

TEACHER'S GUIDE

About this scheme . . .

This scheme for the teaching and learning of numeracy through problem solving consists of a series of modules which provide effective support for teachers of mathematics who wish to introduce into the curriculum a component which enables their students to link their mathematics to the real world in which they live.

It has been developed with students of all abilities in the age range 13–16, and their teachers.

Each module package provides comprehensive materials for both teaching and assessment, related to a practical context which has proved interesting and enjoyable to the students who have taken part in its development. It is accessible to those who normally find mathematics difficult, while at the same time it provides a challenge for the most able.

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Tea	roughout this book, all references prefixed with the letter T refer to pages in acher's Guide, references prefixed M refer to the Masters for Photocopy d unprefixed references refer to the Student's Booklet.	

Authors

This series of module packages has been developed, as part of a joint project on the assessment of numeracy by the Shell Centre and the Joint Matriculation Board. Many teachers and their students have worked with the central team: Alan Bell, Hugh Burkhardt, Rosemary Fraser, Kevin Mansell, Jim Ridgway and Malcolm Swan together with Barbara Binns and John Gillespie who, assisted by Clare Trott, coordinated the project. Building on previous discussions involving the Shell Centre and the JMB, it was conceived and directed as part of the Testing Strategic Skills programme, by

Hugh Burkhardt

This module, Design a Board Game, has been written by

Malcolm Swan, Barbara Binns and John Gillespie

It is a result of successive revision through six versions on evidence collected from the comments and suggestions of the teachers and students involved, and through classroom observation by the team.

The assessment tasks owe much to the advice of John Pitts.

Many contributions to the work of the project have been made by staff and committee members of the Joint Matriculation Board – notably John Mathews, the Chairman of the JMB's Steering Committee and Austin Fearnley, who has played a major role in the organisation of the operational trials.

Among the teachers to whom we are particularly indebted for the contributions to the early development of the module are David Cain, Joanne Cooper, Ray Crayton, Paul Davison, John Edwards, Mick Fitzgerald, Tansy Hardy, Aidan Harrington, Anne Haworth, Sue Marshall, Cath Mottram, Mary Robinson, Norbert Gajda, Bob Smith, Aileen Steven, Glenda Taylor, Margaret Tuck and Dave Wilson.

The later trials involved teachers and students in over 50 schools in many local authorities including Barnsley, Bradford, Bury, Calderdale, Cheshire, Cumbria, Derbyshire, Doncaster, Gateshead, Humberside, Kirklees, Knowsley, Lancashire, Leeds, Leicestershire, Manchester, Newcastle upon Tyne, Northumberland, North Tyneside, North Yorkshire, Nottinghamshire, Rochdale, Rotherham, Salford, Sheffield, Stockport, Sunderland, Tameside, Trafford, Wakefield and Wigan. Consultations with their Mathematics Advisers have made significant contributions to the development of the scheme.

The manuscript was prepared through many revisions by Susan Hatfield, Diane Jackson and Jenny Payne, and the staff of Burgess and Longman.

We are grateful to them all.

Numeracy through problem solving

Design a Board Game is one of a series of modules that have been designed to encourage a new approach to the teaching and learning of numeracy, understood in the original broad sense^(1,2) as

the ability to deploy mathematical and other skills in tackling problems of concern or situations of interest in everyday life.

There is now a general acceptance that people need to learn to *use* the knowledge and skills they acquire at school, and this requires a shift in the balance of the curriculum to include more real problem solving. This is particularly important for the mathematics curriculum, because the power of mathematics in helping people tackle real problems more effectively, is not often realised.

The Cockcroft Report says

'Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much.' (paragraph 34)

and

'Our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation.' (paragraph 39)

TVEI and other recent curricular initiatives have similar aims, emphasising that curricula should contain a strong element concerned with the tackling of practical problems relevant to everyday life and work. The assessment criteria for the GCSE emphasise these aspects too. Employers say that they are primarily interested in people who can use their knowledge sensibly and effectively.

A curriculum component of this kind places new demands on teachers; it needs a broader range of teaching strategies than does the traditional mathematics curriculum, with some new roles for both teachers and students. The scheme has been developed to provide an introduction to such work in the classroom that is both effective and enjoyable for those involved.

What are the skills?

The modules are based on practical contexts which have been chosen to allow students of all abilities to develop general problem solving (or strategic) skills in areas of activity such as designing and making, planning and organising, and choosing.

These strategic skills include:

- understanding general ideas and details
- following instructions precisely
- distinguishing between essential constraints and desirable features
- identifying faults
- correcting faults
- generating and listing viable possibilities (brainstorming)
- developing a rough plan, including: reviewing the prepared suggestions; reaching and recording agreed decisions; maintaining a broad level of description, avoiding excessive detail; identifying needed information and materials; making estimates of quantity and cost; describing, testing and evaluating the plan
- making the final plan, product and/or detailed instructions with comprehensiveness, accuracy, clarity and quality
- implementing the activity with full preparation
- testing and evaluating the plan or product comprehensively.

Various *tactical skills*, more specific to each context, are involved in implementing these strategies; for example, different ways of collecting and recording information are appropriate if you are considering alternative products to buy, or alternative routes to follow.

Technical skills are, of course, required to carry through the solution of problems using the higher level skills described above. Technique is only useful for these purposes insofar as it is *reliable*. This implies much higher standards in this respect than are expected in the traditional curriculum, with a greater emphasis on thorough understanding and checking of whatever

¹ 15–18 A report of the Central Advisory Council for Education (England). HMSO, 1959.

² Mathematics counts. Report of the Committee of Enquiry into the Teaching of Mathematics in Schools under the chairmanship of Dr W H Cockcroft. HMSO, 1982.

techniques are used. Among the mathematical techniques and concepts, of importance in this scheme, are:

- the ability to:
 - carry through simple calculations with suitable accuracy, using a calculator where appropriate,
 - make estimates,
 - make measurements (including number, length and time),
 - draw accurately,
 - interpret and display data in a variety of representations (including graphs, maps, timetables and other tables).
- understanding and using some techniques of
 - probability and statistics,
 - ratio and proportion,
 - geometry in two and three dimensions.
- logical reasoning, including the ability to enumerate alternative possibilities and classify them in various ways.
- research skills, including the collection and evaluation of relevant data.

The relevant mathematical skills are discussed in more detail in each module package. There is also opportunity for the use of other parts of the mathematics curriculum which a student has mastered.

In addition skills from other curriculum areas, such as language and arts, are inevitably called upon, as these are necessary for the presentation of the reasoned arguments which are essential for real problem solving. Since group work is involved, social skills also play their part. Thus, though numeracy is focussed on the deployment of mathematical skills and reasoning in real problem solving, it has a broad cross-curricular aspect.

What is provided?

The scheme is implemented in a modular form, each module being designed to occupy between 10 and 20 hours of teaching time spread over 3 to 6 weeks. Five modules will be available in the first instance. A feature of each module is the importance attached to students working in groups, explaining their ideas and listening to each other, making their own decisions and living with the consequences, reflecting on their experience and learning from it, just as they do in life outside the classroom. While working through the modules, students themselves become responsible for setting and tackling their own problems, rather than simply responding to tasks set by the teacher. Modules are not necessarily staged nor are they dependent upon each other but the sequence which follows is recommended as providing an appropriate progression and a balance of different kinds of context.

The modules in the series are:

- Design a Board Game: in which students design and produce a board game which can be played and evaluated by other members of the class.
- Produce a Quiz Show: in which students devise, schedule, run and evaluate their own classroom quiz.
- Plan a Trip: in which students plan and undertake one or more class trips, and possibly some small group trips.
- Be a Paper Engineer: in which students design, make and evaluate 3-dimensional paper products, such as pop-up cards, envelopes and gift boxes.
- Be a Shrewd Chooser: in which students research and provide expert consumer advice for clients in their class.

Many contexts were considered and tried in the early stages of development, to see which led to the best balance of classroom activities and learned skills. Those that were chosen all have a practical outcome, interesting and relevant to the students' present circumstances. This corresponds with our observation that people best develop the strategic skills of numeracy in the course of solving problems which they see as realistic, stimulating and within their capabilities. The themes selected were found to have general appeal and to require the use of a wide range of skills, whilst not making unreasonable demands on classroom or school organisation.

Discussion with students and observation in the classroom support the expectation that students' problemsolving abilities improve as they work through the series of modules and that skills acquired in one area are subsequently applied in others. Students themselves maintain that they will be able to apply these strategic skills with advantage in tackling further problems as they arise in their lives outside the classroom. Groups of students also suggested many other interesting and worthwhile themes, each of which could form the basis for a further module. These include: planning and running a jumble sale; raising money for charity by sponsored events; planning and running a magazine; setting up a small business; planning a party; designing a bedroom; planning a youth group weekend; making a garden; orienteering; designing and marketing T-shirts.

The scheme provides classroom materials and assessment tasks, together with further support materials to help teachers explore in greater depth the issues and teaching strategies involved. Suggestions for further mathematical development are also included.

Classroom materials, including detailed teaching suggestions, have been developed to offer a proven approach that has worked well for a representative

group of teachers, new to this kind of work, without imposing on them excessive demands of design or implementation. We recognise that, of course, each teacher works in his or her own way in the classroom but most have been found to appreciate detailed, explicit suggestions which they can use, and adapt, in the knowledge that they have worked well for others. Such materials are provided in each module package.

Assessment tasks play an important role in the curriculum, providing targets that help students and teachers recognise objectives more clearly and help them to progress towards them. (The effect of assessment on the curriculum has often been to narrow and distort its aims but, equally, assessment can be used to enhance what is achieved.) In a new curriculum component like this one, assessment is particularly important. Thus assessment tasks are provided throughout these materials.

The suggestions for further mathematical development provide a variety of ideas, together with discussion on how and when they might be introduced and linked to the more traditional teaching of mathematical techniques.

Support materials are designed to help teachers with the new aspects of classroom activity and teaching style that this work involves. The materials relate to the three principal differences between this work and the traditional mathematics curriculum - the broader range of skills involved, the practical priorities of numeracy, and the much greater responsibility of the students for their own work. In the traditional curriculum students are largely *imitative*, here they are *autonomous* in deciding on and carrying through their approach to the task. The primary support is provided by the teaching suggestions in the classroom materials and elsewhere in each Teacher's Guide. The support materials, which form a separate package, take this further, sharpening awareness and tackling more fully and deeply the teaching and assessment issues and skills involved. They may be regarded as a do-it-yourself in-service course, designed to be used either on a distancelearning basis by teachers in a school or within LEA or college courses. This material, which is linked particularly to the 'Be a Paper Engineer' module, includes a video of the modules in use, together with comments from teachers and students on the work, its challenges and its benefits.

Introduction to Design a Board Game

Design is an important area of real problem solving that many people face regularly in their lives at home and at work. The context of 'the design of board games' fits well in a mathematics classroom. Board game designers face a wide variety of problems which have to be solved before a game is complete. These problems involve them in the deployment of a range of strategic skills and a variety of different mathematical techniques if the end product is to be a success. In this module students face and solve such problems in designing their own board games.

The classroom activities are arranged in four stages that are typical of the design process. These are outlined below, together with the main strategic skills that are being developed.

1. Looking at examples. Students play a number of games which have been devised by someone else, discover faults and short-comings and suggest improvements. (This involves the strategic skills of 'understanding a situation' and 'identifying relevant factors in it'.)

2. Developing your own ideas. Students share their ideas within groups, then decide on a rough plan for their own game. (This involves 'listing alternatives', 'estimating resources required', 'making decisions' and 'detailed planning'.)

3. Making your game. Each group of students produces a detailed design, then makes it up and checks the finished version. (This involves 'implementing a plan', 'selecting and using appropriate mathematical techniques' and 'checking and testing'.)

4. Testing and Evaluating. The groups exchange games and test them. When they are returned, each group re-assesses its own game in the light of another group's comments. (This involves 'evaluating the outcome of a plan in action'.)

Chapter 1 provides classroom materials and teaching suggestions for the sequence of activities.

The range of mathematical techniques required will depend on the students' abilities and the demands made by their chosen design. However, the range is likely to include:

- carrying out simple whole number and length calculations,
- estimating,
- drawing simple figures, using drawing instruments appropriately,
- understanding and using ideas of angle and parallelism,
- using simple tessellations to create original board designs,
- writing clear, concise and complete instructions,
- appreciating and using ideas of fairness and bias, randomness and variability.

This aspect is discussed further in Chapter 2, while Chapter 3 is concerned with assessment.



Classroom materials

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Classroom materials

Introduction

This chapter provides a detailed guide to the classroom materials. The lesson outlines are offered in the recognition that every teacher has an individual style in which he or she prefers to work. However, many teachers have found it helpful to be given detailed suggestions which they can then adapt to meet their own needs. This has also enabled us to offer an approach which we have seen working well in a representative range of classrooms.

The classroom materials are centred around the Student's Booklet. This Booklet is important for several reasons:

- it provides students with a coherent structure for their work. At any point, it should help students to have an overview of what they have achieved and where they are going.
- students who are inexperienced in designing a product often latch on to an idea that seems superficially attractive, without carefully considering implications or alternatives. The booklet will help to slow down and stimulate the more impatient or less imaginative students by, for example, inviting them to criticise a range of different designs before embarking upon their own.

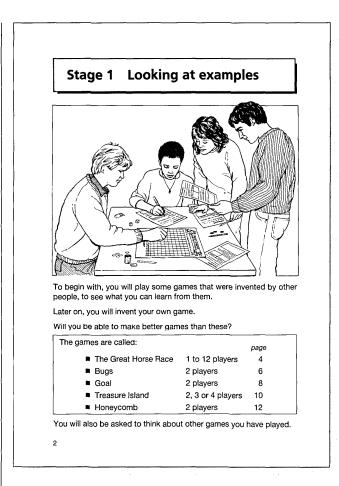
Much of this work involves students working collaboratively in small groups. It is often difficult to assess an individual's contribution to a group's product, so we have also included a number of short assessment tasks which may be given to students at the end of each stage. These should provide further evidence of what a particular student can do on his or her own.

This kind of real problem solving in the classroom demands a different balance of teaching styles and strategies from the traditional mathematics curriculum. For some teachers, the emphasis on student-led decision making will be unfamiliar. In this case, we offer the following suggestions which have been found helpful.

Your role will involve rather less task setting and explaining than you may be used to. Instead, you will be acting more as an adviser and resource, responding to the class rather than directing them. As you tour the room, it is helpful if you can

- listen to students and ask questions which may help them to clarify their own thinking and move beyond superficial discussions.
- encourage students to listen to one another, making sure that the less forceful or articulate are given a fair chance to express their views.
- help students to pace their work, by agreeing target dates by which phases of their work should be completed.
- encourage students to take a pride in their work and aim for a product which is both imaginative and polished.
- avoid 'taking over' by suggesting your ideas and making decisions for them. If you do this, students may feel that they are no longer working on *their* board game and become disenchanted. Intervention is particularly tempting when it is clear to you that a student is using a particularly inappropriate or time-consuming method, but hold back if you possibly can until the students realise *for themselves* that their method is unsuitable.

These suggestions are summarised on the final page of this book for ease of reference.





- Drawing up a rough plan,
- Testing and improving your plan,
- Getting everything ready.

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Summary of activities

Time needed

About 2 to 3 hours is sufficient, though you may choose to spend longer.

Students' activities

- Playing and criticising games with inbuilt faults, in order to identify features which may be taken into account when designing their own game. A group activity with some class discussion.
- Writing about other games, and considering why they like or dislike them. This could be an individual activity, possibly as homework, or it could form the basis for discussion.

The teacher's role

- Facilitating the sharing of ideas.
- Encouraging groups to get beyond a superficial analysis of a game.
- Encouraging groups to suggest and try out improvements to the games.

Time needed

About 2 hours.

Students' activities

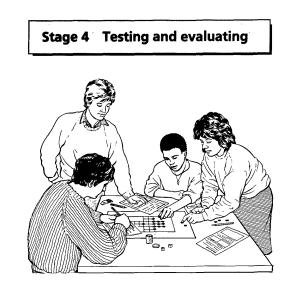
- Brainstorming ideas about different aspects of board games. Mainly an individual activity.
- Reaching agreement on which idea to use for the group's game.
- Drawing up a rough plan for the game, including rules and board design.
- Testing and improving the plan to make sure it works well.
- Making lists of what will be needed to make the game.

These are all group activities.

The teacher's role

Encouraging groups to work through each activity thoroughly, rather than opting for one idea without considering alternatives.





When several games are finished, swap your game with one from another group. You will then

- test their game to see how well it works.
- see what the other group thinks of your game.

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Time needed

About 3 to 4 hours, though students may wish to spend longer. You may wish to set a finishing deadline.

Students' activities

- Making the board accurately and neatly using suitable geometrical techniques and other 'artistic devices'.
- Writing the final version of the rules so that someone else could play their game.

These activities are shared amongst the members of the groups.

The teacher's role

- Making sure necessary equipment is available and organising its use.
- Making sure that all members of each group are involved in some aspect of the production of the game.
- Encouraging and helping students to use mathematical techniques where appropriate.

Time needed

About 1 hour, but you may wish to spend longer.

Students' activities

- Testing the game produced by another group, and making constructive criticisms. A group activity.
- Responding to another group's comments about their own game, perhaps by improving it still further.
 A group activity followed by an individual reflective activity.

The teacher's role

- Organising the exchanging of games.
- Encouraging students to criticise in a constructive, positive manner.

Preparation

This module requires students to work in groups on practical activities. In order for this work to run smoothly, it is advisable to make sure that all the necessary equipment is accessible and to consider the best way of organising the classroom.

Equipment required

When needed	Item	Quantity	Source
Throughout the module	Student's Booklets	1 for each student	supplied
	Envelopes or folders in which to keep work	1 for each student	
	Rough paper	a plentiful supply	
Stage 1	Sets of the five board games with equipment	3 sets for a class of 30	supplied
	'Comments' sheets	at least 3 for each student	master supplied
	'Looking at other games' sheet	1 for each student	master supplied
	Assessment tasks (if required)	1 for each student	masters supplied
Stage 2	'Brainstorming' sheets	1 for each student	masters supplied
	'Rough Plan' sheets	1 for each student	master supplied
	Assessment tasks (if required)	1 for each student	masters supplied
	A variety of different grid paper e.g. squared, isometric, dotty, etc.	a plentiful supply	some masters supplied
Stage 3	'Rough Plan' sheets	several for each group	master supplied
	Assessment tasks (if required)	1 for each student	masters supplied
	Equipment listed on students' own lists (see facing page).		
Stage 4	'Comments' sheets	several for each group	master supplied
	'Evaluating your own game' sheets (this is also used for assessment)	1 for each student	masters supplied

By the end of Stage 2, students are required to write out their own equipment lists, which apply to their own designs. The following list includes items that students have often asked for. Not all of these items are essential, but most will be found useful:

pencils felt tipped pens in assorted colours, including a few broad liners rulers set squares protractors compasses rubbers scissors glue, especially glue sticks sellotape coloured paper in various sizes, including some large sheets gummed paper tracing paper	grid paper card in various sizes, including some large sheets counters in various colours dice a guillotine headless matches or cocktail sticks (for making spinners) some 'Transpaseal', 'Fablon', or an alternative method of laminating the board with transparent plastic a tape recorder (to help some students write their rules) a stapler a box for each group to store things in.
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Classroom organisation

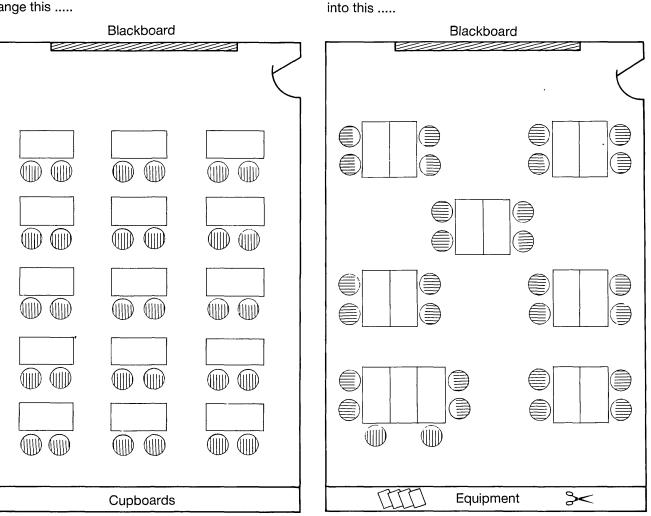
While using this module, it is helpful if the classroom furniture is arranged so that students can comfortably

- work and discuss in small groups,
- collect equipment,
- see the blackboard, which may be used for class discussions.

For example,

change this

Stage 1 may be easier to manage if the games are allocated to tables before the lesson, then when a group has finished playing one game, they move to a different table to play the next. This avoids having to pass the game boards and pieces around the room.



Stage 1 Looking at examples

Introduction

In this stage, students are expected to play at least three games, evaluate and identify any faults in them, and then offer constructive comments on how they may be improved. The games are carefully chosen to offer variety in board design, equipment used, and skill required. They also embody a number of serious design faults. We hope that students will find that they both stimulate ideas and illustrate the kind of mistakes that should be avoided.

Total time needed

About 2 to 3 hours, though you may choose to spend longer.

Organisation and equipment required

The students will need to work in small groups, according to the number required for each game. All the games may be played with two players, although 'Treasure Island' is designed for four players.

Each student will need:

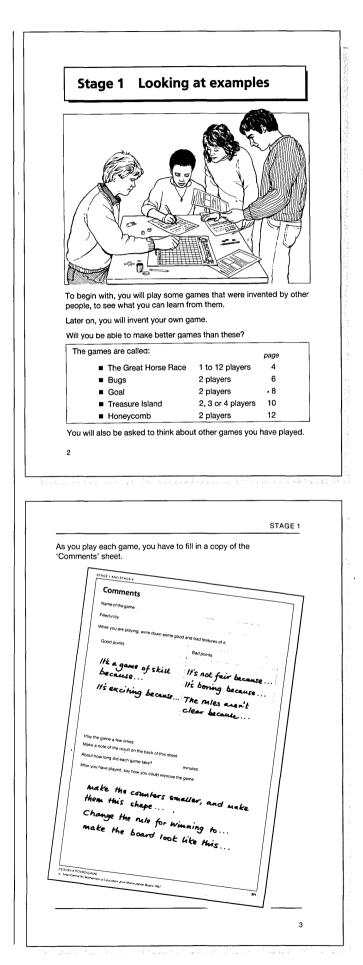
- one Booklet,
- some 'Comments' sheets (one for each game played),
- one 'Other Games' sheet,
- an envelope or folder in which to keep his or her work.

Related assessment criteria

This stage offers students the opportunity to show that they can:

- (i) follow a set of rules,
- (ii) evaluate a design and identify faults in it,
- (iii) devise and evaluate improvements to a design.

Note: assessment tasks are included at the end of each stage and the role of assessment is discussed more fully in Chapter 3.



Purpose

To introduce the module, to explain the purpose of Stage 1, and to involve the students in playing and criticising at least 3 games.

Presentation

An introductory discussion, followed by group work, and concluded with a reflective discussion.

Suggestions and comments

You may wish to introduce the module with a general discussion about board games, perhaps asking for responses to questions like:

Who enjoys playing board games?

Which ones do you like?

What is good about them?

Name a game you find boring.

Why is it boring?

Read through page 1 of the booklet and explain the processes that the students will work through during the next few weeks.

Now look at pages 2 and 3.

Emphasise that

- the games have faults,
- students should note down any criticisms they have, using a 'Comments' sheet,
- students should move beyond superficial comments like

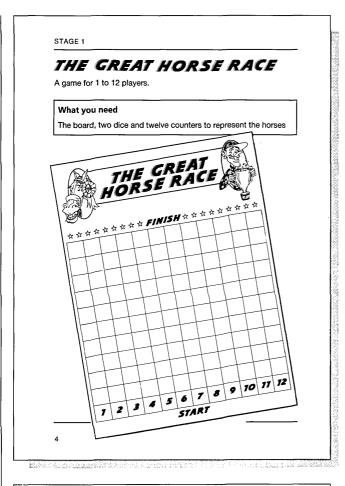
'it's boring'

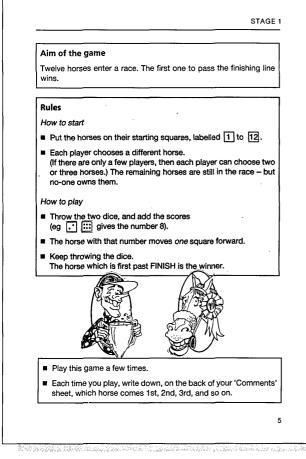
'it's hard to understand'

and explain *why* and suggest ways of improving the games,

a different 'Comments' sheet should be used for each game.

You may like to go through the comments that have already been started (on page 3), asking students to complete them orally with reference to particular games they know.





Throughout the stage, you will need to encourage students to reflect on the games and write constructive comments.

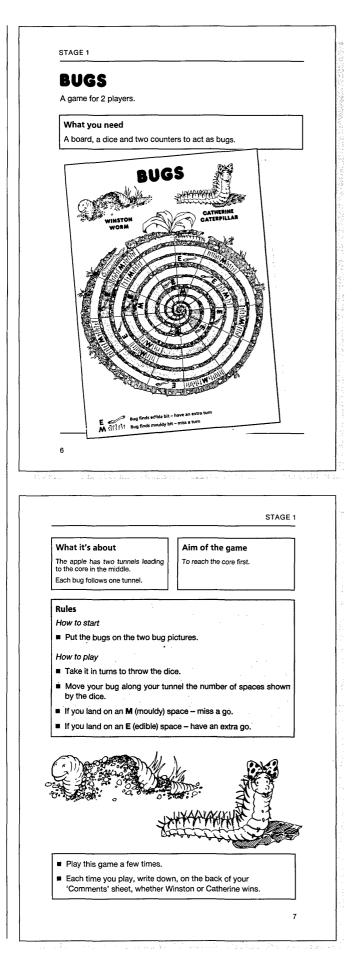
Here are some ideas which promote this:

- Play and discuss one game with the whole class. 'The Great Horse Race' is recommended for this.
- Discuss specific aspects of a game e.g. context, board design, rules, skill or luck factor, fairness.
 Write these headings on the blackboard and suggest that students comment under each.
- Hang large sheets of paper on the wall one for each game. As each group completes a game they add their comments to the sheet. When several groups have done this, the sheets may be used as a focus for a class discussion.
- When a group has suggested improvements to a game ask them to play the game again, incorporating their changes.
- As you go round the class, ask each group to point out and describe the faults they have found. Listen as if you know nothing about the game! If they appear to have misread the rules, ask them to explain each rule and demonstrate its effect. Encourage students to tell you about the game.

Below, we offer a few detailed comments on each game.

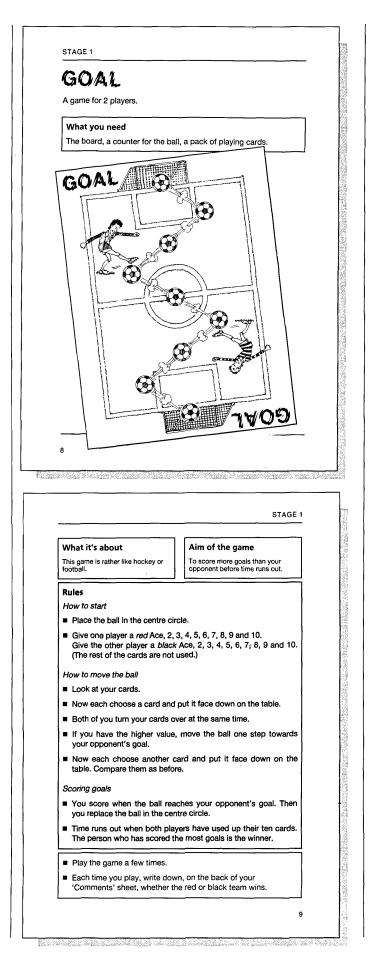
Comments on 'The Great Horse Race'

- A simple game of pure chance.
- The game is unfair as it favours the horses in the middle. It is worth asking students to explain why this is the case.
- Horse number 1 never moves!



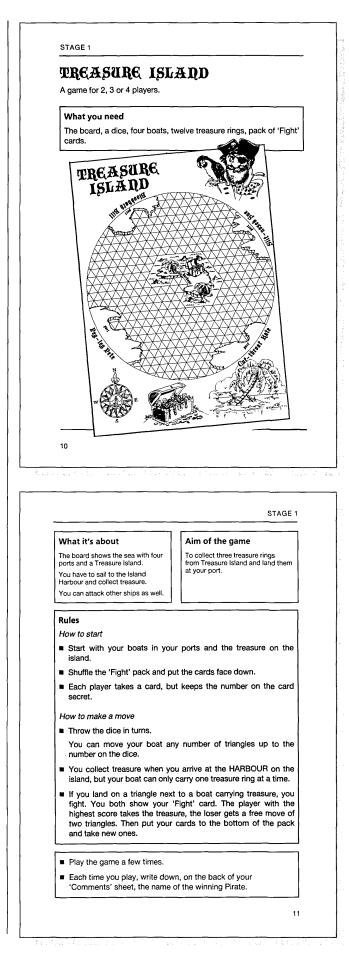
Comments on 'Bugs'

- A simple game of pure chance. No skill is involved.
- It is unfair. Winston worm has further to travel and has more mouldy bits and fewer edible bits to pass through.
- As the game progresses, the counters have to be placed on smaller and smaller spaces. Near the centre this becomes almost impossible!



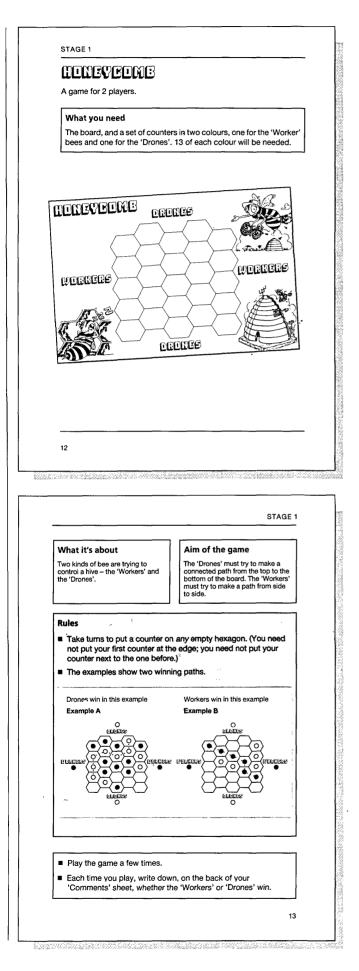
Comments on 'Goal'

- A boring game because it rarely finishes in a result, and if it does, then no more than one goal is usually scored.
- The structure of the board is very simple. The rest is merely for decoration.
- The rules are fairly complex, and are sometimes misinterpreted by students.
- Although the game requires little skill, it is worth trying to remember which cards have been played, and some students enjoy the 'mind reading' element.



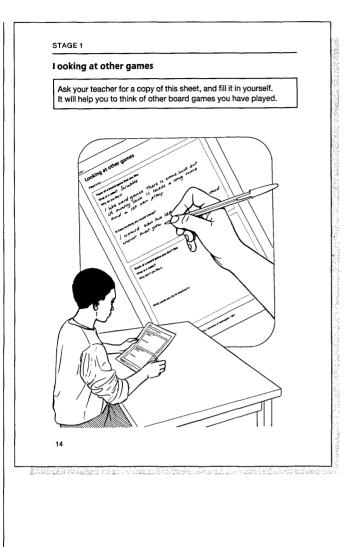
Comments on 'Treasure Island'

- The triangles on the board and the pieces are too small.
- The 'Fight' cards are badly numbered.
- The rules are incomplete. There is no indication of what should happen if the cards drawn in a fight have the same value.
- The rule for moving is ambiguous.
- The rule which restricts a boat to carrying one treasure ring at a time is unfortunate. It prevents two boats which are carrying treasure from engaging in a 'fight'.
- The board is drawn unfairly. 'Peg-Leg Pete' is nearer to the harbour than the other players.



Comments on 'Honeycomb'

- This is a game of strategy which never ends in a draw.
- The board is unfair. The Drones have further to go to create a path when they are blocked. The game thus favours the Workers.



The final activity in this Stage invites the students to think reflectively about other games they have played. This may be used in several ways, for example:

- as homework,
- for the last 5 minutes of a lesson while the equipment is being collected and checked,
- to ease pressure on particular games (this may occur with large classes),
- as the focus for a short lesson, when students could bring into school for discussion some games which are commercially available.

Assessment tasks for Stage 1

On the following pages we offer two assessment tasks which may be helpful when deciding whether or not students can satisfy the Stage 1 criteria.

Assessment task 1 helps to assess whether or not a student can

- (i) follow a set of rules,
- (ii) evaluate a design and identify faults.

Assessment task 2 also relates to these criteria but, in addition, it helps to assess whether or not a student can

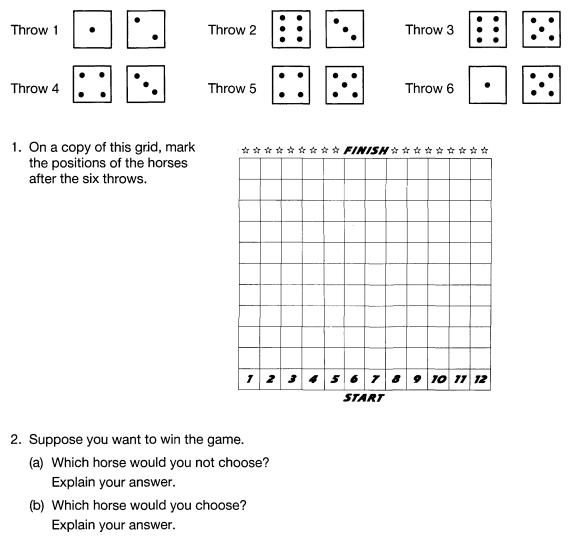
(iii) devise and evaluate improvements to a design.

The Great Horse Race

Turn to pages 4 and 5 of your Student's Booklet.

The 'Great Horse Race' is being played.

The scores on the first six throws of the dice are shown below:



(c) Do you think the game is fair? Explain your answer.

STAGE 1 ASSESSMENT TASK 2

Snakes and Ladders

Read the following description of a game, and answer the questions which follow.

This is a game for 2 players. You will need a coin and two counters.

Rules

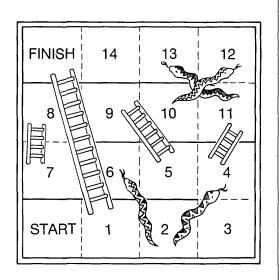
Take it in turns to toss the coin.
 If it is heads, move your counter 2 places forward.

If it is tails, move your counter 1 place forward.

If you reach the foot of a ladder, you must go up it.

If you reach the head of a snake, you *must* go down it.

The winner is the first player to reach 'FINISH'.



- 1. Suppose you start by tossing a head, then a tail, then a head. Where is your counter now?
- 2. List and describe all the faults you notice with the board.
- 3. Using 3 snakes and 3 ladders, design a good game on a copy of this blank board.

FINISH	14	13	12
8	9	10	11
7	6	5	4
START	1	2	3

Stage 2 Developing your own ideas

Introduction

The activities contained in this stage develop a number of important strategic skills which may be applied to almost any design task. These skills include:

- exploring a wide range of alternatives before deciding on a particular solution,
- working through cycles of testing and improving rough designs before making a final version.

As we have already mentioned, such activities are unfamiliar to many students, who tend to embark upon the first idea they find attractive, without considering implications and alternatives.

Total time needed

About 2 hours.

Organisation and equipment required

While actually making the games, small groups of 2, 3 or 4 students will work together. These groups will be formed during the 'reaching agreement' phase.

Each student will need

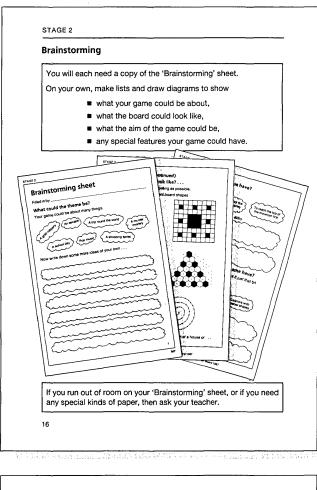
- a 'Brainstorming' sheet
- a 'Rough Plan' sheet
- plenty of rough paper
- access to different kinds of grid paper.
 (e.g. isometric, dotty, etc)

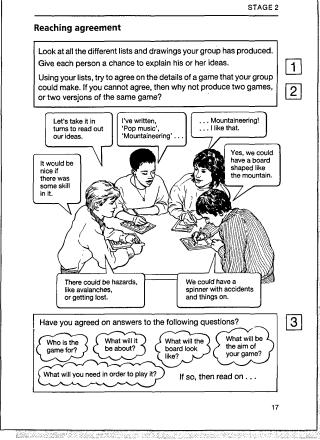
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Related assessment criteria

This stage offers students the opportunity to show that they can:

- (iv) list alternative ideas for design features,
- (v) devise a satisfactory rough plan including spatial layout and instructions,
- (vi) sketch a design to fit given conditions,
- (vii) devise rules to fit given conditions.





Purpose

To give students the opportunity to think of alternatives before discussing their ideas with the group.

Presentation

Class discussion followed by individual work.

Time needed

10-15 minutes.

Suggestions and comments

You may like to begin this activity by asking a few members of the class to suggest one or two ideas for each of the different sections in the 'Brainstorming' sheet, and list these on the blackboard.

Encourage students to fill in all the spaces on the sheet on their own before discussing their ideas. This will ensure that every student has something to contribute to the group discussion which follows.

If students have difficulty with this activity you could ask them to think of games that they have at home, games from stage 1 or even computer games.

Purpose

For everyone to share their ideas in a group, and to reach agreement on which ideas to develop. For students to learn how to listen and discuss constructively with one another.

Presentation

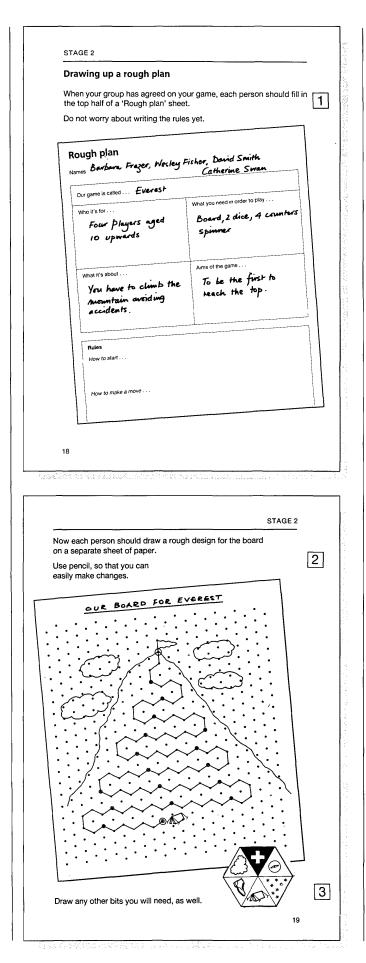
Group discussion.

Time needed

About 10-30 minutes. (This will vary from group to group).

Suggestions and comments

- 1 This may need emphasising, as there is sometimes a tendency for one or two people in a group to dominate the discussion, by pushing their own ideas forward and ignoring those suggested by other students.
- 2 This is an option that can be taken up when negotiations fail, or if a group is too large. A group which contains more than four students will not be fully occupied in the making of a single game, so we suggest that such large groups are broken up at this point.
- 3 The decisions made at this stage are likely to change while the rough plan is being developed.



Purpose

To allow each student the opportunity to record the decisions made by the group, to begin to consider possible spatial layouts for the board, and to write a first draft of the rules.

Presentation

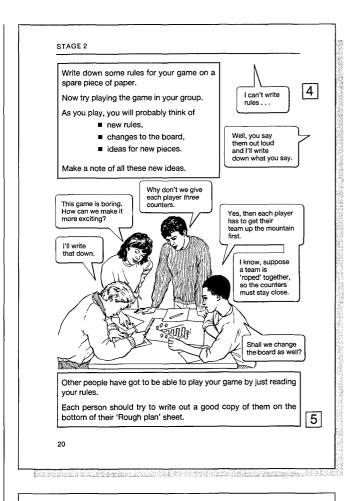
Individual work, interspersed with short group discussions.

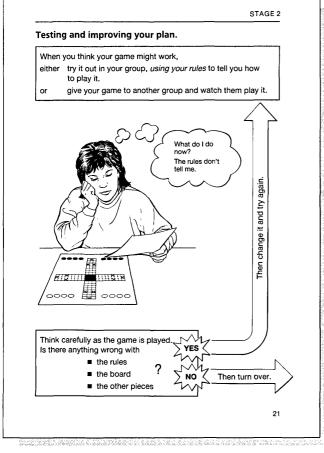
Time needed

About 30 minutes

Suggestions and comments

- 1 In order to involve everyone in the work, we have asked each student to record the group's decisions. Students will either discuss exactly what to write down and thus produce identical versions, or produce different versions and then decide on the best one.
- 2 Students should be encouraged to think about the board layout on their own before sharing their ideas within the group.
- 3 If extra features are designed (for example spinners, or 3-dimensional counters), then, as well as adding to the interest of the finished game, they will promote a wider variety of activities in Stage 3.





The work involved in this page could take much longer than the estimated time, depending on how much ideas are modified.

Encourage groups to develop their ideas as much as possible.

- 4 Many students have difficulty when writing their rules. On this (and on page 21) we encourage students to write them out roughly, then test and refine them until they are complete and coherent.
- 5 You could help students to notice omissions or inconsistencies in their rules by reading them out and trying to interpret them.

Purpose

To check that all aspects of the plan work.

Presentation

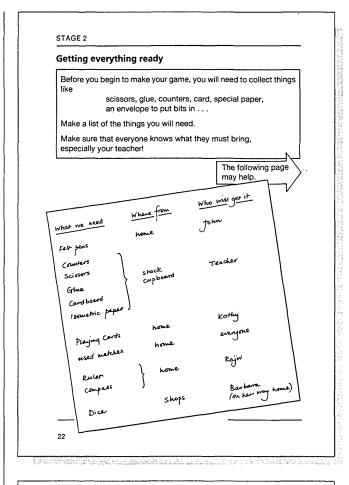
Group (or possibly intergroup) work.

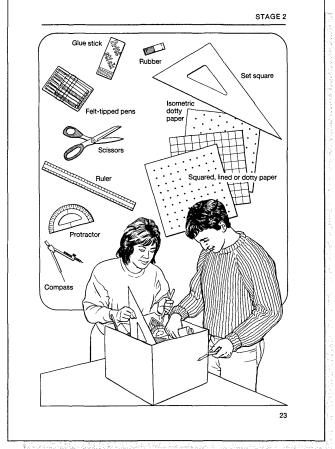
Time needed

15 minutes

Suggestions and comments

It is unlikely that all groups will reach this stage at the same time, but you may be able to arrange some swapping of games between groups that have reached approximately the same stage.





Purpose

To encourage students to think about which resources are needed, where these can be obtained from, and who will be responsible for collecting them.

Presentation

Group discussion.

Time needed

10 minutes

Suggestions and comments

This activity is particularly important if you do not have a permanent classroom base. In these circumstances it may be easier if one member of each group arranges to see you before the next lesson to sort out a box of equipment for his or her group. These people could then be responsible for collecting the boxes before each lesson.

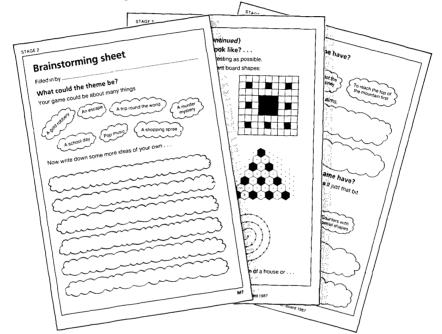
Assessment tasks for Stage 2

On the following pages, we offer four tasks which may help in deciding whether or not students can satisfy the Stage 2 assessment criteria. The first two should already have been completed by students during the normal course of their work, but the remaining two tasks are new, and may be completed at the end of Stage 2.

Stage 2 Assessment task 1

Each student should have completed a 'Brainstorming' sheet at the beginning of the stage (see pages 16 and 17). This may be used to assess whether or not students have shown that they can:

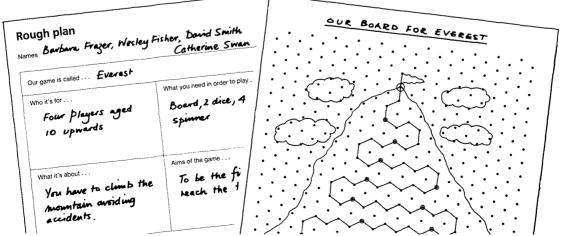
(iv) list alternative ideas for design features.



Stage 2 Assessment task 2

Each student should have completed a 'Rough Plan' sheet (see pages 18, 19 and 20). This may be used to assess whether or not students have shown that they can:

(v) devise a satisfactory rough plan, including spatial layout and instructions.

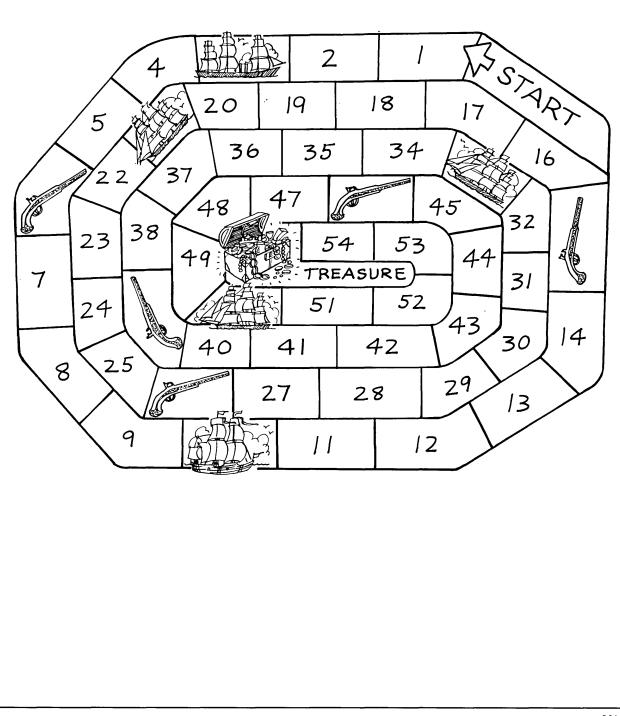


The remaining two tasks may be completed by a student when he or she has completed Stage 2. Assessment task 3 assists in verifying that a student can: sketch a design to fit given conditions. (vi) Assessment task 4 assists in verifying that a student can: (vii) devise rules to fit given conditions. STAGE 2 ASSESSMENT TASK 3 **Deep Sea Diving** One group have been discussing their plans for a board game. Here are some of the things they have decided: Our game will be about The game will be deep sea divers looking for 3 players. for treasure in a sunken shipwreck. The winner will be the first to bring the treasure to the surface. You need to throw a dice to move around the board. There are sharks and other dangers which make the task more difficult. Your task is to design a board for this game. Try to make your design interesting and fair. (You do not need to draw it accurately.)

STAGE 2 ASSESSMENT TASK 4

Treasure

The board drawn below was designed by a group of students, but they haven't written any rules.



STAGE 2 ASSESSMENT TASK 4 (CONTINUED)

Answer sheet for 'Treasure'

Fill in the 'Rough Plan' sheet shown below, so that a young child could understand how to play this game.

Dur game is called	
Vho it's for	What you need in order to play
/hat it's about	Aims of the game
ules ow to start	
How to make a move	
<i>Other rules</i> Make sure you explair	n what happens if you land on a gun or a ship.

Stage 3 Making your game

Introduction

In this stage, the emphasis is on using mathematical and other skills to implement the students' own plans. Each group will need to:

- make the board and other accessories to the highest standard they can achieve,
- write a clear, concise and complete description of the rules of the game.

In addition, the group may also want to design a box to keep the game in.

Total time needed

About 3 to 4 hours.

This stage can take a long time, so you may wish to set a finishing deadline.

Organisation and equipment required

Students are likely to share out tasks according to the particular skills possessed by individuals. If this happens, you will need to ensure that everyone has a task to do and is kept fully involved.

Each group will need:

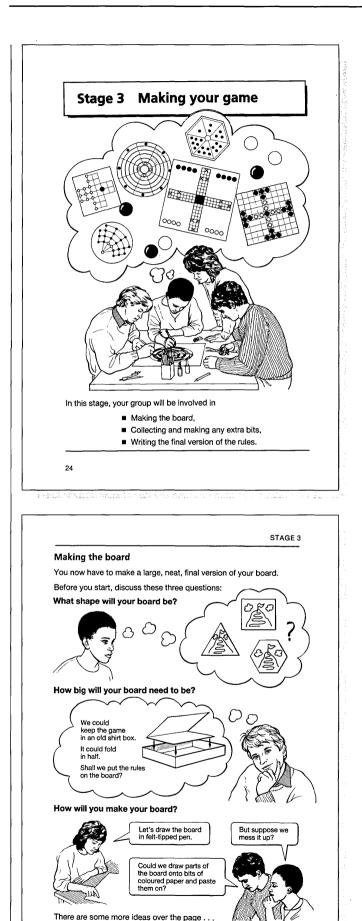
- several copies of the 'Rough plan' sheet,
- the equipment listed by the group at the end of Stage 2.

You may also need to ensure that a supply of basic equipment and a variety of paper and card is constantly available. A suggested list is given at the beginning of this chapter on page T15.

Related assessment criteria

This stage offers students the opportunity to show that they can:

- (viii) estimate the materials that they need to make a final product,
- (ix) draw a design accurately, following geometric and other specifications,
- (x) take an active part in the production process.



Purpose

To encourage students to make a product of which they may justifiably be proud.

Presentation

Group work.

Suggestions and comments

Groups are likely to complete stage 2 at different times. It may therefore be necessary to start each group off on Stage 3 separately.

It is important that throughout this stage, jobs are shared out within groups so that everyone is fully occupied.

If possible ensure that everyone has a task that involves using geometrical techniques. For example, a group of four could divide up the jobs as follows:

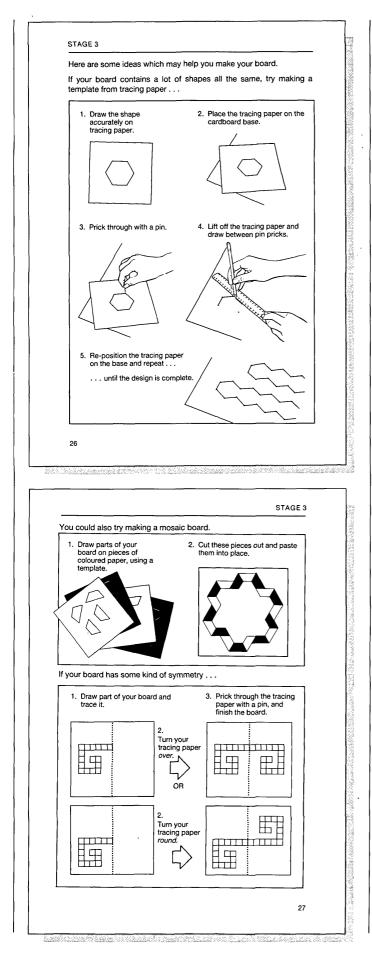
- two people draw the board,
- one person makes a spinner and counters,
- one person cuts up a set of cards and makes a box to keep all the bits in.

Alternatively, a group could perhaps make two copies of the same game.

At the end of each lesson make sure that everyone is aware of the progress that has been made by their group, otherwise unforeseen absence can cause problems. For the same reason, it is better that all the games are kept in school.

Making the board

Before a group shares out tasks, students should discuss the three questions on page 25. It may help some groups if you actually ask the questions and encourage ideas from each student.

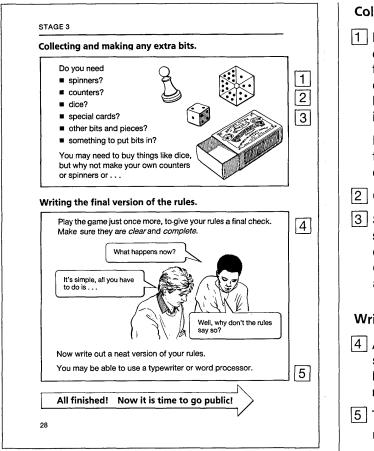


The ideas on these two pages suggest ways of reducing the time and labour involved in drawing and colouring repeated designs on the board, while at the same time making it more attractive and accurate. Students should be encouraged to look carefully at these ideas and choose any which are appropriate for their games.

While students are working through this stage, they may request advice on how to draw particular geometrical shapes (e.g. 'How do you draw a spiral, sir?'). You may also wish to make everyone aware of techniques and equipment that would be particularly helpful.

More help on how to prepare for this is offered in Chapter 2.

Encourage students to make the board look as professionally finished as possible. It may, for example, be covered with some kind of self adhesive transparent plastic.



Collecting and making extra bits

1 It is not always easy to make a fair spinner (or a fair dice). A tally chart, for example, could be used to test whether the numbers generated by a spinner occur in a truly random way.

It may then prove possible to alter it slightly or incorporate the bias into the design of the game.

Hexagonal pencil pieces which may be rolled along the table provide cheap alternatives to spinners or dice.

2 Counters do not need to be 2-dimensional!

3 Students tend to cut their cards from the middle of a sheet thus causing wastage. They should be encouraged to think about the best way of cutting a certain number of cards of given measurement from a large sheet of card.

Writing the final version of the rules

- 4 Although the rules were well tested in Stage 2, some students may wish to amend them still further in the light of their finished board. If this happens, then the rules will need to be checked again.
- 5 The rules could be written
 - on the sheets provided,
 - on the actual board,
 - on a separate piece of card,
 - using a typewriter (if a member of the group can type)
 - using a word processor if one is available.

Assessment tasks for Stage 3

The following three tasks may assist you in deciding whether or not students can satisfy the Stage 3 assessment criteria.

Assessment tasks 1 and 2 help you to verify whether a student can

(viii) estimate the materials needed for a final product.

Assessment task 3 helps you to verify whether a student can

(ix) draw a design accurately, following geometric and other specifications.

Note that criterion (x) which states that students have shown that they can 'take an active part in the production process', may only be assessed by observation of the students during their work in Stages 2 and 3.

Assessment task 1

Show each student a finished board game. It may be a game that has been completed by another group in the class, or it may be a commercially-produced game.

Ask the student to imagine that she is going to make this game at school. Ask her to write out a list showing all the equipment and materials she would need in order to make the game. The student should assume that *every* item apart from the dice, the counters and the box must be made from paper or card. She should include quantities and dimensions wherever possible.

Equipment and materials needed	Quantity or dimensions
Item	
	1 pair
Scissots	40 cm x 60cm
sheet of cardboard	
Land and a second	-

STAGE 3 ASSESSMENT TASK 2

Chasing Packs Here is part of a rough plan for a game called 'Chasing Packs'. Rough plan Names Philip Smith Colette Beaujur Our game is called ... Chasing Packs Who it's for . . . Four Players What you need in order to play , . . Board, pack of playing We will draw the board cards. on a large square sheet 6 counters for each What it's about . . . player of cardboard The players chase each Aims of the game . . . other round the board To win the most cards and try to capture each others cards There mu be 21/2 cm between Rules the track and How to start. Start by dealing out all 52 cards Each player places one country in the board the edge of the board How to make Z'/2 Rough Board Design z'/2 M9 21/2 Each of these small squares nil be 2 cm by 2cm 2'/2

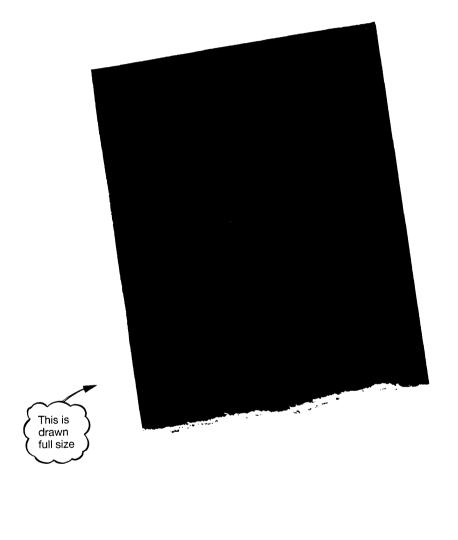
STAGE 3 ASSESSMENT TASK 2 (CONTINUED)

Chasing Packs

Two students are about to make this game.

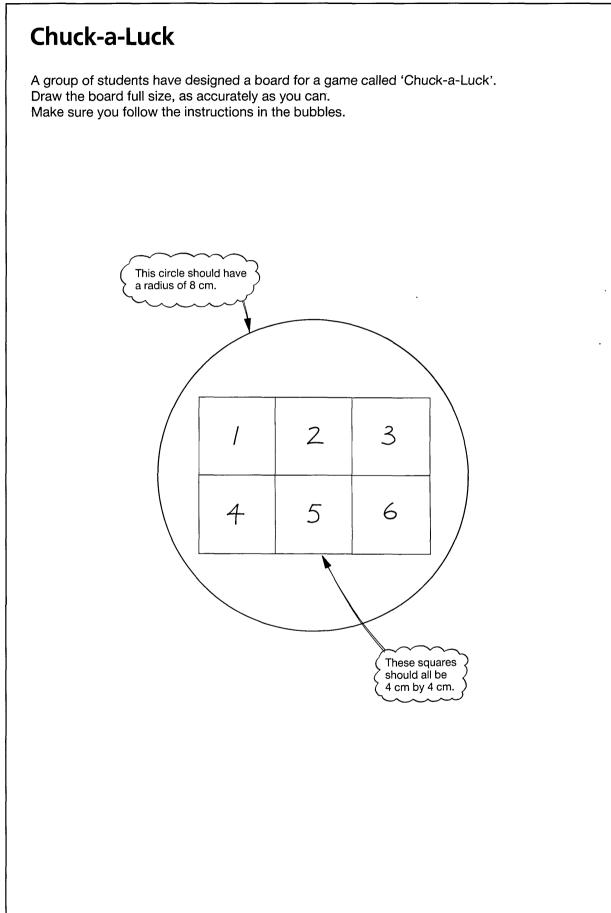
- 1. How many counters will be needed altogether?
- 2. They are going to make a full-size version of the board. Work out how big the large cardboard base should be (use the measurements on the rough plan).

The students have found this scrap of black paper



 They could cut out the black squares on the board from it and then paste them on. How many 2 cm by 2 cm squares can be cut from this piece? (Find the *largest* number you can make).

STAGE 3 ASSESSMENT TASK 3



Stage 4 Testing and evaluating

Introduction

This stage completes the cycle of the design process by inviting students to play and comment on each other's games, using the same method as in Stage 1. They are then asked to react to the comments other groups have made about their game, and encouraged to incorporate any worthwhile suggestions that are practicable.

Total time needed

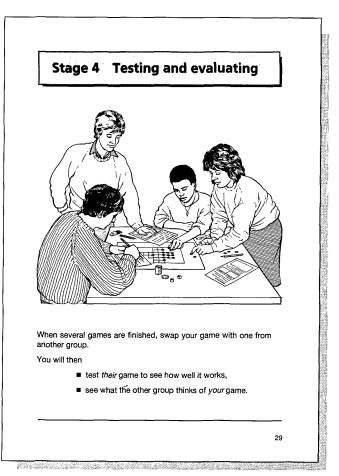
It is only necessary to spend about an hour on this stage, ensuring that each game is played at least twice, although you may wish to spend longer.

Organisation and equipment required

It may prove possible to organise the exchanging of games in a systematic way, but more often, this is governed by the order in which groups finish.

Each group will need:

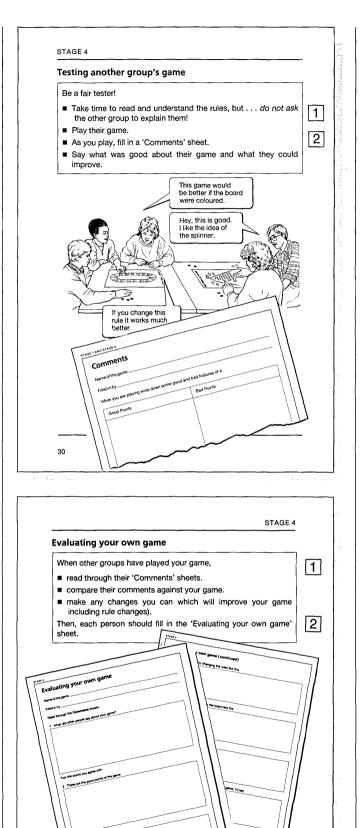
- a 'Comments' sheet for each game they play,
- an 'Evaluating your own game' sheet.



Related assessment criterion

This stage offers students the opportunity to show that they can:

(xi) take part in testing and evaluating their product.



Purpose

To encourage students to make *helpful* comments about a game so that the inventors can improve it still further.

Presentation

A group activity.

Time needed

About 45 minutes

Suggestions and comments

1 Students should attempt to understand the rules without asking for help. If they do need help they should note down the extra explanation that was needed.

2 Students should be encouraged to offer only *constructive* criticism, and no negative comments should be made without a corresponding suggestion for improvement.

Purpose

To incorporate other people's ideas where possible.

Presentation

Group discussion followed by individual work.

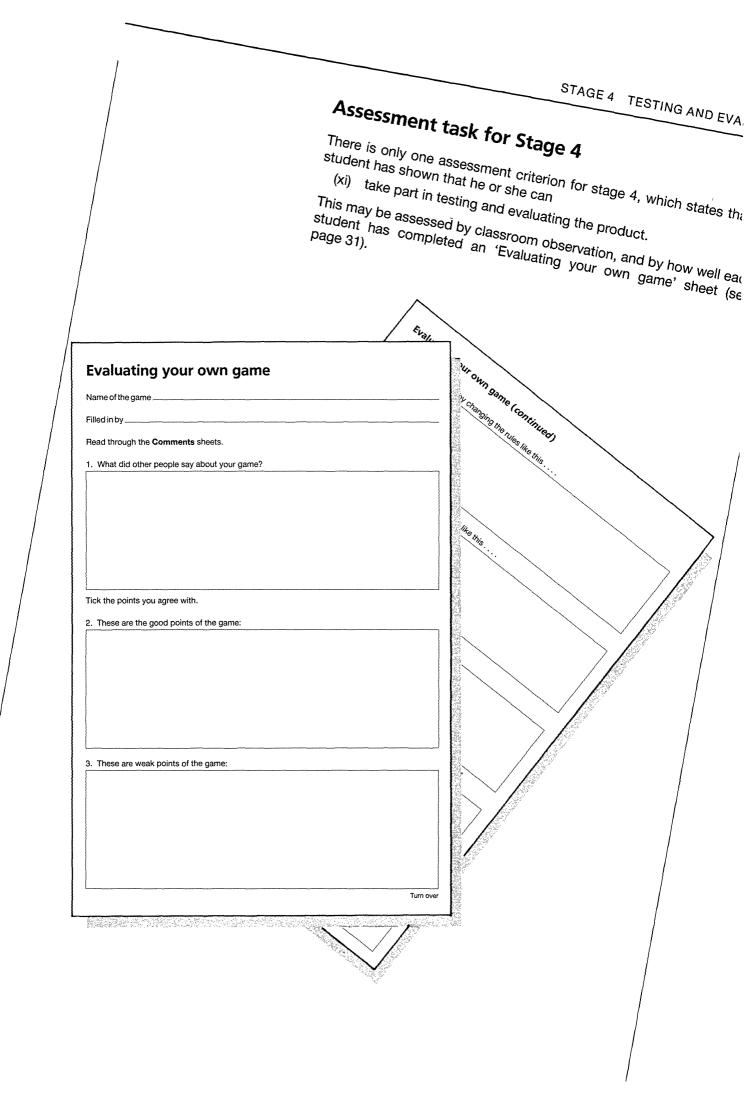
Time needed

15 to 20 minutes

31

Suggestions and comments

- 1 The group should consider other people's comments carefully and make any changes possible to the final version.
- 2 The 'Evaluating your own game' sheet should be filled in by every student. You may wish to make students do this on their own, especially if the work is being used for assessment.



Using the board games

The aim of the classroom material is to produce a game that other people can play. There are, however, ways that the work may be further developed.

Here are some ideas that have been tried.

- Present the games to people outside the class. For example some groups have taken games designed for younger people into local primary schools.
- Look into the possibility of mass producing and selling a game, maybe at a school fête, or even at a local community volunteer shop. This could become a springboard for the consideration of all kinds of manufacturing and marketing issues. For example, the students could look at reprographics, costs, cash flow, materials, production lines, advertising and so on.



Developing the mathematics

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How and when may they be introduced?	52
Some sample worksheets	54
A few solutions to the sample worksheets	61

Developing the mathematics

Introduction

'Design a Board Game' is a rich theme for generating mathematical activity. While students are working on the module, their main objective is to design an attractive, exciting board game – not to develop particular mathematical techniques. However, as a teacher, you may wish to use the module to motivate the learning of mathematics in a more explicit way. This chapter offers some ideas on how this may be achieved without destroying the essential flow of activities contained in the module.

Which skills may be developed?

The table below illustrates some of the mathematical topics related to the context of board games. This list is by no means complete. A group of students may want to design a game within a particular context, and this context may in itself provide opportunities for mathematical development. A game about buying and selling may offer practice at handling money, simple percentages and use of a calculator. An abstract 3-dimensional game will offer more demanding geometrical challenges. A game about the school day may involve using simple timetables and so on.

Mathematical topics	Examples from the module
Number	
 Counting, ordering and combining whole numbers in various ways. 	'How do I combine the scores on these two dice?'
Understanding and using fractional measurement.	'Commercial counters are 1½ cm in diameter, so let's make each square on the board 2 cm by 2 cm. My cardboard base is 29.6 cm wide so I could fit 12 squares in with a space of 2.8 cm each end'
Making estimates.	'How many 'hazard' cards will I need to make? Roughly how big will my board be? How long will the game take on average?'
Spatial Concepts	
 Drawing simple figures to given specifications, and using drawing instruments appropriately 	'How can I make a hexagon template, where each side is 3 cm?'
 Using tessellations to create original designs. 	'Maybe my board could consist of hexagons, squares and equilateral triangles!'
Logic	
Reading and writing instructions	'Are my rules clear and complete?'
Reasoning	'How can I play this game well?'
Statistical concepts	
Understanding and using	
ideas of fairness and bias	'How can I make my game fair?'
 ideas of randomness and variability 	'Is it harder to throw a 6 or a 3?'
simple theoretical and experimental probabilities	'Which horse is most likely to win 'The Great Horse Race'? Why?' Is this what happens in practice?'
measures of average and spread	'How long does the average length game last? Does this vary much?'

How and when may they be introduced?

Mathematical activity may be initiated either by the student or by the teacher. For example:

- a group of students may become aware of their need to acquire a particular skill in order to carry through their own plan. 'How do you draw spirals, Miss?'
- you may wish to use some of the materials to support a more intensive piece of work on a particular topic. 'Today, we're going to do some work on probability, using a game from the 'Design a Board Game' module'.

The first type of situation can lead to valuable learning experiences, because the student *wants* to know something. You can actually 'scratch where they itch'! Such opportunities occur rather unpredictably however and it is difficult to spend a great deal of time helping one group if you have a large class to supervise. One possible solution is to write a few simple 'How to ...' sheets, like the example shown opposite. These may then be used to deal with some of the more common requests for help. Alternatively, a member of the group could describe the group's problem to the whole class and invite help and advice from other students.

Do not expect students to use, autonomously, mathematics that they have only recently been taught. There is a gap, typically of several years, between 'learning' a skill and being able to adapt and use it with flexibility and fluency. Students will only tend to use knowledge or apply those technical skills that they have *mastered*. Narrowing this gap requires a more 'rounded' approach to learning, with a variety of applications and non-routine problem solving to supplement and give meaning to technical exercises.

More intensive work on mathematical techniques relating to the theme of board games may occur before, during or after working on the module:

Before: 'I'll do some work on tessellations now, and then my class will be more able to use tessellations later, when they come on to designing their game.'

This timing has the advantage that the students will have their techniques polished and ready to be used, but it does seem artificial to learn a new technique before seeing a need for it. Students may tend to assume that the module is merely a vehicle for practising these techniques, rather than to develop individual autonomy. During: 'They don't seem to be able to draw as well as I thought. I'll break off for a lesson or two and give them more practice at using rulers and compasses.'

> 'I'll prepare some problem sheets for my class so that I can keep them together. So if one group finishes a stage early, they can do problem sheets until the rest of the class have caught up.'

This timing enables you to respond to needs as they arise, but if students always expect you to produce the method or solution when the going gets difficult, you may reinforce dependence and undermine autonomy.

After: 'When we finish the module, we will spend the rest of the term working on two topics which relate to board games, accurate drawing and probability.'

> 'Now we've finished the module, I'm going to encourage my class to look in depth at one or two simple strategy games. They could explore the effects of changing rules and so on. Maybe they could write up their investigations for GCSE coursework.'

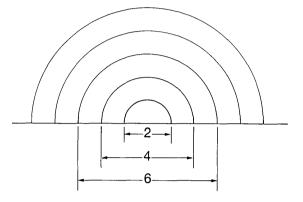
The experience of working on the module may motivate and enable students to perceive the value of techniques when they are taught. However, in this case the techniques still need to be related to a real problem solving activity.

Whatever you decide, it is important to be vigilant about preserving the students' autonomy and strategic control of the problem; it is only too easy to allow them to revert to the imitative role that the traditional curriculum encourages. On the following pages we offer a few resources for developing mathematical techniques on non-routine problems. You may like to devise some more of your own.

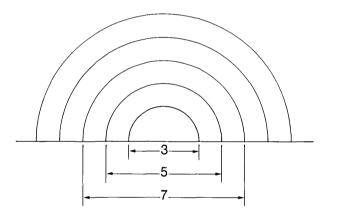
How to . . . draw a spiral

You will need: Pencil, ruler, compasses, scissors, glue, rubber.

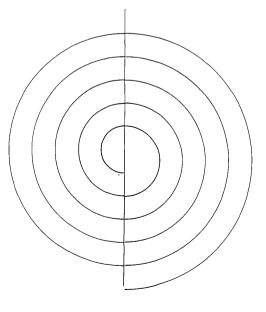
1. Draw some concentric semi-circles, with diameters of 2, 4, 6, 8 . . . units.



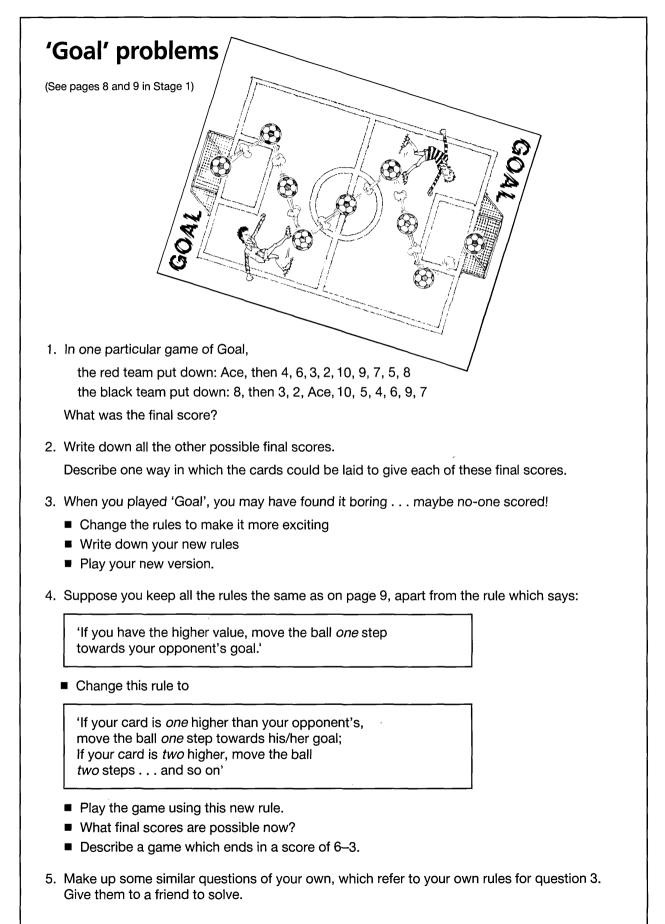
2. Now draw some more, with diameters of 3, 5, 7, 9... units.

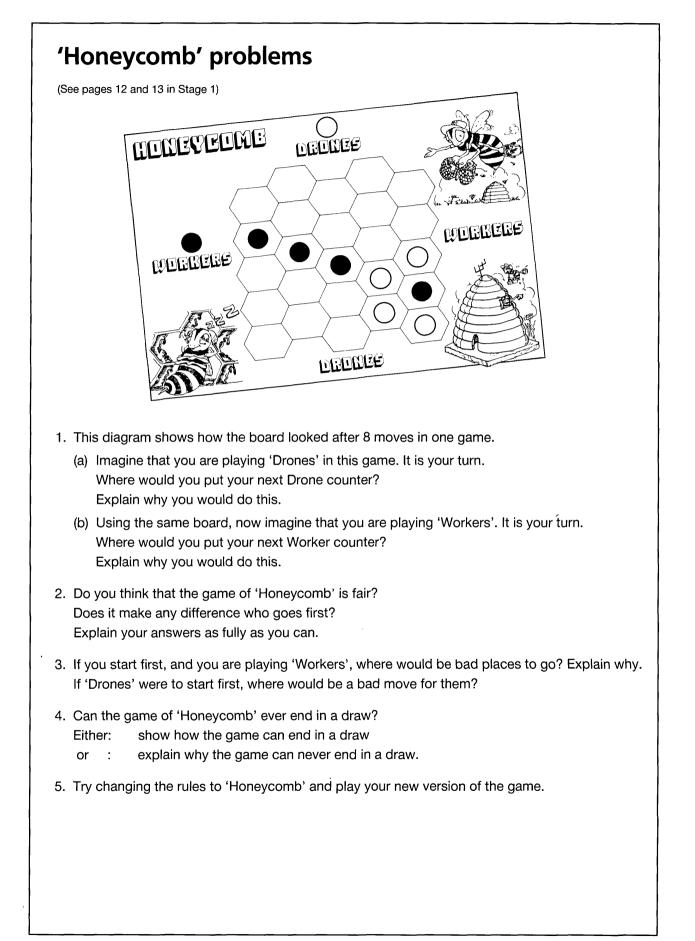


3. Now join the two halves together:



Some sample worksheets





Making and playing board games

Choose a game from those described below. Draw the board, full size, as accurately as you can. Play the game with a friend and comment on it. Now try changing the rules . . .

Ashi – a game for 2 players from Ghana. What you need: 8 counters, 4 of one colour, 4 of another.

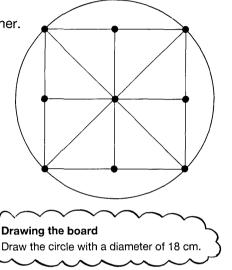
Aim of the game: To get 3 counters of your colour in a straight line, before your opponent does.

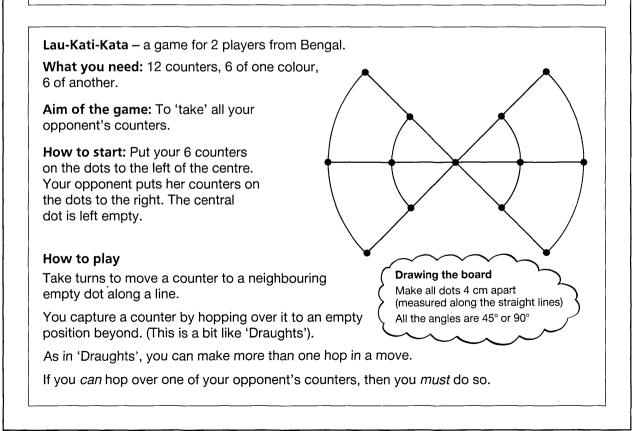
How to play

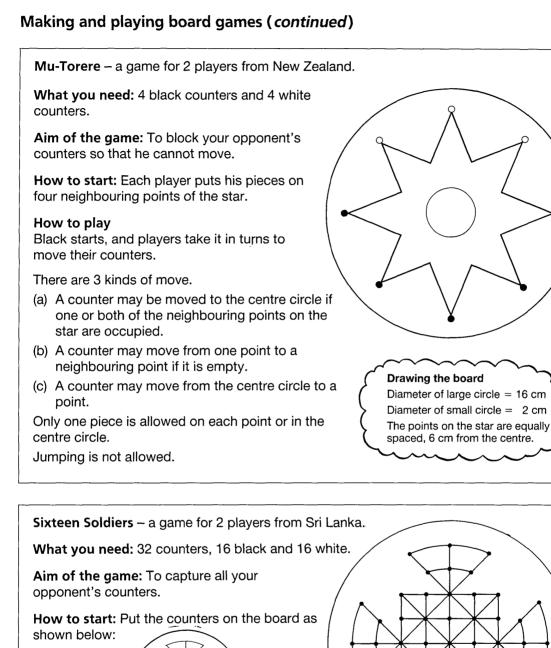
Take turns to put a counter on the board, like Noughts and Crosses.

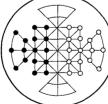
When all the counters are on the board, take turns to move a counter to a neighbouring empty position along the lines.

(You can only move a counter of your own colour).



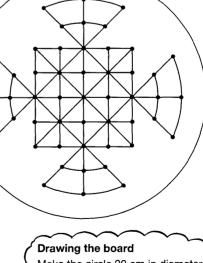






How to play

dot along a line.



Make the circle 20 cm in diameter

As in Draughts, you can make more than one hop in a move.

If you *can* hop over one of your opponent's counters then you *must* do so.

empty position beyond. (This is a bit like 'Draughts').

Take turns to move a counter to a neighbouring empty

You capture a counter by hopping over it to an

Looking for winning strategies

Make one of the following games and play it.

In each game, one player can make sure that he or she always wins.

- Try to find this winning strategy
- Try changing the rules, and see what happens

Now try playing a different game.

The Spiral Game

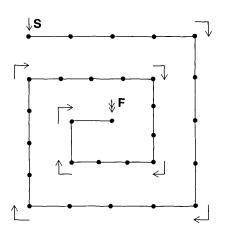
This is a game for 2 players.

Place a counter on the dot marked ' \downarrow S'.

Take it in turns to move the counter inwards, along the spiral.

On your turn you can only move the counter 1, 2, 3, 4, 5 or 6 dots.

The first player to reach the dot marked ' F' wins.



Pin them down!

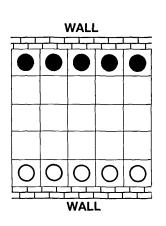
A game for 2 players.

Place the counters on the board as shown.

The players take it in turns to slide one of their counters up or down the board *any* number of spaces.

No jumping is allowed.

The aim is to stop your opponent from being able to move, by pinning all her counters to the wall.



Looking for winning strategies (continued)

First one home

This game is for 2 players.

Place the counter on any square of the grid.

Now take it in turns to slide the counter any number of squares due West, South or Southwest (like the dotted arrows).

The winner is the player who moves the counter to "Finish".

	-			•		_	
			1				
		1					
				+			
_							
INISH	 						·

Domino square

This is a game for 2 players.

You will need a supply of 8 dominoes or 8 paper rectangles.

Each player in turn places a domino on the square grid, so that it covers two squares.

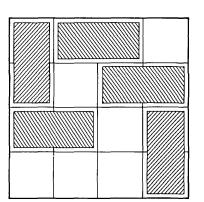
After a domino has been placed, it cannot be moved.

If you cannot place a domino on the grid, you lose the game.

For example:

This board shows the first five moves in one game.

It is the second player's turn. How can she win with her next move?



M28

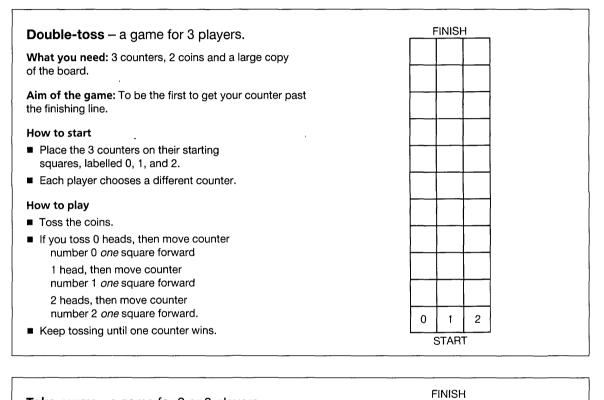
Games and probability

Read through the games described below.

Before you play a game, try to answer the following questions:

- Is the game fair?
- If the game is played a lot of times, who would you expect to win most often? Who do you expect to come second, third, . . . last? Explain your reasoning.

Now play the game and see if your predictions are correct! Try to explain what you notice.



Take-away - a game for 2 or 3 players. What you need: 5 counters, 2 dice and a large copy of the board. Aim of the game: To be the first to get your counter past the finishing line. How to start Place the 6 counters on their starting squares, labelled 0 to 5. Each player chooses 2 or 3 counters (depending on the number of players). How to play Roll the dice. Find the *difference* between the numbers you roll. (e.g. :: • gives a difference of 3) Move the counter which is labelled with this difference, one square forward. (e.g. :: • would mean that you move counter number 3 0 4 5 1 3 2 one square forward) START

A few solutions to the sample worksheets

'Goal' problems (page T54)

- 1. The final score was red team 1, black team 0.
- 2. Other possible final scores are 0-0, 1-0, 0-1, 1-1, 2-0, or 0-2.
- 4. If the red team play 5, 6, 7, 8, 9, 10, 4, 3, 2, Ace and the black team play Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, then the final score is red team 6, black team 3.

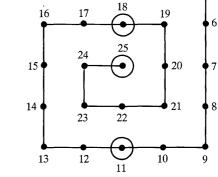
'Honeycomb' problems (page T55)

- 1. (a) If 'Drones' play their next counter on hexagon A, then the 'Workers' cannot prevent the 'Drones' from winning.
 - (b) Similarly, if 'Workers' have the next move, they should play on hexagon A.
- 2. The game is biased in favour of the 'Workers'.
- 4. The game cannot ever end in a draw.

Looking for winning strategies (page T58)

The Spiral Game

If the first player moves to position 4 then, whatever the second player does, the first player can always land on 11, 18 and 25.



DRONES

DECNES

3

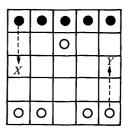
2

Rocases

А

WORBERS

6

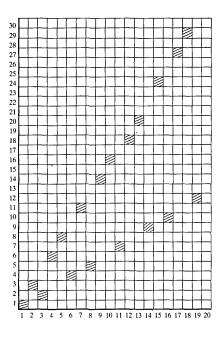


Pin them down!

This game should result in a win for the player who moves first. If white moves the centre piece as far forward as possible, then whatever piece black moves on one side of this column, white can imitate on the other side (e.g. if black moves to X then white can imitate with Y). This kind of symmetrical play will ensure that white will have the last forward move, and then go on to win.

First one home

The player who can first move the counter onto a shaded (^{III}) square must win. Whatever the other player does, he cannot prevent this player from moving from shaded square to shaded square and eventually into the bottom left hand corner.



Domino square

Player 2 can always ensure a win on a 4×4 board by placing his dominoes in such a way that the pattern of dominoes is always kept rotationally symmetrical about the centre point of the board.

For example, if player 1 puts her domino in position A, player 2 should respond by placing his at B; player 1 puts hers at C then player 2 should put his at D etc.

Games and probability (page T60)

Double toss

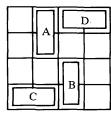
Theoretically, in this game, you would expect one head to occur twice as often as no heads or two heads.

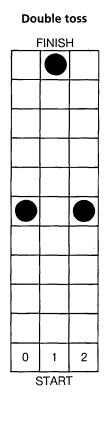
If this happened the board would appear as shown.

Take-away

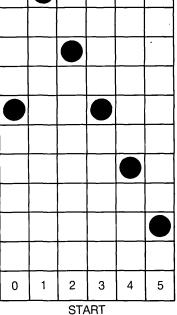
In this game, the counter labelled '1' is the most likely to win.

Again, the theoretical distribution has been illustrated on the board.











The role of assessment

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Assessing a student's contribution to the work of a group	65
Assessing with reference to criteria	65
Recording students' achievements	66
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The role of assessment

Introduction

You may wish to assess your students' work for a variety of reasons. These range from using the module in the context of an externally administered assessment scheme (e.g. GCSE) to using it to measure the students' performance and to make them more aware of their own progress.

This module encourages the development of many different types of skill. These include (with examples)

- technical skills (the ability to measure accurately),
- strategic skills (the ability to plan and design),
- social skills (the ability to discuss and listen).

Each of these poses a very different assessment challenge. In this chapter, we will focus on ways of assessing and recording

- a student's contribution to the work of a group,
- the strategic and technical skills which we have identified in the introduction (pages T5 and T6), and which are described in terms of criteria to be satisfied.

Illustrations will be drawn from the collection of assessment tasks that followed each stage of the material in Chapter 1. In addition, we provide examples of two written examination papers, at different levels of difficulty, which may be used to assess the students' retention and ability to transfer their skills to fresh situations within the same context.

Assessing a student's contribution to the work of a group

The teaching material encourages group work. This is mainly done to develop each student's ability to discuss, listen and cooperate with his or her peers. In addition, a group which works well together will often design a more imaginative product than its individual members would produce in isolation.

However, for most assessment purposes, you are likely to be interested in an individual's contributions to, and understanding of, the work of the group. Two extreme ways of assessing these are:

through one-to-one discussions with each individual student, possibly using a checklist of some kind. This method is time-consuming, particularly with larger classes, and it is difficult to carry out an interview while you are simultaneously trying to supervise the whole class,

through students completing a number of written tasks, on their own, at various stages of the work, these tasks reflecting the work that they have done in the group. However, some skills cannot be assessed in this way, and since students have difficulty in expressing themselves in writing, their responses may not fully reflect their capabilities.

A combination of these methods may provide an optimum solution. Some written tasks may be used, where appropriate, and some follow-up interviews may be conducted to clarify incomplete or confusing responses. With a class of very low attainers, where the numbers are often small, you may prefer to assess using only oral methods.

Assessing with reference to criteria

The following criteria have been used throughout the classroom materials. Students are given the opportunity to show that they can

- (i) follow a set of rules,
- (ii) evaluate a design and identify faults in it,
- (iii) devise and evaluate improvements to a design,
- (iv) list alternative ideas for design features,
- (v) devise a satisfactory rough plan including spatial layout and instructions,
- (vi) sketch a design to fit given conditions,
- (vii) devise rules to fit given conditions,
- (viii) estimate the materials that they need to make a final product,
- (ix) draw a design accurately, following geometric and other specifications,
- (x) take an active part in the production process,
- (xi) take part in testing and evaluating their product.

Such criteria are useful in giving both the teacher and the student a useful profile of relative strengths and weaknesses, but what does it mean to 'satisfy' a particular criterion in any absolute sense? A statement such as 'the student can follow a set of rules' is unhelpful without specifying

the context. Following a set of rules for playing a game is very different from following a set of rules for assembling a car.

- the complexity within the context. Following the rules for playing 'noughts and crosses' is very different from following the rules for playing 'chess'.
- the frequency of success. Has the student 'satisfied' the criterion on many occasions or only once? Has the criterion been satisfied at every attempt or only on a small proportion of the attempts?
- the amount of help that was given. Did the student receive help from others in the group, or from the teacher, or did he or she perform the task completely alone?
- the occasion. When has the student been asked to do the task? Did the student have any recent experience of a similar situation, or was the task administered in isolation?
- the mode of response. Has the student demonstrated that he or she can satisfy the criterion in writing, or do you have only oral evidence?

If students work through several modules in this series, they will be required to demonstrate similar strategic skills in a variety of different contexts. In this event you will be able to make more general statements about a student's ability to satisfy certain criteria. From the assessment of this module alone, you will only be able to make statements like 'the student has shown the ability to follow the rules for a variety of simple board games'.

Recording students' achievements

You may find it helpful to record your students' successes on a grid like the one shown opposite. The criteria associated with this particular module have been listed horizontally across the top, and there are spaces for students' names to be filled in vertically down the side. There are many ways of filling in the cells in this grid. You may wish to record only that a student has satisfied or 'passed' a criterion (for example, by entering a P), or you may wish to qualify this by recording, for example, the amount of help that was given and the frequency of success. Thus,

- P may mean 'pass with no help'
- Ph may mean a 'pass with a little help'
- PH may mean a 'pass with a lot of help'
- PP may mean that a student has 'passed' the criterion on more than one occasion

If a student has not yet shown that he or she can pass a criterion, the cell should be left blank, and the student given a further opportunity to pass it at a later date. We suggest that only positive achievement is recorded and that no cells are labelled with an F (for failure).

An alternative would be to use a numbering system on a three or five point scale. There is always the temptation to add up such numbers so that a single result can be obtained for each student. Although such results are convenient, they are meaningless and miss the whole point of criterion referenced assessment.

Record sheet Criteria satisfied Stage 1 Stage 2 Stage 3 Stage 4 devise a satisfactory rough plan, includingispatial layout and instructions. take an active part in the production process. draw a design accurately, following geometric and other specifications. (viii) estimate the materials needed to make a final product.) devise and evaluate improvements to a design.) take part in testing and evaluating their product. list alternative ideas for design features. evaluate a design and identify faults in it. sketch a design to fit given conditions. (vii) devise rules to fit given conditions. A student has shown that he or she can: follow a set of rules. (x) ▣ X € ŝ Ξ Ē X Name Ξ

Some sample assessment tasks

We will now illustrate some of these ideas, using sample assessment tasks from each stage of the classroom materials. With each task, we give

- the related assessment criteria,
- the type of response which will 'satisfy' the criteria, annotated with examples of students' work. These 'satisfactory' responses are those that we believe are achievable by a substantial majority of the school population. You may wish to set more demanding standards for more able students. (Throughout, we have used the P, Ph or PH notation as defined in the introduction (page T66)).

The tasks included here are:

Stage 1, Assessment task 1	'The Great Horse Race'
Stage 2, Assessment task 3	'The Deep Sea Diving game'
Stage 3, Assessment task 3	'The Chuck-a-Luck game'
Stage 4, Assessment task	The 'Evaluation sheet'.

We follow this by offering two written examination papers, which may serve as a useful resource in assessing a student's ability to retain and transfer the skills to fresh situations in the same context. Paper 1 is intended for nearly all students while paper 2 is intended for approximately the top 30% of students.

9 10 11 12

START

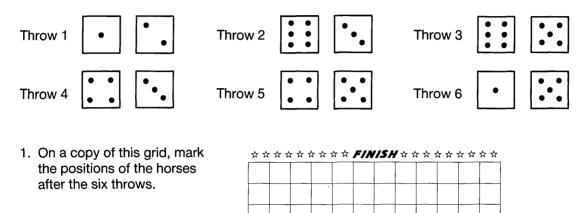
STAGE 1 ASSESSMENT TASK 1

The Great Horse Race

Turn to pages 4 and 5 of your Student's Booklet.

The 'Great Horse Race' is being played.

The scores on the first six throws of the dice are shown below:



1 2 3 4 5 6 7 8

- 2. Suppose you want to win the game.
 - (a) Which horse would you not choose? Explain your answer.
 - (b) Which horse would you choose? Explain your answer.
 - (c) Do you think the game is fair? Explain your answer.

Stage 1 Assessment task 1 question 1

Related criterion: (i) A student can follow a set of rules.

Response required

	START											
	1	2	3	4	5	6	7	8	9	10	<i>11</i>	12
			×			×	×		x		×	
									×			
The correct response is:												

However, if just one or two Xs are omitted or incorrectly positioned, it may still be the case that the student does understand the rules and, if prompted, he or she would be able to correct the error. (Ph could then be entered on the grid.)

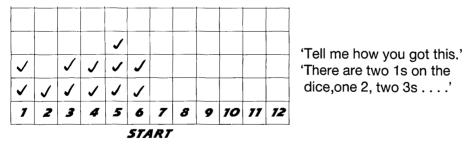
Lee's work, reproduced below, provides one such example.

Lee

												'What is wrong he 'I just slipped up ₁ adding.'
		×			×	×		X	×	×		ladang.
1	2	3	4	5	6	7	8	9	10	17	12	
1	2	3	4	5	6	7	8	9	10	17	12	
					STA	1 <i>RT</i>	,					

If more that two Xs are incorrectly positioned, then it is likely that the student has not understood the rules. Mandy provides one such example.

Mandy



If when asked to re-read the rules, Mandy is able to revise the response satisfactorily, then Ph would be a suitable entry. If more help is needed then PH would be suitable.

Stage 1 Assessment task 1 question 2

Related criterion: (ii) A student can evaluate a design and identify faults.

Response required

- 2. (a) Either 1, because this number is impossible to throw, or 2, 3, 11 or 12, because these numbers are harder to throw.
 (Answers of 4, 5, 9 or 10 may need to be followed up with an interview.)
 - (b) 6, 7 or 8, because these numbers are likely to occur most often.
 - (c) The game is unfair because it favours the middle horses. (You may wish to demand a more complete explanation from more able students.)

To pass the criterion there must be some indication that the fault (given in the answer to part (c)) is understood. Although Richard has not answered part (c) in a satisfactory way, he has indicated that he understands the fault in his answer to part (a). He should be asked to expand on that answer, and a P, Ph or PH awarded accordingly.

Richard

Suppose you want to win the game.

- (a) Which horse would you not choose? I would not choose 10 Explain your answer.
 10 is always harder to get on a throw than 7
 (b) Which horse would you choose? Explain your answer.
 I would choose horse number 7.
- (c) Do you think the game is fair? Explain your answer. No it is not a fair game.

STAGE 2 ASSESSMENT TASK 3

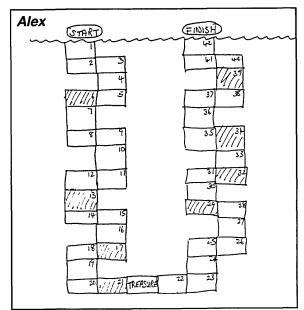


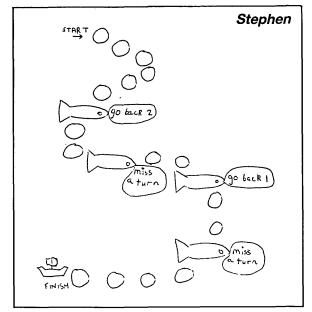
Stage 2 Assessment task 3

Related criterion: (vi) A student can sketch a design to fit given conditions.

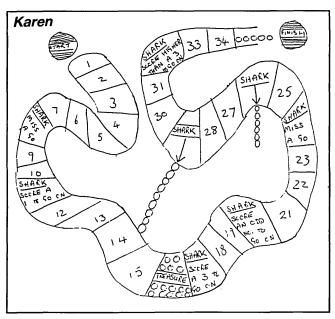
Response required

In order to satisfy the criterion, a student must take all the conditions (given in the speech bubbles) into account. Alex and Stephen have both failed to do this.





Alex has not shown 'sharks and other dangers', although he may have intended the shaded squares to represent these. Stephen has not taken account of the fact that 'the winner will be the first to bring the treasure to the surface'. Both students need to be asked to check their boards against the conditions, and make the necessary changes, before they can be awarded a Ph or PH.

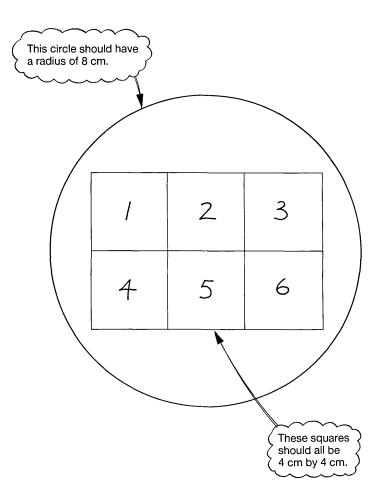


Karen's design is much clearer, except for the discontinuity at square 26. She would almost certainly be able to correct her mistake if she were asked, and so a P should be recorded.

STAGE 3 ASSESSMENT TASK 3

Chuck-a-Luck

A group of students have designed a board for a game called 'Chuck-a-Luck'. Draw the board full size, as accurately as you can. Make sure you follow the instructions in the bubbles.



Stage 3 Assessment task 3

Related criterion: (ix) A student can draw a design accurately, following geometric and other specifications.

Required response

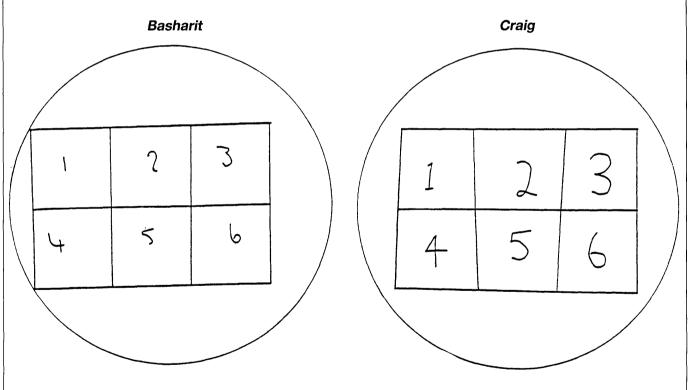
A minimum requirement may be that

- the radius of the circle must be 8 cm (± 2 mm)
- the rectangle must measure 12 cm by 8 cm (± 2 mm on each dimension)
- the board is complete and numbered correctly.

However, for more able students you may also wish to assess the accuracy of

- the subdivisions of the rectangle
- the location of the rectangle within the circle.

Both Basharit's and Craig's boards satisfy our minimum requirements, but Basharit has not centred the rectangle and Craig has not subdivided the rectangle accurately. (Note that these drawings have been reduced from the original size.) It would be helpful to discuss these problems with the students and allow them to attempt the task again.



Stage 4 Assessment task

Related criterion: (xi) A student has taken part in testing and evaluating his or her own product.

Response required

At least four of the questions on the 'Evaluating your own game' sheet should be answered in a reasonable manner consistent with the student's own game.

In the examples shown below, Rasak and Marie have both given very brief answers, but have nonetheless shown that they were involved in the testing and evaluating process. They should both be encouraged to expand upon their answers, either orally or in writing, before a P is entered on the record sheet.

· · · · · · · · · · · · · · · · · · ·	STAGE 4
Evaluating your own game	Evaluating your own game (<i>continued</i>)
ame of the gameGrand Prix	4. I could improve it by changing the rules like this
illedinbyRAZAK	none
lead through the Comments sheets.	
. What did other people say about your game?	
It was well set up. It did not take	5. I could improve it by changing the board like this
so long to play.	
The shapes are not the right size for the cars and it is not very good for the board.	none
ck the points you agree with.	6. Next time I design a board game, I will
These are the good points of the game:	do a different doning but still he
It was well set up and fair	do a different design but still keep it straight-forword.
	 If a friend asked my advice about designing a board game, I'd say
	I would get the idea first and
These are weak points of the game:	write a set of rules then draw
	the design and the numbers.
none	8. Space for further points
	l enjoyed doing it and learned something about that.
Turn over	
NA BOARD GAME M16	DESIGN A BOARD GAME

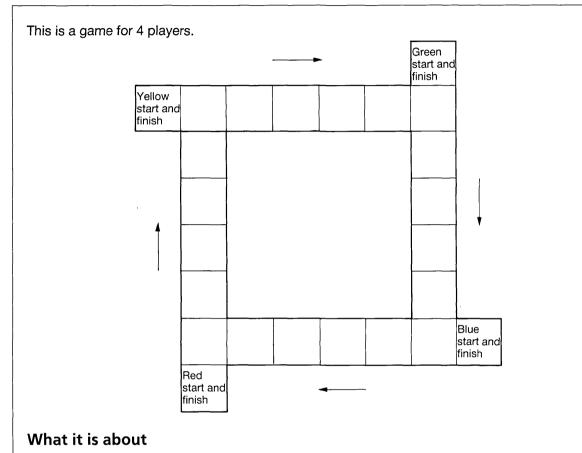
valuating your own game	
ame of the gameHorse Race	
ledinby Marie	
ead through the Comments sheets.	
 What did other people say about your game? 	
It was fast	
Tick the points you agree with.	
2. These are the good points of the game:	
lt was fast	
•	STAGE 4
	Evaluating your own game (<i>continued</i>)
	4. I could improve it by changing the rules like this
3. These are weak points of the game:	
It did not have any playing cards	
	5. I could improve it by changing the board like this
SN A BOARD GAME	Tum over
ell Centre for Mathematical Education/Joint Matriculation Board 1987	6. Next time I design a board game, I will
	think more carefully about all the points
	think more carefully about all the points of the game and try not to make it
	a flop.
	7. If a friend asked my advice about designing a board game, I'd say
	try and find an idea which most people will enjoy and make it so it was not complicated because it spoils the game if it's hard to understand. Go on Luck!
	8. Space for further points
	DESIGN A BOARD GAME

DESIGN A BOARD GAME © Shell Centre for Mathematical Education/Joint Matriculation Board 1987

PAPER 1 ASSESSMENT TASK 1

Overtaking

Here is the description of a game called 'Overtaking'. Read it through and answer the questions which follow.



Each player has a counter.

Players take it in turn to throw a dice.

Each player moves his or her counter around the track the number of squares shown on the dice, in the direction of the arrows.

How to win

The first player to complete the circuit back to their 'home' square is the winner.

Rules

- 1. Red starts first, then Yellow, then Green, then Blue.
- 2. If a counter lands on the same square as another, both counters are removed and those two players are out of the game.

The remaining players continue to play in the same order as before.

continued

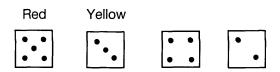
PAPER 1 ASSESSMENT TASK 1 (CONTINUED)

Overtaking

Using counters if you wish, become familiar with the game.

Then answer the questions.

The first 4 throws of the dice give these numbers:



1. On a copy of the diagram, show the position of each counter after the matching moves.

•

The next three throws give

•		•	•	•
•	•	•	•	

2. Show the *final* positions of the counters after all the moves.

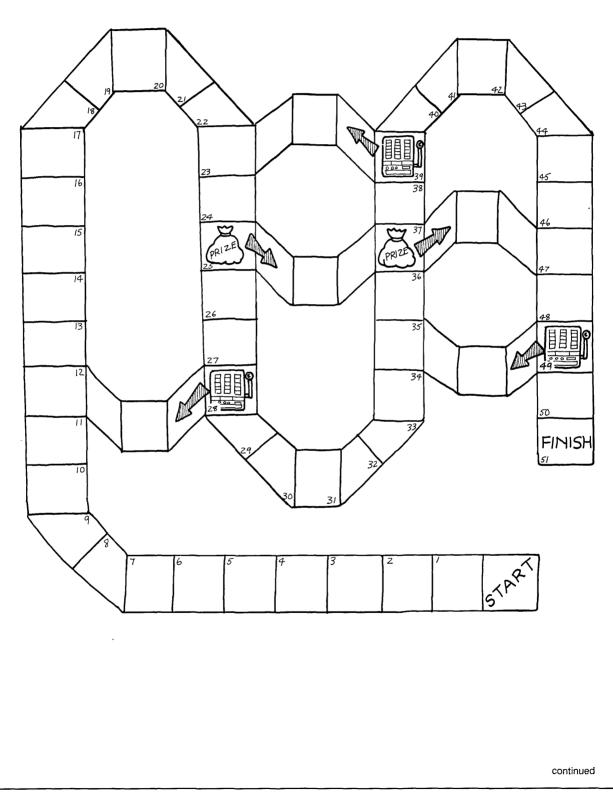
3. Which counter has moved the furthest round the track?

- 4. 'Now look what's happened! No-one can win this game!' Explain how this situation could happen in the next move.
- 5. Change one of the rules so that there will always be a winner. Write down the changed rule.

PAPER 1 ASSESSMENT TASK 2

Arcade

This is the rough plan of a board for the game of 'Arcade'.



PAPER 1 ASSESSMENT TASK 2 (CONTINUED)

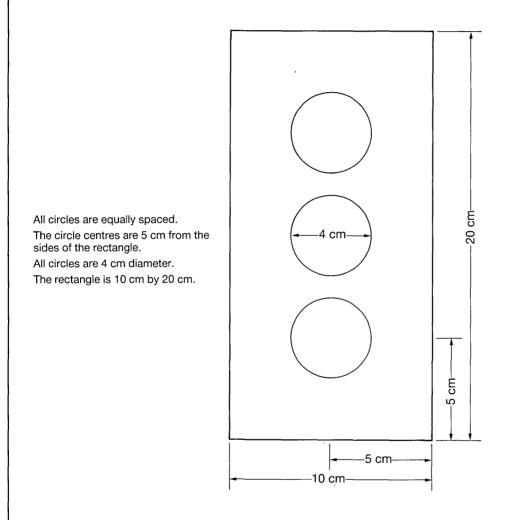
Rough plan	
Names	
Dur game is called Arcade	
Who it's for	What you need in order to play
2-4 players	a counter for each player, a dice
Albet His shout	
What it's about going round an amusement arcade	Aims of the game
	Aims of the game
going round an amusement arcade Rules	Aims of the game

PAPER 1 ASSESSMENT TASK 3

Nsolo

Nsolo is a game for 2 players. Many versions of the game are found in parts of Africa and India with different names.

A small picture of part of the board is shown here, with a row of 3 equal-sized circles inside a rectangle.

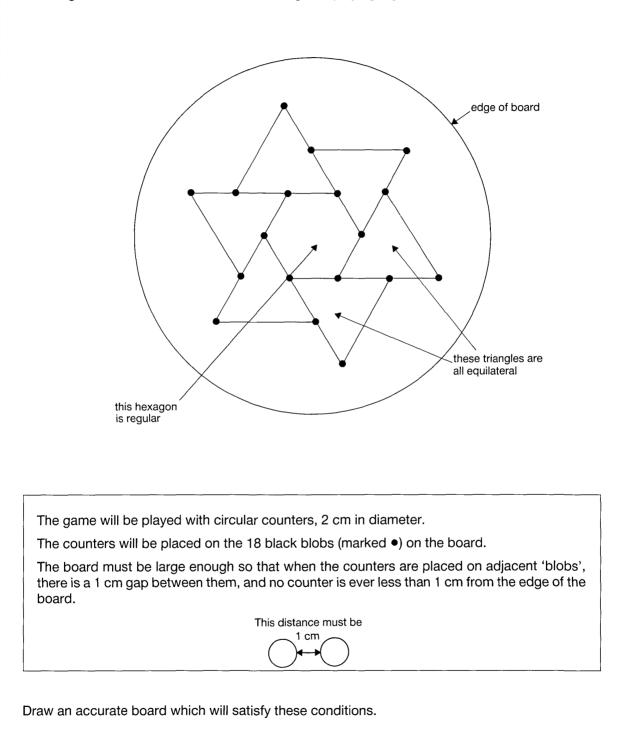


- 1. Draw this part of the board accurately and full-size.
- Using the same spacing as in question 1, how long would the rectangle be if the row had 5 circles instead of 3? (You do not need to draw this.)

PAPER 2 ASSESSMENT TASK 1

Star

The diagram below shows a small board design for playing a game called 'Star'.



PAPER 2 ASSESSMENT TASK 2

Build a Pyramid

This is a game for 2 players

Aim

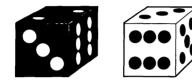
The first player to build a pyramid with 15 counters, wins the game.

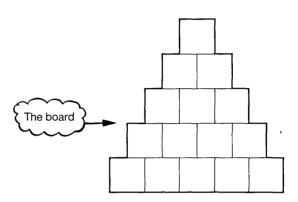
Equipment

One board for each player

15 counters for each player

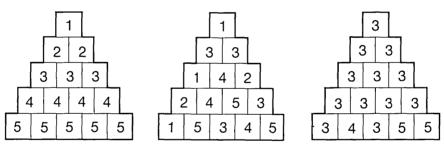
2 dice, one black and one white.





Preparation

Each player fills her board with numbers. A number from the set $\{1, 2, 3, 4, 5\}$ must be written in each square. A player can choose to place some or all of these numbers in any position. For example, here are three possible ways:



Rules

- Decide which player will be 'BLACK' and which will be 'WHITE'.
- Throw the two dice.
- If the number on the black dice is greater than the number on the white dice, then BLACK calculates the *difference* between the two numbers on the dice and covers any correspondingly numbered square on her board with a counter.

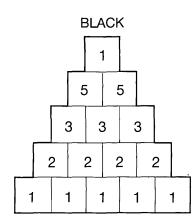


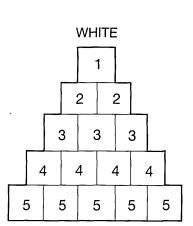
- Similarly, if the number on the white dice is greater than the number on the black dice, then WHITE calculates the difference and covers a corresponding square on her board.
- If both numbers on the dice are equal, neither player covers a number.
- The first player to cover all 15 squares and complete the pyramid wins the game.

PAPER 2 ASSESSMENT TASK 2 (CONTINUED)

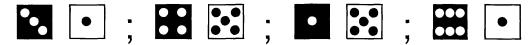
Build a Pyramid

Suppose that the two boards are filled in like this:





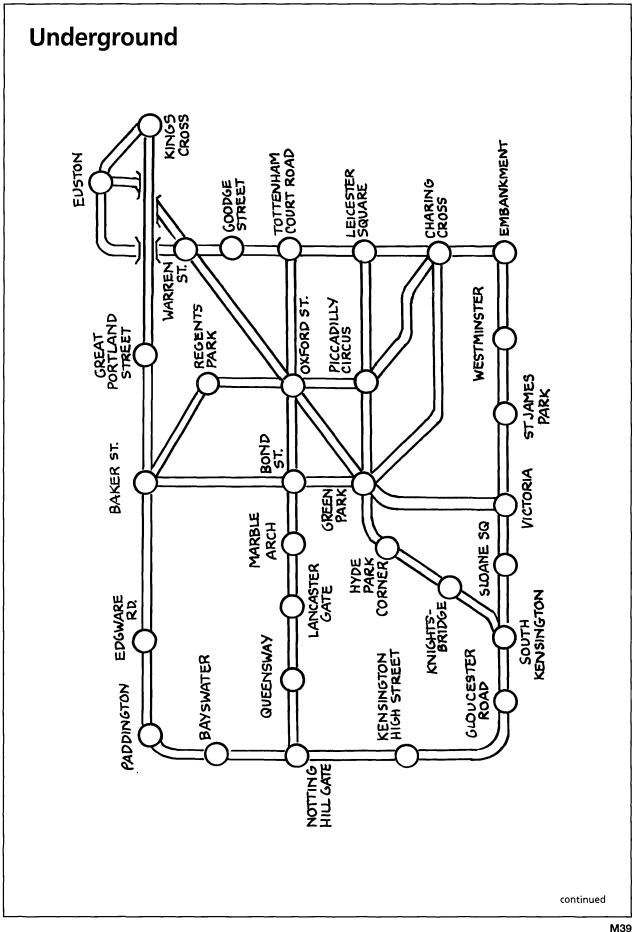
1. The first four throws of the dice are:



On copies of the boards, show clearly which numbers have been covered.

- Which player do you think is most likely to win?
 (If you think that both players are equally likely to win, write 'You can't tell'). Give a reason to support your answer.
- 3. Now suppose that you were playing the game.
 - (a) Show how you would number your board to give the best chance of winning.
 - (b) Explain *why* you think this would give you the best chance of winning.
- 4. Suppose that the game is to be adapted for 3 players using 3 dice and 3 boards. Describe how you would adapt the rules, so that the game will still work.

PAPER 2 ASSESSMENT TASK 3



PAPER 2 ASSESSMENT TASK 3 (CONTINUED)

Underground

A group is making up a game for 2 players. It is about travelling on the London Underground. The group want to make up rules so that the winner is the first person to travel through or change trains at every station on the map. Make up a complete workable set of instructions for their game. You can put extra details on the board if you wish.

Who it's for	What you need in order to play
What it's about	Aims of the game

Rules	
How to start	
How to make a move	
Other rules	
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NOTES

Checklist for the teacher

Real problem solving demands a different balance of teaching styles and strategies from the traditional curriculum. Students need to learn how to

- assume more control of and responsibility for their work
- share and develop ideas with each other
- give and accept constructive criticism
- turn their own ideas into actions, and learn from the consequences of their own decisions.

To effect this change of roles, it is helpful if the teacher can:

Frequently	
praise achievement	Those seem to be really good ideas.
 encourage groups to think further and deeper 	Can you make a game that uses all your ideas?
 ask clarifying questions 	Can you explain that to me again? I didn't quite understand.
 agree targets 	Do you think you could finish your rough plan by Monday?
 encourage students who lack confidence 	I'm sure you can do it; I'll come back and see you in a little while.
Occasionally	Have you thought about using some-
make a suggestion if a group is running out of ideas	thing other than dice?
 divert a group from a particularly unhelpful idea 	Are you sure a game that needs 20 players is a good idea?
Avoid	
taking over a problem	Why don't you try doing it like this?
making negative comments	Do you <i>really</i> think that's the best way?
 determining whose view is accepted. 	Sarinda's idea seems to be the best.