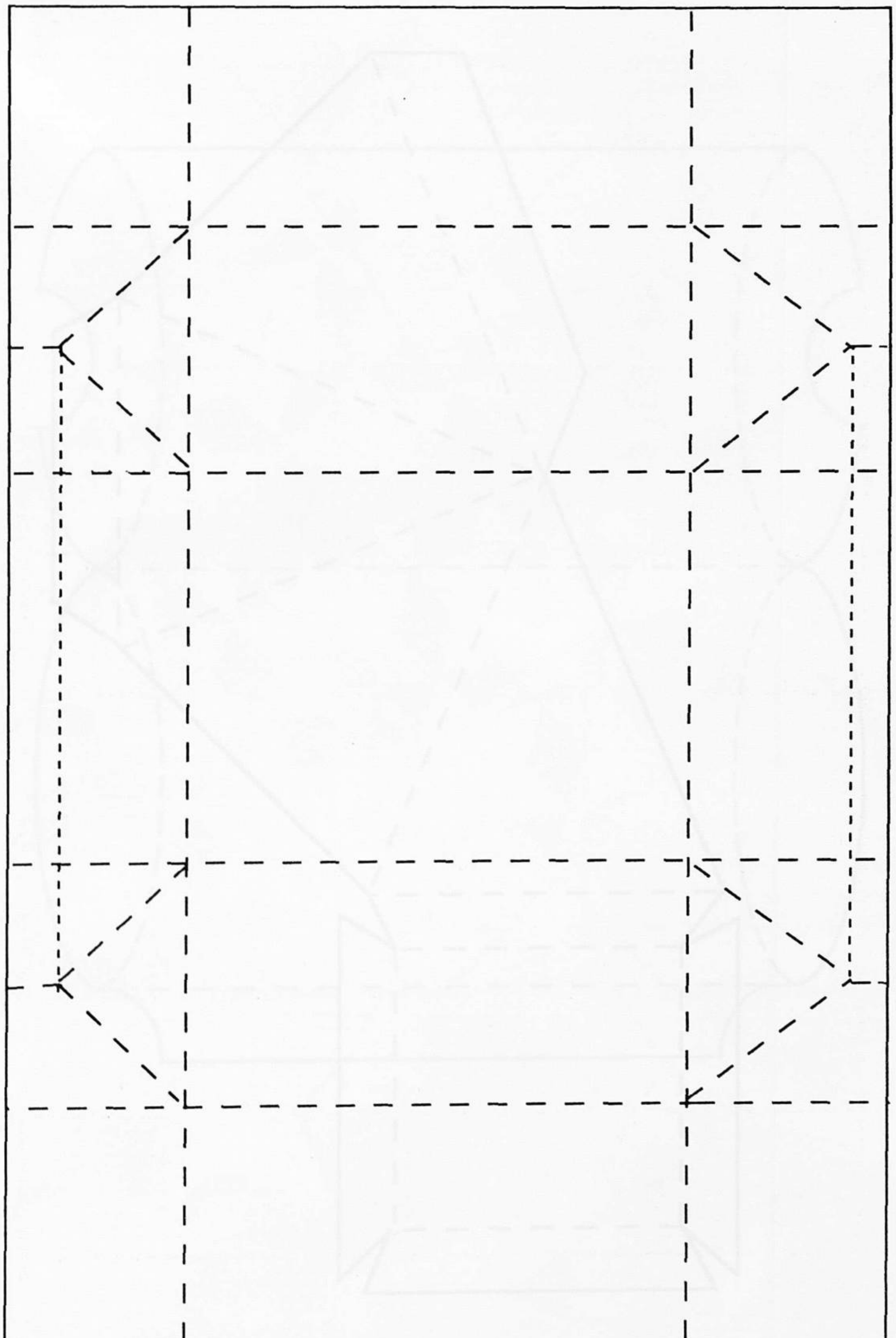
PACKAGING : continued

DRINK CARTON



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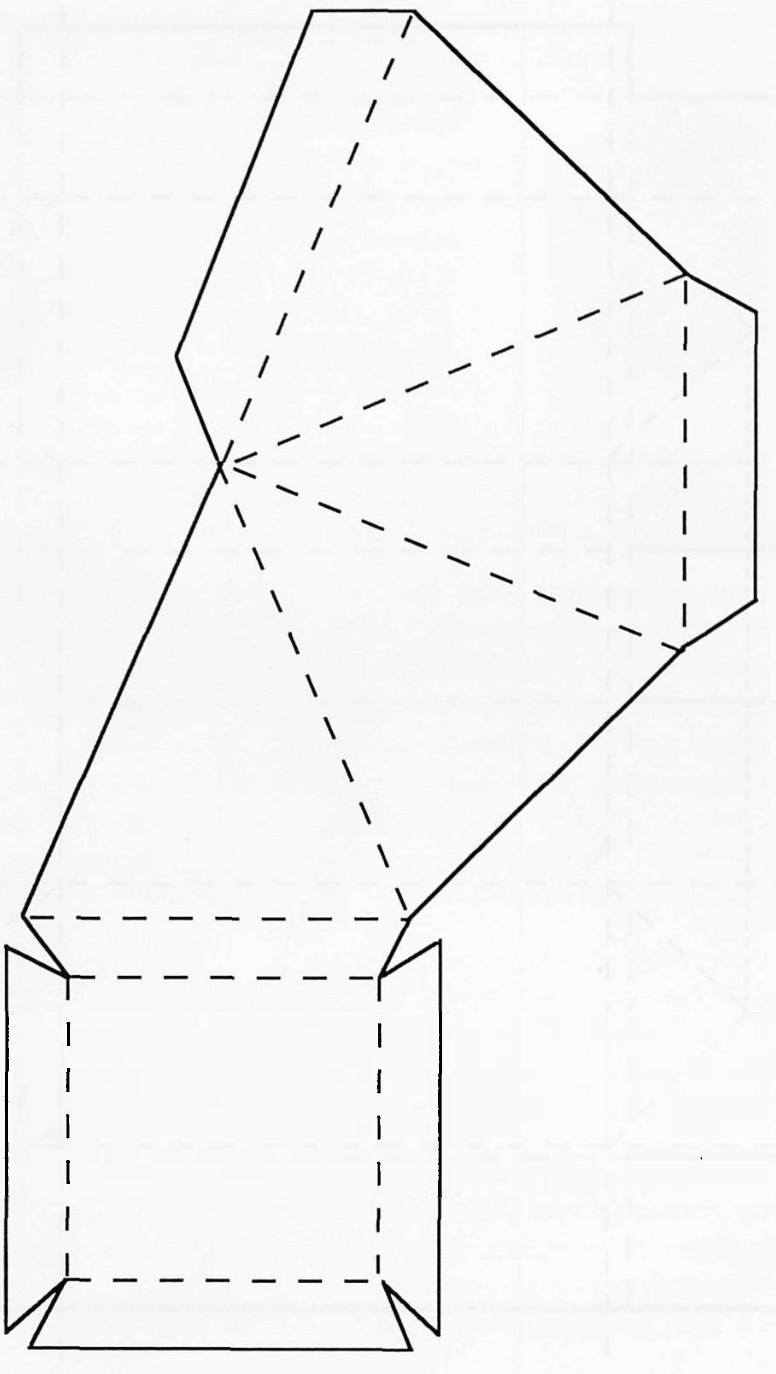
PACKAGING : continued

PYRAMID BOX



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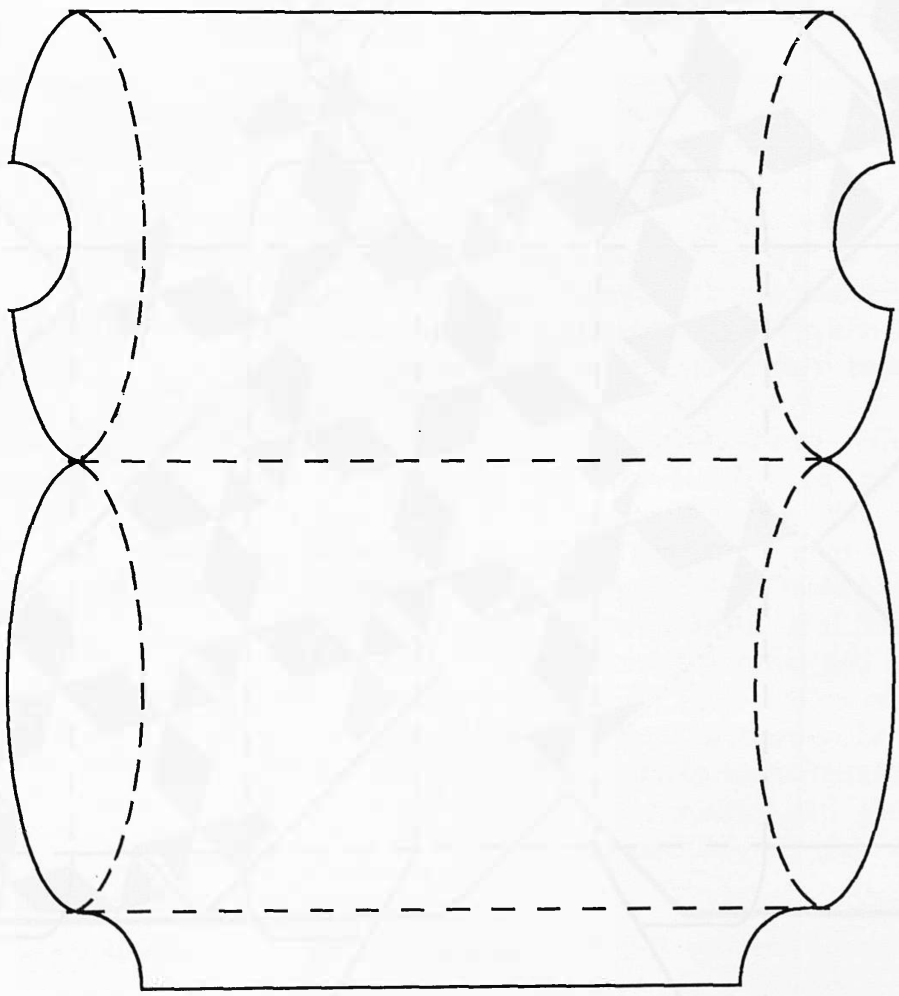
PACKAGING : continued

GIFT BOX



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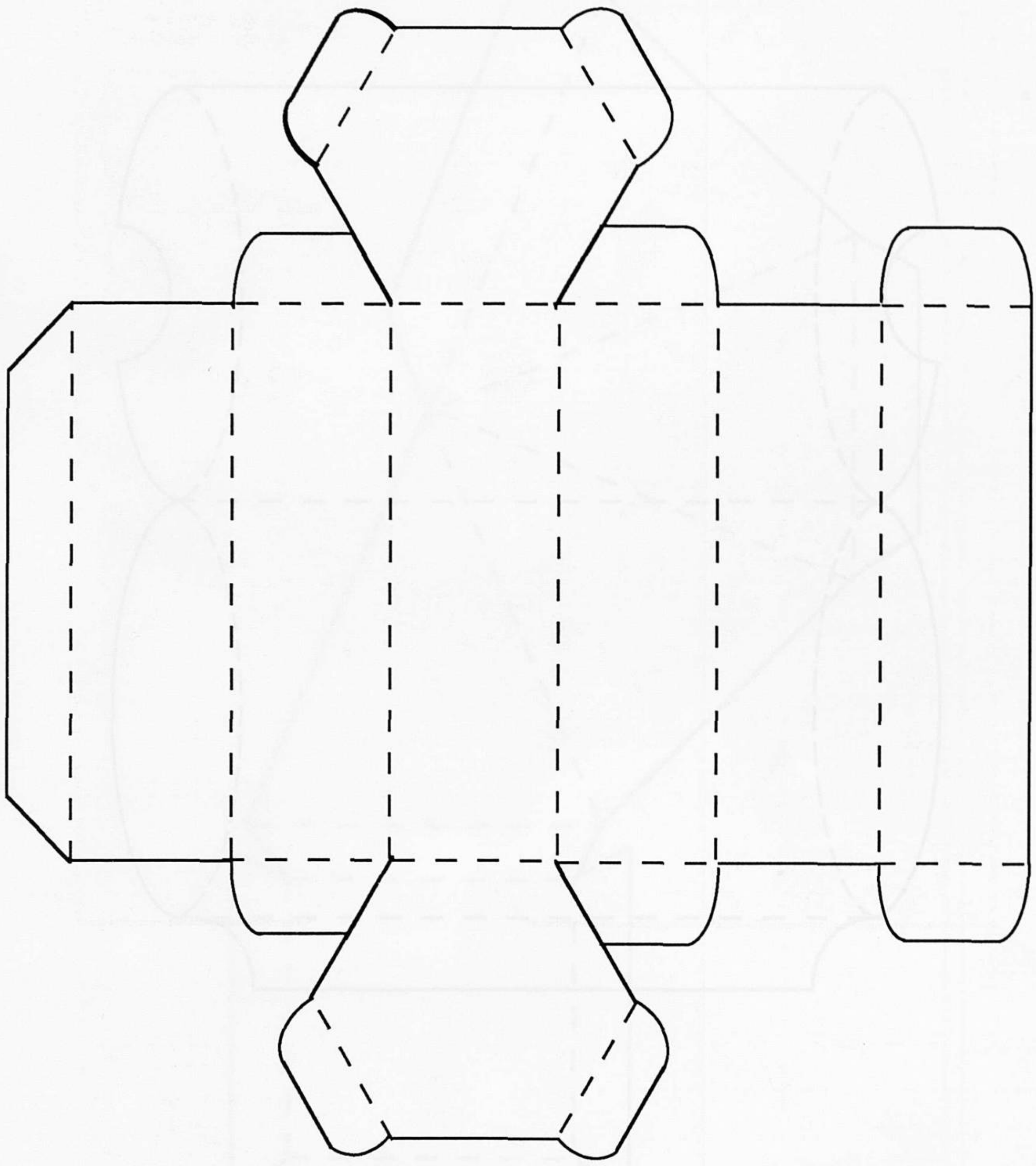
PACKAGING : continued

HEXAGONAL BOX



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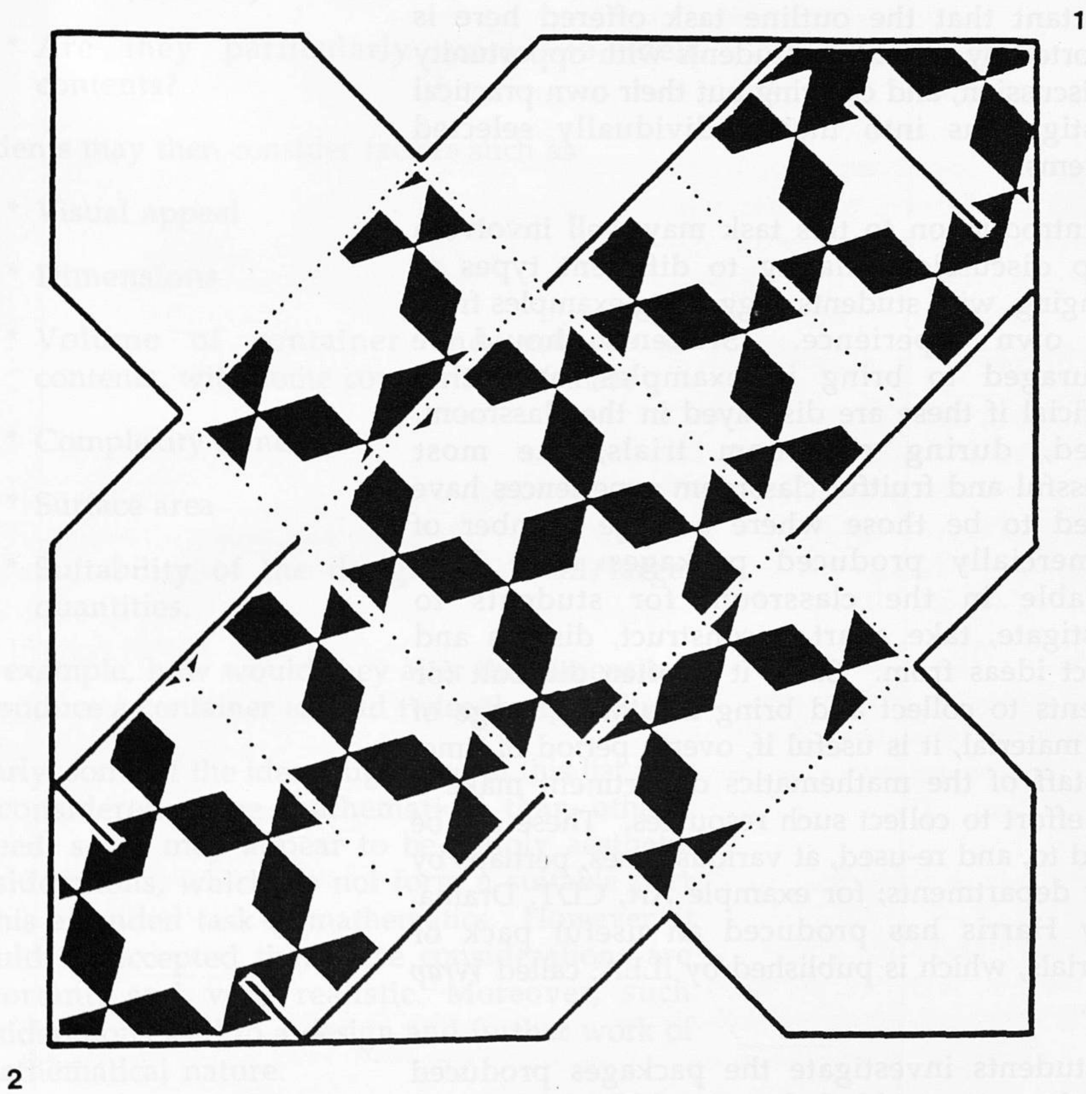
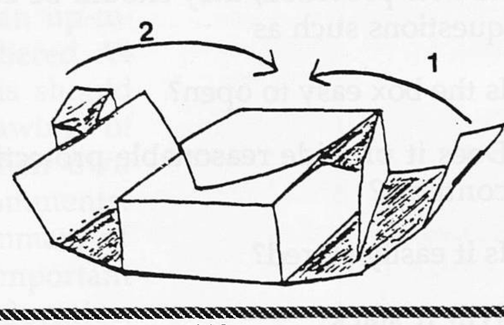
PACKAGING : continued

GIFT BOX 2





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Packaging - Teacher's Notes

This task is, essentially, a practical geometry task. Students should be involved in practical model making, geometrical thinking, mathematical design and evaluation of their products. It is important that the outline task offered here is supported by providing students with opportunity for discussion, and carrying out their own practical investigations into their individually selected problems.

The introduction to this task may well involve a group discussion relating to different types of packaging, with students suggesting examples from their own experience. Students should be encouraged to bring in examples, and it is beneficial if these are displayed in the classroom. Indeed, during classroom trials, the most successful and fruitful classroom experiences have proved to be those where a large number of commercially produced packages have been available in the classroom for students to investigate, take apart, reconstruct, discuss and extract ideas from. Since it is often difficult for students to collect and bring in a wide range of such material, it is useful if, over a period of time, the staff of the mathematics department make a joint effort to collect such resources. These can be added to, and re-used, at various times, perhaps by other departments; for example Art, CDT, Drama. Mary Harris has produced an useful pack of materials, which is published by ILEA, called *Wrap It Up*.

As students investigate the packages produced from the nets provided, they should be encouraged to ask questions such as

* Is the box easy to open?
* Does it provide reasonable protection for the contents?
* Is it easily stored?
* Will it stack?
* Is it easy to produce?

* Is it an attractive design?

* Does the quality of the packaging match the quality of the product?

* Which packs are similar? Why?

* Which are very different?

* Are they particularly suited to their contents?

Students may then consider factors such as

* Visual appeal

* Dimensions

* Volume of container and volume of contents, with some comparison of these

* Complexity of net

* Surface area

* Suitability of the design for small/larger quantities.

For example, how would they alter the dimensions to produce a container to hold twice the quantity?

Clearly, some of the ideas suggested in this list may be considered more mathematical than others. Indeed, some may appear to be simply aesthetic considerations, which do not form a suitable part of this extended task in mathematics. However, it should be accepted that these considerations are important, and very realistic. Moreover, such considerations lead to a design and further work of a mathematical nature.

Students should be encouraged to keep an up-to-date notebook, probably a series of numbered A4 sheets in a file or an exercise book. This should include sketches, and perhaps accurate drawings of the nets they considered, together with their own questions, their answers and general comments. This notebook will provide a valuable summary of their work and progress, and will be an important aspect of the assessment and moderation procedures. Alternatively, the notebook may form an appendix to, and an aid to the production of the final written report of the work undertaken.

The work discussed so far should be seen as only an introductory stage; one which simply allows the student to consider the problem and formulate ideas. Students' own ideas should form the major part of the work submitted for the assessment. Students should then select, and look in more detail at, a few examples of packaging used for the same type of commodity. They could compare and contrast a variety of packages for this commodity, which should be one of their own choice, for example, chocolates, drink, cereals. Students' discussions and conclusions should be presented in their notebooks, together with an accurate net for each of the examples of the packaging used. Here again, they will use the packaging available in the classroom, since they may well apply some of the techniques which they have seen and studied.

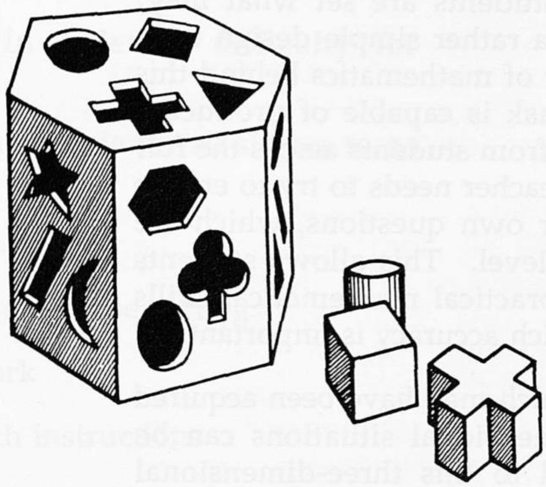
A reporting back session could be included here, when students are asked to tell the rest of the group about the packaging they have investigated and the conclusions they have arrived at. It is useful to encourage others to comment upon, and be criticial of, the ideas put forward.

The final stage is for the students to be given the task of choosing to package a specific item from the commodity type they have investigated. They could design and make, say, two different packages for this item. The design and production of these models should draw upon all the knowledge and experience gained in the first two stages.

Students should show all their working, calculations, sketches, etc in their notebooks. They should be encouraged to investigate their designs fully, looking at strengths, weaknesses, suitability, different sizes of packs, etc. Accurate nets for each design should be included in their files, together with instructions on how to assemble each container.

In their final report, students should evaluate their packages. This should include a comparison of the two packages produced and a recommendation about which one they think is the most suitable together with reasons which support their comments. A display of students' final packages adds extra impact to the task.

SORTING SHAPES



Shape sorting toys are very popular with young children. These toys are designed so that each shape will fit into, or pass through, only one hole. You may have seen toys like this in shops, or you may have watched your younger sisters and brothers, or your friends' children, playing with them.

Your task is to design and make a model of your own shape sorter, which is suitable for a young child.

It need not look anything like the one drawn above.

You can introduce any ideas you like. Try to ask yourself lots of different questions about your work as you go along. This will help you to understand more fully what you are doing.

You should keep notes and diagrams of everything you do. These notes will help you to write your final report. This report should contain ideas you had, any errors you made, any changes that were necessary, how you made the shapes, together with anything else you think is interesting or important. Your report, together with your shape sorter must be handed in for assessment.

ENJOY YOURSELF!

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Sorting Shapes - Teacher's Notes

Sorting shapes has been designed to allow students to tackle a GCSE coursework task involving practical geometry. Students are set what may, initially, appear to be a rather simple design task. However, there is a lot of mathematics behind this starting point. This task is capable of producing responses and designs from students across the full range of ability. The teacher needs to try to ensure that students ask their own questions, which are relevant at their own level. This allows students to demonstrate their practical mathematical skills within a context in which accuracy is important.

Concepts and skills which may have been acquired in relation to two-dimensional situations can be extended and applied to this three-dimensional task.

The main task within this investigation is to design and construct a shape sorter which involves several different types of shapes. Each shape, however, has to fit one, and only one, hole in the sorter. Naturally, there are many possible directions from this starting point, and students should be encouraged to think of possibilities and to explore them for themselves. Some possible areas for further consideration by students may well include

* The shape of the container
* The shapes to sort
* The dimensions of the sorting shapes
* The variety of shapes to sort
* Symmetry
* Nets and models
* How many different nets are there for each solid?
* Areas and volumes
* Fitting the nets on card for economy and minimum wastage

* Mathematical calculations relating to some of the above decisions
* Using more than one of each shape
* Euler's relationship
* Placing solids in order of difficulty for sorting
* Making a minimum hole for all shapes to go through
* Shape classification
* Learning curves for shape sorting
* Construction work
* Making a kit with instructions
* What could I have considered to help me meet the constraints immediately?
* How could I have avoided my problems?

and many more.

The intention is that students should introduce specific areas of mathematics, as the need arises, at appropriate stages within their design process. It should be emphasised that it is mathematical thinking and skills we wish to assess here, rather than the creative, artistic or aesthetic qualities of the product. However, it is to be expected that these issues will be discussed during the work, and they may well help in the design process, even if the assessment does not take account of some aspects of them. The line between art and mathematics has always been difficult to draw.

One fruitful way to overcome this problem is to consider the possibility of a cross-curricular task. This task could well be linked with assessment within other curriculum areas, say CDT, where the work is also assessed in relation to criteria appropriate to that particular subject, using an entirely different set of assessment objectives.

All students should be encouraged to keep a record of their work as they proceed through this task. Their notes should indicate what they have tried, found out and decided. They need to be encouraged to ask questions of the type

'WHAT IF ....?'

These questions may be directed towards anything relating to their individual design problem.

One of the skills which we are keen to develop and encourage through GCSE coursework is the students' ability to review their work critically and improve upon it. This skill is, perhaps, particularly highlighted within this task because of the need for a unique relationship between shape and hole. Any improvements or alterations made during the move towards this uniqueness should be indicated within the student's report.

As with any GCSE extended task, it is difficult to give more than a broad outline concerning what may happen in the classroom when this task is tackled. However, it is often useful to have some guidelines about its possibilities, and that is what is offered here.

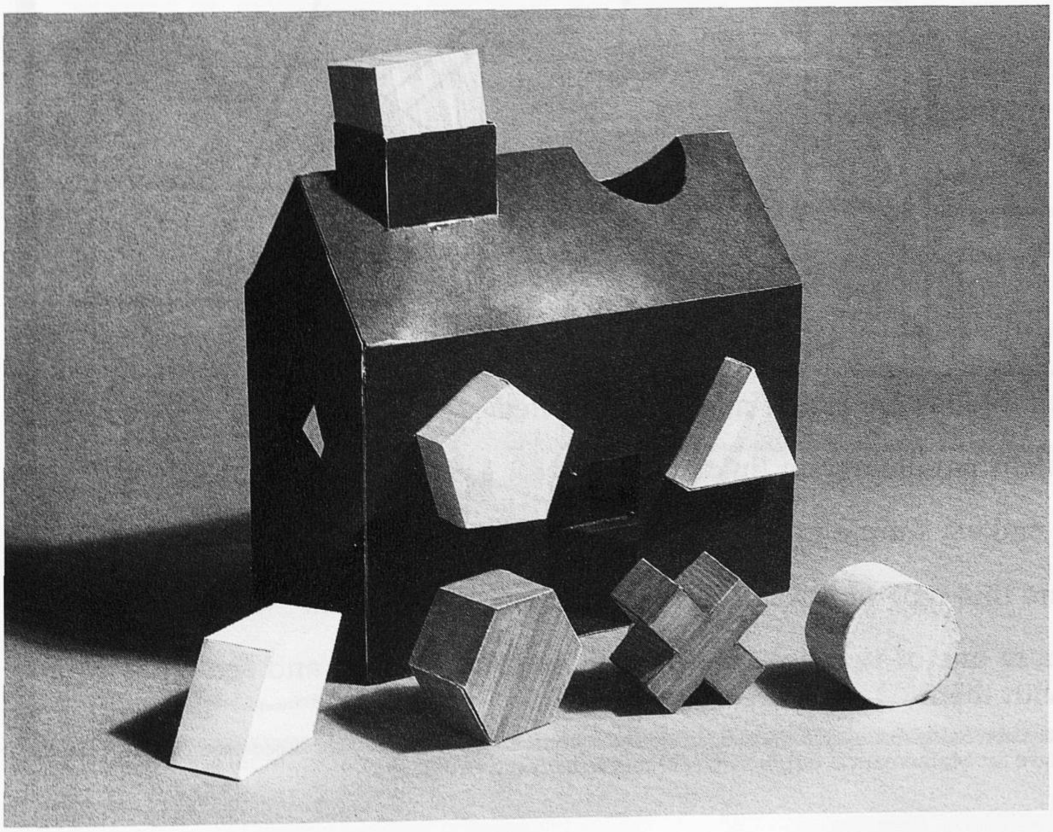
Practical activity is the basis of this work, and students of all abilities will probably benefit from diving straight into the problem. This will take a considerable time, and should provoke much informal discussion concerning the chosen shapes, the number of shapes, dimensions etc. You should ensure that your students keep notes of decisions taken, questions posed, as well as sketches and rough models. During the early stages of tackling this problem, you may prefer to allow your students to work with paper rather than card. Clearly, it is not an ideal medium to experiment with, but the process is much more cost-effective; a point which most students will appreciate.

This task is one that every student can tackle individually, even if there is considerable inter-student discussion. When the majority of students have produced an initial product, it may be profitable to have a whole class discussion about the problems which individuals have encountered, and how the problems might be given further consideration.

Clearly, there will be a wide range of final products in relation to mathematical quality, further considerations and extension work. During the latter part of the work some of the areas already suggested as possibilities may be considered. Students ought to be encouraged to look at just one or two ideas in depth, rather than many superficially. It is important that students explain in their report, all the stages they have gone through in producing their final product.

The assessment should be based upon the final product together with the student's design report. However, account ought to be taken of the teacher's observations of each student's work and actions throughout the task, as well as their discussions concerning this.

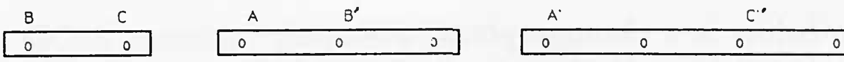
The illustration below is a photograph of a shape sorting toy produced by a student in one of our trial schools.



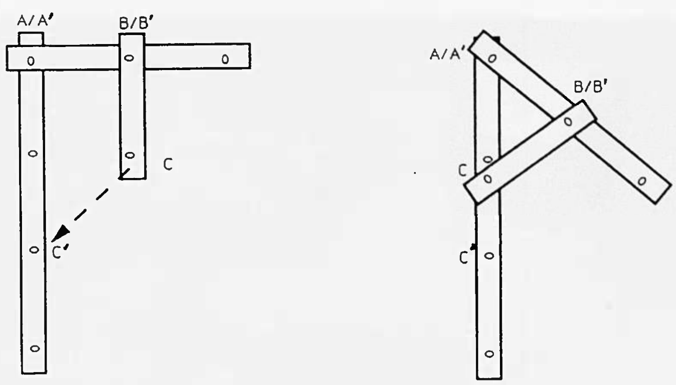
LINKAGES

Have you ever thought carefully about how an umbrella works?

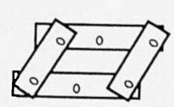
You can begin to investigate this problem using three strips of card, or plastic, and two paper fasteners.



Fix one paper fastener through the two holes labelled A and A', and another fastener through the two holes labelled B and B'. Place C on top of C', and then slide C along the longer strip towards A/A'.



Folding umbrellas are rather more complicated.



They contain parallelogram linkages.

Can you make a simple model for a folding umbrella?

Investigate this type of linkage further in any way you wish.

Now choose one or two linkages from the resource sheets and see what you can do with them.

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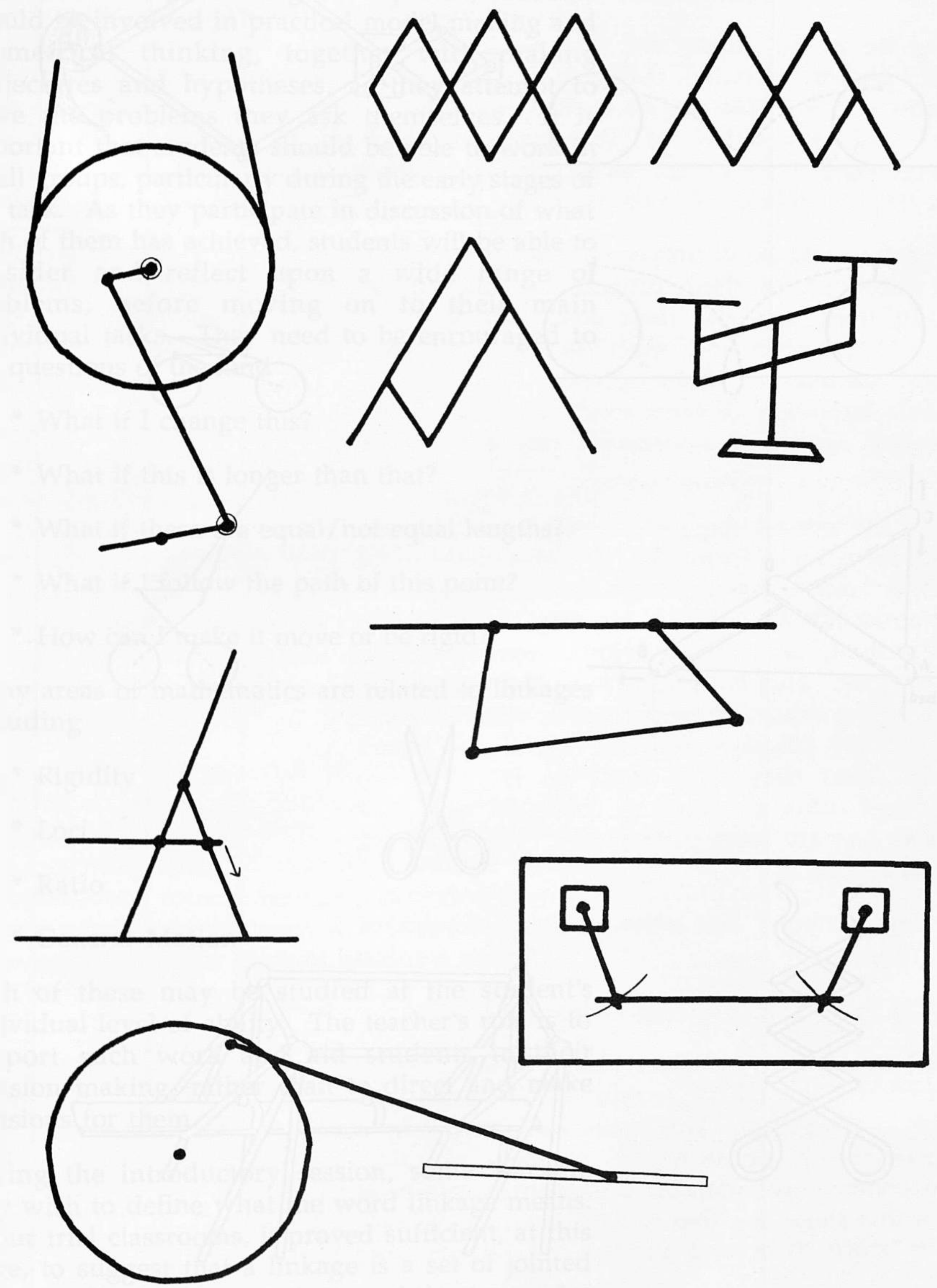
LINKAGES : continued



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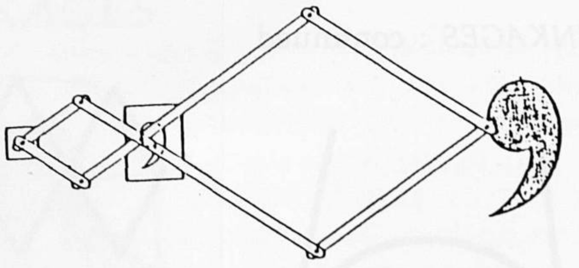
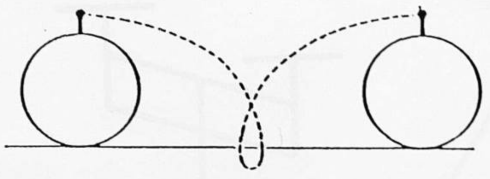
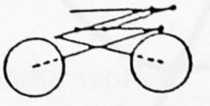
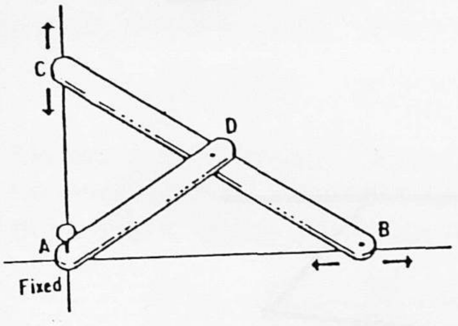
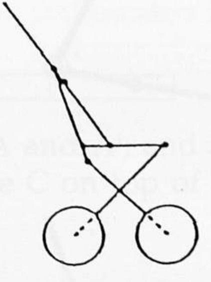
LINKAGES : continued

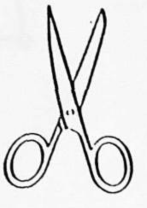


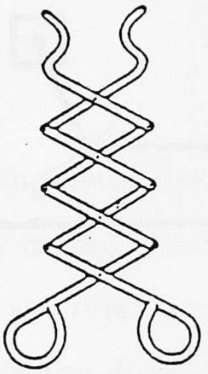


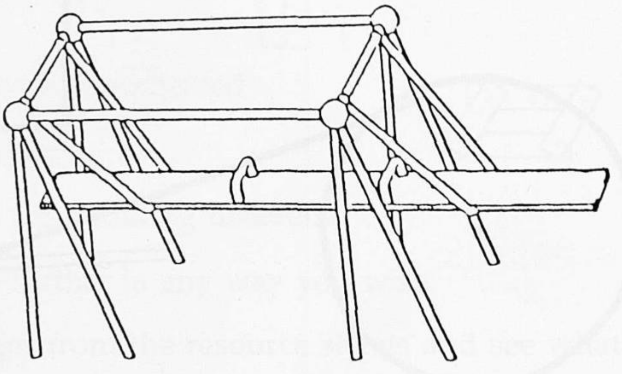












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Linkages - Teacher's Notes

Since this is a practical geometry task, students should be involved in practical model making and geometrical thinking, together with making conjectures and hypotheses, as they attempt to solve the problems they ask themselves. It is important that students should be able to work in small groups, particularly during the early stages of the task. As they participate in discussion of what each of them has achieved, students will be able to consider and reflect upon a wide range of problems, before moving on to their main individual tasks. They need to be encouraged to ask questions of the kind

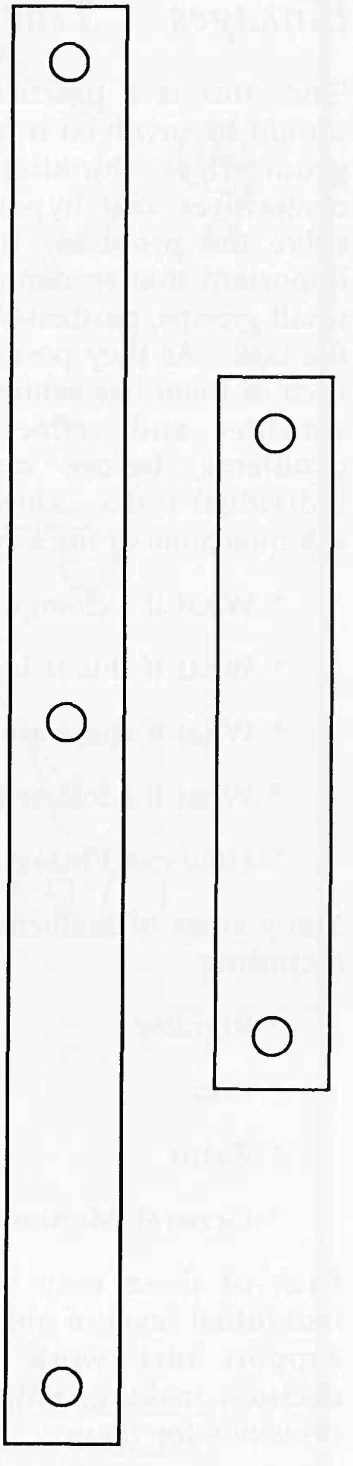
* What if I change this?
* What if this is longer than that?
* What if these are equal/not equal lengths?
* What if I follow the path of this point?
* How can I make it move or be rigid?

Many areas of mathematics are related to linkages including

* Rigidity
* Loci
* Ratio
* General Motion

Each of these may be studied at the student's individual level of ability. The teacher's role is to support such work and aid students in their decision making, rather than to direct and make decisions for them.

During the introductory session, some teachers may wish to define what the word linkage means. In our trial classrooms, it proved sufficient, at this stage, to suggest that a linkage is a set of jointed bars or rods. The first student worksheet provides an easy entry into the subject. Some schools may have resources which include Meccano or Geostrips, but strips of card containing holes produced using a hole punch, together with a few paper fasteners, can get one a long way. For your convenience, we include examples of such strips in the right hand margin of these notes.



It may be useful during the first week to use a blank poster entitled LINKAGES on your classroom wall. This can be used by students as they find examples of linkages in their environment. They can write on it or stick photographs, pictures etc, onto it.

Students should be encouraged to keep an up-to-date notebook of their progress. This should include their models, their own 'What if ....' questions together with their own answers and general comments. This notebook will form a valuable summary of their work, and will be an important aspect of assessment and moderation procedures.

During the second stage of this topic, as they use the resource sheets provided, students ought to be looking more specifically at one of the mathematical topics, i.e rigidity, loci, ratio, enlargement, as well as trying some practical applications. This is perhaps best encouraged by organising a reporting back session, where several students are asked to tell the rest of the group briefly what they have done so far. Try to encourage other students to comment upon the ideas as they are being put forward. You could ask questions such as

* Has anyone else done anything similar to this?

* Is this a useful linkage or just fun?

Some students may, of course, bring in some aspect of mathematics which would be of interest to others in the group. This could be followed by a brainstorming session on further practical applications. The classroom poster may again be of use at this stage, together with a reminder of ideas which were discussed during the introductory session.

Students should then be allowed to continue their own investigations, looking at both the mathematical topics and practical applications.

Again, they may need reminding about their notebooks.

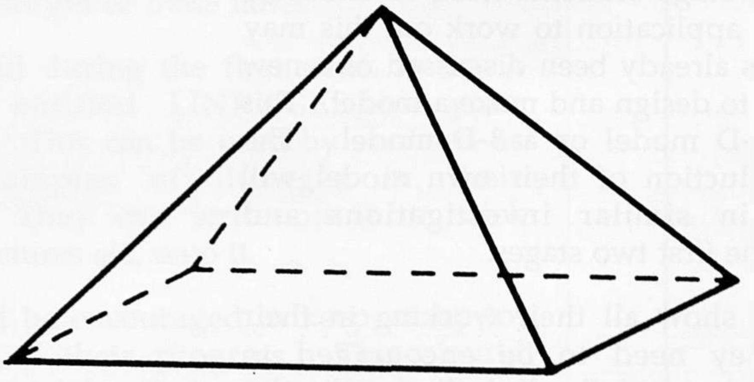
During the third stage students need to choose a specific practical application to work on, this may be one which has already been discussed or a new idea. They need to design and make a model. This may be a flat 2-D model or a 3-D model. The design and production of their own model will involve them in similar investigations and thinking as did the first two stages.

Students should show all their working in their notebooks. They need to be encouraged to investigate their design fully, including strengths, weaknesses, variations, other applications of the same structure etc.

There is, of course, a fourth stage which is closely linked to stage three. This involves the student in evaluating their final products. Assessment will be based on students' final reports, including notebook items which will include their final product, together with any other models they have made. Students' oral ability to communicate with their fellow students, as well as with their teacher, about what they have done, will also be taken into consideration.

This outline is intended to be flexible. The individual needs of both yourself and your students ought to be built into the programme. The essential thing is that all students should experience and record their own extended practical investigation into linkages, and that they should have used their own ideas in making a model.

PYRAMID HOME



The great pyramids of Egypt were built by thousands of slaves, in order to provide burial places for the ancient pharoahs.

Forty miles outside Chicago stands the Gold Pyramid of Wadsworth, Illinois. This was built by one man, in order to provide a luxury home for his family of seven. This *Pyramid Home* is five storeys high, with an observatory on the top floor, and its roof is covered with twenty four caret gold-plated tiles.

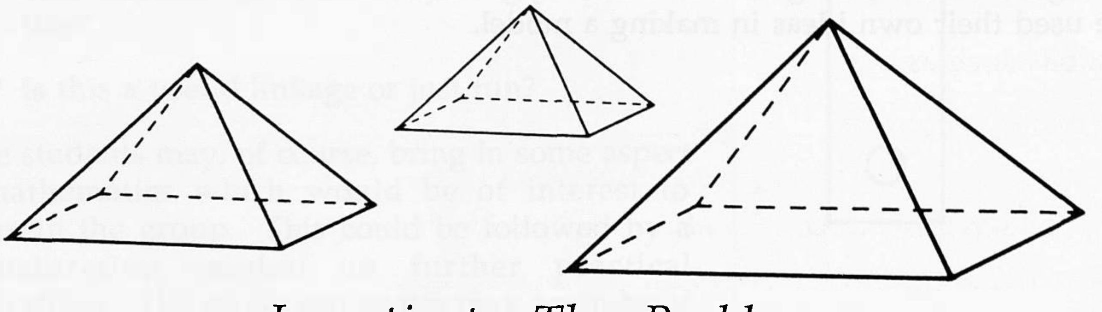
It has a total floor area of approximately 2 000m^2, five bedrooms and six bathrooms. Three linked pyramids provide garaging for the four family cars.

* Make models of the house and garage.

* How do you think you would arrange the rooms in this ideal home?

* This builder now plans to construct a *pyramid home* village!

* What shape would your ideal home be?



Investigate The Problem

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Pyramid Home : Teacher's Notes

This task is intended to be tackled practically. As students complete this task, it is anticipated that they will bring together a wide variety of experiences relating to the world of shape and space in which they live.

During the introductory session, the student worksheet could simply be handed out. As they work in small groups, students could be encouraged to discuss what they perceive to be the advantages and disadvantages of a *pyramid home,* compared with a conventional home. They should be encouraged to produce model homes, so that they can support their arguments with accurate facts and numbers.

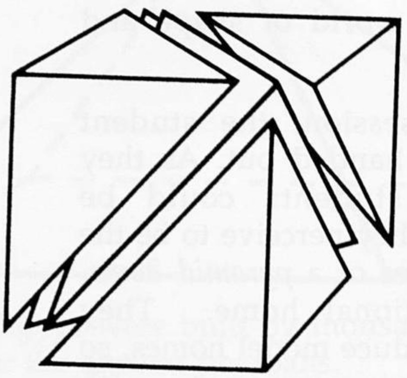
Such activities provide a need for accurate drawing, measuring and model making, as well as the use of trigometrical concepts. Notions of scale drawing, similarity, ratio, area and volume, will all be called into action.

During this task the emphasis is on students using their spatial imagination and making conjectures. At first, students will test their ideas practically, learning from their mistakes. Gradually, they will need to begin to use other mathematical skills, as they produce designs using calculations, as well as experience and practical model making.

Trying to imagine what it might be like to be *inside* a pyramid, can develop students' spatial sense quite considerably as they attempt to answer questions such as

* What angle do the sides of the pyramid make with the base?
* Is there likely to be a lot of wasted space?
* Where would you position the stairs, bathrooms, kitchen ...?
* Would you buy a pyramid house?
* What shape is *your* ideal home?

CHOP IT UP

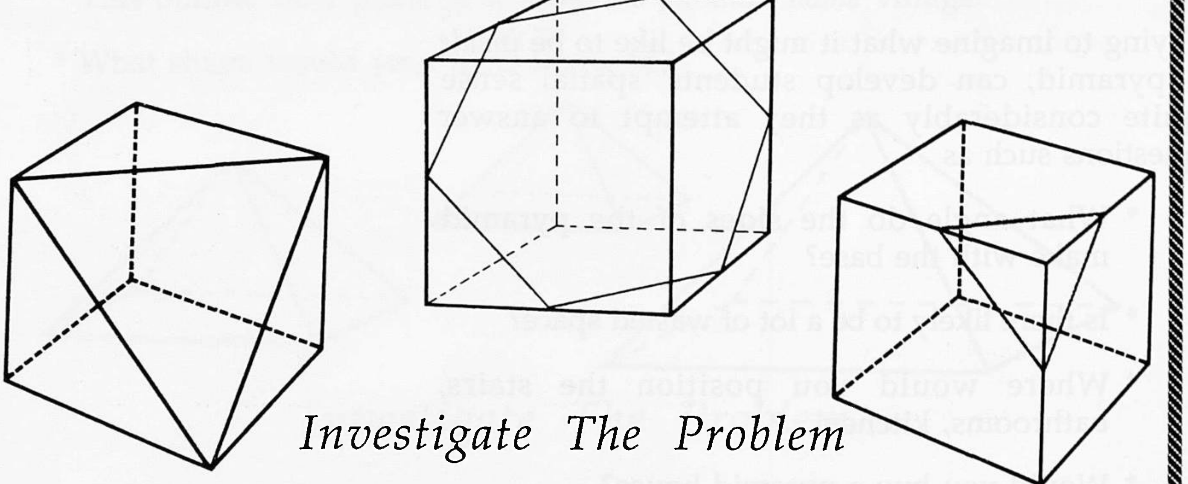


Puzzles which involve fitting shapes together to make other shapes are a very popular pastime.

On the resource sheets provided you are given the nets of a variety of solids, and you are challenged to put several of them together to make another different solid.

You may find it helpful to work in small groups as you explore the resource sheets provided.

Your final task is to design a puzzle which you think is suitable for marketing commercially.



Investigate The Problem

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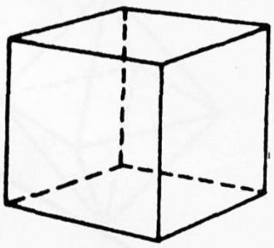
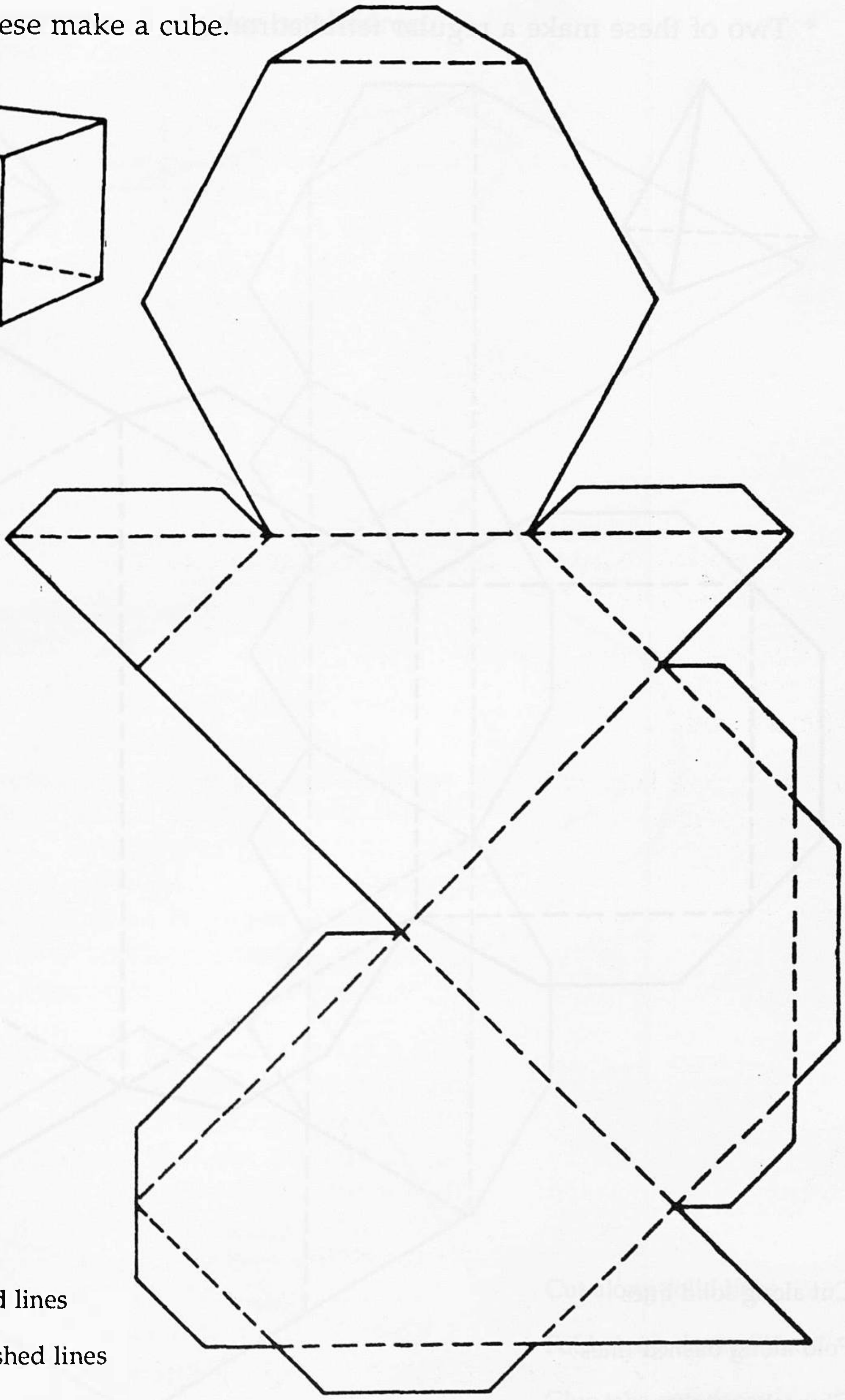
CHOP IT UP : continued

* Two of these make a cube.





Cut along the solid lines

Fold along the dashed lines

Glue tabs onto faces

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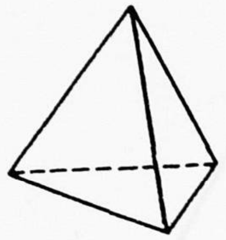
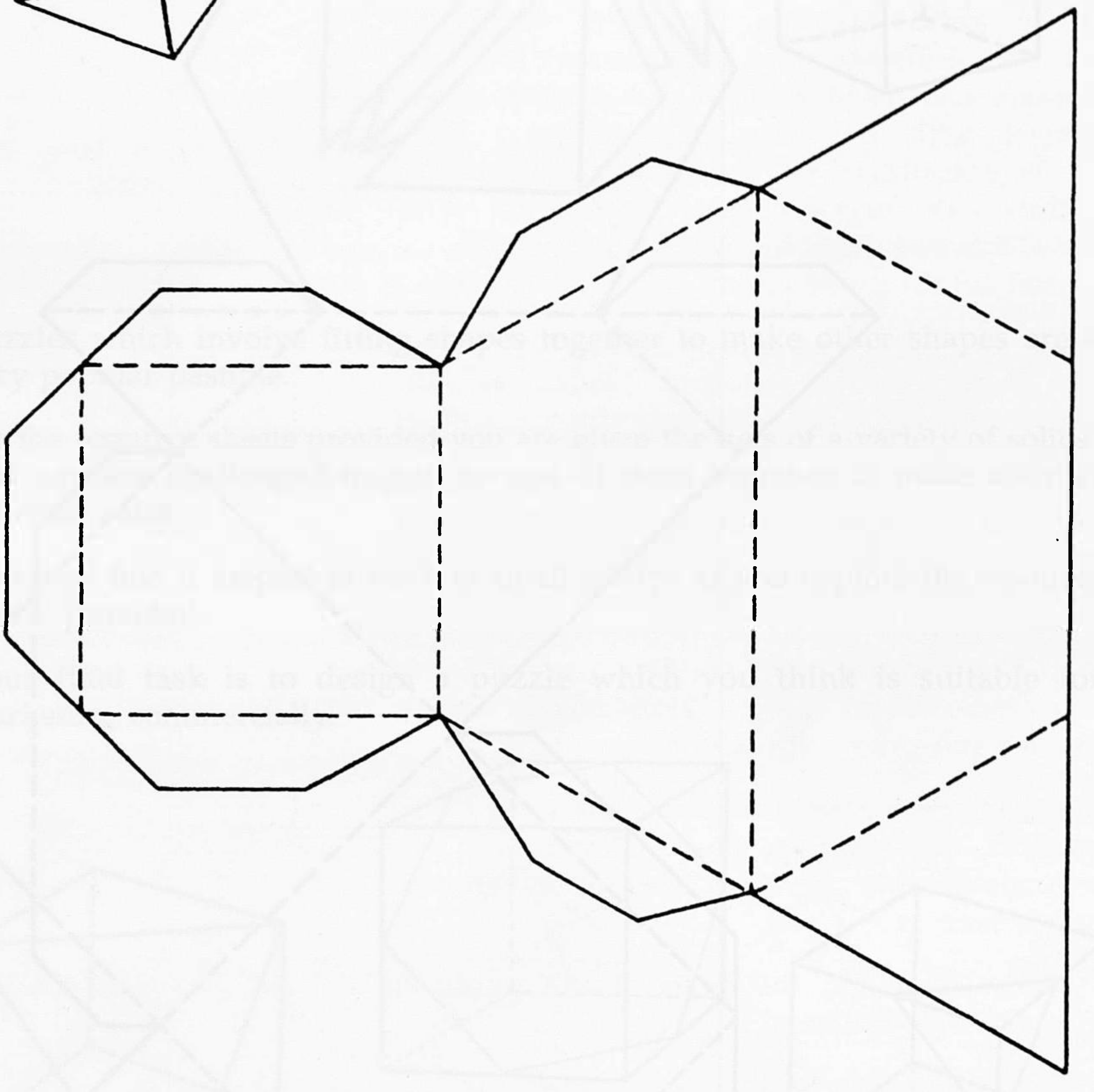
CHOP IT UP : continued

* Two of these make a regular tetrahedron.





Cut along solid lines

Fold along dashed lines

Glue tabs onto faces

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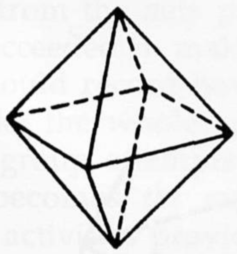
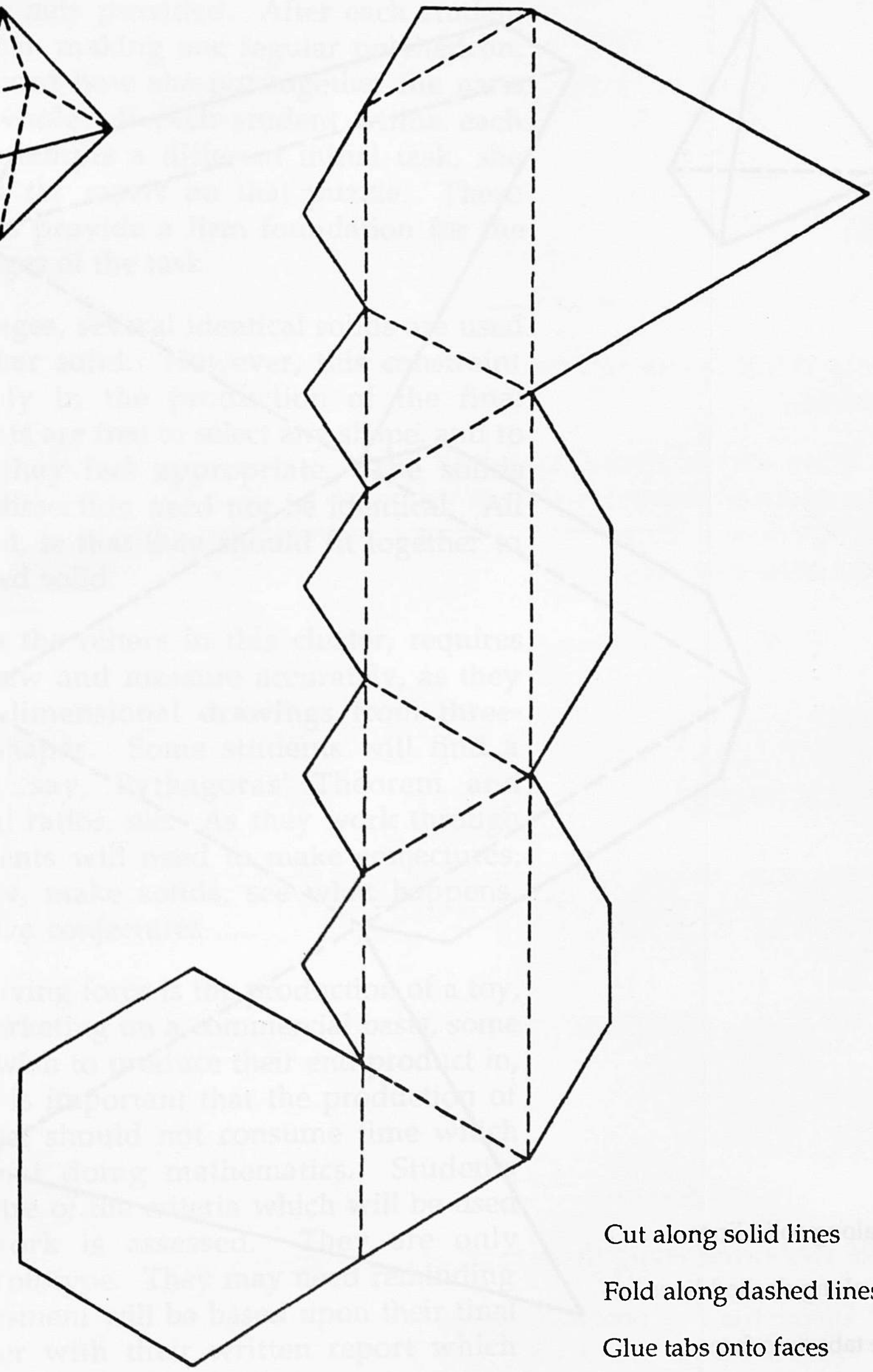
CHOP IT UP : continued

* Two of these make a regular octahedron.





Cut along solid lines

Fold along dashed lines

Glue tabs onto faces

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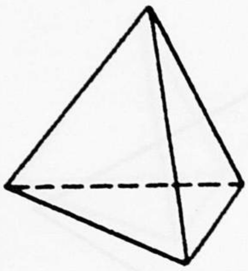
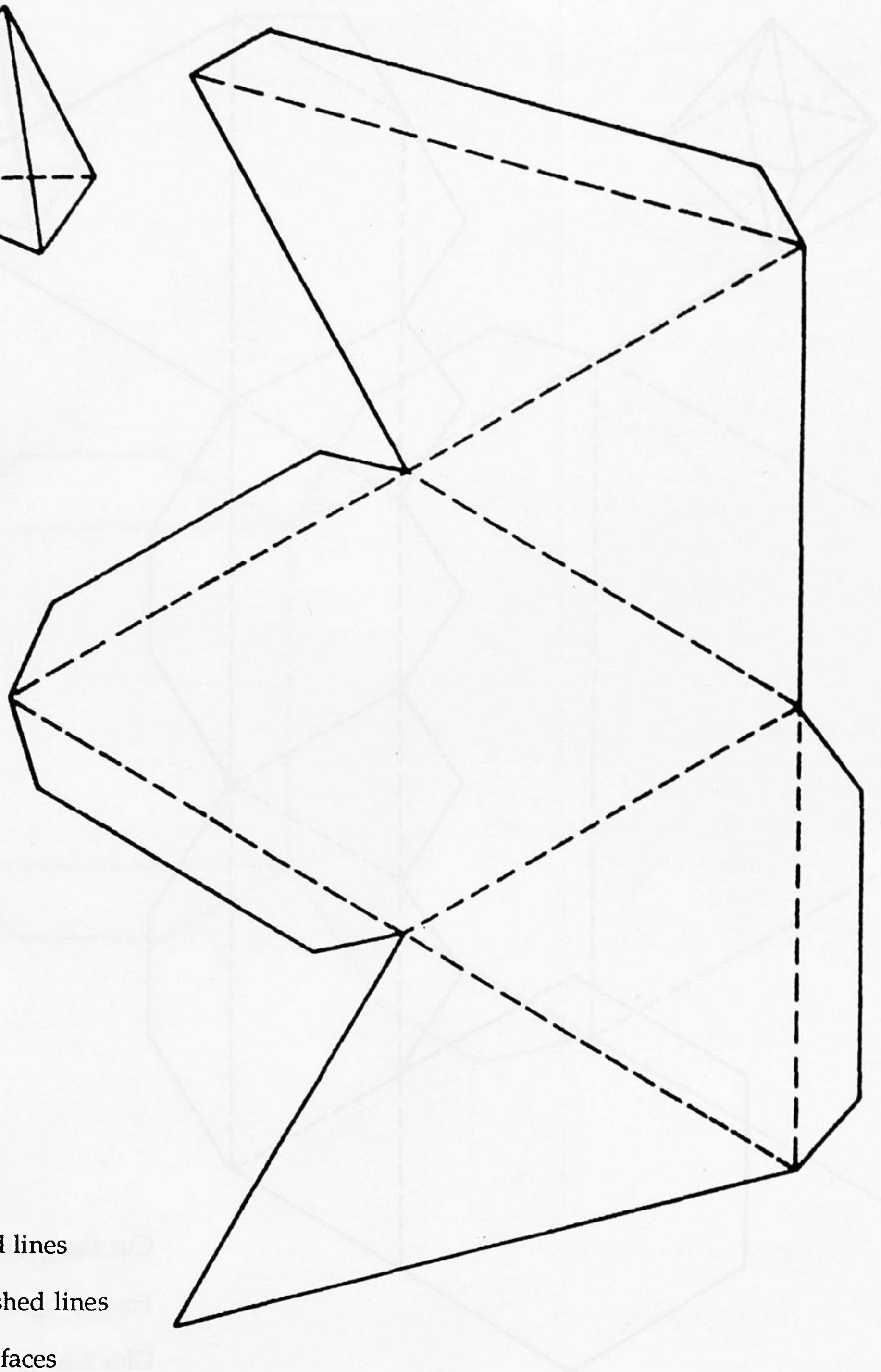
CHOP IT UP : continued

* Four of these make a regular tetrahedron.





Cut along solid lines

Fold along dashed lines

Glue tabs onto faces

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Chop It Up - Teacher's Notes

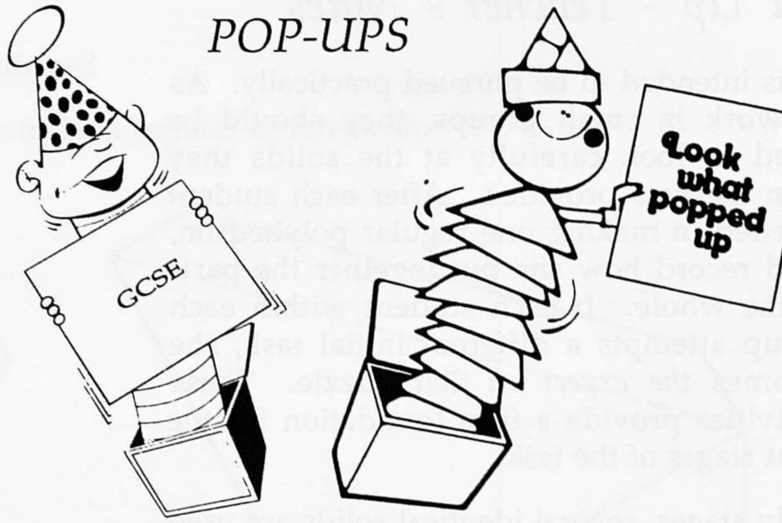
This task is intended to be pursued practically. As students work in small groups, they should be encouraged to look carefully at the solids they make from the nets provided. After each student has succeeded in making one regular polyhedron, she should record how she put together the parts to make the whole. If each student within each small group attempts a different initial task, she then becomes *the expert* on that puzzle. These initial activities provide a firm foundation for the subsequent stages of the task.

In the early stages, several identical solids are used to make another solid. However, this constraint does not apply in the production of the final model. Students are free to select any shape, and to dissect it as they feel appropriate. The solids produced by dissection need not be identical. All that is required, is that they should fit together to make a specified solid.

This task, like the others in this cluster, requires students to draw and measure accurately, as they produce two-dimensional drawings from three-dimensional shapes. Some students will find a need to use, say, Pythagoras' Theorem and trigonometrical ratios, etc. As they work through this task students will need to make conjectures, calculate, draw, make solids, see what happens, make alternative conjectures .....

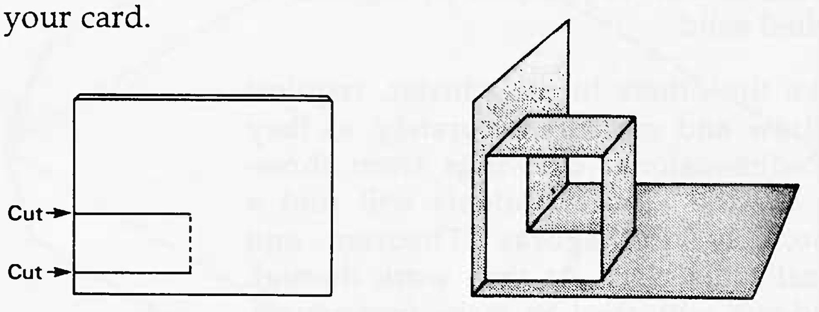
Because the driving force is the production of a toy, suitable for marketing on a commercial basis, some students may wish to produce their end product in, say, wood. It is important that the production of the end-product should not consume time which should be spent doing mathematics. Students need to be aware of the criteria which will be used when their work is assessed. They are only producing a prototype. They may need reminding that their assessment will be based upon their final model, together with their written report which should contain drawings, calculations, reasoning, and details of their investigations. Their assessment should also take account of their ability to explain orally, what they did and why they did it.

POP-UPS

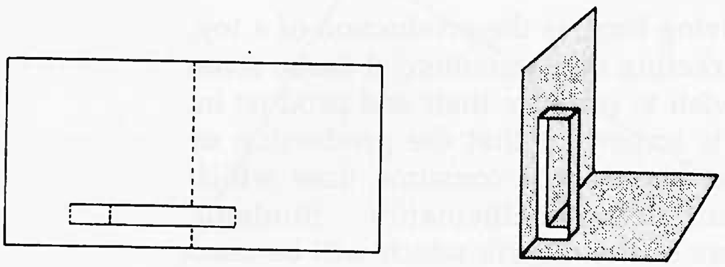


Pop-up cards, books and toys are very popular. They can also be extremely amusing.

It is quite easy to make your own pop-up cards. One simple method is to fold a piece of paper, as shown below. Cut along the solid lines, and fold along the dotted lines. Paste a picture on the folded paper, and it will *pop-up* as you open your card.



The diagram below shows another way of making a pop-up card. You should cut your paper while it is flat, then fold it, and finally paste on your picture.



Make two cards using these methods, and discuss any problems which arise.

* Using the two resource sheets provided, make the pop-up cards *THE FOOTBALLER* and *THE TRAIN*.

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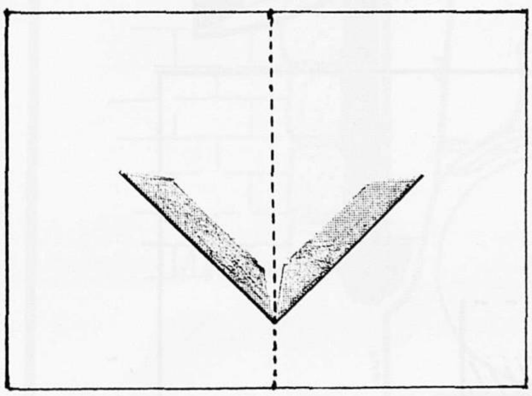
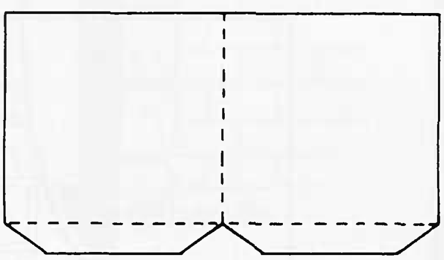
POP-UPS : continued

You can produce different results, which are also suitable for pop-up books, using the diagrams below. Pop-up books often open through 180 degrees, so that the book lies flat on a table.

The tabs on B should be glued onto the shaded parts of A. B should be symmetrical about the fold in A.



A



B

When the book is open, A is flat and B is upright. When the book is closed, A folds along its dotted line, both A and B lie flat.

Make a paper model which is suitable for a page in a book. You can obtain even more interesting effects if you attach pictures to B as you did previously.

Try to use this method to produce some interesting effects.

Explain how your model works.

- Make the card DRACULA.
- Try to explain how it works.
- Investigate how other pop-up cards and books work.

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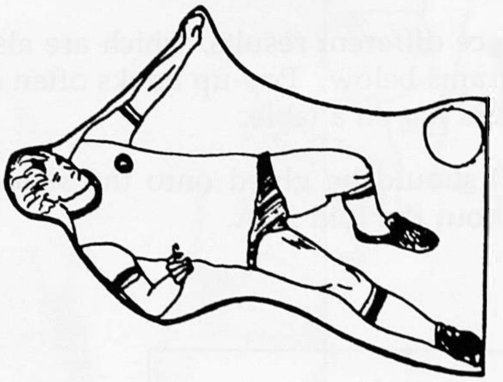
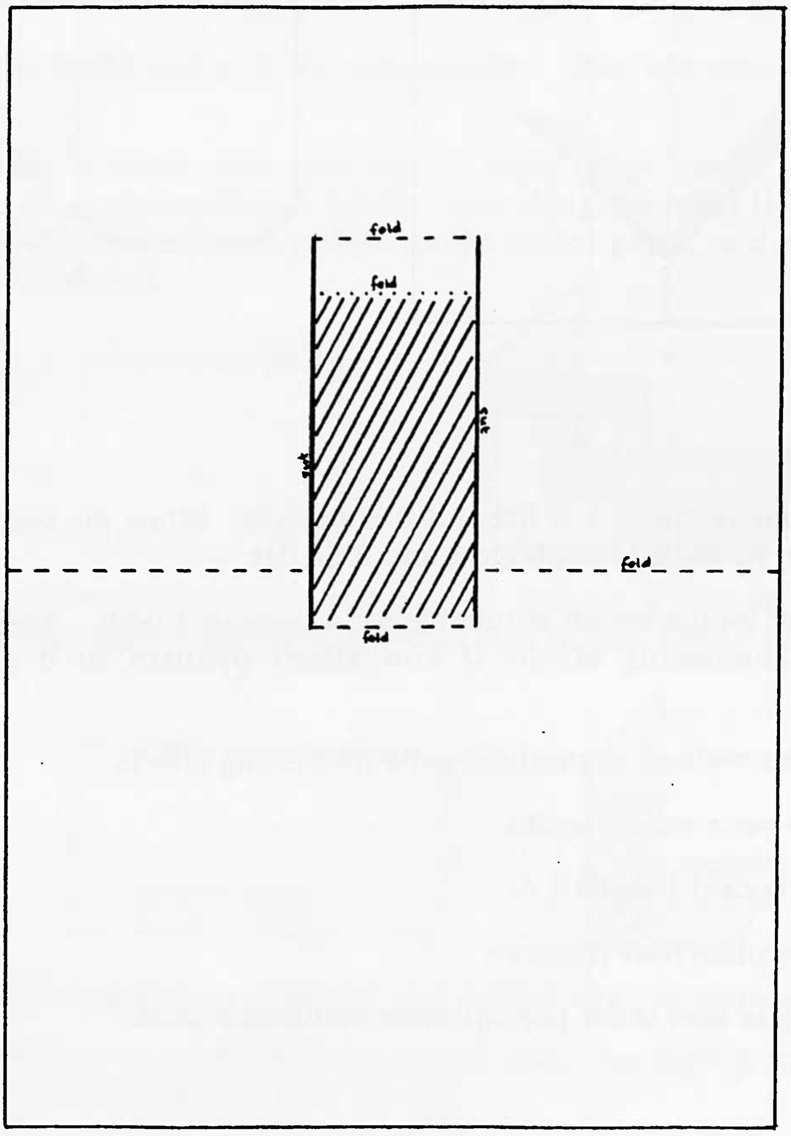
THE FOOTBALLER





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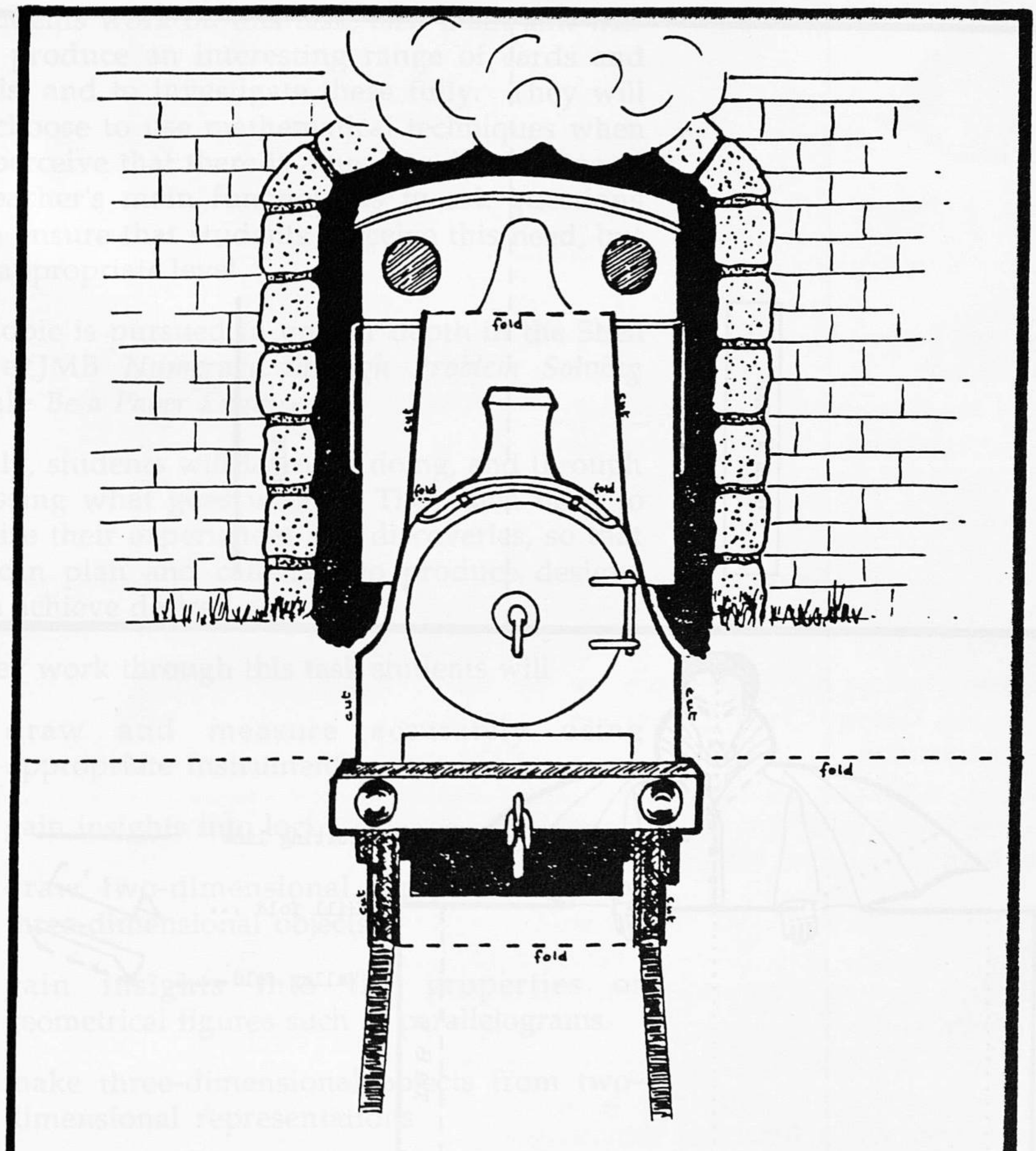
THE TRAIN



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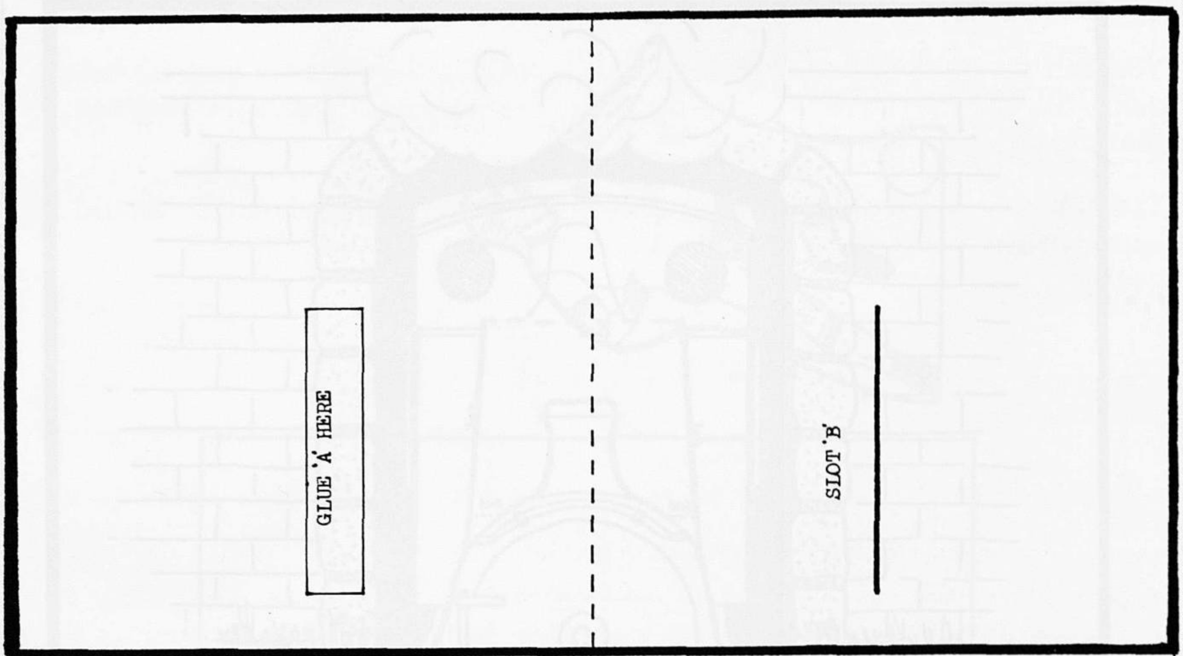
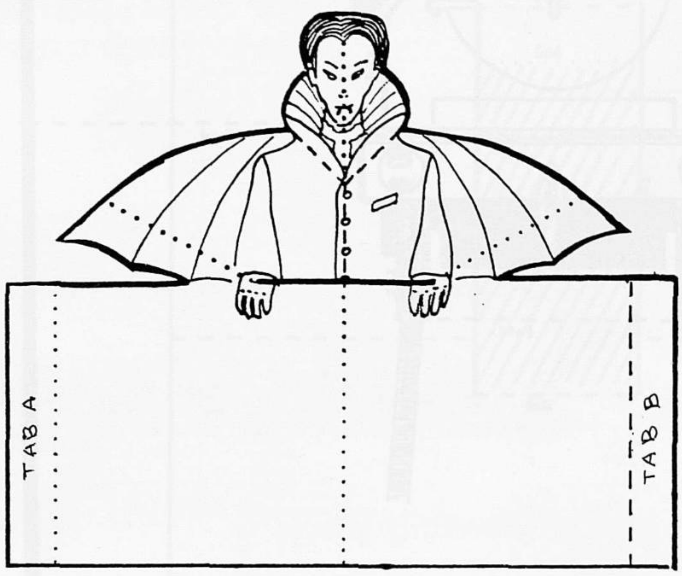
DRACULA



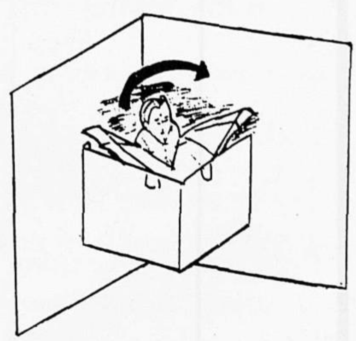


Cutting line

Hill fold ...

Valley fold - - -

- Cut out base sheet
- Cut slot B and fold sheet
- Colour Dracula and cut out
- Score, fold up and press well
- Glue tab A in place on sheet
- Push tab B through slot and glue on the back



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Pop-Ups : Teacher's Notes

As students work on this task, *their* main aim will be to produce an interesting range of cards and models, and to investigate these fully. They will only choose to use mathematical techniques when they perceive that there is a need to do so. One of the teacher's main functions is to ask questions which ensure that students perceive this need, but at an appropriate level.

This topic is pursued in greater depth in the Shell Centre/JMB *Numeracy through Problem Solving* module *Be a Paper Engineer*.

Initially, students will learn by doing, and through discussing what goes wrong. They then need to organise their experiences and discoveries, so that they can plan and calculate to produce designs which achieve desired effects.

As they work through this task students will

* draw and measure accurately using appropriate instruments
* gain insights into loci
* draw two-dimensional representations of three-dimensional objects
* gain insights into the properties of geometrical figures such as parallelograms
* make three-dimensional objects from two-dimensional representations
* use concepts of symmetry
* gain insights into linkages
* construct 'proofs'
* use trigonometrical concepts etc.

The assessment of students' work will be based upon their use of such mathematical concepts and skills, as well as upon their ability to explain to their teacher and fellow students how their models work.

5 *Students' Work*

These six pieces of work cover a wide range of achievement. Two pieces of work are offered at each of the three levels of GCSE study; Foundation, Intermediate and Higher. These three levels are common to all GCSE schemes although the level titles differ.

The six pieces are in rank order of attainment and finish with the piece which is considered the best from the set. In Chapter 6, you will find detailed comments made on each piece by the Midland Examining Group Chief Coursework Moderator. We recommend that you should consider each piece of work in detail, make a few written comments and attempt to grade each student's work, before you read the moderator's comments.

For identification purposes, the six student's scripts are labelled G1/1 to G1/6. Because of space constraints the project team decided to reduce the size of the student's scripts, in order to include a wide range of student achievement. In addition to the loss of quality through the reduction in size, some scripts suffer from the loss of colour which originally added emphasis and clarity to the arguments presented. Nevertheless, we are hopeful that much of the strength inherent in the original scripts will become apparent as you read through the following pages.

G1/1

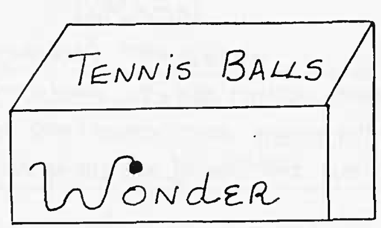
Anyone for Tennis.

Introduction.

We have been asked to choose a design for The Wonderland Sports Co The two designs we were given were a cuboid box and a cylindrical box.

These are the aspects which we had to consider

* What are the internal dimensions of the box
* What is the minimum required volume for three tennis balls.
* What is the volume of each box.
* How much space is wasted in each box.
* How much card is required for each box.
* How can you cut out these boxes from the standard sheet of card measuring 569 mm by 841 mm
A



The CUBOID Box

The CYLINDRICAL Box

Wonderland Sports Company Marketing Report.

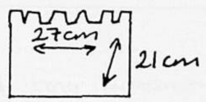
Project Description : Design of packaging for three "Wonder Bounce" tennis Balls.

Specification	CUBOID Box	CYLINDRICAL Box
Dimensions	19 x 7 x 7 = 931	21cm x 27cm = 567 ends (diameter) 76·930
Volume	954. 56 cm^3	3·14 x 35 x 3.5 x 20 = 769.3 cm^3
Excess Volume	945.56 - 538.50999 = 416.05001	769.3 538.50999 = 230.79001
Card required	513 cm^2	27x 21 = 567 cm^2
No. from standard sheet	squares = 9 circles = 96	squares = 25 circles = 96 out of two standered sheets

Anyone for Tennis.

Internal Dimensions.

First of all we found out the length and width of the box, which was 27cm by 21cm We then multiplied 27 x 21 which was 567 So our internal dimension was 567cm



Volume

Radius

3.14 x 3.5 x 3.5 x 20 = 769.3 cm^3

Excess volume

4 x 3.14 x 3.5 x 3.5 x 3.5 ÷ 3 = 17950333.

3 tennis Balls ÷ 179.50333

179.50333
179.50333
179.50333
538.50999
221

769.3
− 538.50999
230 .79000

Card Required.

27 x 21 = 567 cm^2

No. from two Standered Sheet

1 sheet measuring 569 mm by 841 mm = 25 squares.

another sheat measuring 569 mm by 841 mm = 96 circles.

first we measured. the length and width which was 145mm by 110mm. Then we seen how many 145mm went into 841mm which was 5. Then we seen how many 110 mm went into 569 which was 5 with 145 mm left over. We will be able to make 25 cylindrical boxes of the 541 mm by 145 mm paper.

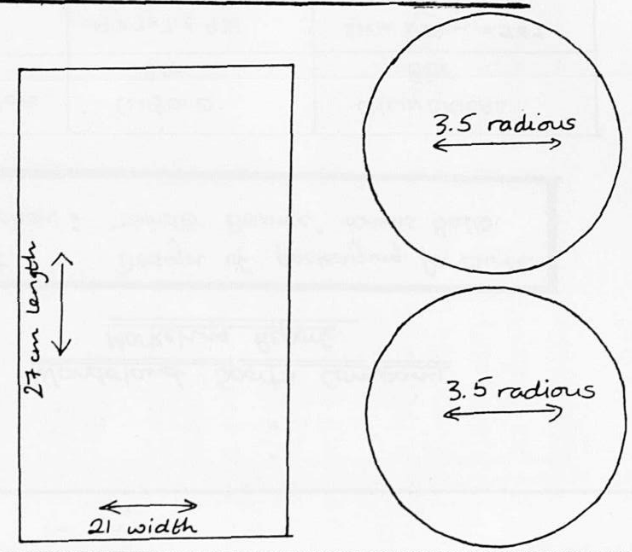
Then ends for the cylindrical box

569 ÷ 70 = 8
541 ÷ 70 = 12
12 x 8 = 96

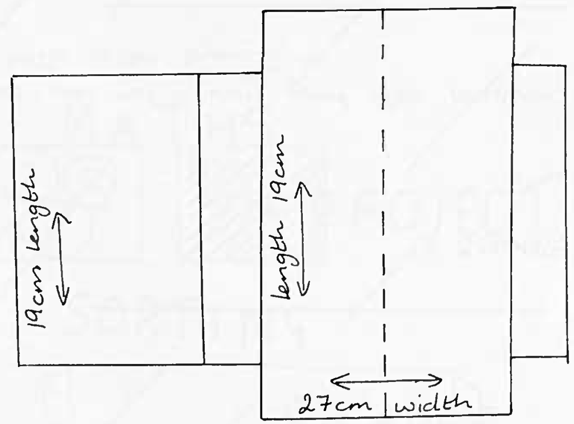
We will be able to get. 96 ends out of another sheet of paper measuring 541 mm by 145 mm

Nets

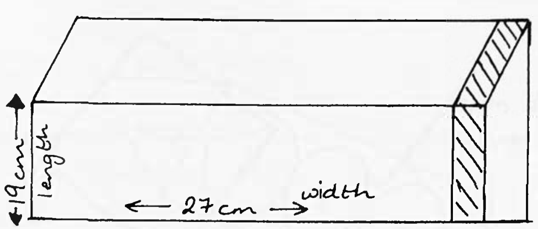
Cylindrical Box



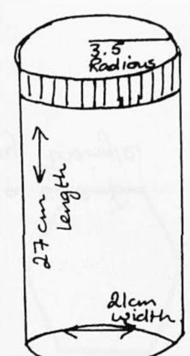
Nets
Cuboid
Box



Dimensioned diagrams.



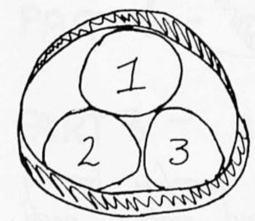
The Cuboid box.



IDEAS FOR MY OWN
Box

This was mine and mandy's idea
It wasn't a good idea to hold
3 balls because we had a lot of
excess volume.

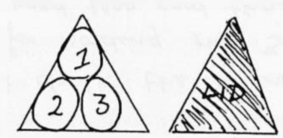
This idea would be useful for 6 or more
balls.



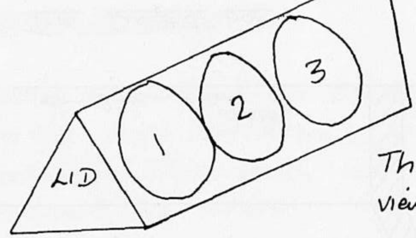
A circle shaped box
design to hold 3 balls
it had alot of excess
volume

LID - a little bigger than
the box itself So it could
Slot together

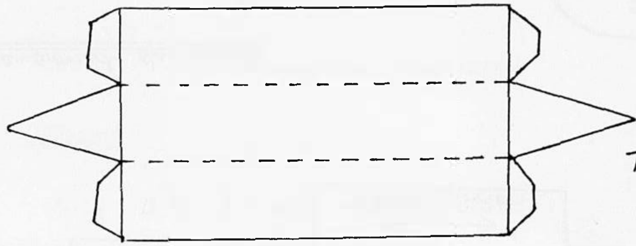
TRIANGLE



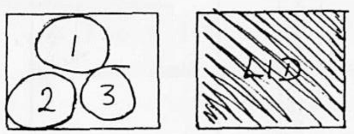
This was quite a good idea. 3 balls fit in
through the ~~#~~ side then the lid ~~[illegible]~~ shut.



This is a side ~~t~~ view of my Triangle



The NET



A SQUARE

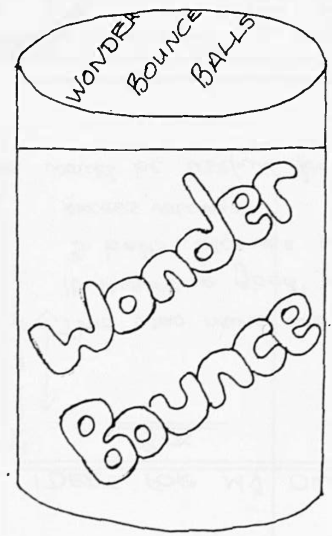
This was not a very good idea because it held 4 balls better than 3.

My Best Design

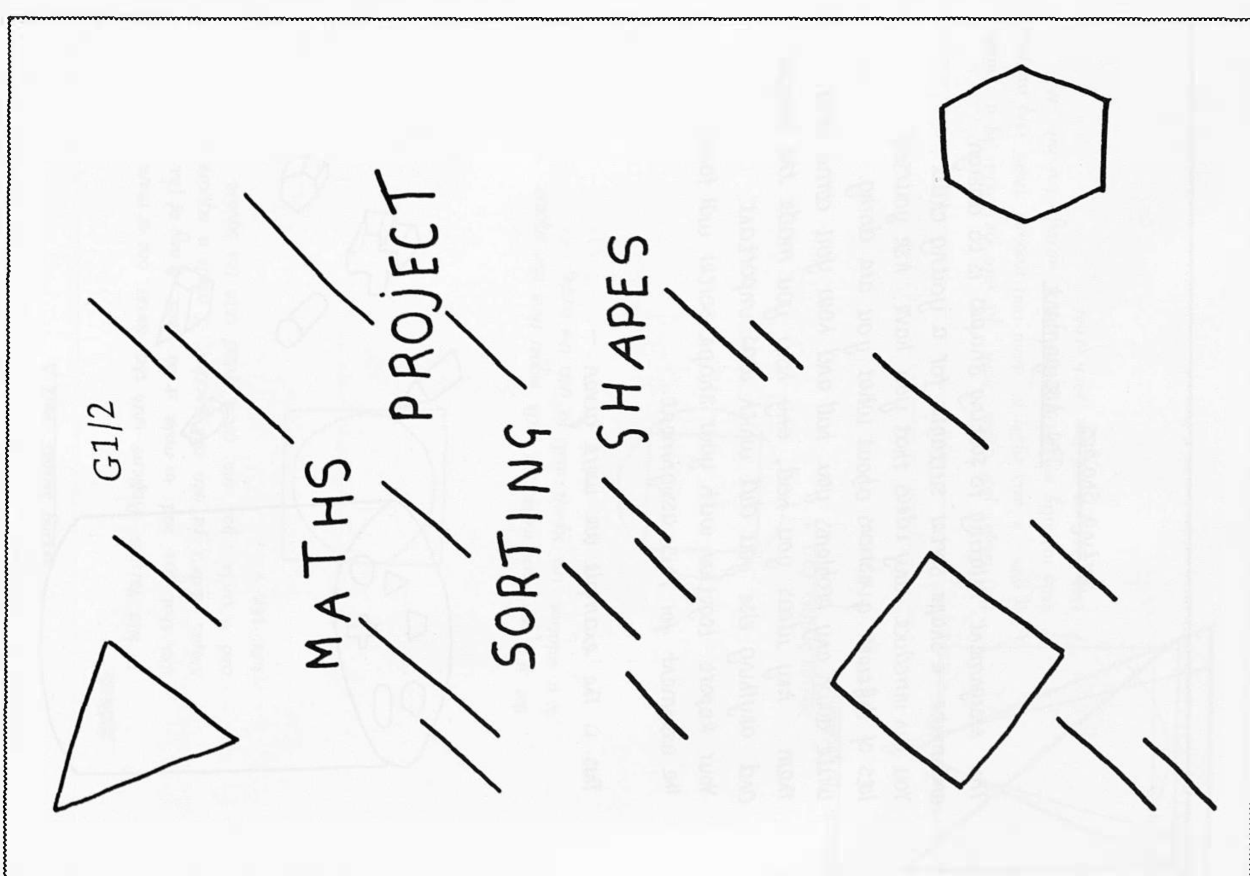
I think my best design was the triangle because the 3 balls fitted in nicely and their was not alot of excess volume.

The best idea.

I think the cylindrical box was the best idea for holding the 3 tennis balls because it used less card than the cuboid and it had less excess volume than the cuboid It was quite easy to make.
It was harder to find the measurements excess volume, volume and diameter than the cuboid but in the end we had a good useful product to hold the balls tidely together.



Our END producc



INTRODUCTION

PART 1 - THE ASSIGNMENT

PART 2 - COLORS + DESIGN

PART 3 - ANY PROBLEMS?

PART 4 - SOLUTIONS

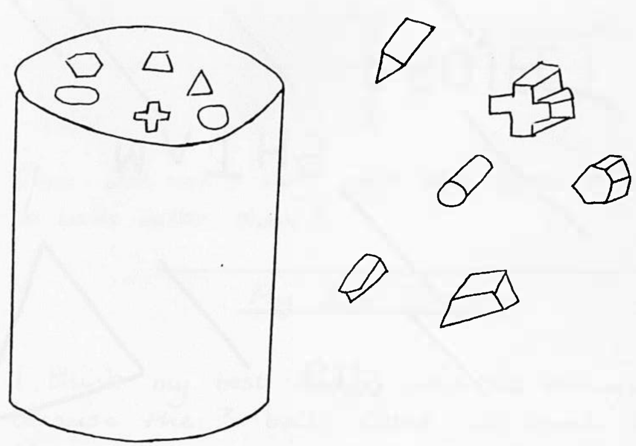
PART 5 - THE AREAS

Sorting Shapes

The Assignment

The Assignment relating to sorting shapes is to design and make a shape sorter suitable for a young child. You can introduce any ideas that you have. Ask yourself lots of different questions about what you are doing. Write down any problems you had and how you come over them. Any ideas you had, ~~any~~ how you made the shapes and anything else you did which was important. Your Report together with your shape sorter will form the assesment for this assignment.

This is the example we were given:–



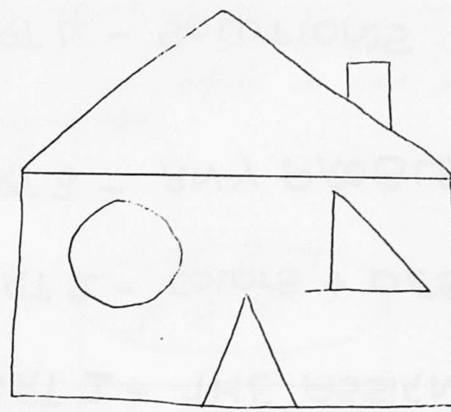
Sorting Shapes Part 2.

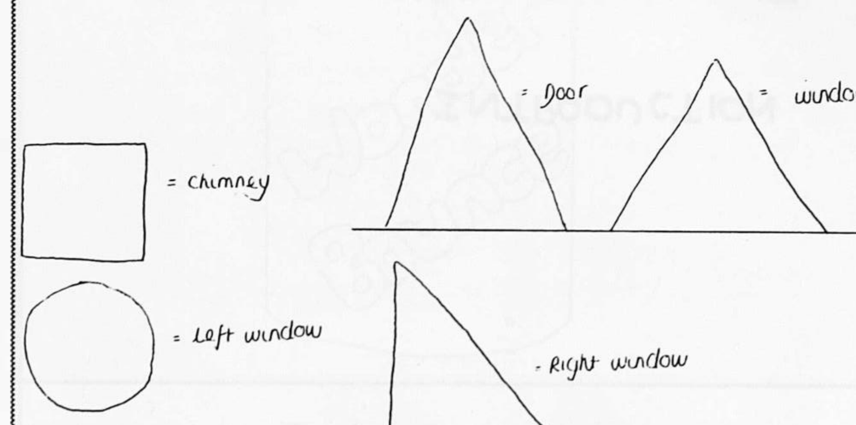
Colours –

Obviously a child is attracted to bright colours such as red & yellow which I have chose if the object was dull the child would not take a second glance.

Design –

The design of the sorter has to be catchy too so I chose to make the sorter out of a house

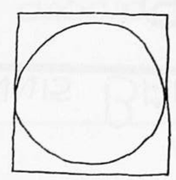




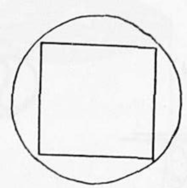
Solving Shapes Part 3

Problems

I had various Problems with this sorter, one of them was that you had to make it so that any of the shapes didn't fit into one another. I made a square and a circle, but the circle fited into the square like this -

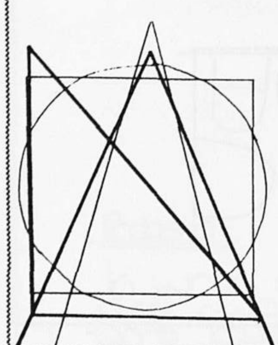


But if you make the circle bigger than the square as a solution the square will fit into the circle



Solving Shapes Part 4

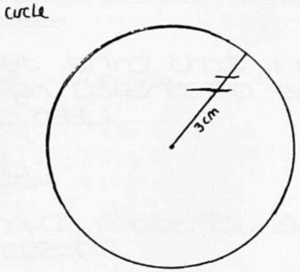
Solutions to problems



The shapes just about DONT fit into each other by a matter of mm. It was easier to draw and measure them very carefully than actually make them and get it wrong. You can see that they just overlap each other.

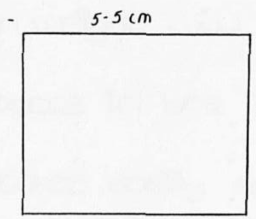
The Areas of the Shapes

The circle



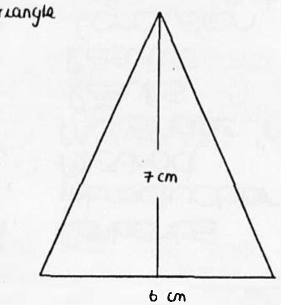
Area = 3·14 × 3 × 3
= 26·26

The Square -

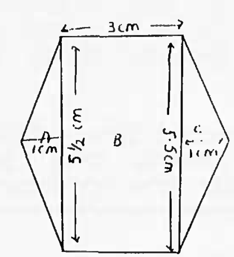


Area = L × W = 5·5 × 5·5
= 30·25

Triangle



Area = 7cm × 6 cm ÷ 2
= 21 cm²



A Area = 5½ × 1 ÷ 2
= 2·75 cm

B Area = L × W
= 3 × 5 ½
= 16·5 cm

C Area = 5·5 × 1 ÷ 2
= 2·75 cm

The same as area A.

G1/3

PACKAGING

By –

Rachel Harris

C1

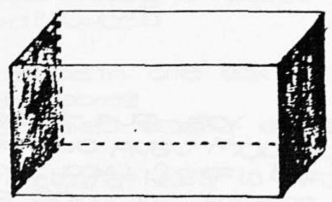
TENNIS BALLS

Contents!

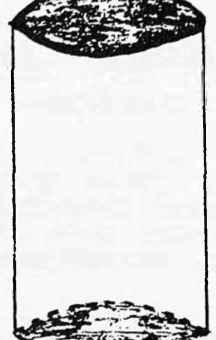
Packaging. Pg2

Introduction.

A sports company have decided to produce a box for three tennis balls.. They have decided on two designs:



Cuboid



Cylindrical

Problem

In what ever way possible I had to find out which box would hold the tennis balls the best.

Pg3

METHOD

The first thing that I did was to draw up a design criteria to see which box would be the best:

Criteria:

1. How much material is needed and what is it's cost?
2. How much weight will the box be able to stand?
3. Is the space in the box made good use of?
4. Are the boxes easily stored?
5. Is the container easy to make/assemble
6. when in the container, are the balls easily accessable

when I had decided what to put in my criteria, I had to decide on how to work out which was the best for each criterion. I decided to do one at a time.

1, I found out all the measurements of a tennisball and used these measurements to draw up two diagrams. one of the cuboid and one of the cylinder. I measured them both up and found the NET, to see which was best. (See fig 1 for an example.)

2, I made the 2 boxes in 3.D, put in the tennis balls and looked to see if 1 was better at holding the tennis balls than the other (Fig 1½)

My Shape! Pg4.

When we had found out which box was the best we had the choice to make and design our own box with up to 5 balls to hold. My box was T-shaped box (see fig 4.)

It took me quite a long time to decide on what to do so to find it easier I wrote down a few questions.

1, Will it be different?

2, Will it stack easy?

3, Will it be quick to make?

So in the end I chose the T-shaped box because this answered the questions well.

The next problem was to see how to make the box, the first thing that I did was to draw the box in 3-D (see fig 5.) When I had done this I tried to make it up but the first t[ime] I did this I went wrong! (Fig 6)

If you look at Fig 4 you will realise what I did wrong. I forgot about the top box and it will not fold at right angles unless you cut along the dashed red lines. Looking at the first box I [(]Fig 6) I realised that I had drawn 1 to many boxes on the top and I had also not put the two side pieces on. I drew it up in a box again and this time it worked (fig 7).

When I had done this I decided to make it full size so first I did it in paper, the piece of paper wasn't big enough which was a problem but I decided to miss off a tab on the side (Fig 8)

After doing this I made a big card board version but this was taken out of the maths room.

3/ All that I did for this one was to look at the tennis balls in the box to see if there was much of a gap or not between the balls.

4, This was the easiest because I just thought about the boxes being on a shelf and seeing which would stack the best when on the shelf. (see fig 2) (+ fig 3)

5, When I was making the box for number 2 I could see which was the easiest to make because one was made quicker than the other to make.

6, All I did for this one was to see which can be got at easiest when the balls are in the box. I just put the balls in the box and then worked out which would come out of the box the easiest.

Pg 5

Results.

Criteria

1, How much material is needed and what is it's cost? Cuboid

2, How much weight will the box be able to stand? Cuboid

3, Is the space in the box made good use of? Both the same

4, Are the boxes easily stored? Cuboid

5, Is the container easy to make / assemble? Cuboid

6, When in the container, are the balls easily accessable? Cuboid.

Measurements of a tennis ball:

Diameter of 1 Tennis Ball = 6·2cm.

Radius of 1 Tennis Ball = 3·1 cm.

Diameter of 3 Tennis Balls = 18·6cm.

Circumference of 1 Tennis Ball = 6·2 x 3·14 (π) = 19

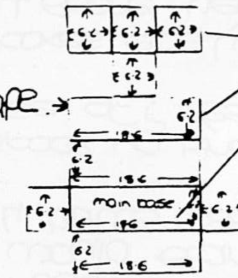
Doing this criteria helped me to find out which box was the best.

The best box was the Cuboid.

My own box.

The only main problem was that the box wouldn't fit on the piece of paper. I worked out this. like this

My Shape →



measure ments of box whe flat.

Length of sheet of paper = 42·1cm

Length of T-shaped box = 6·2 37·1cm

$$\begin{array}{r} 6·2 \\ 6·2 \\ 6·2 \\ 6·2 \\ 6·2 \\ +6·2 \\ \hline 37·2 \end{array}$$

The model is long enough to put on the paper.

The widths of the paper = 29·8cm

The width of the T-Shape = 18·6 31/0

$$\begin{array}{r} 18·6 \\ 6·2 \\ +6·2 \\ \hline 31·0 \end{array}$$

The model is to wide to fit on the paper.

My box =

Area 1 = 18·6 x 24·8 = 461·28 + 76·88 = 538·16 } main Base.

Area 2 = 6·2x6·2 = 38·44 } sides of box

x2 = 76·88

Pg6.

RESULTS:

Area3 - 6.2×6.2 = 38.44 } 1 square

Area4 = 38.44×3 = 115.32 } 4 squares.

Whole area of Net = 615.04 + 115.32 = 730.36

$$\begin{array}{r} 615.04 \\ +\ 115.32 \\ \hline 730.36 \end{array}$$

Cuboids Net = 538.16.

Cylinders Net = 466.1

Pg7.

Conclusion

After having done all of that work I had to pick 1 to be the best I think that I would chose my box not because I built it or because I thought of it but because it suits all the criteria I did in the beginning quite well:

1, How much material is needed and what is it's cost?
If you had made the cylinder to fit four balls then it would have probably been more than the T-shape and the cuboid would have been but I dont think there would have been much between the cuboid and the T-shape. So the T-shape wouldn't have cost as much.

2, How much will the box be able to stand?
My box would be very sturdy and it would stand very well holding 4 balls with out bursting

3, Is the space in the box made good use of? Yes.

4, Are the boxes easily stored?
They will store very well, just as good as the cuboid because once the T-shape box is placed

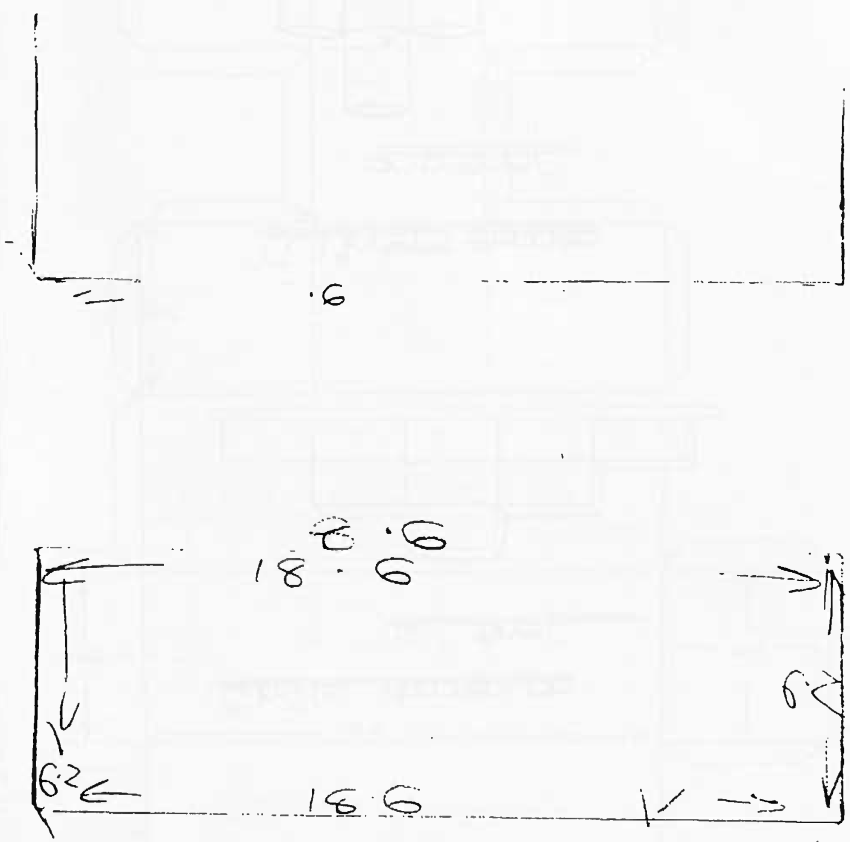
up-side-down it will tesselate and stack verywell.

5, Is the container easy to make base be? The box I made was the quickest to make so surly it will be the easiest to assemble

6, When in the container are the balls easily accessable? When the balls are in the box and you open the flap they come out very easily and when in the box they are very snug

If I could change any thing in my design I would put the lid at the top and make the box about 2mm bigger so that they don't fit as snug. I would also plan out what I am doing more because I rushed the last part of the assessment a lot so that I got it done.

Fig 7½



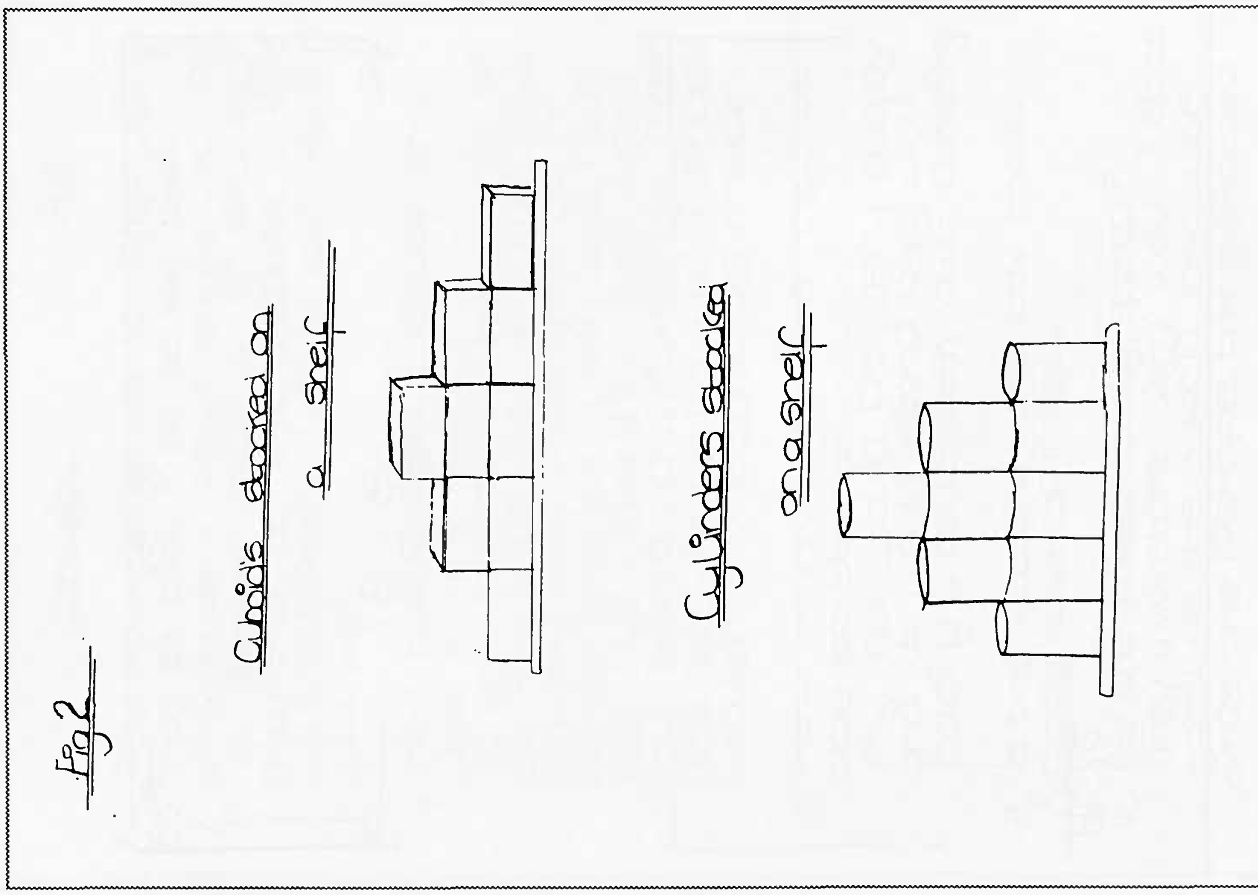


Fig 2
Cuboids stacked on a shelf
Cylinders stacked on a shelf

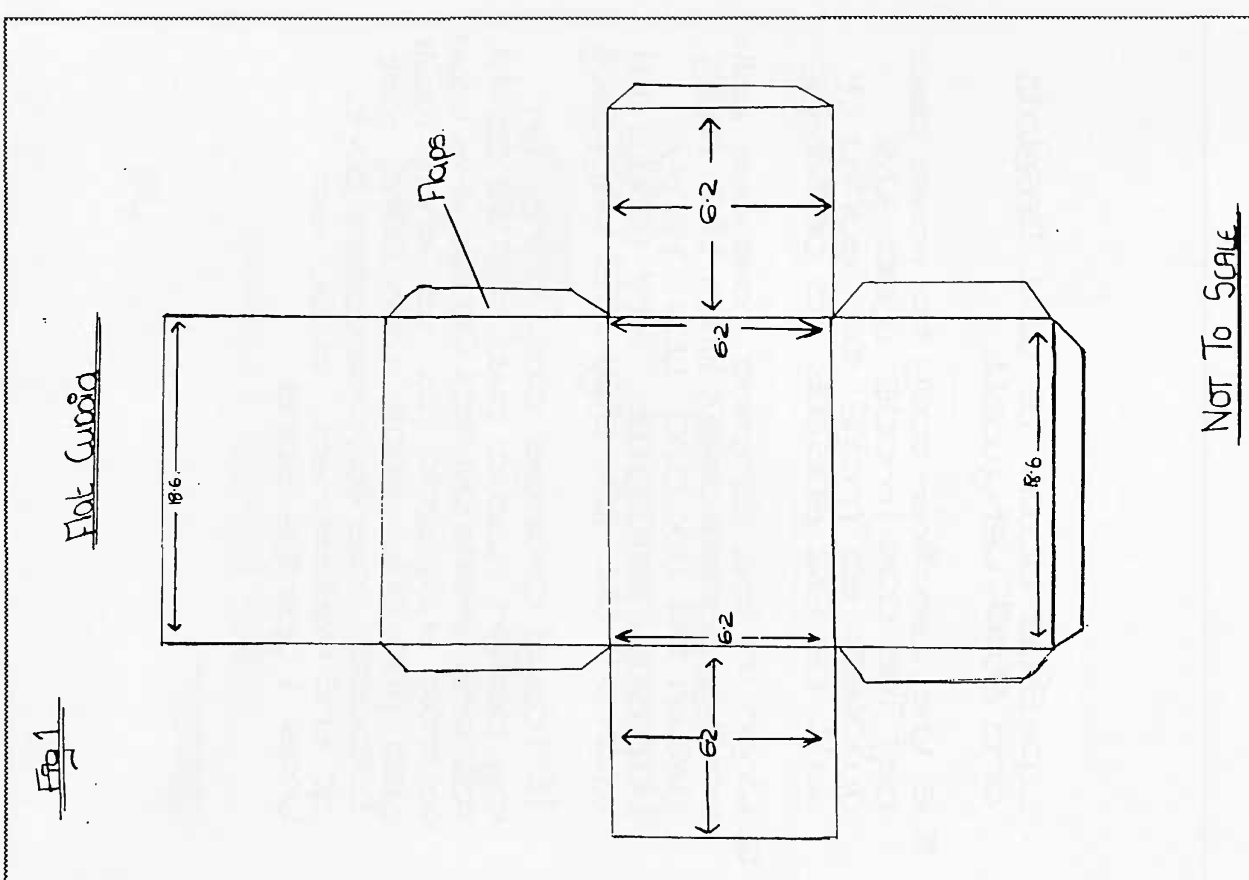


Fig 1
Flat Cuboid
Flaps
6·2
6·2
6·2
6·2
18·6
18·6
Not To Scale

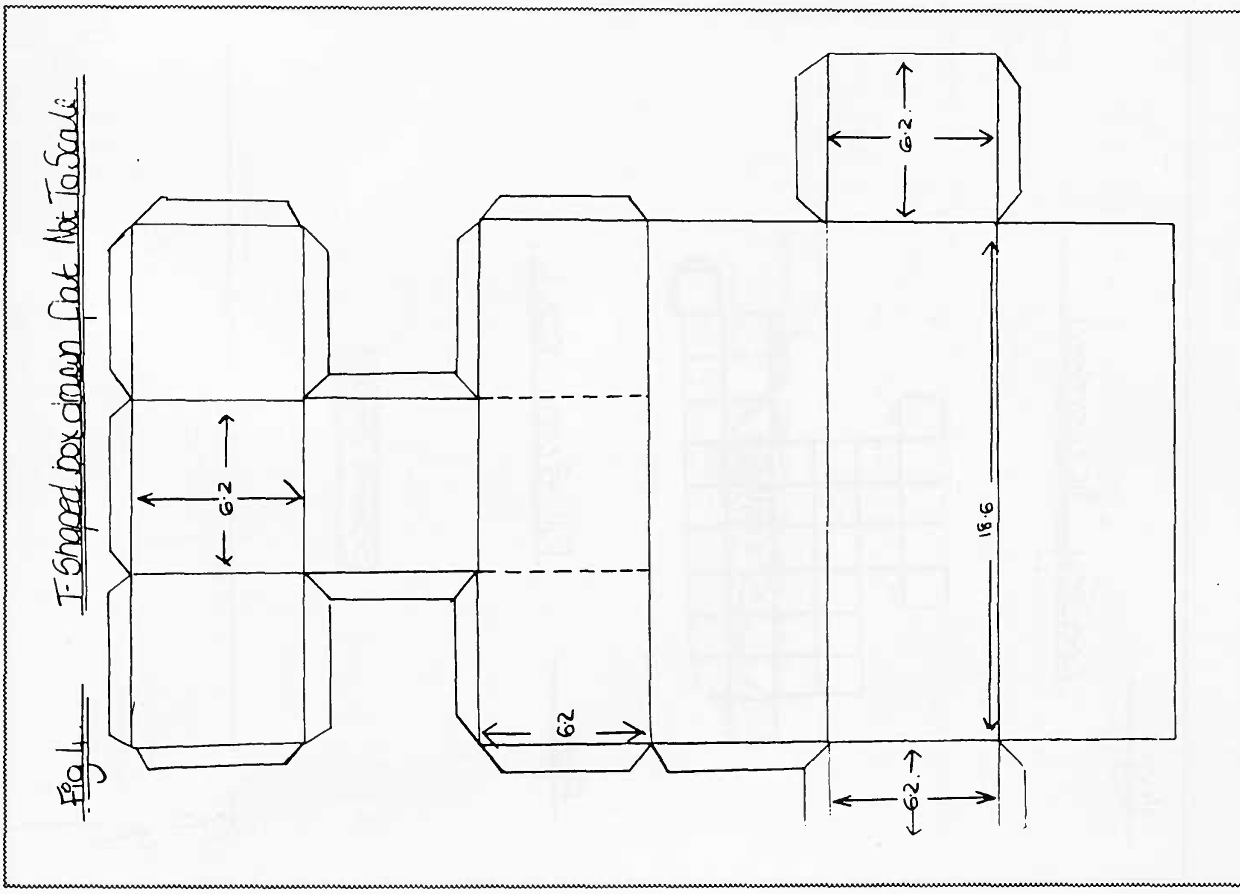


Fig 1
T-Shaped box drawn Out Not To Scale
6·2
6·2
6·2
6·2
18·6

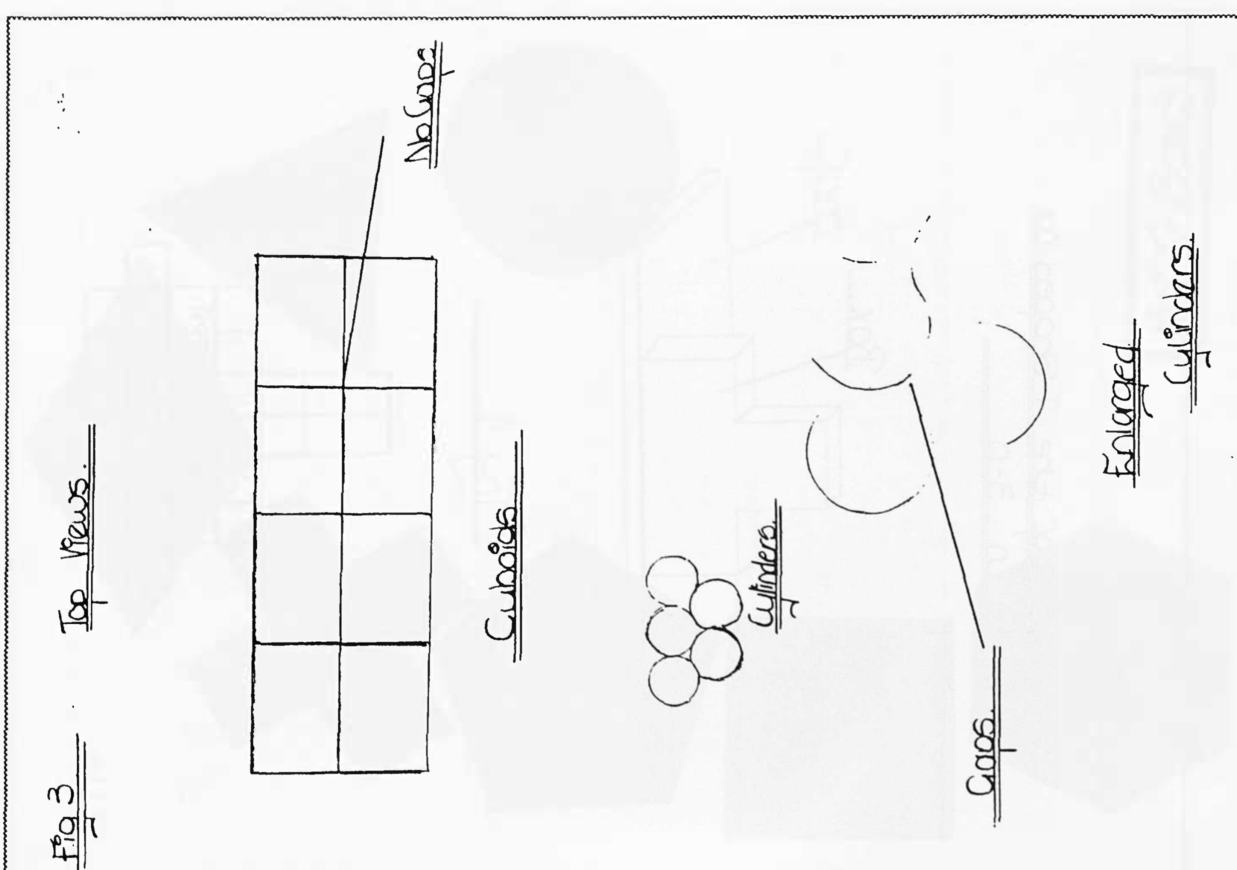


Fig 3
Top Views.
No Gaps
Cuboids
Cylinders
Gaps
Enlarged Cylinders

Fig 5

Drawing of the T-Shaped box in 3-D

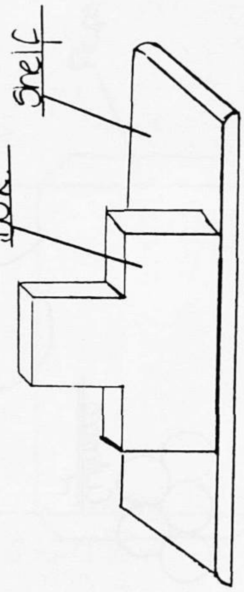


Fig 6

Try 1

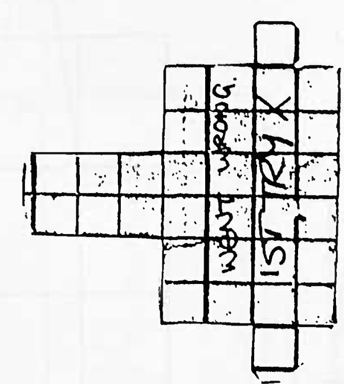


Fig 7

2nd Try, "it worked!"

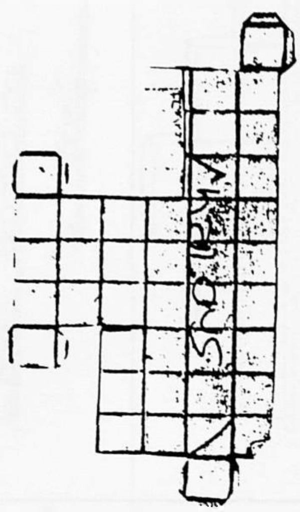
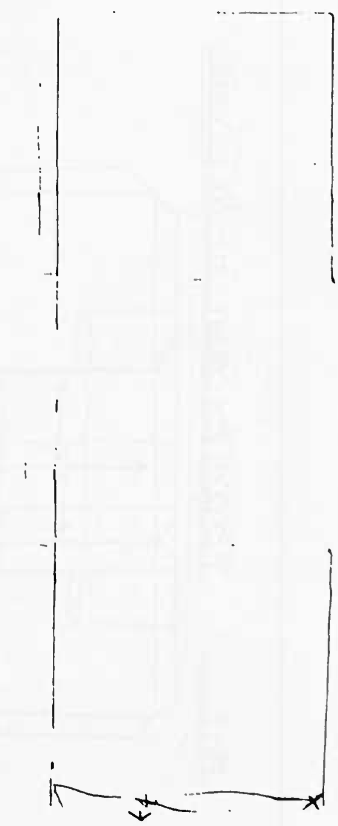
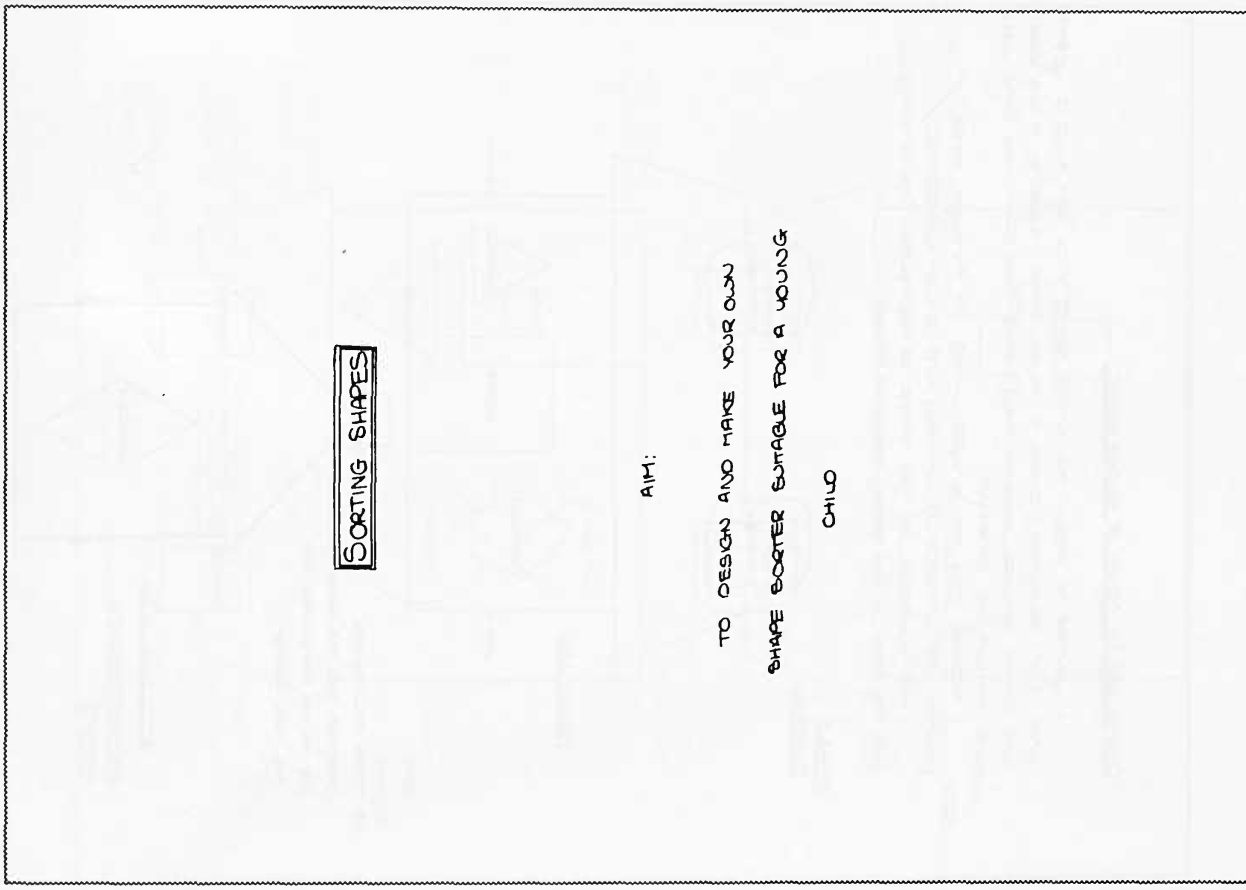
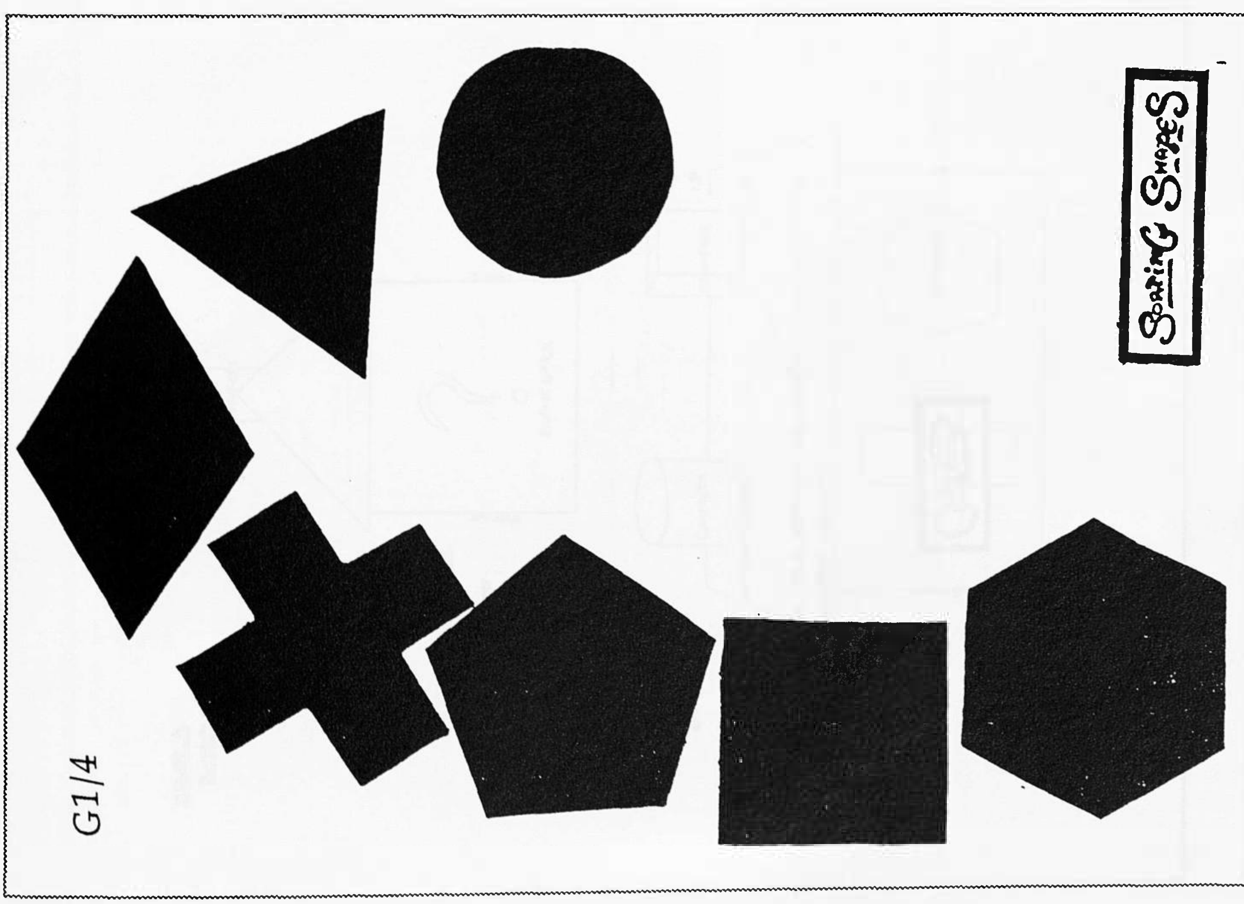


Fig 8

Full sized box!




SORTING SHAPES
AIM:
TO DESIGN AND MAKE YOUR OWN
SHAPE SORTER SUITABLE FOR A YOUNG
CHILD


SORTING SHAPES

G1/4

Contents

Original idea for nature of Shape Sorter:

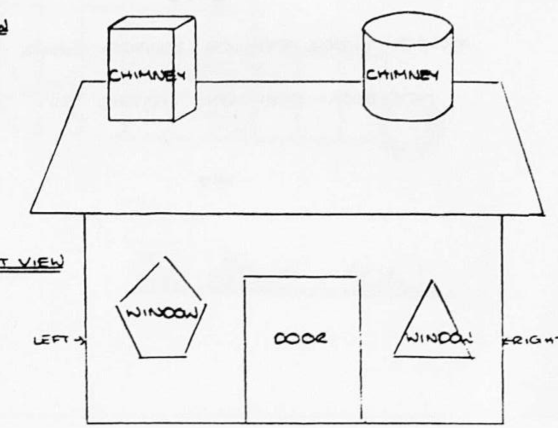
I wanted to design the shape sorter in a form that a young child can recognise. I chose a house, after rejecting a big wheel and animal shapes because they would have been too complicated and difficult to construct.

I decided that the shapes would fit through holes in the position of windows, a chimney and one or two doors.

I had two ideas for the shape of the house. One, a bungalow and the other, a two storey detached house.

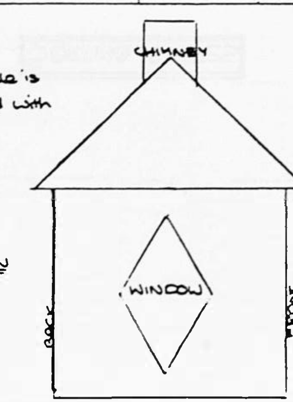
HOUSE A
BUNGALOW

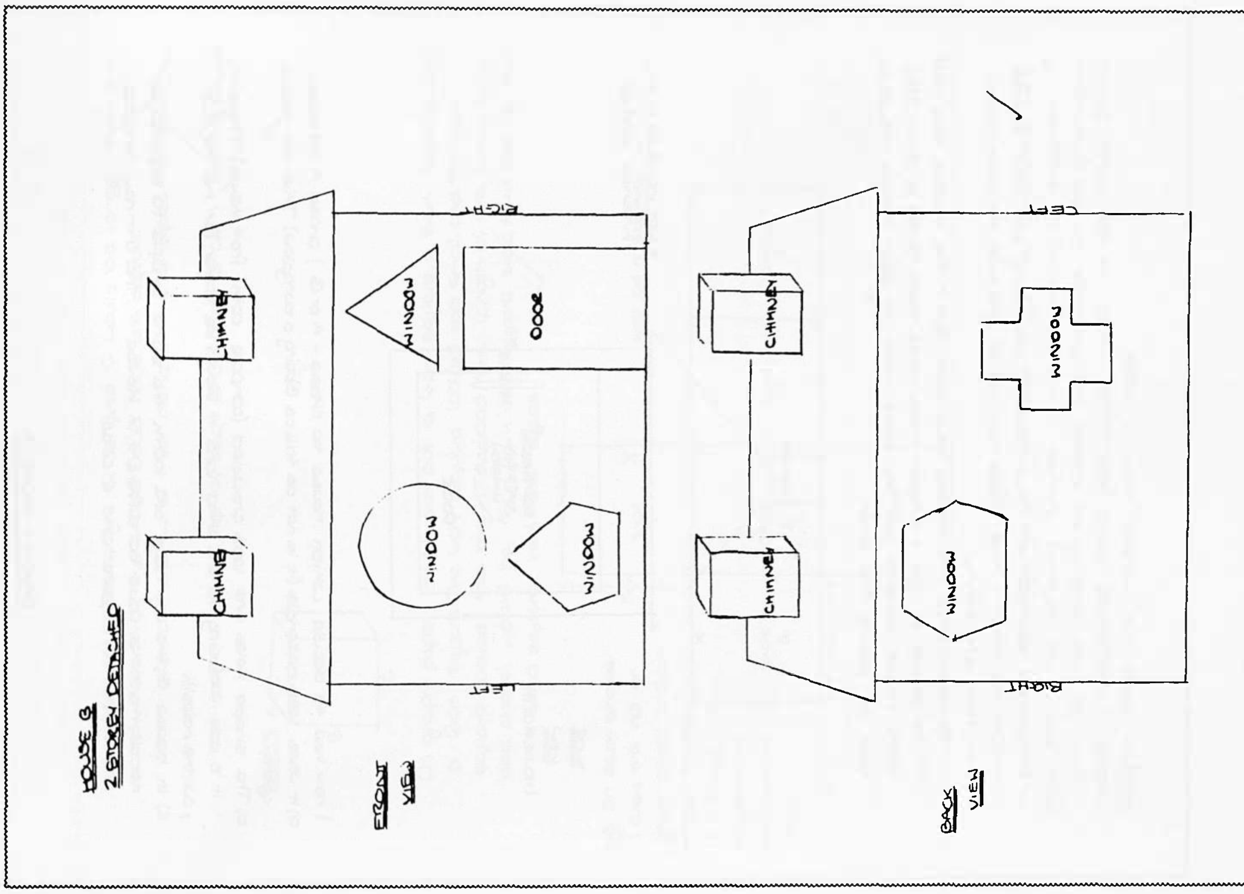
FRONT VIEW

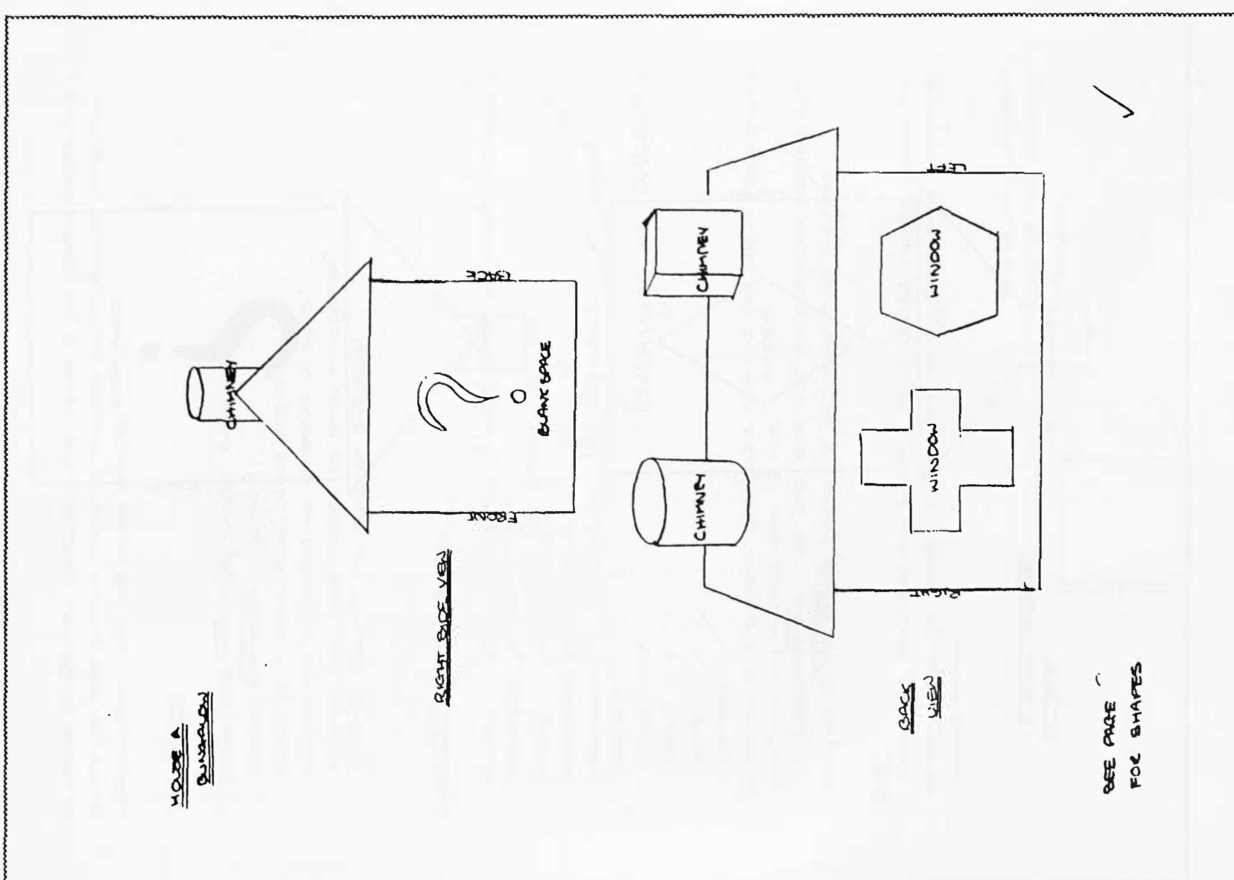


NB: These drawings are sketches, and the scale is not to be compared with the real design.

LEFT SIDE VIEW

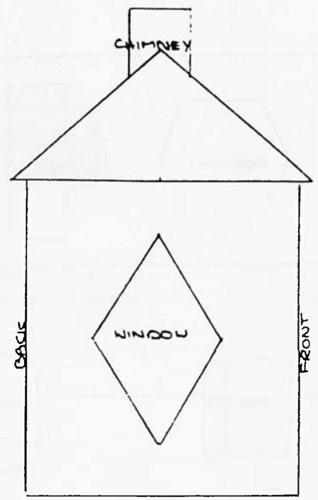



HOUSE B
2 STOREY DETACHED
FRONT VIEW
CHIMNEY
CHIMNEY
WINDOW
WINDOW
WINDOW
DOOR
LEFT
RIGHT
BACK VIEW
CHIMNEY
CHIMNEY
WINDOW
WINDOW
LEFT
RIGHT

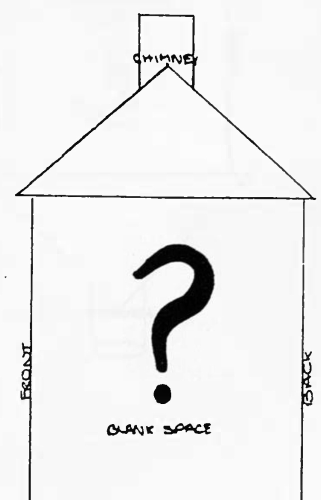

HOUSE A
BUNGALOW
RIGHT SIDE VIEW
CHIMNEY
BLANK SPACE
FRONT
BACK VIEW
CHIMNEY
CHIMNEY
WINDOW
WINDOW
LEFT
RIGHT
SEE PAGE FOR SHAPES

HOUSE B
2 STOREY DETACHED

LEFT SIDE VIEW



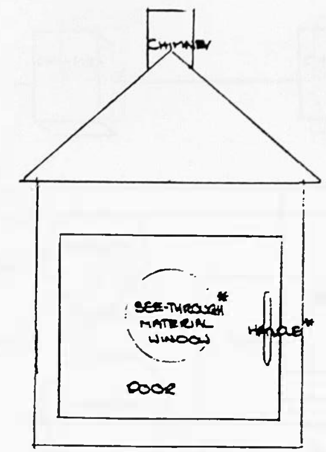
RIGHT SIDE VIEW



As you will see later on, I had chosen how many and what shapes I would use. When I had fitted them all on, one side on each house had been left blank.

Obviously, once the shapes have been put in the house they need to be allowed to get out again: there would have to be a door that opens to let shapes out. The blank walls on each house would be ideal for placing this door.

SIDE VIEW (HOUSE A OR B)



* Not included in the finished sorter

I now had to decide which house to choose – A or B. I chose A because
a) It uses less materials (it is not as tall as B being a bungalow) This will reduce costs.
b) The shape holes are less crowded (compare both front views). Therefore it is less confusing to the child (and it would be easier for me to cut out the holes!)
c) In house B, the chimneys are both the same shape (a square) so not all the holes would be different. In house A, the chimney shapes are different (a square and a circle)

DESIGN : HOUSE A

All I have to do now concerning the house is the measurements, but I cannot do this without knowing the size of the shapes that I will use, so now I will design the shapes

THE SHAPES

a) What, and how many shapes shall I do?
b) What size shall I do them?
c) How do I draw them? (What angles etc.)
d) What net is needed to construct a solid?
e) Where are flaps needed to stick it together?
f) What colour? (The same or different)

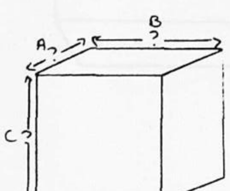
A List Of Possible Shapes:

SQUARE ✓
RECTANGLE ✓
CIRCLE ✓
PENTAGON ✓
HEXAGON ✓
TRIANGLE ✓
CROSS ✓
OVAL ×
STAR ×
DIAMOND ✓

Are there the correct "names" for solid objects? ie Square or cube? Rectangle or cuboid?

On house A, B shapes are used. From the list above I selected & and those are marked with a tick.
I discarded the oval and star for no particular reason, but they would be slightly harder to construct than the rest.

SIZE:
Considering that this toy is for a child whose hands would not be very large, the shapes cannot be so big that they cannot be grasped easily.

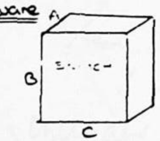


This square represents any of the selected shapes

9

I decided that the length of B and C would be longer than A - would mean that the shape would have less depth.
Also I had to make sure that none of the shapes could fit in each other.
Now I had to do the net of each shape. First of all I did the square

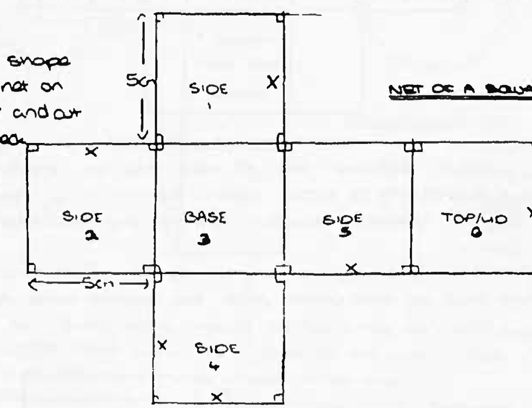
Square



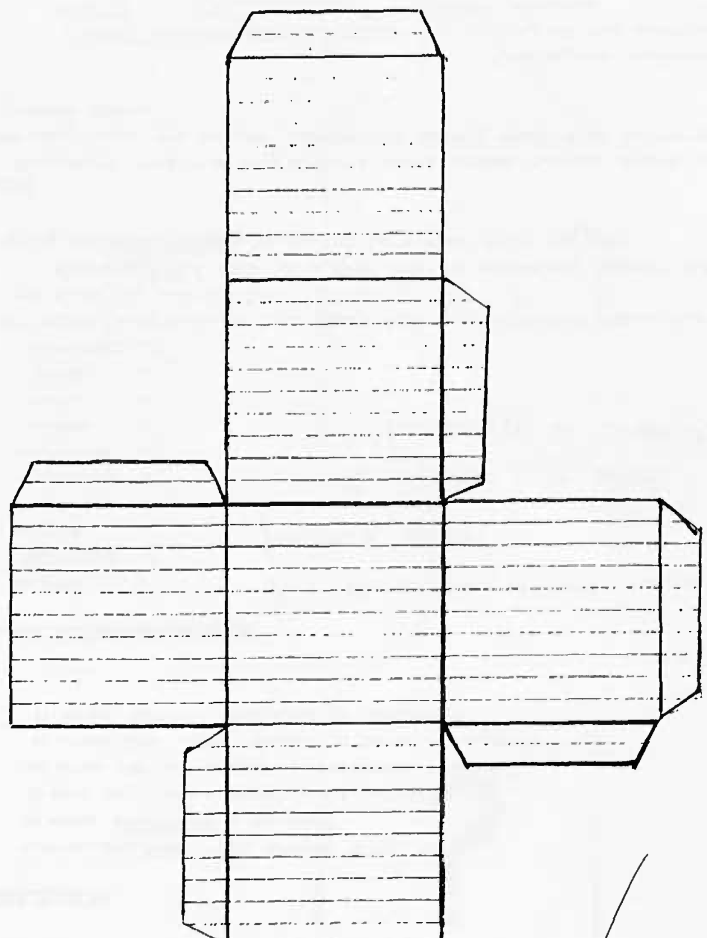
On graph paper, I tried out out a 4cm square, a 5cm square, and a 6cm square. The 4cm square looked too small, and the 6cm square looked too big. I decided the length of the sides B+C (see above) would be 5cm. As it is a square side A will also be 5cm
A solid square has six sides.

NB. On each shape I drew the net on graph paper and cut it out to check if it fitted together. In all cases it did.

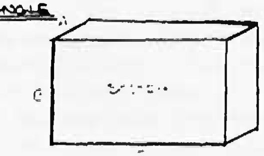
NET OF A SQUARE (SKETCH)



In this case, the shape would look better if all the dimensions were equal, but in the following shapes I have not made an exception.
Now all that have to be added are the flaps. On the graph paper model I discovered where joins would have to be made. On the diagram these are marked with a cross.



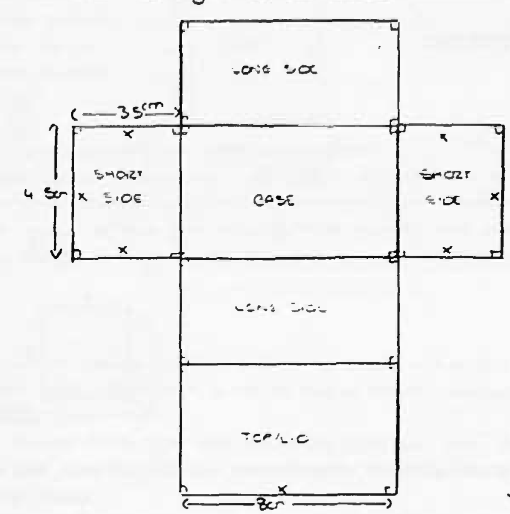
RECTANGLE



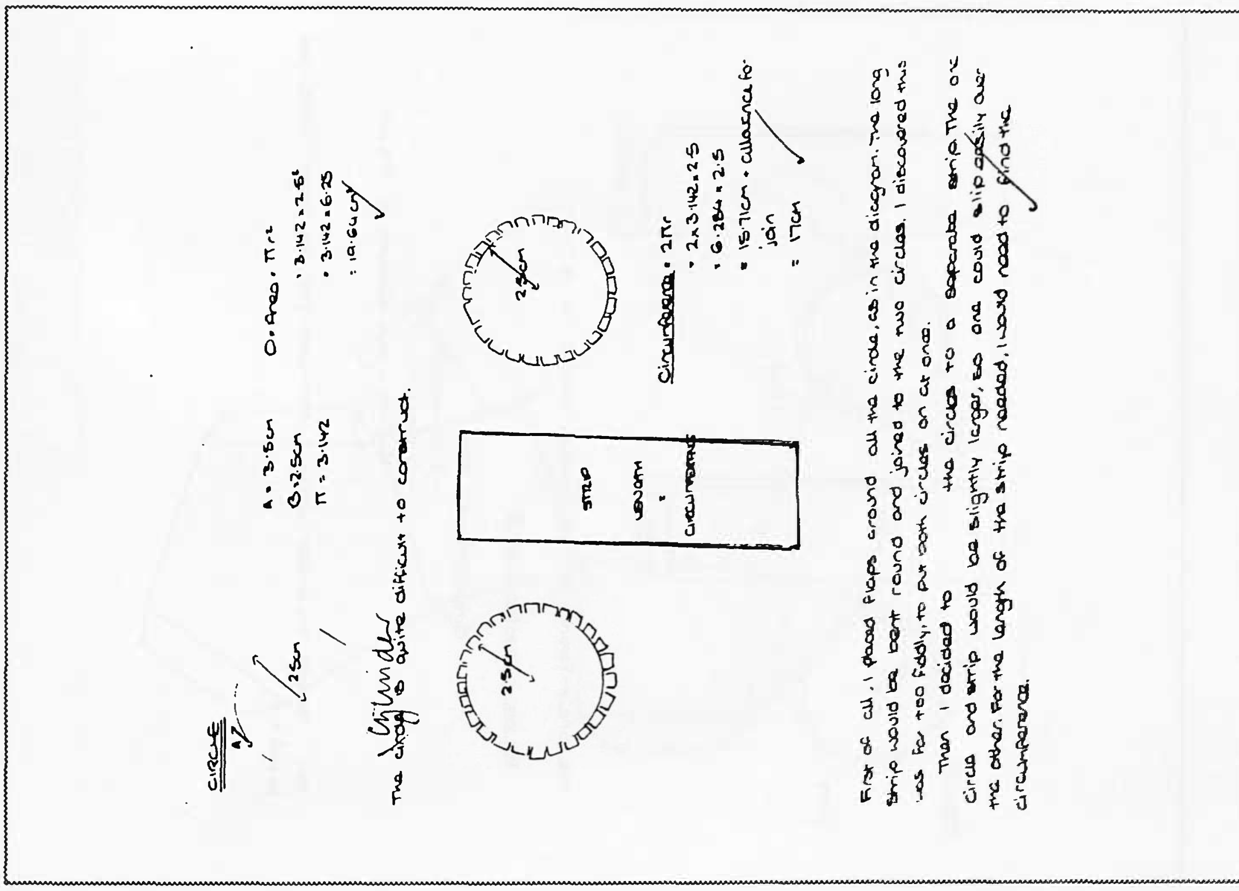
To keep each shape so that each one fit into only one hole, I drew the length of B ½ cm smaller than the square to place this.

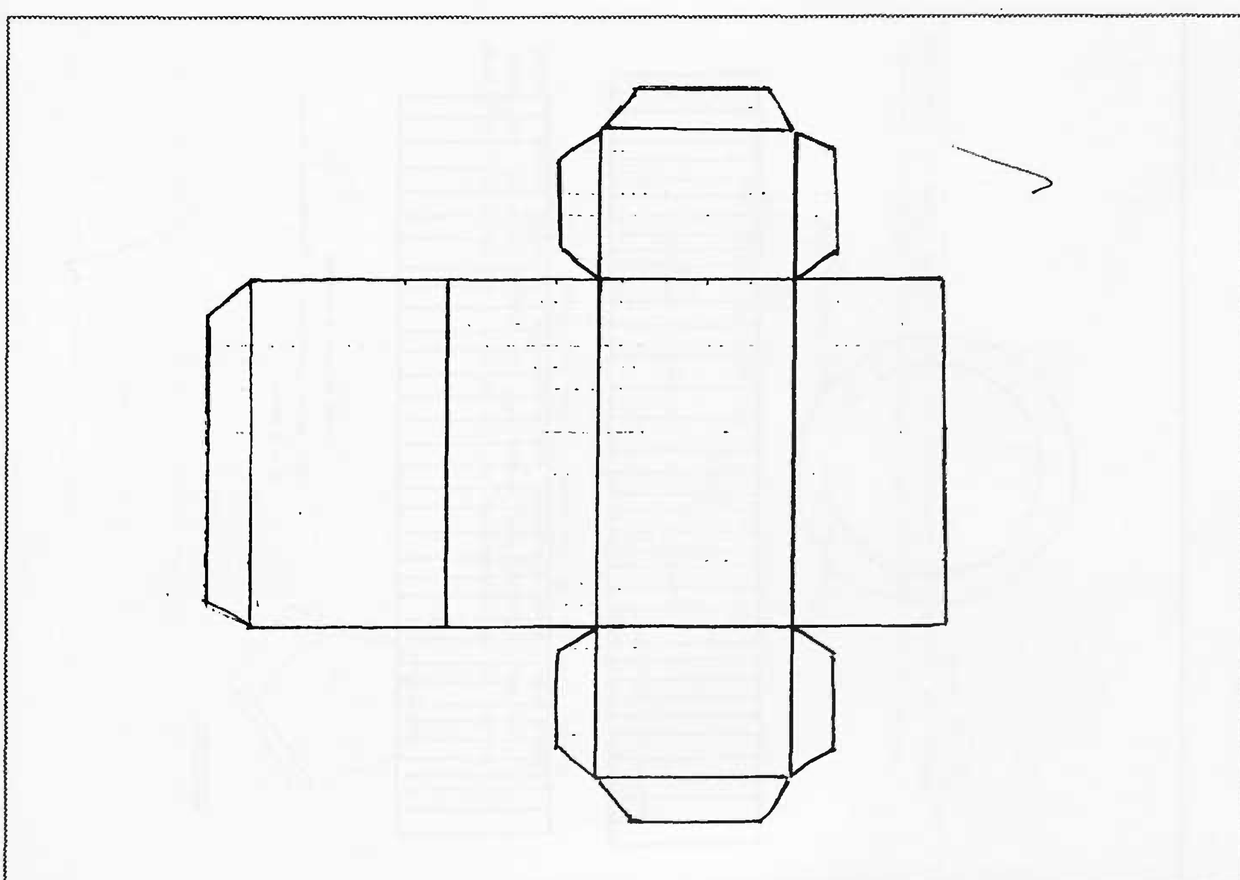
To separate the square and the rectangle as two different shapes in a childs mind I made C quite long. (8cm) I made A, as throughout all the rest of the shapes, 3·5cm.

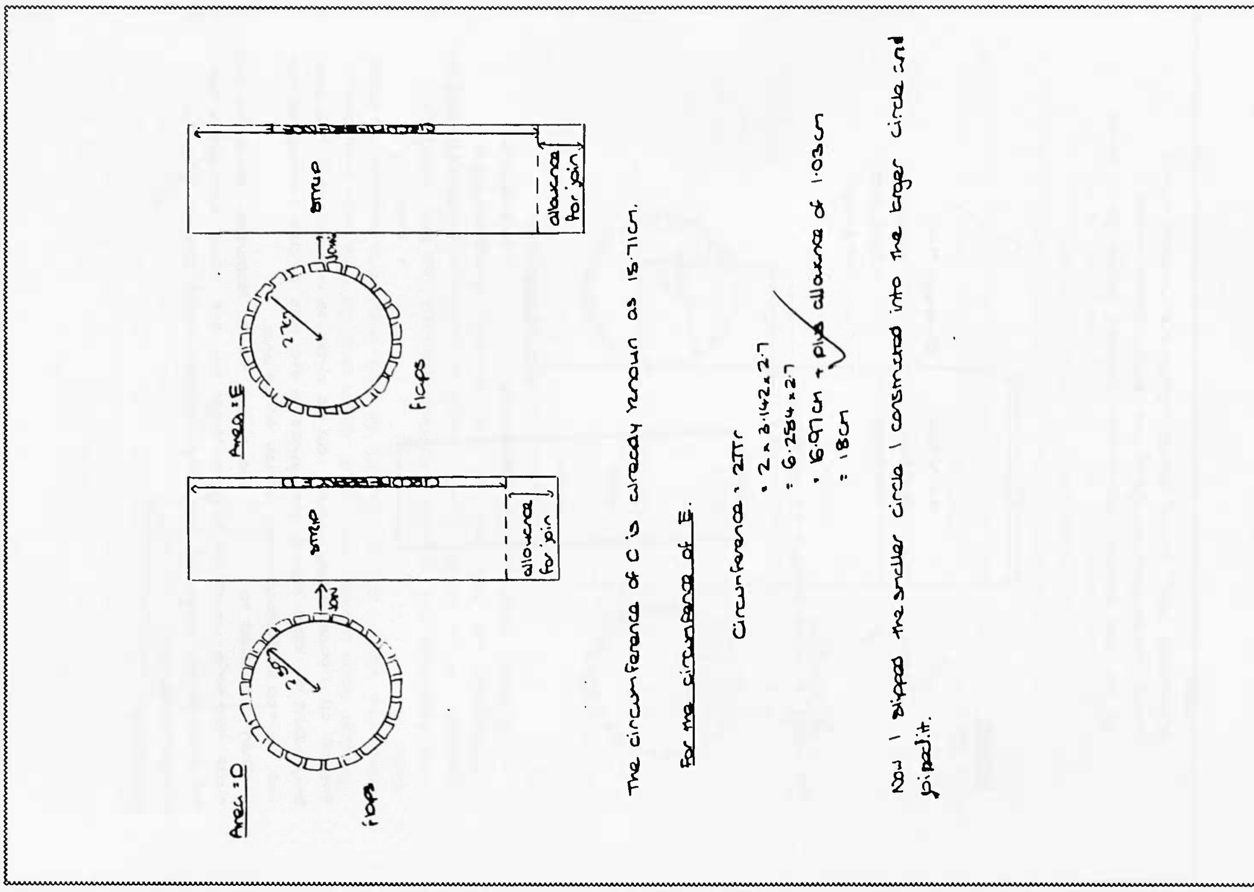
A solid rectangle has six sides:

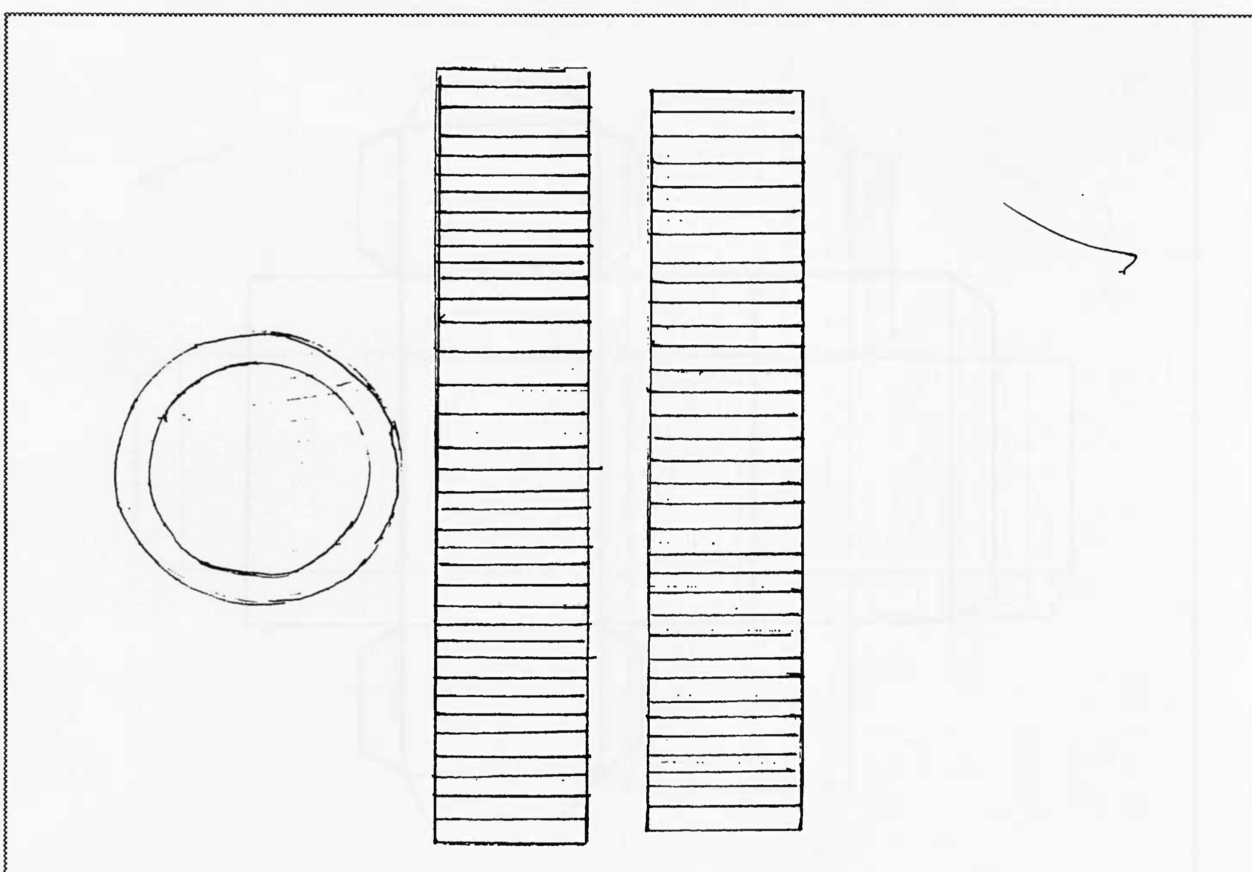


As for the square, the shape needed flaps for it to be easily joined together. Using the graph paper model, I discovered the joining points, which are marked with a cross.

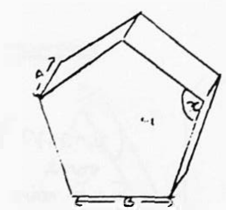

CIRCLE
A = 3·5cm
B = 2·5cm
π = 3·142
O Area = πr²
= 3·142 × 2·5²
= 3·142 × 6·25
= 19·64cm
2·5cm
Cylinder
The circle is quite difficult to construct.
2·5cm
STRIP
LENGTH
=
CIRCUMFERENCE
2·5cm
Circumference = 2πr
= 2×3·142×2·5
= 6·284 × 2·5
= 15·71cm + allowance for join
= 17cm
First of all, I placed flaps around all the circle, as in the diagram. The long strip would be bent round and joined to the two circles. I discovered this was far too fiddly, to put both circles on at once.
Then I decided to ... the circles to a separate strip. The one circle and strip would be slightly larger, so one could slip easily over the other. For the length of the strip needed, I would need to find the circumference.




Area: D
flaps
strip
allowance for join
Area: E
flaps
strip
allowance for join
The circumference of C is already known as 15.71cm.
For the circumference of E.
Circumference = 2πr
= 2 x 3.142 x 2.7
= 6.284 x 2.7
= 16.97cm + plus allowance of 1.03cm
= 18cm
Now I slipped the smaller circle I constructed into the larger circle and piped it.



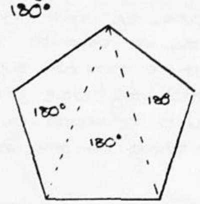
PENTAGON



A = 3·5cm
B = 8cm - This was in proportion to the rest of the shapes

I decided on the measurements for A + B, but I did not know what angle C was. So I divided the shape into 3 triangles, each of 180°* (see below) when I had calculated the angles for the whole shape, I could divide this by 5 for each of the 5 angles.

* The angles of a triangle always add up to 180°

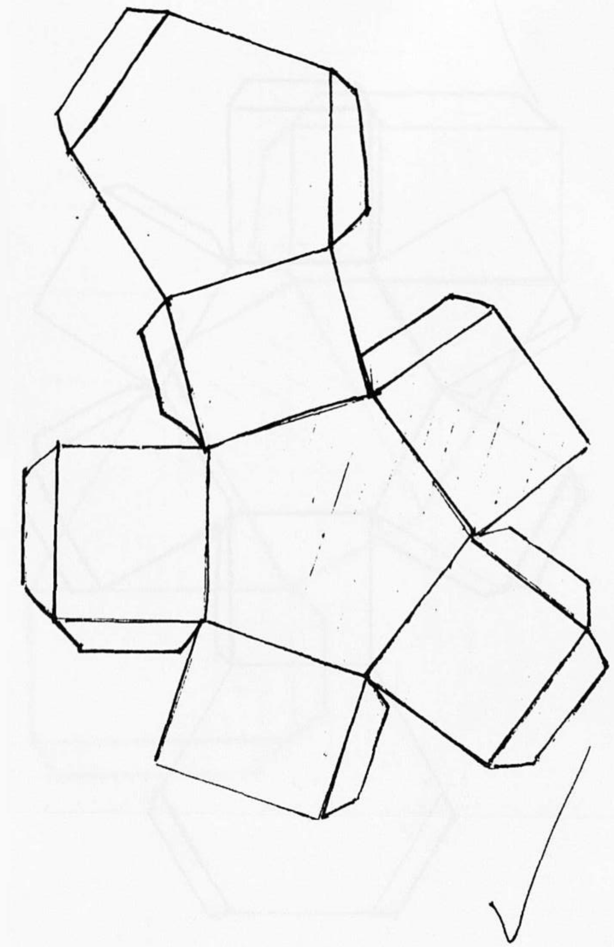


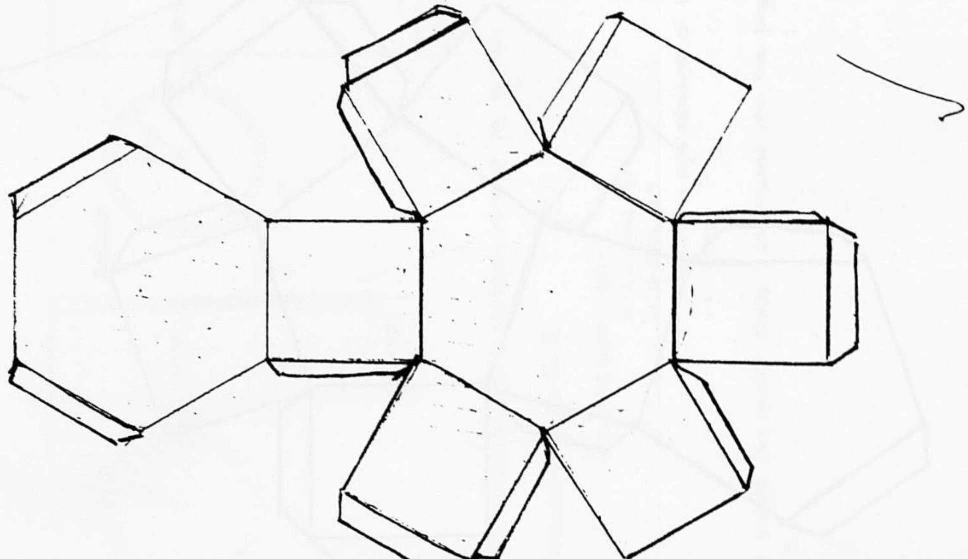
angles = 180° × 3
= 540°

540° ÷ 5 angles (c)
C = $\frac{540}{5}$ = 108°

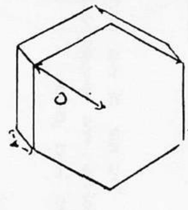
<u>Each angle = 108°</u>

The net of the pentagon is shown over the page for lack of space. As before, the shape was constructed on graph paper first to check and to find joins.



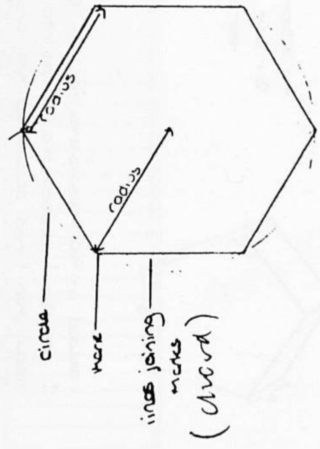


HEXAGON



A = 3·5 cm
B = 3·3 cm

I chose A so that the shape would be in proportion to the others. The easiest way to construct a hexagon is to draw a circle, in this case with a radius of 3·3cm. Then place the point of the compass on the circumference line and make a mark on the same line. Then place the point of the compass on the mark and draw another mark. Do the same until the marks are round the circle. You should have 6 marks for the six points of the hexagon. Make sure when you are drawing the marks you do not alter the radius. Then join up the marks.

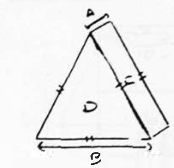


Each line joining the marks is the radius.

The net of the hexagon is shown over the page for lack of space

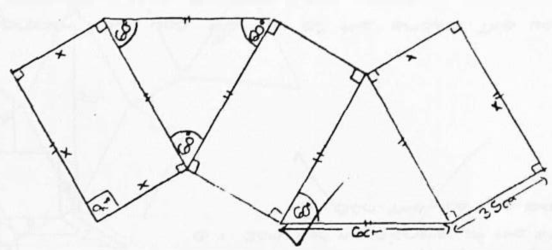
As before, the shape was constructed on graph paper first to check and then to find the areas needing joins.

TRIANGLE

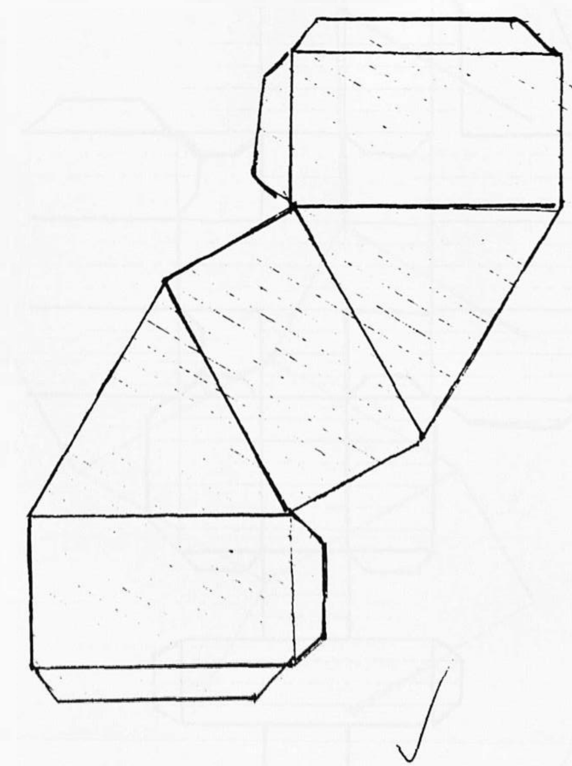


A = 3·5cm as on other shapes
B = 6cm – the smaller sizes were too small.

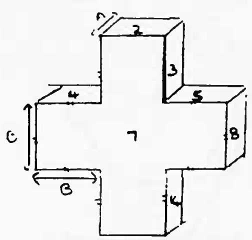
This triangle is an equilateral triangle, so each side and each angle are the same.
If a whole triangle equals 180°, and there are 3 angles, so each angle = 180 ÷ 3 = 60° each.
To construct the triangle I used a protractor.
A solid triangle has 5 sides 3 of them are 3·5 × 6cm (marked C etc) and 2 are D surfaces.



Now all I had to do ~~now~~ was add the flaps. This was checked with a graph paper model. The flaps are joined marked with a cross



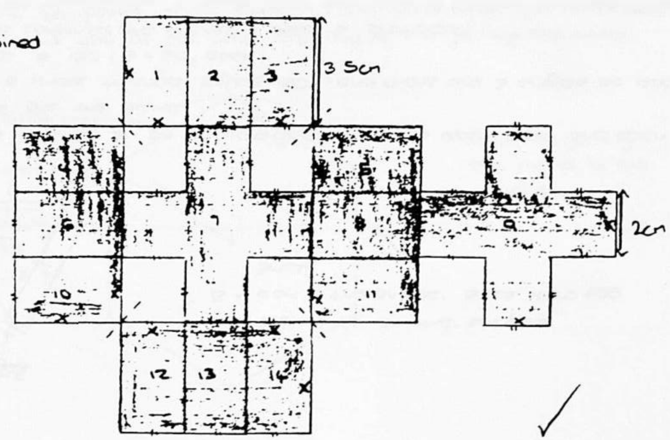
Cross



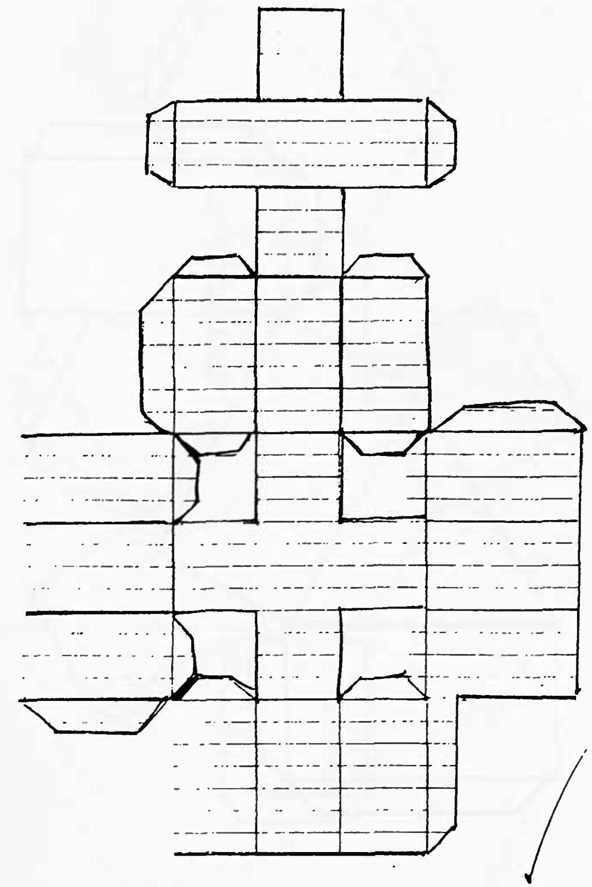
A = 3.5cm (as before)
B = 2cm - so the diameter of the shape is 6cm. This was the best size

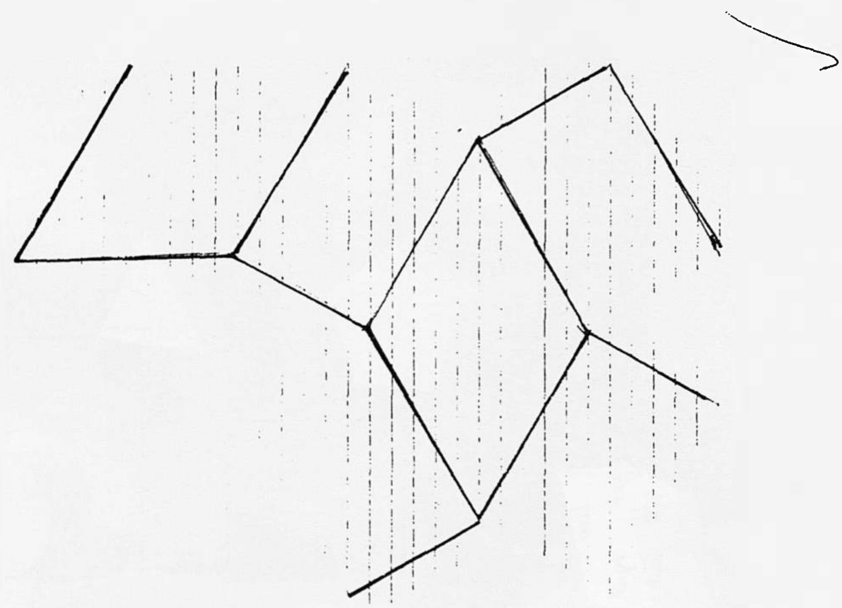
The only problem now was the net of the shape. This was very complicated, especially when the flaps were added.
A cross in solid form has 14 sides.

- - - - not joined

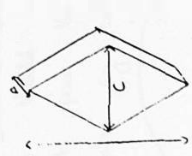


For the flaps, a graph paper model was made, and the joins were found.



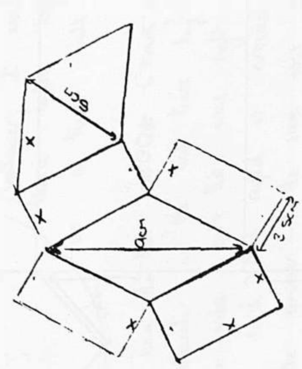


DIAMOND

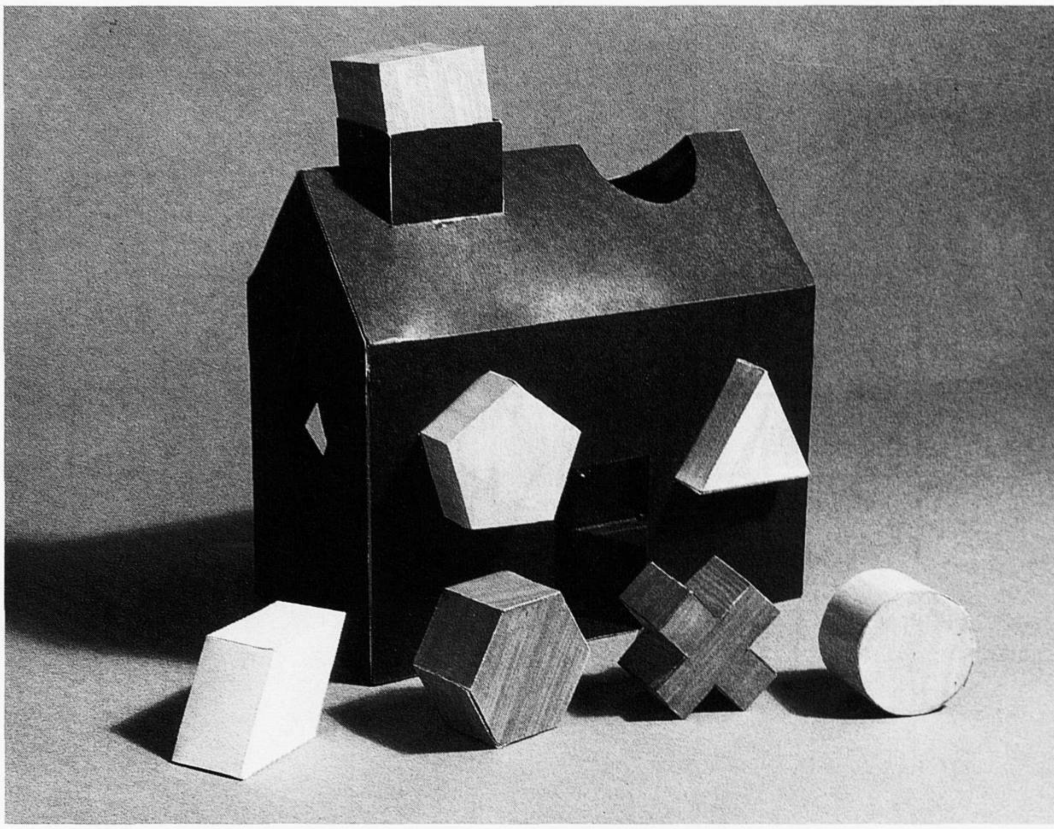


A : 3.5cm
B : 9cm } best size in proportion to rest of
C : 5cm } shapes

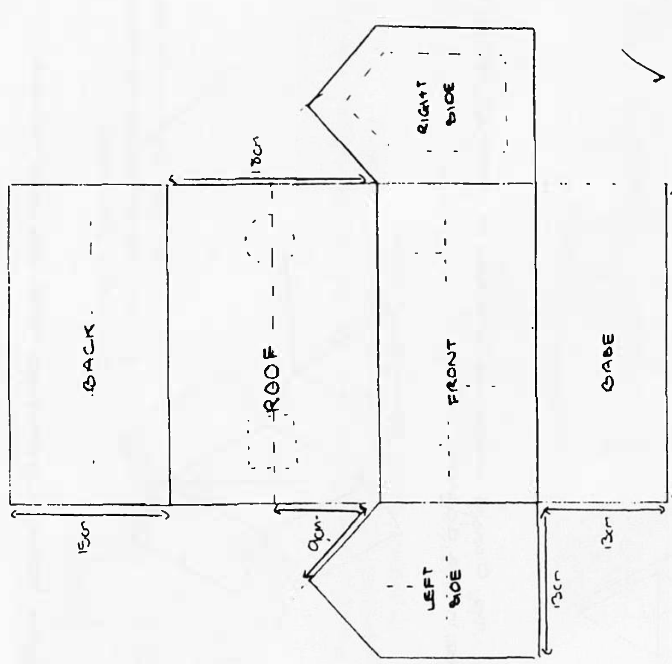
The Diamond shape was fairly easy to construct, as you can see in the diagram.



As before, a graph paper model was used to check.



Now all the shapes are done, all I have to do now is design the net of the house and determine the size.



These measurements were found to suit best the size of the shapes.

For the holes for the shapes to go through, the holes were made also 2 or 3mm² bigger than the area, so that the child would have no trouble getting the shapes through.

<u>CONCLUSION</u> - AS YOU WILL SEE WHEN YOU TRY OUT THE MODEL, TO MAKE SURE THAT THE SHAPES WOULD FIT THROUGH EASILY, I MADE THE HOLES SLIGHTLY BIGGER. THIS ALSO ALLOWED SHAPES TO FIT IN MORE THAN ONE HOLE. ALSO, IN MAKING THE DEPTH 23CM, THE SHAPES COULD BE SIDEWAYS-TURNED AND PUSHED THROUGH THE WRONG HOLES THIS WAS ESPECIALLY TRUE OF THE RECTANGLE. I WAS UNABLE TO MAKE THE CIRCULAR CHIMNEY FOR LACK OF CARD, SO I JUST CUT A HOLE. ALSO THE DOOR WOULD HAVE BE IMPROVED.

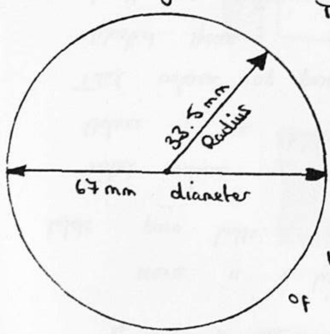
Good

Tennis balls.

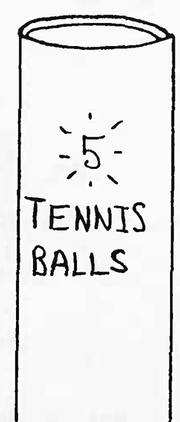
G1/5

Brief : The brief that I had to work to was, Design a container to hold 3,4 or 5 tennis balls leaving as little space as possible.

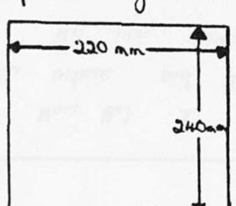
My first thought was that 3 or 4 tennis balls was to little, So I decided 5 balls was the minimum. To leave as little wasted space as possible. I thought a round container should be used because a ball is round. Next I had to find the size of a tennis ball to know how big to make my container.



This is a life size drawing of a tennis ball, it has a diameter of 67mm. My design of container is going to be a large tube with a lid on each end, I have done prototype drawing below. To build the container I need a square net 5 tennis balls high and the circumference of a tennis ball wide.



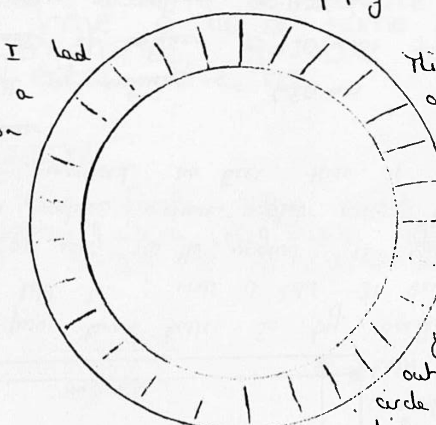
A sum can do this it is 67mm x 3.14 This is the diameter of the ball times by pie, but first to make sure this was right I cut a piece of card and raped it around a tennis ball, the answer to the sum was 210 mm, my experiment was 215 mm, I did not build a proper one yet I built one out of paper to hold three tennis balls. The net for this one was like this :-



I made it abit bigger So that I could stick it better.

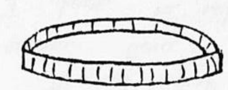
This prototype worked so far the card one I kept the width the same but made it longer to hold five balls. The Net was 240 mm by 240 mm. A problem I found when I was sticking the card into a tube shape was that it creased, so for this one I lightly stuck the ends and then filled the container with tennis balls to stop it collapsing when I stuck it properly with cellotape.

Next I had to put a bottom on It.



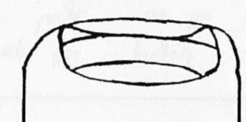
This is a scale net of the bottom and for the lid. For both you cut out a circle 83mm across and mark on it a circle 67mm across, and then you cut from the out side to the smaller circle to make a sort of windmill. Then you fold up the tags so that they fit inside one end of the container and cellotape them in place. For the lid you stick the tags together when they are folded up and this looks abit like this. I also put a handle on the top to help you pull the lid of. But then I found that if you bend the top of the container over the lid you can pick the container up with the handle.

lid



bottom

This is a picture of the container with the top folded over.



Now that I had built the container I had to find the volume and the wasted space.

This sum works out the volume of my container.

Radius × Radius × Pie × Height = Answer

3·25 × 3·25 × 3·14 × 34 = 1127·6 cm^2

This sum works out the volume of one tennis ball.

4/3 × 3·14 × $3{\cdot}25^3$ = 143·7 cm^2

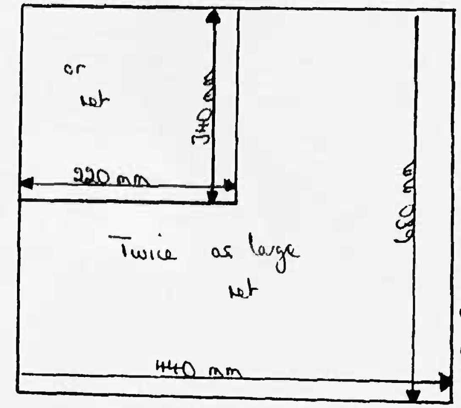
4 ÷ 3 × Pie × Radius = Answer.

Here is a list of all the data for my container that holds five balls.

Total volume of container	=	1127·6 cm^2
Volume of one ball	=	143·7 cm^2
Total volume of five balls	=	718·5 cm^2
Wasted space	=	409·1 cm^2
length of container	=	34 cm
Radius of ball	=	3.25 cm
diameter of ball	=	6·5 cm
circumference of ball	=	21 cm
Percent of wasted space	=	36%

My container had 36% wasted space inside the container which is a very small amount compared with some of the other containers. So my idea of a round container was a good idea. It does has one problem though. If I packaged lots of them for transport there would be a lot of wasted space because of the curves, other designs link together.

The next part of this project was to make another container twice as large as the original. this means that I have to make my square net twice as large.

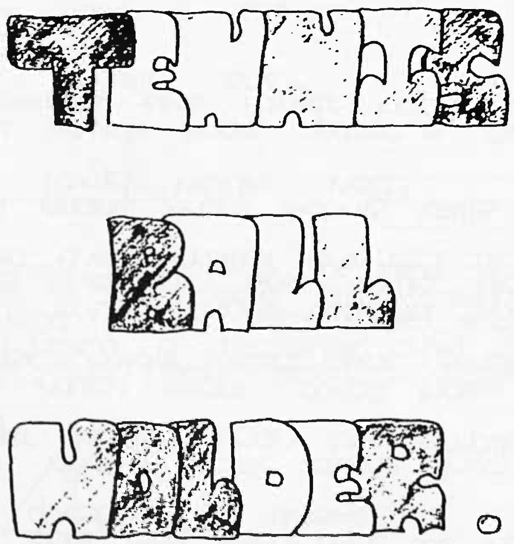


I built the bigger version out of the same thin card that the original was built out of and this was a little thin so the big one creased and did not hold its shape very well when it was crammed full of tennis balls. The original held five tennis balls so by doubling the size it should hold ten, well it held 31 tennis balls, over six times as many as the original. If you double the sizes the volume increases alot more than twice, in most cases it increased six times. Here is a list of data for the big container.

length of container	=	680 mm
diameter of container	=	130 mm
circumference of container	=	425 mm
volume of container	=	9021·2 cm^2
Total volume of balls	=	4454·7 cm^2
wasted volume	=	4567 cm^2
Percent wasted	=	50·6%
Number of balls held	=	31

There is a larger percent of space wasted in the large container.

G1/6



COURSEWORK

MAKING A TENNIS BALL HOLDER

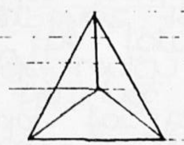
PROBLEM

The problem that we were given was to design and make a tennis ball holder. The holder had to contain either 3, 4 or 5 tennis balls. The holder couldn't be square but it had to have a lid or some other means of enterance for the tennis balls to enter the box.

SOLUTION

Our first idea that we came accross was to have the holder in the shape of a pyramid

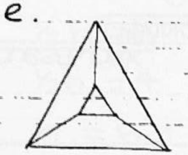
i.e.



BIRDS EYE VIEW

We tried to make this shape using paper but we could not find a suitable rance for the tennis balls to enter the holder, so instead of starting again from scratch, we folded over each top corners to get a new shape

i.e.



This new shape is called a truncated tetrahedron. It is a much better design than the original pyramid as there is a much better enterance. It has five sides.

Once the holder had been made, we had to make the same shape again but this time we had to double all of the dimensions. When this had been done, we had five questions to answer.

1. How much more volume has the larger holder than the smaller one?
2. How much more area has the larger holder than the smaller one?
3. How much more wasted space was there in the larger holder than the smaller one?
4. How many more tennis balls will the larger holder hold?
5. How much more paper is needed to make the larger holder than the smaller one?

The first thing I tried to do was to find the area of the two holders.

But before this could be done I had to number each of the five sides of both holders between 1-5.

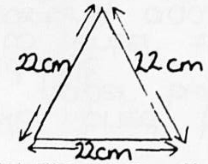
This was done to help me to find out the area by numbering the sides in order to clearly show the area of each of the five sides of the tennis ball holder.

COURSEWORK 17TH MAY 19?

MAKING A TENNIS BALL HOLDER

AREA

1.



$\frac{1}{2} b \times h$ = formula for the area of a triangle

Even though we have the formula for the area of a triangle, we do not have the height of the triangle which is required to work out the formula for the area. However, there is a formula for finding out the height, by knowing the base.

i.e. $H^2 = b^2 - \frac{1}{2} b^2$

Knowing this, we can work out the height of the triangle.

$H^2 = b^2 - \frac{1}{2} b^2$

$b^2 = 22^2$
$= 484$
$\frac{1}{2} b^2 = 121$

$484 - 121 = 363$

$363 = H^2$

$\sqrt{363} = 19$ ✓

The height of the triangle is 19cm. ✓

Knowing this we can now work out the area for side number 1 of the box.

$\frac{1}{2} b \times h$ = area of triangle.

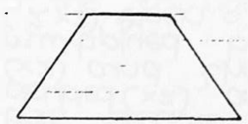
$\frac{1}{2} b = 11$

$h = 19$

$11 \times 19 = 209$

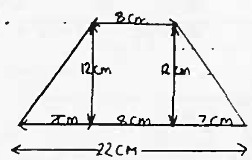
The area of the triangle (which is the base) is $209 cm^2$ ✓

2.



The only way to find the area of the above shape is by splitting it up into three sections, one being a rectangle, and the other two both being triangles.

i.e.



The rectangle $= 8 \times 12$
$= 96 cm^2$

The triangles $= \frac{1}{2} b \times h \times 2$
$= b \times h$
$= 7 \times 12$
$= 84 cm^2$

Total area for side 2 is $180 cm^2$ ✓

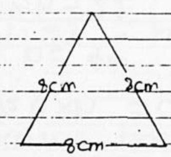
COURSEWORK

MAKING A TENNIS BALL HOLDER

AREA

Sides 3 and 4 are identically to side 2. That is $180 cm^2$

5.



As we can see, this is an equilateral triangle which is proportional to the side which is numbered 1. ✓

could you use this to find its area?

So we first need to find the height of the triangle.

$H^2 = b^2 - \frac{1}{2} b^2$

$b^2 = 8^2$
$= 64$

$\frac{1}{2} b = 4$
$4^2 = 16$

$64 - 16 = 48$

$\sqrt{48} = 6.9$

$h = 6.9$

We then have to find the area of the triangle.

$\frac{1}{2} b \times h$

$\frac{1}{2} b = 4$

$h = 6.9$

$4 \times 6 \cdot 9 = 27 \cdot 6\ cm^2$ ✓

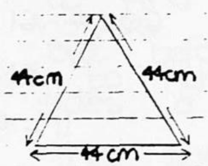
To find the total area, we add together the areas of the five sides

$$180 \times 3 = \begin{array}{r} 209 \\ 540 \\ + \ 27 \cdot 6 \\ \hline 776 \cdot 6\ cm^2 \end{array} = \text{Total Area}$$

Now that I know the total area of the smaller of the two holders, I can now try to work out the area of the larger holder. When both areas become known, the area factor can then be worked out.

To guess the area factor, I would say that it was four I thought this because if the length is doubled (x2) and the width is doubled (x2) and the extra width was multiplyed by the extra length (2x2) then the answer is four. Though to check my estimated answer, I will find out what the total area of the larger holder is.

1



$\frac{1}{2}\, b \times h$ = formula for the area of a triangle.

Like before we still have the height to find and therefore we use the formula: $H^2 = b^2 - \frac{1}{2}b^2$

COURSEWORK

MAKING A TENNIS BALL HOLDER

1. So, $h^2 = b^2 - \frac{1}{2}b^2$

$b = 44$

$b^2 = 1936$

$\frac{1}{2}b = 22$

$\frac{1}{2}b^2 = 484$

$1936 - 484 = 1452$

$h^2 = 1452$

$\sqrt{1452} = 38$

$h = 38\ cm$ ✓

Knowing this we can then find the area of the triangle.

$\frac{1}{2}\, b \times h$ = area of triangle

$b = 44$

$\frac{1}{2}b = 22$

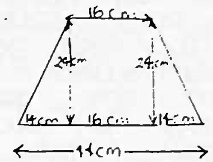
$h = 38$

$\frac{1}{2}\, b \times h = 22 \times 38$

$= 836$ ✓

The area of the triangle (which is the base of the holder) is $836\ cm^2$ ✓

2.



The rectangle $= 16 \times 24$

$= 384\ cm^2$

The triangles $= \frac{1}{2} b \times h \times 2$

$= b \times h$

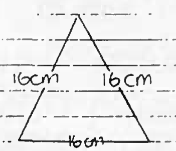
$= 14 \times 24$

$= 336\ cm^2$

The total area for side 2 is $720\ cm^2$

Sides 3 and 4 are identically to side 2. That is, $720\ cm^2$

5.



This equilateral triangle is proportional in size to the triangle which is shape 1, so the same working out is required.

so first we find the height.

$H^2 = b^2 - \frac{1}{2} b^2$

$b^2 = 16^2$

$= 256$

COURSEWORK

MAKING A TENNIS BALL HOLDER

$\frac{1}{2}\ b = 8$

$8^2 = 64$

$256 - 64 = 192$

$H^2 = 192$

$\sqrt{192} = h$

$= 13.8$

Knowing the height we can then find the area.

$\frac{1}{2}\ b \times h$

$\frac{1}{2}\ b = 8$

$h = 13.8$

$\frac{1}{2}\ b \times h = 8 \times 13.8$

$= 110.8$

The area of the top triangle is $110.8\ cm^2$

To find the total area we add up all the five sides of the holder.

$720 \times 3 =$

$$\begin{array}{r} 836 \\ 2160 \\ +\ 110.4 \\ \hline 3106.4 \end{array} = \text{total area}$$

Knowing the total area, we

can now find out the area factor. I thought it was four, but to check if I was right or not I would need to divide the total area of the larger of the two holders by the total area of the smaller of the two holders. If the answer is four (my estimated guess) then I know that the area scale factor is four, and that my guess was correct.

To check;

$$\frac{3106.4}{776.6} = 4 \checkmark$$

The answer to the above question was four, so this proves two things. Firstly that the area scale factor is four, and secondly, that my guess was correct. Of course though, this will only work if the larger holder was twice the size of the original holder which is true in our case.

Now that we know the scale factor for the area, we can then try to find the scale factor between the volume of the smaller holder and the volume of the larger holder.

First, I intend on finding the volume of the smaller holder, but whilst we were trying to find the volume, we became slightly stuck as the holder that we had designed was an irregular shape. A way to make the volume easier to find is by dividing the shape of the holder up into

COURSEWORK

MAKING A TENNIS BALL HOLDER

rectangles, though as our shape was nearer to a pyramid than what it was to a rectangle, I decided that it must be easier to work out the volume of a pyramid and then minus the part which isn't on the holder, but would (if attached) make the holder into a pyramid. ✓

The formula for the volume of a pyramid is:

Volume $= \frac{1}{3}$ base area x height

Area of base $= 209\ cm^2$

$$\frac{209}{3} = 69.666^{\circ} \checkmark$$

The height of the pyramid is:

height squared = base squared − distance between centre point on base and corner of base squared

height squared $= h^2$

base squared $= b^2$

distance between centre point on base and corner of base squared $= Y^2$

$h^2 = b^2 - Y^2$

$b = 22$

$b^2 = 22^2$

$= 484$

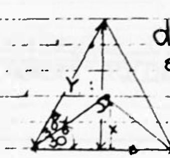


diagram to show the various distances required.

Y = height of slant − distance of part of base. (positions shown on diagram on other side).

$Y = S - x$

$S = 19.05$

$x = \frac{1}{2} b \times \tan 30°$

$= 11 \times 0.5773502$

$= 6.350853$

$x = 6.350853$ cm

$Y = 19.05\cancel{259} - 6.350853 = 12.701706$

$Y = 12.701706$ cm ✓

$h^2 = b^2 - Y^2$

$h^2 = 484 - 12.701706^2$

$= 484 - 161.33334$

$= 322.66666$

$h^2 = 322.66666$

$\sqrt{322.66°} = 17.962925$

$h = 17.962925$ cm ✓

knowing the height, we can now work out the volume for the smaller of the two holders.

volume = ⅓ base area × height

base area = 209 cm²

⅓ base area = $\frac{209}{3}$

COURSEWORK

MAKING A TENNIS BALL HOLDER

$\frac{209}{3} = 69.666°$

$h = 17.962925$ cm

⅓ base area × height = $69.666° \times 17.962$

$= 1251.4171$

1251.4171 = volume of pyramid cm³ ≈ 1250 cm³

1251.4171 cm³ is the volume of a pyramid, though as our holder is not a pyramid but a truncated tetrahedron, we still need to find the volume for a pyramid which is needed to make our holder into a pyramid.

volume = ⅓ base area × height

Area of base = 27.6 cm²

$\frac{27.6}{3} = 9.2$

The height of the pyramid is: why not use proportionals

height squared = base squared − distance between centre point on base and corner of base squared

height squared = h^2

base squared = b^2

distance between centre point on base and corner of base squared = Y^2

$h^2 = b^2 - Y^2$

COURSEWORK

MAKING A TENNIS BALL HOLDER

base area = 27.6 cm²

⅓ base area = $\frac{27.6}{3}$

= 9.2

h = 6.5518243 cm

⅓ base area x height = 9.2 x 6.5518243

= 60.2767836

volume of pyramid = 60.2767936 cm³ ✓

To find the volume of the smaller holder you have to minus the volume of the smaller pyramid from the volume of the larger pyramid.

1251.4171
− 60.276783
1191.140316 ✓

The volume of the smaller holder is 1191.140316cm³ ≈ 1190 cm³

To find the volume scale factor we now have to find the volume of the larger holder, by using the same method.

The formula for the volume of a pyramid is:

Volume = ⅓ base area x height.

Area of base = 836 cm²

b = 8

b² = 8²

= 64

Y = height of slant − distance of part of base.

Y = s − x

s = 6.9

x = ½ b x tan 30°

= 4 x 0.5773502

= 2.3094

x = 2.3094

Y = 6.9 − 2.3094

Y = 4.5906

h² = b² − Y²

h² = 64 − 4.5906²

= 64 − 21.07360

= 42.9264015

h² = 42.9264015

√42.9264015 = 6.55182429 ✓

h = 6.55182429 cm

knowing the height we can now work out the volume of the pyramid

Volume = ⅓ base area x height

$\frac{836}{3} = 278 \cdot 666^{\circ}$

The height of the pyramid is:

height squared = base squared − distance between centre point on base and corner of base squared.

height squared $= h^2$

base squared $= b^2$

distance between centre point on base and corner of base squared $= Y^2$

$h^2 = b^2 - Y^2$

$b = 44$

$b^2 = 44^2$

$= 1936$

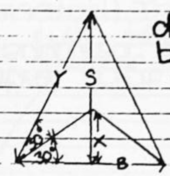


diagram to show the various distances required.

Y = height of slant − distance of part of base (position shown on diagram above).

$Y = S - X$

$S = 38 \cdot 105118$ cm

$X = \frac{1}{2} b \times \tan 30^{\circ}$

$= 22 \times 0 \cdot 5773502$

$= 12 \cdot 701706$

$x = 12 \cdot 701706$

$Y = 38 \cdot 105118 - 12 \cdot 701706$

$Y = 25 \cdot 403412$ cm

$h^2 = b^2 - Y^2$

COURSEWORK

MAKING A TENNIS BALL HOLDER

$h^2 = 1936 - 25 \cdot 403412^2$

$= 1936 - 645 \cdot 33334$

$= 1290 \cdot 666^{\circ}$

$\sqrt{1290 \cdot 666^{\circ}} = 35 \cdot 92585$

(35·92585 = height of pyramid for smaller holder x two.)

$h = 35 \cdot 92585$ cm

Knowing the height, we can now work out the volume of the pyramid

volume = 1/3 base area x height.

base area = 836 cm²

1/3 base area $= \frac{836}{3} = 278 \cdot 666^{\circ}$

height = 35·92585 cm

volume = 278·666° x 35·92585

= 10011·337 cm³

volume of larger pyramid = 10011·337 cm³

To find the volume of the larger holder, we now have to find the volume of the pyramid which is needed to make the larger holder into a pyramid. We then minus this from our answer above, and this will give us the answer for the volume of the larger of the two holders.

Volume $= \frac{1}{3}$ base area $\times$ height

Area of base $= 110 \cdot 8\ cm^2$

$\frac{1}{3}$ base area $= \frac{110 \cdot 8}{3}$

$= 36 \cdot 9333^{\circ}\ cm^2$

The height of the pyramid is:

height squared = base squared − distance between centre point on base and corner of base squared

height squared $= h^2$

base squared $= b^2$

distance between centre point on base and corner of base squared $= Y^2$

$h^2 = b^2 - Y^2$

$b = 16$

$b^2 = 16^2$

$= 252$

Y = height of slant − distance of part of base

$Y = s - x$

$s = 13 \cdot 8$

$x = \frac{1}{2} b \times \tan 30^{\circ}$

$= 8 \times 0 \cdot 5773502$

$= 4 \cdot 6188$

$x = 4 \cdot 6188$

COURSEWORK

MAKING A TENNIS BALL HOLDER

$Y = 13 \cdot 8 - 4 \cdot 6188$

$Y = 9 \cdot 1812$

$h^2 = b^2 - Y^2$

$h^2 = 252 - 9 \cdot 1812^2$

$= 252 - 84 \cdot 29443$

$= 167 \cdot 70557$

$h^2 = 167 \cdot 70557$

$\sqrt{167 \cdot 70557} = 12 \cdot 95012$

$h = 12 \cdot 95012\ cm$

Knowing the height we can now work out the volumte of the pyramid

volume $= \frac{1}{3}$ base area $\times$ height

base area $= 110 \cdot 8\ cm^2$

$\frac{1}{3}$ base area $= \frac{110 \cdot 8}{3}$

$= 36 \cdot 9333^{\circ}$

$h = 12 \cdot 95012\ cm$

$\frac{1}{3}$ base area $\times$ height $= 36 \cdot 9333^{\circ} \times 12 \cdot 95012$

$= 478 \cdot 2911$

volume of pyramid $= 478 \cdot 2911\ cm^3$

To find the volume of the larger pyramid, you have to minus

COURSEWORK

MAKING A TENNIS BALL HOLDER

This can be worked out without using any mathematical equation, as the amount of paper used is almost the same as the area, (thus is because the area which has been calculated is the surface area) except that the area does not take the paper inside the holder (i.e. flaps) into consideration, but as the area has a scale factor of four, then the flaps also will have, so the amount of paper needed for the larger holder is four times the amount that is needed for the smaller holder.

V. G.

The next question that I attempted to answer was how many extra tennis balls would be needed to fit into the larger holder? I choose this question to answer next as the answer to this question would lead nicely into the next question, how much more wasted space was there in the larger holder than what there was in the smaller holder?

I knew that the volume scale factor was eight but the volume of extra tennis balls would only be eight if the holder was a regular shape. As it isn't a regular shape, then the extra amount of tennis balls required for the larger holder would not be eight, but there is another way of working it out which will be used later.

To start with we need to know how many tennis balls the

the volume of the pyramid which is required to make the holder into a pyramid (i.e. answer on other side) from the volume of the larger pyramid. The answer for the above sum will give us the volume for the larger of the two holders.

$$\begin{array}{r} 10011.337 \\ -\ 478.2911 \\ \hline 9533.0459 \end{array}$$

The volume of the larger pyramid is 9533.0459 cm³

To find the volume scale factor, we have to divide 9533.0459 (numerator) by the volume of the smaller holder (1191.140316 [denominator]). The answer to this sum will then give us the volume scale factor.

$$\frac{9533.0459}{1191.140316} = 8$$

This tells us that the volume scale factor between a three dimensional shape, and another shape which is proportional to the first, when all dimensions have been doubled, is eight.

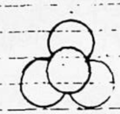
Two of the questions that I set myself have now been answered. These were the scale factor for the area, and the scale factor for the volume.

My third question that I shall answer is how much extra paper is needed to make the larger holder.

smaller holder held. The easiest way to work this out is by placing the tennis balls inside the holder, and then count how many the holder held.

The smaller holder was able to hold four tennis balls. They were packed, three balls on the bottom and one ball on top.

i.e.



✓

There are two different methods which can be used to calculate the number of tennis balls which are needed to fill the larger holder.

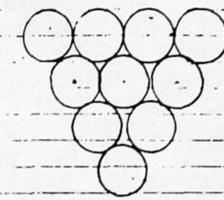
The first is that as the volume scale factor was eight, then I might expect the amount of tennis balls that the larger of the two holds to hold four x eight, which is thirty-two tennis balls.

The second method is to calculate the number of tennis balls which are needed by taking the positions of the balls in the smaller holder, and multiplying each side by two, which will give the bottom layer. Then, add a extra ball inbetween each gap, so that each side of the same layer contains the same number of balls. so the total number of balls that the larger holder will hold would be twenty when using the above method. Here

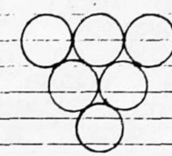
COURSEWORK

MAKING A TENNIS BALL HOLDER

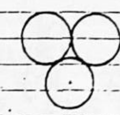
is a diagram to explain their positions.



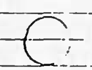
- Bottom layer of tennis balls in the holder.



- Second layer of tennis balls in the holder.



- Third layer of tennis balls in the holder.



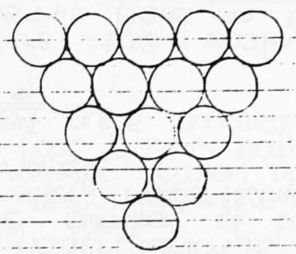
- Top layer of tennis balls in the holder. ✓

Each set of balls lie on the gaps between the balls in the layer below. When all the above balls are added together, the total number of tennis balls is twenty.

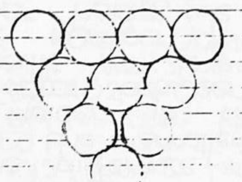
✓

i.e. $10 + 6 + 3 + 1 = 20$

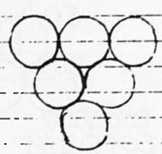
So to find out which method was correct, we placed the necessary amount of tennis balls into the holder and added up the amount which it took. It held thirty-one tennis balls, though there was a reason for this. It was that the three identical sides were all two centimetres short of what they were supposed to be. The thirty-one balls that fitted into the larger holder were positioned like this:



- Bottom layer of tennis balls in the holder



- second layer of tennis balls in the holder



- Top layer of tennis balls in the holder

As you can see there are only three rows of tennis balls, whereas there should be four because there were two rows of tennis balls in the smaller holder, though this was due to the

COURSEWORK

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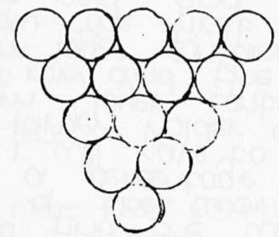
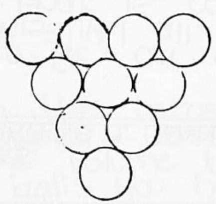
larger holder being two centimetres shorter than what it should have been. Even so, it can clearly be said that in the case of a pyramid, the number of balls in the smaller holder, times the volume scale factor does not equal the number of balls in the larger holder. (Even though the larger of the two holders has a measurement which is slightly inaccurate, this will not affect my results for the area and the volume as the measurements which I used were taken from the smaller holder, and doubled, when necessary to give the measurements for the larger holder.)

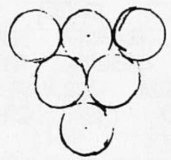
If the larger holder had been made accurately, then it would have held thirty-four balls. The balls would have been positioned like this:



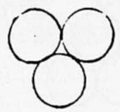
- Bottom layer of tennis balls in the holder.



- Second layer of tennis balls in the holder.



- Third layer of tennis balls in holder.



- Top layer of tennis balls in holder.

The difference in tennis balls between the smaller holder and the larger holder is thirty. To find the scale factor between the number of tennis balls in the smaller holder and the number of tennis balls in the larger holder, we have to divide the number of tennis balls in the larger holder (which is 34 [the numerator]) by the number of tennis balls in the smaller holder (which is 4 [the denominator]).

$$\frac{34}{4} = 8{\cdot}5$$

Here the scale factor between the tennis balls in the smaller holder and the tennis balls in the larger holder is 8·5.

The reason for the bottom layer of tennis balls containing five balls in a row, as opposed to four is because of the amount of wasted space in the larger holder. It was impossible to fit an extra tennis ball into the smaller holder, but when that extra space was doubled, (in the larger holder), there was the room to fit an extra row of tennis balls. This is all due to the wasted space that is contained in both of the holders. What I intend to do now is to find the wasted space in each of the two holders, and then find a percentage for them both, but I will have to do it twice for the larger holder. Once with the bottom layer containing five balls in a row, and the other with the bottom layer containing four balls in a row. The first being what the holder held and the second being what the holder was predicted to hold without taking the wasted space into consideration.

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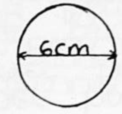
To find the amount of wasted space, we have to calculate the volume of space which is taken up by the tennis balls and then minus it from the total amount of volume in the holder.

So firstly we have to find the volume of one tennis ball, and multiply it by how ever many tennis balls are in the holder.



- A tennis ball with the given measurement.

Volume of sphere $= \frac{4}{3}\pi r^3$

$r = 3$

$r^3 = 3^3$

$3^3 = 27$

$\frac{4}{3} = 1.333^\circ$

$\frac{4}{3} \times \pi r^3 = 1.333^\circ \times 3.1415927 \times 27$

$= 4.1888 \times 27$

$= 113.1$ ✓ units!

In the smaller holder there were four tennis balls, so to find the volume taken up by the tennis balls, you have to multiply 113.1 by four.

$113.1 \times 4 = 452.4$

The total volume taken up by the tennis balls in the smaller holder is 452.4 cm³. ✓ To find the volume of wasted space, we now have to minus the volume of the tennis balls, from the volume of the holder.

$$\begin{array}{r} 1191.1403 \\ -\ 452.4 \\ \hline 738.7403 \end{array}$$

The wasted space in the smaller holder is 738.7403 cm³. Knowing this we can now work out what percentage of the volume in the holder is wasted space.

$\frac{738.7403}{1191.1403} \times 100 = 62\%$

The wasted space as a

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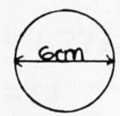
percentage in the ✓ small holder is 62 %.

Next we will work out the amount of wasted space present in a holder which has dimensions which are twice the size of the ones in the smaller holder, and which holds twenty tennis balls, using the same method as before.



– A tennis ball with the given measurement.

volume of sphere $= \frac{4}{3} \pi r^3$

$r = 3$

$r^3 = 3^3$

$r^3 = 27$

$\frac{4}{3} = 1.333^\circ$

$\frac{4}{3} \times \pi r^3 = 1.333^\circ \times 3.1415927 \times 27$

$= 4.1888 \times 27$

$= 113.1$

In the larger holder there were twenty tennis balls, so to find the volume taken up by the tennis balls, you have to multiply 113.1 by twenty.

$113.1 \times 20 = 2262$

The total volume taken up by the tennis balls in the larger holder is 2262 cm³. To find the volume of wasted space, we now have to minus the volume of the tennis balls, from the volume of the holder.

$$\begin{array}{r} 9533.0459 \\ -\ 2262 \\ \hline 7271.0459 \end{array}$$

The wasted space in the larger holder is 7271.0459 cm³. Knowing this we can work out what percentage of the volume in the holder is wasted space.

$$\frac{7271.0459}{9533.0459} \times 100 = 76.27\%$$

The wasted space as a percentage in the larger holder is 76.27%.

The reason why this percentage for the larger holder is so high is because in the holder there was room for an extra fourteen tennis balls. These are not supposed to be present, but because of the vast amount of wasted space in the holder, there is room for the extra balls.

The next thing that needs to be worked out is the amount of wasted space in the larger holder when the extra fourteen tennis balls have been added.

So firstly we have to find the volume of one tennis ball, and then multiply it by thirty-four, (the number

COURSEWORK

MAKING A TENNIS BALL HOLDER

of tennis balls that will fit inside a holder which has dimensions which are twice the size of the smaller holder). This will give us the total volume of the tennis balls.

– A tennis ball with the given measurement.

volume of sphere $= \frac{4}{3} \pi r^3$

$r = 3$

$r^3 = 3^3$

$r^3 = 27$

$\frac{4}{3} = 1.333^{\circ}$

$\frac{4}{3} \times \pi r^3 = 1.333^{\circ} \times 3.1415927 \times 27$

$= 4.1888 \times 27$

$= 113.1$

In the larger holder there were thirty-four tennis balls, so to find the volume taken up by the tennis balls, you have to multiply 113.1 by thirty-four.

$113.1 \times 34 = 3845.4$

The total volume taken up by the tennis balls in the larger holder is 3845·4 cm³. To find the volume of wasted space, we now have to minus the volume of the tennis balls from the volume of the holder.

$$\begin{array}{r} 9533{\cdot}0459 \\ -\ 3845{\cdot}4 \\ \hline 5687{\cdot}6459 \end{array}$$

The wasted space in the larger holder is 5687·6459 cm³. Knowing this we can now work out what percentage of the volume in the holder is wasted space.

$$\frac{5687{\cdot}6459}{9533{\cdot}0459} \times 100 = 59{\cdot}66\%$$

The wasted space as a percentage in the larger holder is 59·66%. This percentage is lower than the other percentage for the larger holder because this time I was able to fit an extr fourteen tennis balls in the holder, and so bringing the percentage of wasted space down.

Having now answered all of the questions that I set myself at the beginning of this piece of coursework, there is still one more question that I would like an answer to and that is, 'How can we cut down the amount of wasted space in each holder?' This question arised whilst I was working out the percentage of wasted space in each holder as at that time I couldn't believe that the amount of wasted space could be so high.

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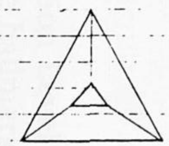
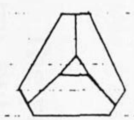
If I was to redesign the tennis ball holder there might be two changes that I would make to it. One idea was that as our holder was not quite a pyramid as it had a smaller pyramid missing from the top of it,

i.e.



then we could do the same every other corner as what we did to the top of our holder. This would make the new holder look like this:



This was an idea as the wasted space which was in the corners would no longer be there.

Another example which would bring down the amount of wasted space is to make each side a centimetre smaller in length. The same amount of tennis balls would still be able to fit into the smaller holder, but the wasted space would be less as there will be a reduce in the amount of volume that the holder can contain. The reason for the same amount of tennis balls being able to fit into the holder is because whilst designing the original holder we allowed extra space incase the tennis balls would not fit into the holder.

No matter which one of these methods (or even both) we use, there will always be a certain amount of wasted space as spheres (i.e. tennis balls) will not tesselate and so leaving gaps inbetween each tennis ball. These gaps may not seem to be much, but when added together they create a vast amount of wasted space.

6 Moderator's Comments

Anyone For Tennis G1/1

Foundation Level

Grade F

This is a distinctly Foundation Level piece of work. The presentation is neat, but lacks precision. It bears the hallmark of an honest try, but seems to contain some of the work of another person. Note the references to 'we'.

The construction of the piece is not logical - the report follows immediately upon the copied introduction, and the supporting 'evidence' follows later. In fact the support is quite slender, for there is no real method detailed, and the origins of the measurement are never made clear.

Some of the following calculations are accurate, but the reason for doing them has not been presented. There is also confusion in the students mind over what is meant by 'dimensions'. The calculation made on the third side seems to have been done on the evidence of an incorrect net. Since the net is so at fault, one must wonder whether any testing of the cuboid has been done. The bulk of the project, in fact, consists of a number of calculations and a few comments. This does not contribute a written method nor a cohesive plan. The student has seen this as a single stage 'argument', but has not argued it at all.

There is negligible development, as the 'ideas' presented at the end have not been founded on any mathematical reasoning, and lack precision and testing.

However, sufficient work has been done to enable the assessor to deduce that the candidate has been capable of performing a relatively simple set of tasks without refinement, and to write a brief record of the work done.

Sorting Shapes G1/2

Foundation Level

Grade E

This is an interesting piece as it does not seem to be typical of a Foundation Level candidate. There are hints here that the candidate has greater insight into the problem than would be expected at this level. Upon reading the task, I feel that the candidate has written a good start to a project but never completed it.

Although the introduction is copied, there is an immediately personal input on the second page (though the drawings are certainly not precise). However, despite the lack of drawing precision, she solves the key issue here of not letting the shapes fit through the wrong hole very well. The method of overlaying the shapes is well on course for a good grade but then it all breaks down. A few, seemingly irrelevant area calculations are done, but then the project dies.

The candidate has answered one single stage in the problem. This has been done well, but with no follow up, can never account for more than a Foundation Level grade. The misfortune of this type of piece, is that it seems to denigrate the work of the honest student at this level, who will strive hard to achieve this grade.

Anyone For Tennis G1/3

Intermediate Level

Grade D

This submission is somewhat above Foundation Level and yet it lacks the stamp of a Higher Level coursework item. The presentation is neat and easy to follow and seems to owe a lot to the teaching of C.D.T in the school. The idea of setting out the criteria by which the model is to be judged is good but the judgements are made, in many cases, on very non-mathematical grounds - see judgement 3! There is also a lack of logic in the layout of the evidence for the work. Why should the nets be at the end and why is there no reference to the derivation of the measurements of the box until later in the project? Even at this level there is still no coherent argument of the case to be made. The pupil follows the instructions given and makes some valid comments upon the work but still sees the problem as a basic 'task' with little refinement.

In order for this pupil to have improved the grade of the work, a less 'essayist' style needs to be adopted with greater emphasis on the mathematical reasoning behind the judgements made. Diagrams may also be drawn to scale. The concept of 'stacking' is taken in a very trivial sense, and has not been developed to look at packing these boxes into large areas. However, a nice touch was added by picking up the error in the construction of the box, and indicating how it may be rectified.

This is a nicely presented project, yet one in which there is little development of the mathematical concepts available in the problem.

Sorting Shapes G1/4

Intermediate Level

Grade C

My first reaction upon reading this item was that it was an excellent piece and deserved full recognition for its mathematical strengths. After all, it does represent a clear and coherent argument in answer to a problem. However it is a specious argument. There is a distinctly 'personal' approach to the task. The sketches are precise, choices of box are made logically and with good reasons.

The designs and careful nets of each of the solids to be sorted are set out in precise detail and all the calculations are set out in exemplary fashion. The language used is, in most cases precise, though the names of solids are not correctly used. So why was my first reaction incorrect?

The key to the entire issue is in the very last paragraph - the essential problem has not been answered!

What is the use of a shape sorter which does not sort shapes? Since the 'solution' does not do the task it is set to solve, much of the work done has to be invalidated - it is therefore inaccurate. The candidate has to be given recognition for the skills demonstrated, but she should have addressed this central problem at the start of the project, and the fact that she did not indicates a lack of true planning. This is a harsh judgement but one which needs to be made.

The photograph included in the Teacher's Notes shows the shape sorter produced by this student.

Anyone For Tennis G1/5

Higher Level

Grade C

This project fits nicely into the Higher Level pattern of coursework and demonstrates that it is not necessary to write at great lengths to achieve a reasonable grade. It is a pity that the models produced are not able to be included with this submission.

The argument encompasses more than one stage and, though it is well buried in the essay that has been presented, the crucial points of method are all there : the measurement of the ball -> circumference, prototype, problems of construction, calculation, conclusion, deduction and development.

The initial problem is solved well, though there are gaps in the calculations presented, i.e the percentage of wasted space, and there are no references to 3-D tessellation which show insight into the problem. It is a shame that the same clarity is not applied to the development of the task. A reliance upon placing tennis balls into his enlarged container to check volumes seems to be 'beneath' his grasp of the task. I am surprised that he did not go on to check his results which would seem perhaps to be at variance with the other findings.

I must again make the comment that the essay style of writing does not, in general, do justice to this type of problem and that short notes, diagrams and varied mathematical techniques are more appropriate to explain such tasks.

Anyone For Tennis   G1/6

Higher Level

Grade A

The candidate has, in this piece of work, completed a cohesive and precise commentary upon the task and developed it with accurate deductions based upon her findings. If I must criticise aspects of the work, then it would be to say that the work is unnecessarily laboured, and some of the area calculations might have been reduced. There are also some errors in notation with omission of brackets from $({}^1/_2\, b)^2$, and use of degree signs where cm are needed, though it is obvious that she knows how to use these results. It is to be hoped that candidates will eliminate such errors from their work and that they will also cut down upon the commitment to time that such a lengthy presentation requires.

The scope of the 'mathematics' employed and understood by the candidate indicates a clear grasp of the necessary skills to solve the problem. In fact, she brings in new aspects of the subject as and when she requires them. Overall the work is clearly dedicated to accuracy, borne out by the precise values of the scale factor calculations for both area and volume.

One danger of a piece of work of this calibre is that if it becomes divorced from reality, it cannot be entered as *Practical* Geometry after all. However, it is nice to see in the commentary that Triangular Numbers are used to explain the stacking of the balls in the box and also attempts to produce such results. There is also consideration of the discrepancy in results.

Over the whole project, there is a pleasing consideration of a number of aspects of the problem, and there is even a suggestion of developments still to come at the end.

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