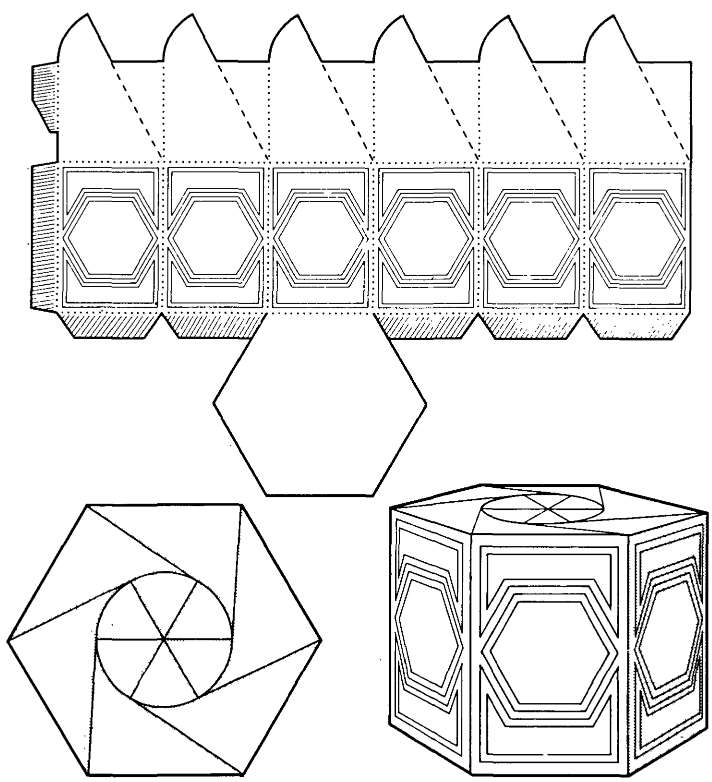
NUMERACY ▸ THROUGH ▸ PROBLEM ▸ SOLVING

Be a Paper Engineer



Shell Centre for Mathematical Education

Joint Matriculation Board

TEACHER'S GUIDE

About this scheme . . .

This scheme for the teaching, learning and assessment of numeracy through problem solving consists of a series of modules which provide effective support for teachers of mathematics who wish to introduce into the curriculum a component which enables their students to link their mathematics to the real world in which they live.

It has been developed with students of all abilities in the age range 13–16, and their teachers.

Each module package provides comprehensive materials for both teaching and assessment, related to a practical context which has proved interesting and enjoyable to the students who have taken part in its development. It is accessible to those who normally find mathematics difficult, while at the same time it provides a challenge for the most able.

A Scheme of Assessment based on these modules is offered by the Joint Matriculation Board. It includes both coursework and examination components of the kind illustrated in this book. Successful candidates receive a Statement of Achievement on each module at Basic, Standard or Extension level and, subject to certain conditions, the JMB Certificate of Numeracy through Problem Solving.

The Scheme relates to the GCSE National Criteria in Mathematics.

Contents

Please note:

Throughout this book, all references prefixed with the letter T refer to pages in the Teacher's Guide, references prefixed M refer to the Masters for Photocopying and unprefixed references refer to the Student's Booklet.

Authors

This series of Modules has been developed, as part of a joint project on the assessment of numeracy, by the Shell Centre and the Joint Matriculation Board. Many teachers and their students have worked with the central team: Alan Bell, Barbara Binns, Hugh Burkhardt, Rita Crust, Rosemary Fraser, John Gillespie, Steve Maddern, Kevin Mansell, Jim Ridgway, Malcolm Swan, and Clare Trott. Building on previous discussions involving the Shell Centre and the JMB, it was conceived and directed as part of the Testing Strategic Skills programme, by

Hugh Burkhardt

This module, Be a Paper Engineer, has been written by

Malcolm Swan, Barbara Binns and **John Gillespie**

assisted by Kevin Mansell and Dorothy Martin.

It is a result of successive revision through four versions on evidence collected from the comments and suggestions of the teachers and students involved, and through classroom observation by the team.

The assessment tasks owe much to the advice of John Pitts.

Many contributions to the work of the project have been made by staff and committee members of the Joint Matriculation Board – notably John Mathews, the Chairman of the JMB's Steering Committee and Austin Fearnley, who has played a major role in the organisation of the operational trials.

The later trials involved teachers and students in over 50 schools in many local authorities including Barnsley, Bradford, Bury, Calderdale, Cheshire, Cumbria, Derbyshire, Doncaster, Gateshead, Humberside, Kirklees, Knowsley, Lancashire, Leeds, Leicestershire, Manchester, Newcastle upon Tyne, Northumberland, North Tyneside, North Yorkshire, Nottinghamshire, Rochdale, Rotherham, Salford, Sheffield, Stockport, Sunderland, Tameside, Trafford, Wakefield and Wigan. Consultations with their Mathematics Advisers have made significant contributions to the development of the scheme.

The manuscript was prepared through many revisions by Susan Hatfield, Diane Jackson and Jenny Payne, and the staff of Burgess and of Longman. The technical drawings were made by Mike Gibbons.

We are grateful to them all.

We acknowledge with gratitude the inspiration we have drawn from the following publications.

- 'Paper Engineering for pop-up books and cards' by Mark Hiner, Tarquin Publications, Stradbroke, Diss, Norfolk, IP21 5JP. ISBN 0 906212 499.
- 'Wrap it Up!' a collection of packaging activities and 'Cardboard Engineering' a projected pack both by Mary Harris of the Maths in Work Project, Department of Mathematics, Statistics and Computing, University of London Institute of Education. 'Wrap it Up' is published by Macmillan Education, Houndmills, Basingstoke, Hampshire, RG21 2XS.
- 'The Gift Box Book' by Gerald Jenkins and Anne Wild, Tarquin Publications, Stradbroke, Diss, Norfolk, IP21 5JP. ISBN 0 906212 367.

These are invaluable complements to this pack.

We also acknowledge with gratitude the assistance given by the following companies in permitting us to use copyright designs:

McDonalds Hamburgers Ltd, 11–59 High Road, East Finchley, London, N2.
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Brooke Bond OXO Ltd, Leon House, Croydon, Surrey.
Disking International, 1 Royal Parade, Hindhead, Surrey.

Numeracy through problem solving

Be a Paper Engineer is one of a series of modules that have been designed to encourage a new approach to the teaching and learning of numeracy, understood in the original broad sense[1,2] as

> the ability to deploy mathematical and other skills in tackling problems of concern or situations of interest in everyday life.

There is now a general acceptance that people need to learn to *use* the knowledge and skills they acquire at school, and that this requires a shift in the balance of the curriculum to include more real problem solving. This is particularly important for the mathematics curriculum, because the power of mathematics in helping people tackle real problems more effectively is not often realised.

The Cockcroft Report says

> 'Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much.' (paragraph 34)

and

> 'Our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation.' (paragraph 39)

TVEI and other recent curricular initiatives have similar aims, emphasising that curricula should contain a strong element concerned with the tackling of practical problems relevant to everyday life and work. The assessment criteria for the GCSE emphasise these aspects too. Employers say that they are primarily interested in people who can use their knowledge sensibly and effectively.

A curriculum component of this kind places new demands on teachers; it needs a broader range of teaching strategies than does the traditional mathematics curriculum, with some new roles for both teachers and students. The scheme has been developed to provide an introduction to such work in the classroom that is both effective and enjoyable for those involved.

1 15–18 A report of the Central Advisory Council for Education (England). HMSO, 1959.
2 Mathematics counts. Report of the Committee of Enquiry into the Teaching of Mathematics in Schools under the chairmanship of Dr W H Cockcroft, HMSO, 1982.

What are the skills?

The modules are based on practical contexts which have been chosen to allow students of all abilities to develop general problem solving (or *strategic*) skills in areas of activity such as designing and making, planning and organising, and choosing.

These *strategic skills* include:

- understanding general ideas and details
- following instructions precisely
- distinguishing between essential constraints and desirable features
- identifying faults
- correcting faults
- generating and listing possibilities (brainstorming)
- developing a rough plan, including: reviewing the prepared suggestions; reaching and recording agreed decisions; maintaining a broad level of description, avoiding excessive detail; identifying needed information and materials; making estimates of quantity and cost; describing, testing and evaluating the plan
- making the final plan, product and/or detailed instructions with comprehensiveness, accuracy, clarity and quality
- implementing the activity with full preparation
- testing and evaluating the plan or product comprehensively.

Various *tactical skills*, more specific to each context, are involved in implementing these strategies. For example, different ways of collecting and recording information are appropriate if you are considering alternative products to buy, or alternative routes to follow.

Technical skills are, of course, required to carry through the solution of problems using the higher level skills described above. Technique is only useful for these purposes in so far as it is *reliable*. This implies much higher standards in this respect than are expected in the traditional curriculum, with a greater emphasis on thorough understanding and checking of whatever techniques are used.

Among the mathematical techniques and concepts, of importance in this scheme, are:

- the ability to
  - carry through simple calculations with suitable accuracy, using a calculator where appropriate
  - make estimates
  - make measurements (including number, length and time)
  - draw accurately
  - interpret and display data in a variety of representations (including graphs, maps, timetables and other tables).
- understanding and using some techniques of
  - probability and statistics
  - ratio and proportion
  - geometry in two and three dimensions.
- logical reasoning, including the ability to enumerate alternative possibilities and classify them in various ways.
- research skills, including the collection and evaluation of relevant data.

The relevant mathematical skills are discussed in more detail in each module package. There is also opportunity for the use of other parts of the mathematics curriculum which a student has mastered.

In addition skills from other curriculum areas, such as language and arts, are inevitably called upon, as these are necessary for the presentation of the reasoned arguments which are essential for real problem solving. Since group work is involved, social skills also play their part. Thus, though numeracy is focussed on the deployment of mathematical skills and reasoning in real problem solving, it has a broad cross-curricular aspect.

What is provided?

The scheme is implemented in a modular form, each module being designed to occupy between 10 and 20 hours of teaching time spread over 3 to 6 weeks. Five modules will be available in the first instance. A feature of each module is the importance attached to students working in groups, explaining their ideas and listening to each other, making their own decisions and living with the consequences, reflecting on their experience and learning from it, just as they do in life outside the classroom. While working through the modules, students themselves become responsible for setting and tackling their own problems, rather than simply responding to tasks set by the teacher. Modules are not necessarily staged nor are they dependent upon each other but the sequence which follows is recommended as providing an appropriate progression and a balance of different kinds of context.

The modules in the series are:

- **Design a Board Game:** in which students design and produce a board game which can be played and evaluated by other members of the class.
- **Produce a Quiz Show:** in which students devise, schedule, run and evaluate their own classroom quizzes.
- **Plan a Trip:** in which students plan and undertake one or more class trips, and possibly some small group trips.
- **Be a Paper Engineer:** in which students design, make and evaluate 3-dimensional paper products, such as pop-up cards, envelopes and gift boxes.
- **Be a Shrewd Chooser:** in which students research and produce consumer reports which inform people on how to make better choices.

Many contexts were considered and tried in the early stages of development, to see which led to the best balance of classroom activities and learned skills. Those that were chosen all have a practical outcome, interesting and relevant to the students' present circumstances. This corresponds with our observation that people best develop the strategic skills of numeracy in the course of solving problems which they see as realistic, stimulating and within their capabilities. The themes selected were found to have general appeal and to require the use of a wide range of skills, whilst not making unreasonable demands on classroom or school organisation.

Discussion with students and observation in the classroom support the expectation that students' problem-solving abilities improve as they work through the series of modules and that skills acquired in one area are subsequently applied in others. Students themselves maintain that they will be able to apply these strategic skills with advantage in tackling further problems as they arise in their lives outside the classroom. Groups of students also suggested many other interesting and worthwhile themes, each of which could form the basis for a further module. These include: planning and running a jumble sale; raising money for charity by sponsored events; planning and running a magazine; setting up a small business; planning a party; designing a bedroom; planning a youth group weekend; making a garden; orienteering; designing and marketing T-shirts.

The scheme provides classroom materials and assessment tasks, together with further support materials to help teachers explore in greater depth the issues and teaching strategies involved. Suggestions for further mathematical development are also included.

Classroom materials, including detailed teaching suggestions, have been developed to offer a proven approach that has worked well for a representative group of teachers, new to this kind of work, without imposing on them excessive demands of design or implementation. We recognise that, of course, each teacher works in his or her own way in the classroom but most have been found to appreciate detailed, explicit suggestions which they can use, and adapt, in the knowledge that they have worked well for others. Such materials are provided in each module package.

Assessment tasks play an important role in the curriculum, providing targets that help students and teachers recognise objectives more clearly and help them to progress towards them. (The effect of assessment on the curriculum has often been to narrow and distort its aims but, equally, assessment can be used to enhance what is achieved.) In a new curriculum component like this one, assessment is particularly important. Thus assessment tasks are provided throughout these materials. They relate closely to the Scheme of Assessment for the Certificate of Numeracy through Problem Solving offered by the Joint Matriculation Board, but may be used more widely.

The suggestions for further mathematical development provide a variety of ideas, together with discussion on how and when they might be introduced and linked to the more traditional teaching of mathematical techniques.

Support materials are designed to help teachers with the new aspects of classroom activity and teaching style that this work involves. The materials relate to the three principal differences between this work and the traditional mathematics curriculum – the broader range of skills involved, the practical priorities of numeracy, and the much greater responsibility of the students for their own work. In the traditional curriculum students are largely *imitative*, here they are *autonomous* in deciding on and carrying through their approach to the task. The primary support is provided by the teaching suggestions in the classroom materials and elsewhere in each Teacher's Guide. The support materials, which form a separate package, take this further, sharpening awareness and tackling more fully and deeply the teaching and assessment issues and skills involved. They may be regarded as a do-it-yourself in-service course, designed to be used either on a distance-learning basis by teachers in a school or within LEA or college courses. This material, which is linked particularly to this module, includes a video of the modules in use, together with comments from teachers and students on the work, its challenges and its benefits.

Introduction to Be a Paper Engineer

Examples of paper products abound in everyday life: gift boxes, pop-up greetings cards or books, envelopes, lampshades and so on. The materials are cheap, and the design techniques involved are usually simple but ingenious. This context therefore provides an ideal opportunity for students to develop geometrical and other design skills in an open-ended way. In this module, students attempt to design and make a product from paper or thin card and then produce a 'kit' containing full instructions so that someone else can make it.

The design process is arranged in four stages.

1. Looking at examples. In groups, students make a wide variety of pop-up cards, gift boxes and envelopes in order to familiarise themselves with the techniques involved. (This involves 'following instructions', 'cutting, folding and gluing accurately', and 'recognising structural features of a design'.)

2. Exploring techniques. Students investigate, in greater depth, a few of the techniques illustrated in Stage 1. (This involves 'drawing a 2-dimensional representation of a 3-dimensional product', 'explaining design features', 'making 3-dimensional objects from 2-dimensional representations', 'identifying and correcting design faults' and 'developing existing ideas for paper products'.)

3. Making your own original. Groups pool ideas for paper products and then, individually, students attempt to design and make an accurate version of one of the products. (This involves 'generating possibilities for a design with original features', 'drawing a design to an acceptable degree of accuracy' and 'constructing a prototype'.)

4. Going into production. Students now attempt to produce 'kits' of their designs so that other people can make the products. This stage involves the use of school reprographics. (This involves 'devising instructions'.)

The range of mathematical techniques required, and the tactical skills needed for their deployment, will depend on the students' abilities and on the demands made by their chosen designs. The range is, however, likely to include

- understanding and using ideas of angle, parallelism and symmetry
- estimating and measuring lengths and angles
- following instructions presented in words, diagrams or symbolic notations
- making, testing, explaining and proving conjectures
- visualising and creating a 3-dimensional shape from a 2-dimensional plan
- drawing a 2-dimensional net of a 3-dimensional object
- making and using simple levers and linkages
- writing clear, concise and complete instructions, using diagrams or photographs where appropriate.

These aspects are discussed further in Chapter 2, while Chapter 3 is concerned with assessment.

1

Classroom materials

Classroom materials

Introduction

This chapter provides a detailed guide to the classroom materials. The lesson outlines are offered in the recognition that every teacher has an individual style in which he or she prefers to work. Many teachers have found it helpful, however, to be given detailed suggestions which they can then adapt to meet their own needs. This has also enabled us to offer an approach which we have seen working well in a representative range of classrooms.

The classroom materials are centred on the Student's Booklet which is important for the following reasons.

- It provides students with a coherent structure for their work. At any point, it should help students to have an overview of what they have achieved and where they are going.
- Students who are inexperienced in designing a product often latch on to an idea that seems superficially attractive, without carefully considering implications or alternatives. The booklet will help to slow down and stimulate the more impatient or less imaginative students by, for example, inviting them to make a range of products which introduce them to possibilities and to the techniques involved, before they embark on their own designs.

The large pack of worksheet masters allows for flexibility in the use of the classroom materials. Some of the suggested activities involve sophisticated mathematics and are therefore not appropriate for all students. It is important that students are not given work that is so far beyond their capabilities that they become disillusioned. On the other hand, motivated students are capable of working at a level well beyond normal expectation.

Assessment may be carried out by observation of the work in progress and by looking at the students' final products. This is possible because, unlike other modules in this series, most students will be working on their own individual projects for much of the time. More detailed suggestions on assessment are given in Chapter 3.

Real problem solving in the classroom demands a different balance of teaching styles and strategies from that in the traditional mathematics curriculum. The emphasis on student-led decision-making will be unfamiliar to some teachers. For this reason, we offer the following suggestions which have been found helpful.

Your role will involve rather less task-setting and explaining than you may be used to. Instead, you will be acting more as an adviser and resource, responding to students in the class rather than directing them. It is helpful if you can

- listen to students and ask questions which may help them to clarify their own thinking
- encourage students to pace their work by agreeing target dates by which phases of their work should be completed
- encourage students to take pride in their work and aim to make products which are both imaginative and polished
- avoid 'taking over' by suggesting your own ideas and making decisions for students. If you do this, students may feel that they are no longer working on their ideas and may become disenchanted.

These suggestions are amplified on the final page of this book for ease of reference.

Stage 1 Looking at examples

In this stage, you will work in a small group.

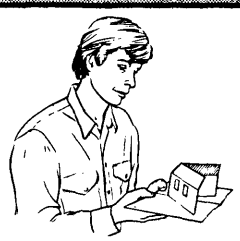
To get you started, each person will make a Cat pop-up card and a Birthday envelope to keep it in.

Your group will then work together to make a collection of cards, boxes and envelopes. You will compare all these items and see how they work.

Finally, you will be asked to look for other examples of paper engineering at home or in the shops. These should give you some more good ideas.

2

Stage 2 Exploring techniques



In this stage you will work mostly on your own using some of the Exploring techniques sheets.

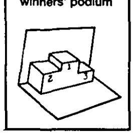
Investigations
Try these . . .

Make up your own examples as well

You will try some

- **investigations**, which ask you to explore what happens when you fold, cut or stick things in different ways,
- **challenges**, which show you pictures of finished articles that you can try to make.

Challenges
Try to make this winners' podium



You will also

- **keep a record** of everything you discover for use later on, when you come to design your own item.
(Page 14 describes how to do this in more detail.)

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Summary of activities

Time needed for Stage 1

About 3 hours.

Students' activities

- Making up an example of a pop-up card and an example of a box. This gives students experience of following instructions and the opportunity to learn the notation with support from their friends and the teacher. A class activity.
- Producing a large collection of cards, envelopes and boxes which illustrates a variety of design techniques. A group activity.
- Playing a game which encourages them to classify the collection according to the mechanisms involved. A group activity.
- Analysing commercially produced paper products to see how they are made and to stimulate a wide range of ideas for original products. A group activity.

The teacher's role

- Explaining the notation and assisting students who have difficulties in following the instructions.
- Helping students to distinguish between the technical aspects of the mechanisms and the decorative features of the cards.
- Encouraging students to help and cooperate with each other.

Time needed for Stage 2

At least 3 hours.

Students' activities

- Working on a selection of 'Exploring techniques' sheets. This gives them the opportunity to look at some of the techniques in more depth and to begin to experiment with their own ideas. Mainly individual work, with support from a group.

The teacher's role

- Ensuring that the necessary equipment is available.
- Helping students to select tasks at an appropriate level of difficulty.
- Encouraging students to keep a full record of everything they discover – including ideas that fail to work.

Stage 3 Making your own original

In this stage you will

- brainstorm to find a good idea for a pop-up card, box etc, of your own,
- make a rough version of your own idea in paper,
- make a 'best' version of your idea in card.

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Stage 4 Going into production

In this stage, your aim is to produce a few complete kits, so that other people can make your paper product.

Each kit will contain

- a copy of your design
- a set of clear instructions for making your product.

The stage is in 5 sections:

- Finding out about copying ('reprographics')
- Preparing your instructions
- Testing your instructions
- Putting your kits together
- Using your kits.

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Time needed for Stage 3

About 2 to 4 hours, though students may wish to spend longer. You may wish to set a finishing deadline.

Students' activities

- Brainstorming ideas for paper products, including items which groups may work on together. An individual activity followed by a group discussion.
- Choosing one idea to develop.
- Making a rough 'prototype' of the product from paper. This involves much trial and error in order to produce a product that works properly.
- Making an accurate version of the product from thin cardboard, using appropriate geometric techniques.

The teacher's role

- Encouraging students to consider a variety of ideas before deciding on one to develop.
- Ensuring that the necessary equipment is available.
- Encouraging students to use mathematical techniques where appropriate.

Time needed for Stage 4

About 1 to 2 hours.

Students' activities

- Finding out alternative ways of mass-producing their products.
- Preparing instructions so that someone else can recreate their products.
- Testing the instructions to check that they are clear and complete.
- Assembling a 'kit', which contains everything needed to make the product.

The teacher's role

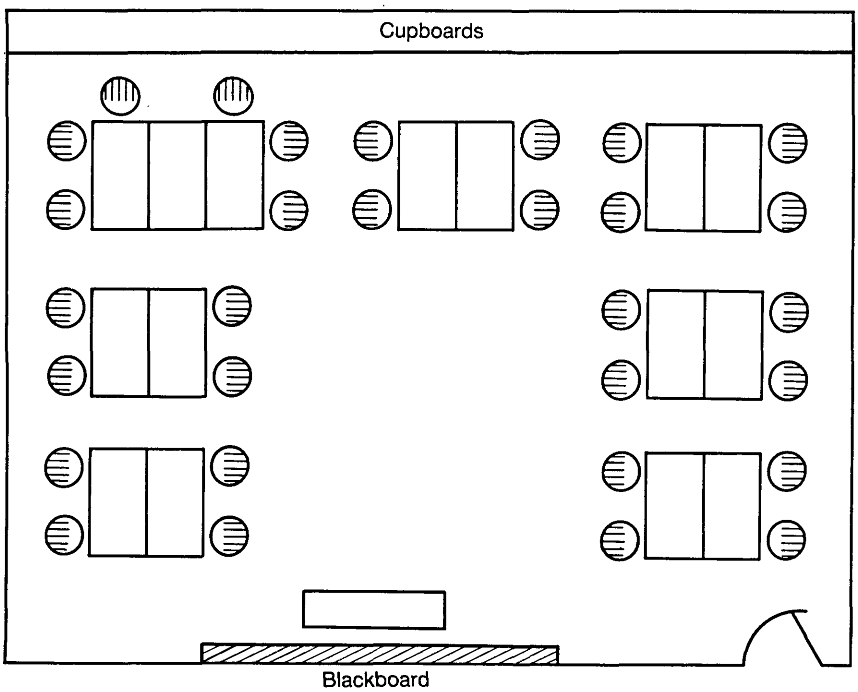
- Ensuring that students are aware of the reprographic facilities within the school.
- Organising the duplication of the paper products.
- Facilitating the production and testing of instructions.
- Encouraging students to aim at a product of which they may be proud.

Preparation

Classroom organisation

This module requires students to work both on their own and in groups on practical activities. The tables should therefore be arranged to encourage group discussion and to facilitate the sharing of equipment. Even when students are working on their own, they may still find it valuable to have the support of a group with which they can talk through ideas. We therefore suggest that, throughout the module, tables are grouped in blocks, with 3 or 4 students sitting at each block.

For example:



Resources required

In addition to the essential resources listed on the next page, the following items have been found useful:

- newspaper or sugar paper to protect the desks
- pencils
- felt tipped pens in assorted colours, including a few broad liners
- pencil erasers
- rulers
- set squares
- protractors
- pairs of compasses
- a few pins
- 'Letraset' or other rub down lettering
- lettering stencils
- a typewriter or word processor
- coloured paper in various sizes, including some large sheets
- thin coloured card in various sizes, including some large sheets
- isometric and squared, lined or dotted paper
- gummed paper
- a guillotine
- sellotape
- a stapler
- 'Polaroid' camera and tape recorder (to help students prepare instructions for making their products).

Equipment needed

When needed	Item	Quantity	Source
Throughout the module	Student's Booklet	1 for each student	supplied
	Envelope or folder in which to keep work	1 for each student	
	Box in which to keep work	1 for each group	
	Rough paper	a plentiful supply	
	Scissors	1 pair for each student	
	Glue sticks	at least 1 for each group	
	Dustbin liners	1 or 2	
Stage 1	Commercially made paper products (pop-up cards and gift boxes)	just a few to provide inspiration	students may supply these
	The 'Cat' and the 'Birthday Envelope' plans	1 copy of each plan for each student	masters supplied (M1, M2)
	Set of 29 other 'plans'	1 set for each student	masters supplied (M3–32)
	Stage 1 Instructions pack	1 set for each group	masters supplied (M33–48)
	'Helpful Hints' sheet	1 for each student	master supplied (M49)
	'Flowchart' sheet	1 for each group	master supplied (M50)
Stage 2	'Exploring techniques' sheets	about 10 copies of each sheet	masters supplied (M51–66)
	Old magazines, comics, or colour supplements	at least 10, to be used as sources for pictures	students may supply these
	Isometric and squared, lined or dotted paper	a plentiful supply	masters supplied (M67–70)
Stage 3	'Brainstorming' sheet	1 for each student	master supplied (M71)
	Thin white card	2 or 3 A4 sheets for each student	
Stage 4	'Cartoons for instructions' sheet	several copies	master supplied (M72)
	Access to reprographic facilities		
	A4 envelopes	at least 2 or 3 for each student	

Stage 1 Looking at examples

Introduction

In this stage students work cooperatively to make a collection of envelopes and gift boxes. The products are chosen to introduce students to a variety of techniques and ways of presenting instructions, and to stimulate their imaginations as to possibilities for their own designs.

Total time needed

About 3 hours, though with some groups you may wish to spend longer.

Organisation and equipment required

The students need to work in small groups of three or four to produce a set of products between them. With some groups, you may prefer to spend longer on this stage and allow groups to produce more than one of each product.

Each student will need

- a Student's Booklet*
- the 'Cat' and 'Birthday Envelope' plans
- a pair of scissors*
- an envelope or folder in which to keep the work*

Each group will need

- a set of plans
- at least one glue stick*
- a Stage 1 Instructions pack
- a 'Flowchart' sheet
- a box in which to keep the folders and any models which do not fold flat*

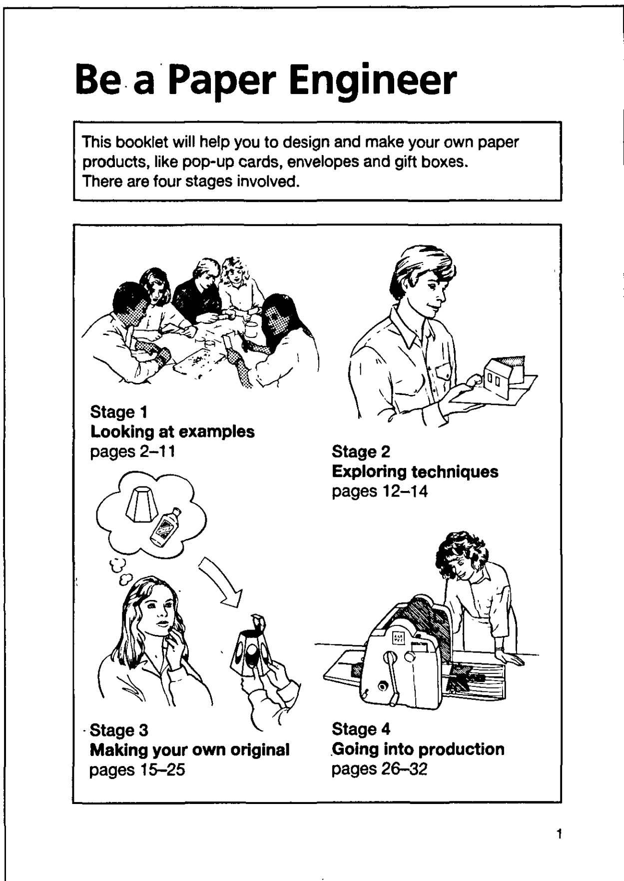
*These items will be required throughout the module.

Related assessment criteria

This stage offers students the opportunity to show that they can

(i) follow instructions,
(ii) cut, fold and glue accurately to assemble a 3-dimensional product,
(iii) recognise structural features of a design.

In contrast to other modules in this series, there are no separate coursework assessment tasks. In Chapter 3 we have included some suggestions for assessing whether or not students have satisfied the criteria, though these suggestions will need to be adapted to fit your class and the particular aspects of the work you wish to accentuate.



Be a Paper Engineer

This booklet will help you to design and make your own paper products, like pop-up cards, envelopes and gift boxes. There are four stages involved.

Stage 1
Looking at examples
pages 2–11

Stage 2
Exploring techniques
pages 12–14

Stage 3
Making your own original
pages 15–25

Stage 4
Going into production
pages 26–32

1

Purpose

To introduce the module.

Presentation

A class discussion.

Suggestions and comments

You may like to introduce the module by asking students to describe the variety of paper products that are sold in gift shops. Most students will have seen pop-up birthday cards or books and received gifts packaged nicely (for example, cosmetics or chocolates).

If possible, have a few examples at hand. This will aid the discussion and provide an opportunity to see how such products are made. Discuss such questions as the following.

- Do they look hard to make?
- How much do such things cost?
- What are you paying for? (materials? labour?)
- Do you think that you could design something like this?

Encourage students to look out for further examples and bring them into school for analysis in future lessons.

Issue the 'Be a Paper Engineer' Student's Booklets and explain that the purpose of the module is for students to discover the mathematical techniques used in the manufacture of paper products and then use this knowledge to produce their own original, marketable designs. Briefly describe the four stages that will help them to achieve this.

If there is any possibility of students using their finished products in some way (they may like to mass-produce and then sell them in aid of a worthwhile cause, for example) then it is motivating to mention this at the outset – referring them to page 32 of the Student's Booklet.

Stage 1 Looking at Examples

In this stage, you will work in a small group.

To get you started, each person will make a Cat pop-up card and a Birthday envelope to keep it in. 1

Your group will then work together to make a collection of cards, boxes and envelopes. You will compare all these items and see how they work.

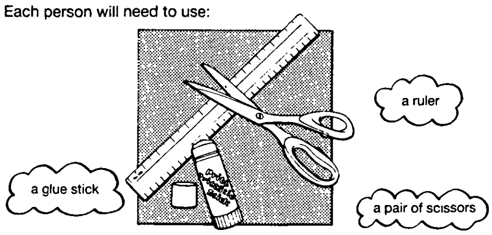
Finally, you will be asked to look for other examples of paper engineering at home or in the shops. These should give you some more good ideas. 2

2

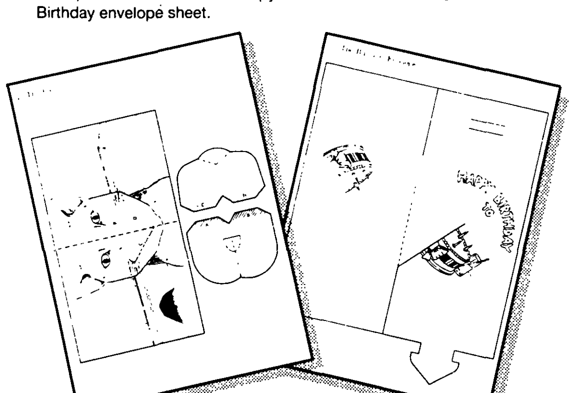
STAGE 1

Making the Cat card and the Birthday envelope

Each person will need to use:



Each person will also need a copy of the Cat sheet and the Birthday envelope sheet.



3

Purpose

To introduce students to some of the techniques involved in making paper products, notably pop-up cards, gift boxes and envelopes.

Presentation

A class discussion followed by individual work (with the support of the teacher and fellow students).

Suggestions and comments

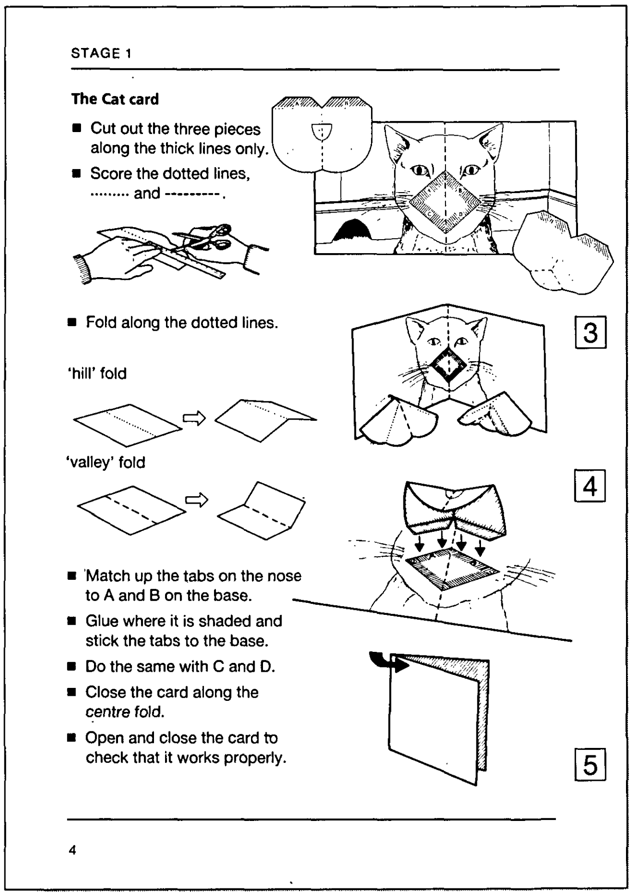
Refer students to pages 2 and 3 and explain the purpose of this stage (given above).

1 We suggest that the 'Cat' card and the 'Birthday Envelope' are made together by everyone in the class because these two examples illustrate the notation and techniques used in making hill and valley folds, cutting edges and slots, and gluing. If students can cope with these, then they should be able to make up the remaining plans without too much difficulty.

If, however, you feel that you need to begin with a few easier examples, introducing the notation and techniques more gradually, we suggest that students begin by making a few of the following:

3. The Pop Star
7. Frankenstein
9. The Flying Kite
11. The Dove of Peace
14. The Settling Butterfly
20. The Gorilla
23. The Box of Tricks
25. The Window Envelope.

2 The final activity in this stage involves the students in looking out for further examples of paper engineering in shops or at home. Encourage each student to start looking for at least one example to bring to school for analysis. (It is sometimes necessary to pull a 'pop-up' apart to see how it works. Students should obtain permission from brothers or sisters in case they destroy treasured possessions!)


STAGE 1

The Cat card

- Cut out the three pieces along the thick lines only.
- Score the dotted lines, ········ and ---------.
- Fold along the dotted lines.

'hill' fold

'valley' fold

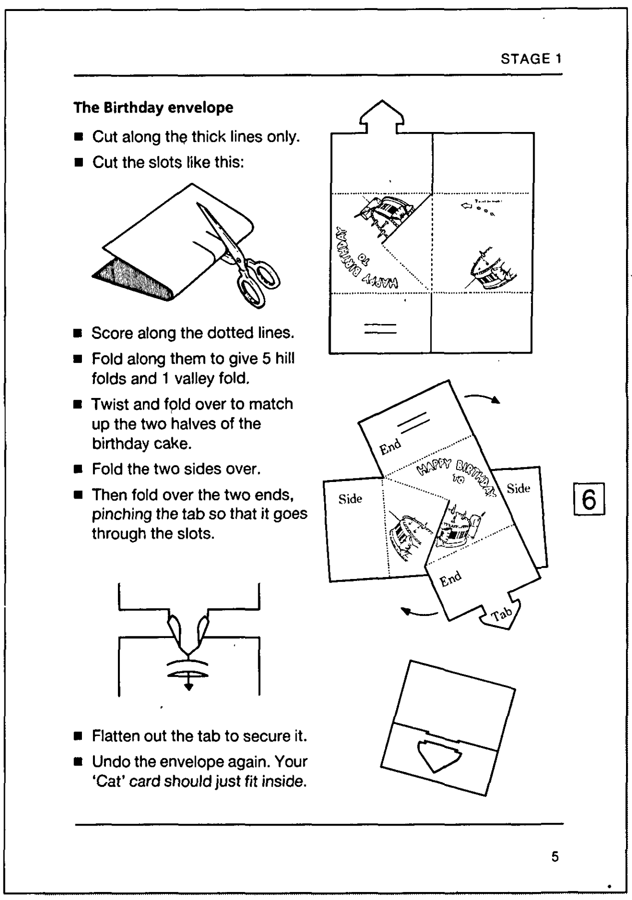
- Match up the tabs on the nose to A and B on the base.
- Glue where it is shaded and stick the tabs to the base.
- Do the same with C and D.
- Close the card along the centre fold.
- Open and close the card to check that it works properly.

[3]

[4]

[5]

4


STAGE 1

The Birthday envelope

- Cut along the thick lines only.
- Cut the slots like this:
- Score along the dotted lines.
- Fold along them to give 5 hill folds and 1 valley fold.
- Twist and fold over to match up the two halves of the birthday cake.
- Fold the two sides over.
- Then fold over the two ends, pinching the tab so that it goes through the slots.
- Flatten out the tab to secure it.
- Undo the envelope again. Your 'Cat' card should just fit inside.

[6]

5

The 'Helpful hints' sheet (M49) summarises the notation and suggests some useful techniques for making the products. We recommend that you familiarise yourself with it now to ensure that the advice students receive remains consistent throughout Stage 1. You may wish to hand out the sheet at this stage.

[3] Go through the method of scoring as described on the 'Helpful hints' sheet. This will help students to make their folds exactly along the dotted lines.

[4] The distinction between 'hill' and 'valley' folds often causes confusion. It may help if you show the class an example of each made from large scraps of paper. A diagram on the board may also help. The jaws of the cat are difficult to fold as they contain quite an intricate arrangement of hill and valley folds. You could help with the following advice.

'To make good creases, fold along the line of symmetry first, and then (without opening the fold) fold along the rest of the dotted lines both ways. Open the piece out and now make the folds go in the right directions'.

[5] It is unlikely that students will have made the card perfectly but, by closing it firmly and pressing it flat, the folds usually become creased in exactly the right place.

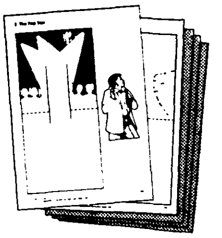
[6] This causes some difficulty but, with some trial and error, students work out what to do.

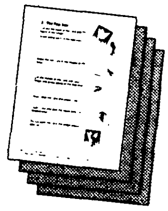
STAGE 1

Making a collection of cards, boxes and envelopes

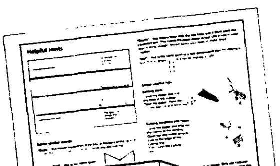
Your group will now need . . .

a set of 29 plans which may be cut out to make pop-up cards, boxes . . .



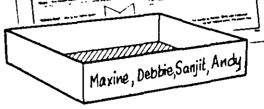


an Instructions pack which tells you how to make each item



a Helpful hints sheet

and a tray or box lid for storing your finished items.



Each person in your group should choose a different card, box or envelope to make, and find the matching instructions.
Make the item you have chosen.
When you've finished one item, choose another . . .

6

Purpose

To enable students to create a large collection of paper products, developing their ability to follow instructions and their manual dexterity. These products illustrate a range of techniques for subsequent classification and analysis.

Presentation

Group work.

Suggestions and comments

You may wish to continue to pick out the easier 'plans', rather than give all of them out at once.

Some students would prefer to make the same item as their neighbours. Discourage this by emphasising that the aim of the activity is for each *group* to end up with a wide variety of ideas to look at.

Students may have some difficulty for the first half hour or so while they are getting used to the techniques and to the form of the instructions. Within their groups, the students usually develop enough expertise to overcome most of the difficulties themselves.

STAGE 1

Looking at the things you have made

When you have made a large collection of items, try playing the following game in your group.

- Put all your cards, boxes and envelopes in the middle of your table.
- One person, the 'chooser', secretly writes down the name of one item.
- The other players try to find out which it is by asking questions in turn.

but . . .

- the 'chooser' can only answer 'Yes' or 'No',
- the questions must not mention pictures or designs, (so you *cannot* ask 'Has it got an animal on it?'),
- only *one* guess is allowed at the name of the secret item.

Here's how one game went . . .

Is it a card? — Yes!

Does it have a centre fold? — Yes!

Are all the fold lines parallel to each other? — No!

Do you need glue to make it? — Yes!

Does the picture pop up when the card is fully open? — Yes!

Which cards could it be?

7

Purpose

To help students to reflect on the structural differences between the items they have made.

Presentation

Class introduction followed by group work.

Suggestions and comments

Play the game with the whole class to clarify the rules. Choose a card and invite students to ask you questions. This will give you an opportunity to make clear which kinds of questions are allowed. Questions should just refer to structure, not to decoration. In finding questions which distinguish between cards, students will, we hope, begin to realise that some mechanisms are the same.

At each 'yes' or 'no' answer, students should sort the remaining cards and put to one side those that have been eliminated.

After each answer it is worth checking that each group has the same cards remaining and discussing any discrepancies. (There are often no 'right answers', but it is important to try to agree.)

Students should now play through the game a few times in their groups to begin to focus on the mechanisms and techniques involved.

STAGE 1

Classifying the things you have made

Ask your teacher for a large copy of the Flowchart sheet and a counter. This will help you to sort out (or classify) the things you have made.

How to use the Flowchart sheet

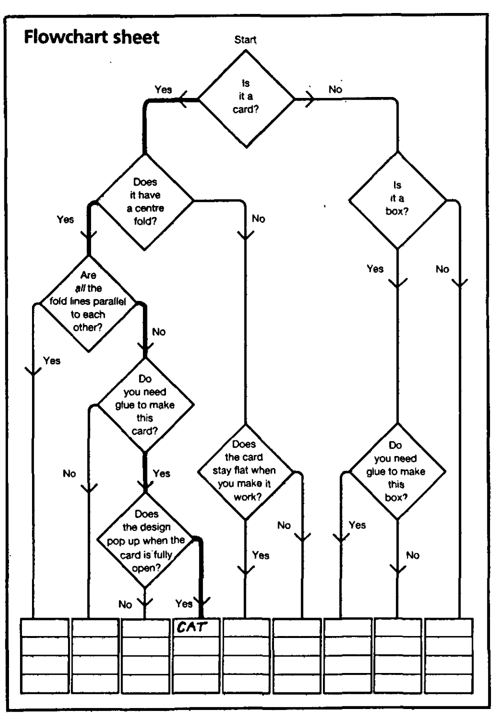
- Choose a card, envelope or box that you have made.
- Now place a counter on the first question: 'Is it a card?'
- If the answer is 'Yes' then move your counter along the 'Yes' line to the next question.
  If the answer is 'No' then follow the 'No' line.
- Now answer the next question . . . and so on.
- When your counter reaches the bottom of the page, write down the name of the card, envelope or box in an empty space.

The page opposite shows the route taken by the counter when the Cat card was chosen.

Now make up your own Flowchart sheet, using your own questions.

8

STAGE 1



9

Purpose

To help students to classify the things they have made, according to structure.

Presentation

Group work.

Suggestions and comments

It is worth comparing the items that different groups have allocated to each category. The more awareness students acquire of different mechanisms, the more creative they will be able to be when they design their own products.

This activity should build on the experience of the game. In some classes, students have devised their own flowcharts, using questions that were asked in the game. (A microcomputer program such as SEEK, may be used to facilitate this*.) We do, however, suggest that this activity is not laboured too much at this stage.

*SEEK is included in Module 4, Building in your ideas, in the Micros in the Primary Classroom series, Longman, 1983.

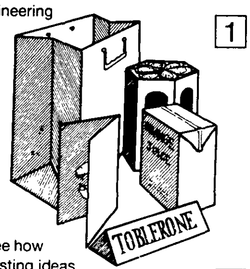
STAGE 1

Looking at other examples

Search for other examples of paper engineering in shops or at home, like

- pop-up books or cards
- carrier bags
- Easter egg boxes
- boxes for scent bottles
- paper lampshades

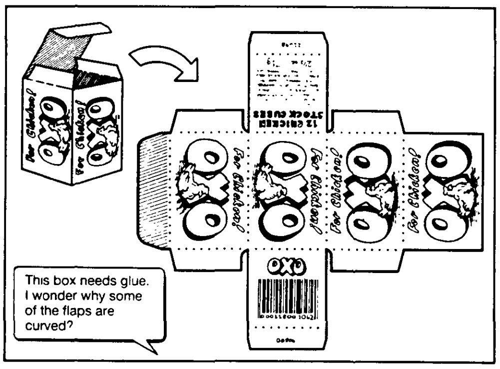
. . . and so on.



1

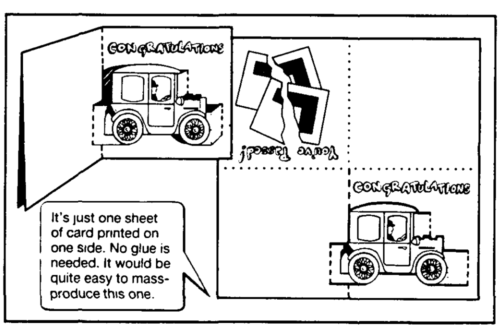
Bring some to school.
If you can, carefully take them apart to see how they are made. Keep a note of any interesting ideas . . .

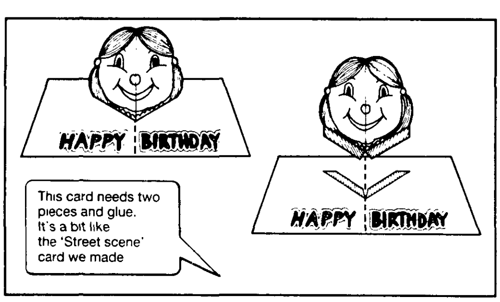
2



10

STAGE 1





Perhaps you will be able to use some of the ideas when you design your own items, later on.

11

Purpose

To encourage students to relate what they have been doing to a broader range of paper items from everyday life. This activity should also give students a bank of ideas which they can subsequently draw upon when designing their own items.

Presentation

Individual and group work.

Suggestions and comments

1 Encourage each student to bring in at least one item if he or she has not done so already.

2 As students look at items, encourage them to make sketches and notes on how the items are made. You may be able to provoke discussion by asking students to imagine that the position of a fold or cut is slightly changed. Would the design still work? Why?

One teacher wrote down the following list of design features for boxes.

- 'Glue flaps' must be cut to about 60°.
- Flaps must be at least 1 cm wide.
- Lid flaps must have rounded edges.
- There should be a small slit each side of the lid flap.
- The dimension of the lid must be about 1 mm bigger than the dimensions of the bottom of the box.

She then asked the class to find out how true these statements were, in general, and asked them to try to explain why such design features may or may not be important.

Stage 2 Exploring techniques

Introduction

In this stage students explore and develop, in more depth, some of the techniques that have already been introduced. This is done by inviting them to discover what happens when they fold, cut and stick things in different ways and attempt to make or adapt finished items without the kind of detailed guidance offered in Stage 1. It is intended that students keep a full, written record of everything they do in this stage.

Total time needed

At least 3 hours.

Organisation and equipment required

Although students will be working mostly on their own, they should be encouraged to consult with other members of their groups when they need help or have discoveries to share.

Each student will need

- some 'Exploring techniques' sheets
  (These are single sheets of A4 paper, folded to form 4-page A5 'booklets'. They are reproduced on pages T27 to T34 of this guide.)
- a pair of scissors
- access to a glue stick
- a pencil, ruler and eraser
- a plentiful supply of paper or thin card
- an exercise book or folder to record what is discovered.

You may also require a back-up supply of old magazines, squared and isometric dotty paper and protractors.

Related assessment criteria

Stage 2 offers students the opportunity to show that they can

(iv) make a 3-dimensional object from a 2-dimensional representation,
(v) draw a 2-dimensional representation of a 3-dimensional product,
(vi) give a reasoned explanation for design features,
(vii) identify and correct design faults,
(viii) develop an existing idea for a paper product.

Their written records may be used for this assessment.

Stage 2 Exploring techniques

In this stage you will work mostly on your own using some of the Exploring techniques sheets.

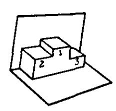
Investigations
Try these . . .

Make up your own examples as well

You will try some

- **investigations**, which ask you to explore what happens when you fold, cut or stick things in different ways,
- **challenges**, which show you pictures of finished articles that you can try to make.

Challenges
Try to make this winners' podium



You will also

- **keep a record** of everything you discover for use later on, when you come to design your own item.
  (Page 14 describes how to do this in more detail.)

12

Purpose

To give students the opportunity to explore a range of different techniques in depth, and to encourage creative thought.

Presentation

Mainly individual work, but with the support of the group.

Suggestions and comments

Talk through pages 12, 13 and 14 with the class, explaining how the 'Exploring techniques' sheets and equipment are organised, and describing exactly how students are expected to record their discoveries.

Emphasise that, although students will be mainly working on their own, they should see it as a co-operative exercise and share ideas and discoveries.

The 'Exploring techniques' sheets. There are 8 sheets covering 7 different technique 'areas' for exploration. We recommend that each student works on at least 3 areas, and spends about 1 hour on each area.

It may be helpful to ask students to choose a new area at the beginning of each lesson, thus limiting the time they spend on each one.

Students do not need to work through every item on each sheet; they can select the investigations and challenges that interest them. In general, however, the sheets begin with easier investigations and progress through to harder challenges, and you may decide to restrict the choice initially for some groups.

Some students may find it hard to move from the more structured Stage 1 to this stage, where there are no detailed instructions to follow. If this is the case then, to start with, students in the same group could be asked to work from the same sheet so that they can offer each other support. (They could, for example, share out the work on the sheet so that different aspects of that technique are covered.)

You could, of course, go even further and begin by starting the whole class off together on the first investigation, so that everyone has a chance to discuss how a record of the discoveries should be kept.

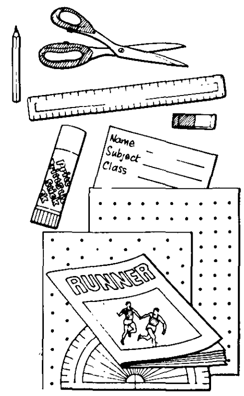
STAGE 2

You will need

- some Exploring techniques sheets,
- a pair of scissors,
- a glue stick,
- a pencil, ruler and rubber,
- lots of paper or card,
- an exercise book or folder to record what you find out.

You may also need

- some old magazines – for extra pictures,
- some squared or isometric dotty paper,
- a protractor – to measure fold angles.



There are seven different 'techniques' to explore:

- cards with parallel folds
- cards with parallel 'stick-ons'
- cards with angled folds
- cards with angled 'stick-ons'
- cards with slides or pivots
- boxes
- cubes

Choose one technique to work on.

You do not have to try everything on the sheets.

You should aim to explore at least three of the seven 'techniques'.

13

Equipment. Encourage students to use paper of an appropriate size. There is a tendency to use A4 paper when A5 would be more suitable.

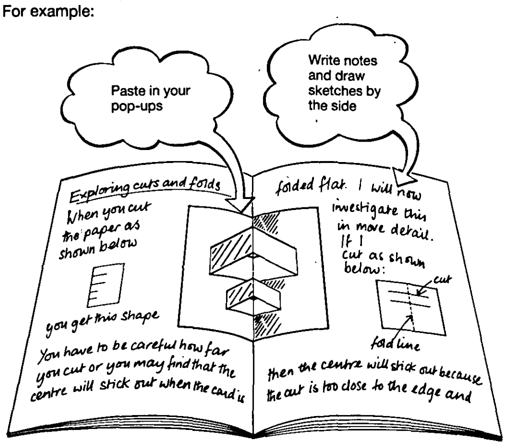
An ordinary exercise book is suitable for record keeping, together with a box to store items that do not fold flat. Items that are not included in the exercise books will need to be labelled with both the students' names and numbers or title which can be cross-referenced to the written records.

It is important to have a back-up supply of magazines. Although students should be encouraged to bring in their own so they can use pictures that interest them, this should not be relied upon.

STAGE 2

Keeping a record

Record all you do in an exercise book or in a folder. This will help you to remember what you've found out.
For example:



Keep everything you make – even the things that go wrong.

Make a note of

- what you tried to do,
- what happened,
- what you learnt.

You'll need all this information for Stage 3.

Remember
You can learn a lot from failures, so include these as well.

14

Keeping a record. Emphasise the need for students to keep a record of everything they try, even unsuccessful attempts. This is unlikely to come naturally.

One teacher prepared for this by exploring a few ideas herself and then writing up her discoveries in the style required of the students. The students were encouraged to see that even the teacher made mistakes and wrote about them.

Remind students that assessment for this stage will give credit for a full account of their explorations.

For the last few minutes of each lesson, ask groups to share discoveries among themselves, or with the rest of the class.

Throughout this stage, students will probably generate a great many ideas for cards and boxes of their own. These may be used to help with the brainstorming activity at the beginning of Stage 3.

'Exploring techniques' sheets

The following pages contain reproductions of the 'Exploring techniques' sheets.

There are 7 sections:

	References to Masters	Pages in Teacher's Guide
Cards with parallel folds	M51, 52	T27
Cards with parallel 'stick-ons'	M53, 54	T28
Cards with angled folds	M55, 56, 57, 58	T29, 30
Cards with angled 'stick-ons'	M59, 60	T31
Cards with slides or pivots	M61, 62	T32
Boxes	M63, 64	T33
Cubes	M65, 66	T34

These may be tackled in any order. It should be noted that M55, 56 are easier than M57, 58. Some of the ideas contained in the sheets are pursued further in Chapter 2, Developing the mathematics.

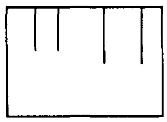
STAGE 2

Cards with parallel folds

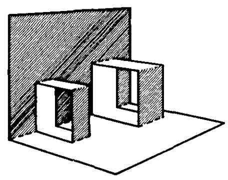
(These are cards like 3. The Pop Star and 4. The Rolls Royce.)

- Fold a piece of paper in half.

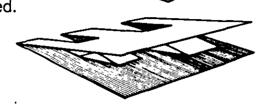
  Make pairs of equal cuts across the fold.



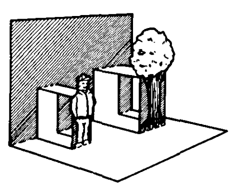
- Open the paper to an angle of 90°. Make the shapes stand out.



- Make the shapes fold flat when the paper is closed.



- Cut pictures out a magazine and stick them onto the vertical faces.



- Look at the card you have made. Does it work properly? Are the pictures completely hidden when the card is closed or do they stick out? Could you have forecast this before you made it? How?

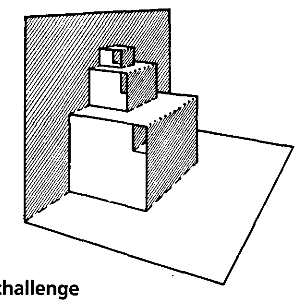
1

M51

STAGE 2

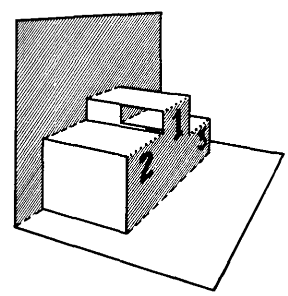
Tower challenge

Try to make this pop-up tower from one sheet of paper without using any glue.



Podium challenge

Make this Winners' podium from a single sheet, without using any glue.



4

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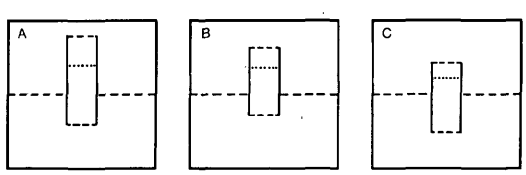
STAGE 2

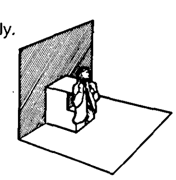
Cards with parallel folds (continued)

Try to make a pop-up card like the pop star card . . .

- Before you start, think carefully.

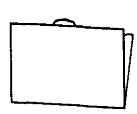


Only one of these three cards will work properly.
Which is it? How can you tell?
What is wrong with the other two?

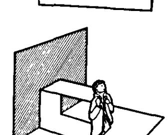


- Now make a card.
  - Make the cuts *before* you fold.
  - Paste on a picture.

- How can you be sure that the picture will *not* stick out of the card when it is shut?



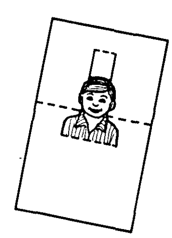
- How could you make the picture stand out *more* when the card is opened at 90°?



2

Try to make a pop-up card like the Rolls Royce . . . . .

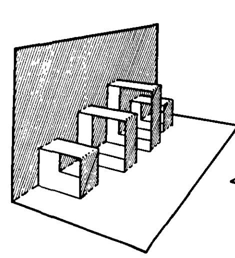
- Start by sticking a picture onto your flat card.



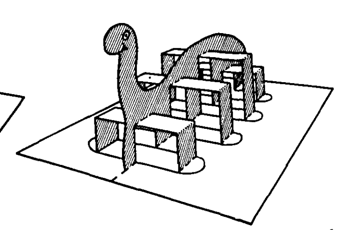
- Now think carefully . . .
  - how far should you cut round the outline?
  - where should you put the folds?

- Test your ideas to see if they work.

Glue two of these together,. back to back . . . . . .



. . . and try to make this creature!



3

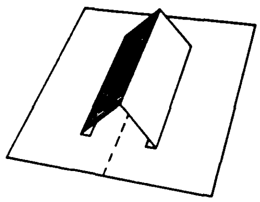
BE A PAPER ENGINEER

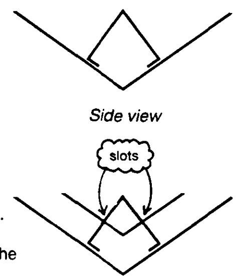
M52

STAGE 2

Cards with parallel 'stick-ons'

(These are like 5. The Indian Palace and 6. Staircase and Pillars.)

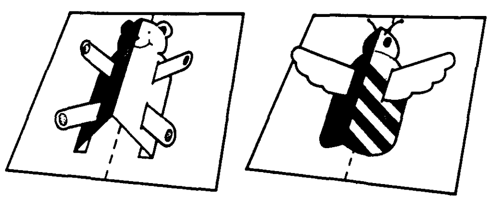




Try making the basic tent shape.

Add slots and a strip of paper in the shape of wings, arms, legs . . .

to make a teddy bear, an insect, an aeroplane or . . .

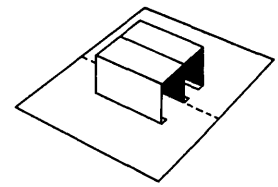


Explore your own ideas.

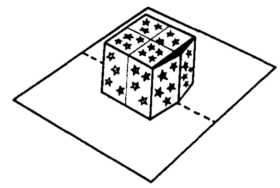
STAGE 2

A house or a parcel or . . .

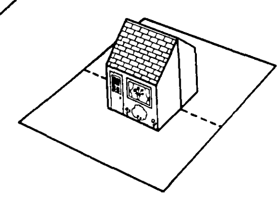
This box is like diagram H on page 3 of this leaflet.



Try making it into a 'pop-up' parcel . . . . .



This would make a good birthday card!



or a 'pop-up' house . . .

or a pop-up 'birthday cake' or . . . what?

Explore your own ideas.
Try making something from diagram I on page 3.

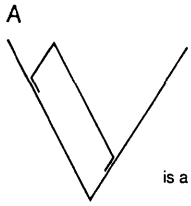
STAGE 2

Cards with parallel 'stick-ons' (continued)

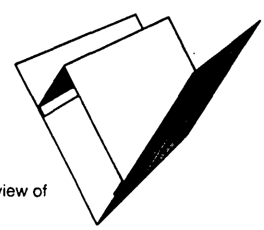
A puzzle to think about

Below are some side views of different cards. For example,

A

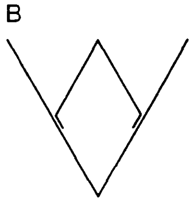


is a side view of

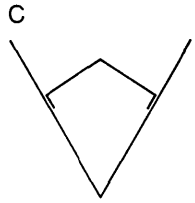


1. Which of the cards can be closed without creasing in the wrong place? Explain how you can tell without having to make the cards.
2. Which of the cards can be opened completely flat on the table without tearing? Again, explain how you can tell.

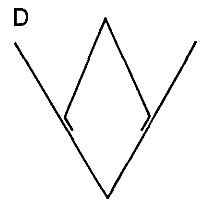
B



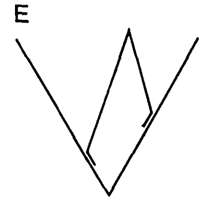
C



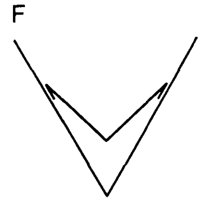
D



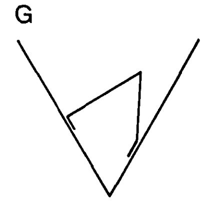
E



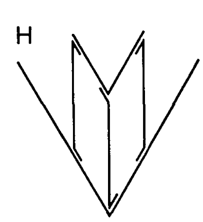
F



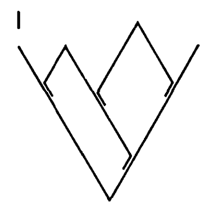
G



H



I



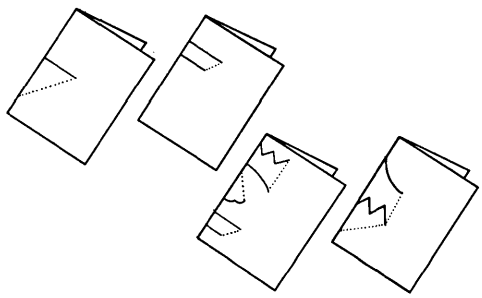
Now try making some, and see if you were right!

STAGE 2

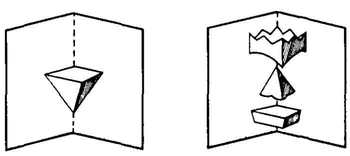
Cards with angled folds . . . part 1

(These are like 7. Frankenstein, 8. A Barbecue Invitation, 11. The Dove of Peace and 12. The Ghost.)

Fold a piece of paper in half.
Try cutting and folding the paper in different ways.
For example:



Now open the cards and make the shapes stand out.



What happens?

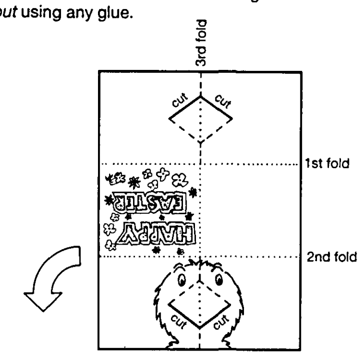
1

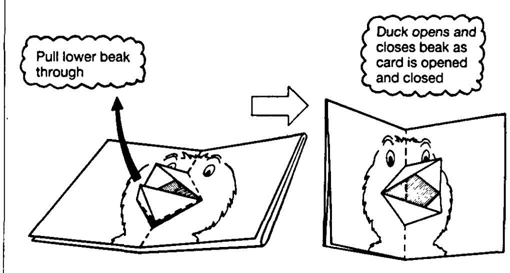
M55

STAGE 2

A Duck challenge

Try to make this Easter card from a single sheet of A4 paper, *without* using any glue.





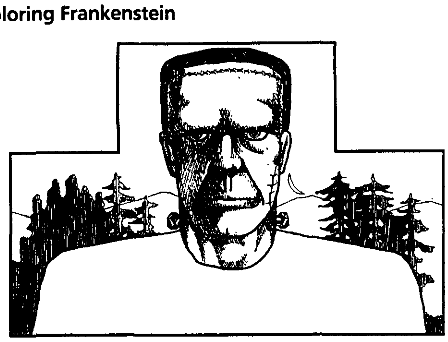
This sort of card only needs to be printed on one side of the paper. Look out for examples of this technique in the shops.

4

STAGE 2

Cards with angled folds (continued)

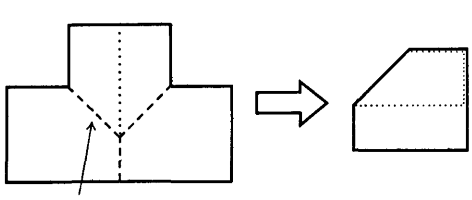
Exploring Frankenstein



The two diagrams below show how the card folds flat.

Open Card

Folded card



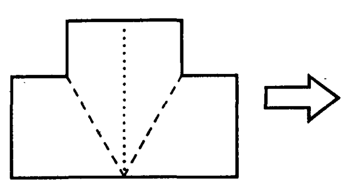
Fold lines

the dotted line shows how the inside folds down.

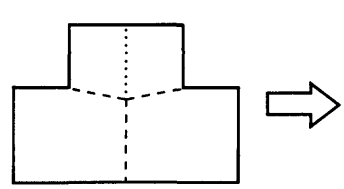
2

Using the *same* card (you don't need to keep making different ones) try changing the angle of fold and see what happens. Draw sketches to show the open card and folded card in each case.

For example:



draw a sketch of the folded card



draw a sketch of the folded card.

The Barbecue invitation uses the same idea, but you can see part of the inside of the card when it is shut. Try to think of other ways of using this idea.

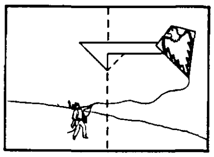
3

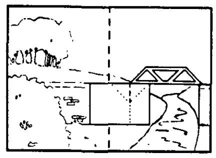
M56

STAGE 2

Cards with angled folds . . . part 2

(These are like 9. The Flying Kite and 10. The Swing Bridge.)

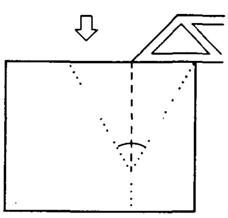


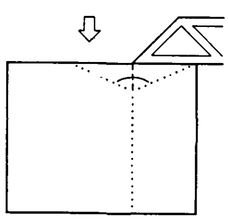


Compare these cards.

- In what ways are they similar?
- In what ways are they different?
- What angle does the bridge turn through as the card is closed?
- What happens if the V shaped folds are changed so that the angle between them . . . .

  . . . gets smaller? . . . gets larger?





1

M57

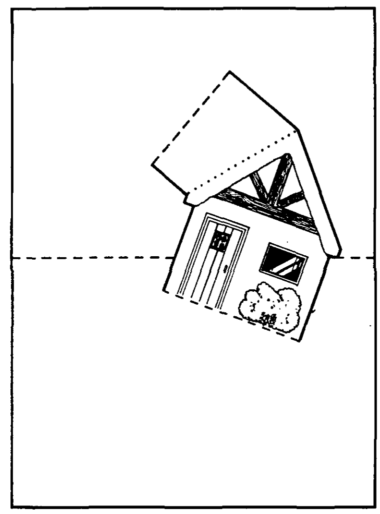
STAGE 2

The 'Angled House'

(You should try page 3 before doing this page.)
Try making a larger version of the pop-up card shown below.

If this card is to work properly, what can you say about

- the directions of the fold lines?
- the angles between the 4 fold lines?



4 Can you make the house pop out at a different angle? How?

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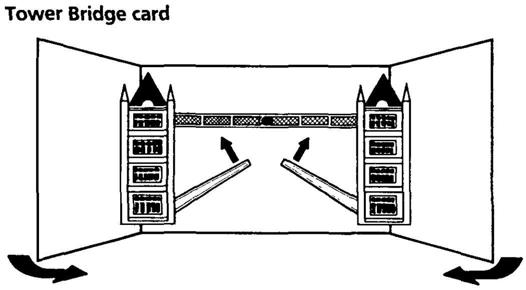
STAGE 2

Cards with angled folds (continued)

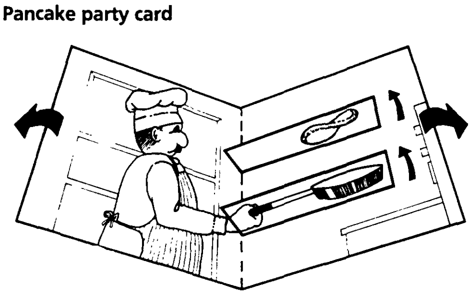
Try to make these cards . . .

Tower Bridge card



As the card closes, the bridge is raised.

Pancake party card

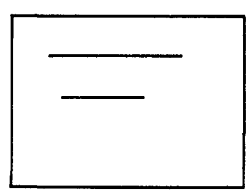


As the card is opened, the pan rises slowly, and the pancake rises quickly.

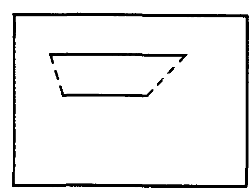
2

A useful result

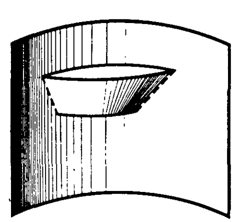
Make 2 slits, anywhere.



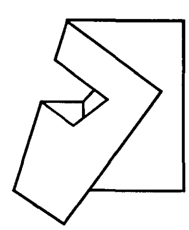
Make 2 valley folds between the ends of the slits.



Bend the paper towards you.



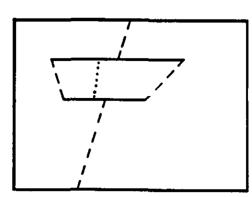
Fold flat and crease.



Open up your paper.
There are four fold lines.
What do you notice about

- the *directions* of these folds?
- the *angles* between these folds?

Will this always work? Why?



3

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M58

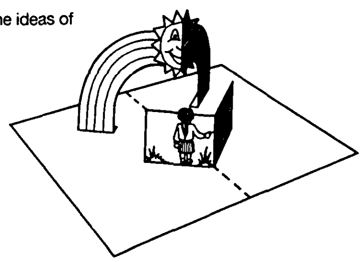
STAGE 2

Cards with angled 'stick-ons'

(These are like 1. The Cat, 13. A Street Scene, 14. The Settling Butterfly and 15. "Rise and Shine!")

The Rainbow card

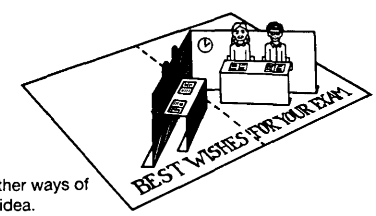
- Try making this rainbow card. (You can use the same idea to make bridges, doorways etc.)
- Try out some ideas of you own.



Exam card

- Try making this card.

(Notice how each half of the wall is like a separate pop-up card.)



- Think of other ways of using this idea.

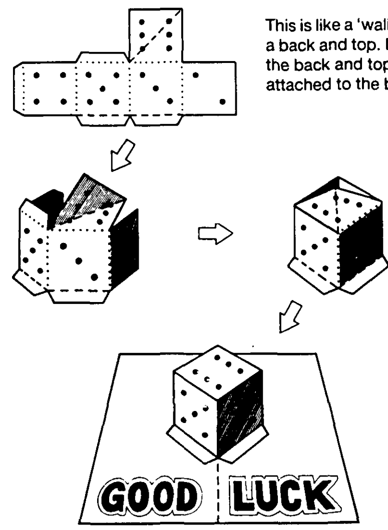
1

M59

STAGE 2

Good Luck card

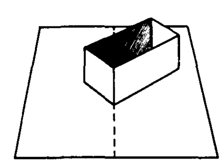
This is like a 'wall card' with a back and top. Notice that the back and top are not attached to the base.



Does the shape you paste on have to be a cube?

For example, is this open topped box possible?

Can it have a lid . . . or a roof?



4

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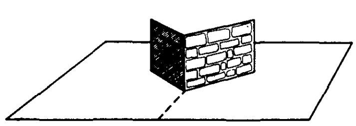
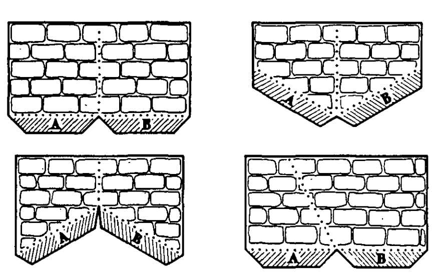
STAGE 2

Cards with angled 'stick-ons' (continued)

Walls investigation



Cut out and fold some different walls like these . . .



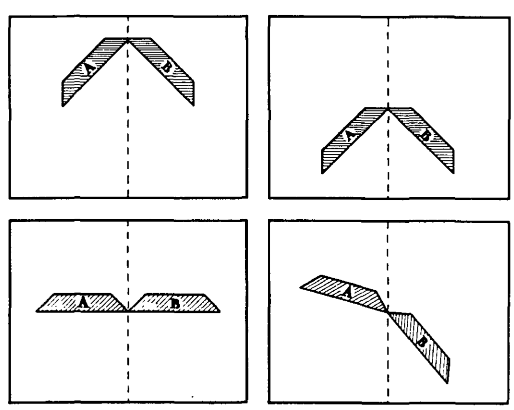
Try pasting them down onto the base sheets at different angles, and in different positions.

See the next page for examples

Try to make them work as pop-up cards.

Make sure they will close without creasing in the wrong place.

2



Write about what happens . . .

- How can you make a wall stand up vertically?
- How can you make a wall lean forwards or backwards or sideways?
- *Do all the cards work properly? How can you tell if a card will work, without having to make it?*
- Do any of the walls stick out of the cards when they are shut?
- How can you predict when this will happen?

3

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M60

STAGE 2

Cards with slides and pivots

(These are like 16. The Magic Rabbit, 17. The Mole, 18. "Say aah!", 20. The Gorilla, 21. The Sleeping Sheep.)

A sliding challenge

Make one of these . . . or invent an idea of your own.

You pull the tab and the cork explodes out of the bottle.

You pull the tab and Santa moves up and down the chimney!

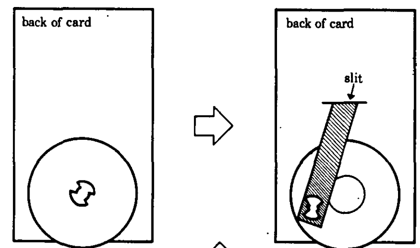
1

M61

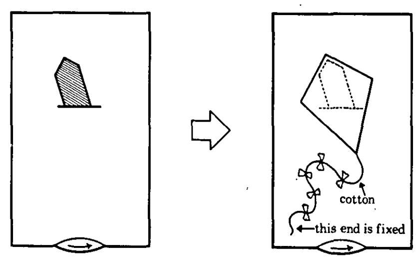
STAGE 2

A Kite challenge

1. Make a wheel like you did for the 'Spiral' . . . .
2. Cover the pivot and attach an arm to the wheel with a second pivot . . . .



3. The front of the card should look like this . . .
4. Stick a kite onto the top of the shaded arm.



5. As the wheel is turned, the kite should move around the card in an interesting way.

4

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STAGE 2

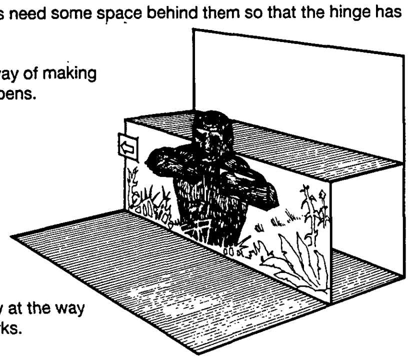
Cards with slides and pivots (continued)

A Gorilla challenge

Cards like this need some space behind them so that the hinge has room to turn.

Here is one way of making sure this happens.

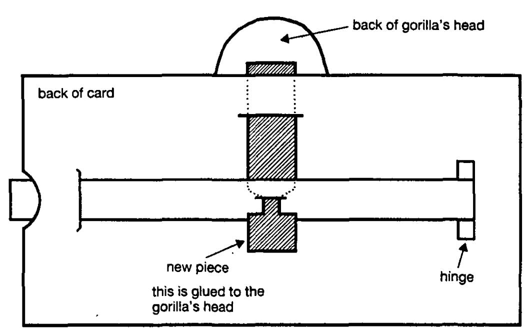
Try it!



Look carefully at the way the hinge works.

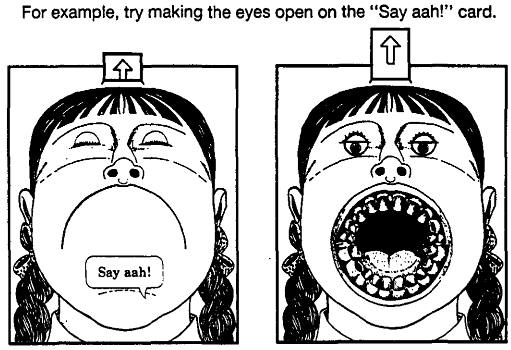
Can you change this card so that the gorilla moves *both* of his arms when the tab is pulled?

2



- This diagram shows how you can also make the gorilla's head rock from side to side as the tab is pulled. Try to make it!
- You may also like to try making small changes to other cards.

  For example, try making the eyes open on the "Say aah!" card.



3

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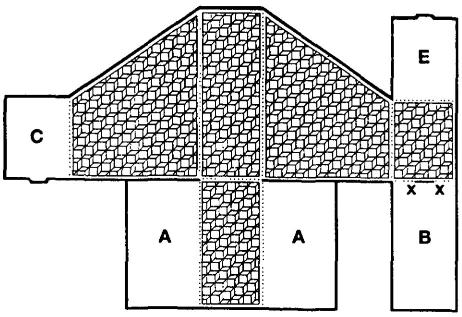
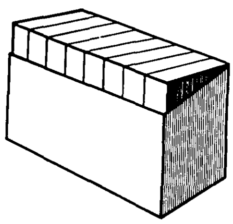
M62

STAGE 2

Boxes

Look at 28. The Library Box from Stage 1.





Either
try to adapt the design to make a box for storing

- cassette tapes
- paperback novels
- or items of your own choice.

Or

try adapting other boxes from Stage 1, so that they fulfil a purpose.

1

M63

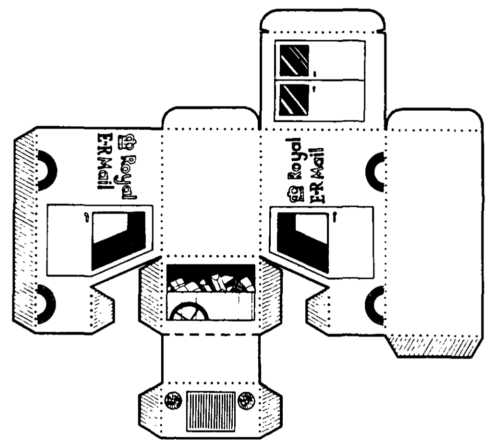
STAGE 2

A novelty box challenge

Sometimes, sweet boxes, scent boxes, soap boxes, etc., are made into novel shapes.

(The example below shows a 'Post Office van' box. You might like to try making it.)

Try to design your own novelty box.



4

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STAGE 2

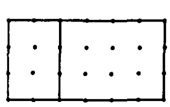
Boxes (continued)

Inventing nets for boxes

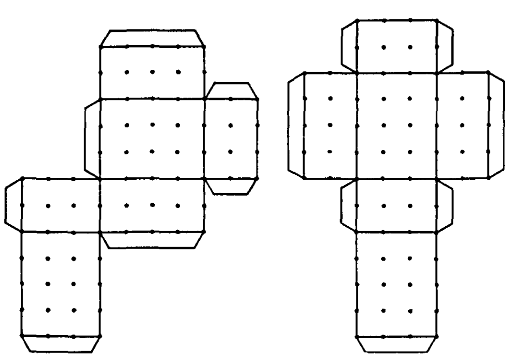
Here is a partly finished drawing of a net for a box.

It will need to be glued together.

(Dotty paper has been used to make the drawing easier.)



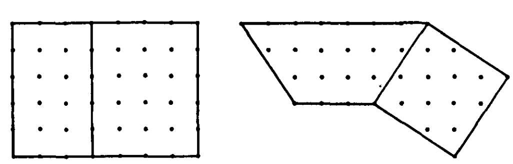
It can be completed in many different ways. For example:



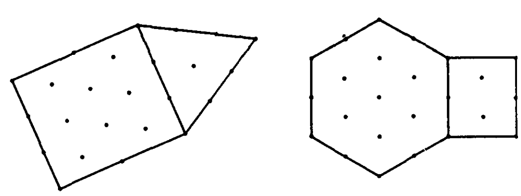
Think of a third way of completing the net, and draw it accurately on dotty paper.
Put flaps on for gluing.
Now make it.

Here are some other partly finished nets. Using dotty paper, try to finish them off. Add on flaps for gluing or fastening. Check that they work by making them.

Use square dotty paper for these two:



Use isometric dotty paper for these two:



Now invent some more of your own . . .

2

3

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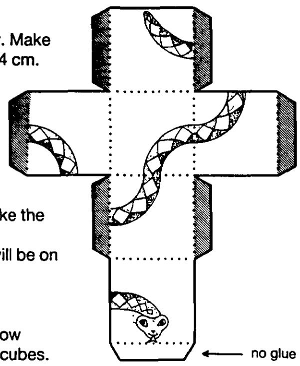
M64

STAGE 2

Cubes

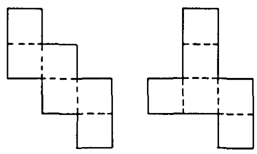
The Snake cube

Draw this net accurately. Make all the squares 4 cm by 4 cm.



Copy the snake and make the cube.
The head of the snake will be on the lid of the box.

The two nets shown below will also fold up to form cubes.



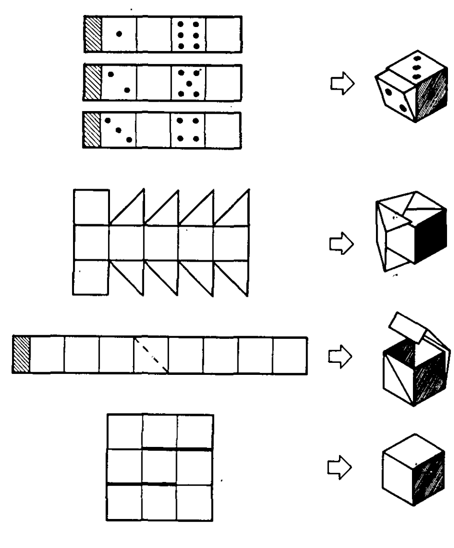
Choose one, show how you would add glue flaps and a snake so that it will make another snake cube, exactly the same.

Now make it to see if you were right!

STAGE 2

Unusual methods . . .

Here are some more imaginative ways of making a cube . . . try some! Do they need extra flaps? Do they need glue?



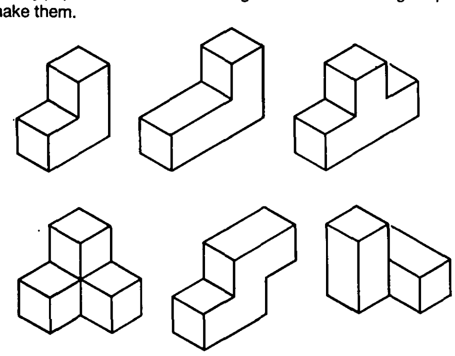
Can you find any other interesting ways?

STAGE 2

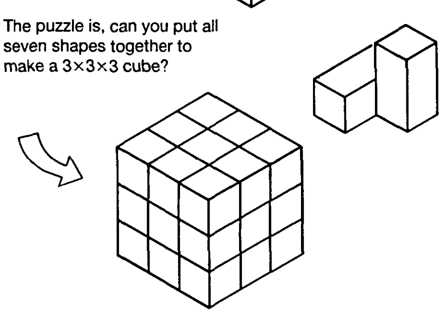
Cubes (continued)

The Soma cube puzzle

Using dotty paper, draw nets for making each of the following shapes, and make them.



The puzzle is, can you put all seven shapes together to make a 3×3×3 cube?



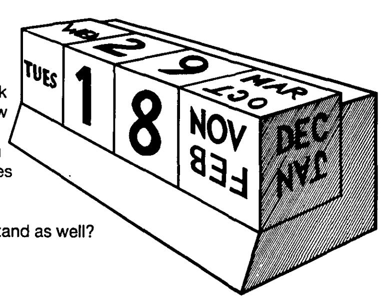
What else can you make with cubes?

Here are two ideas . . . can you think of any more?

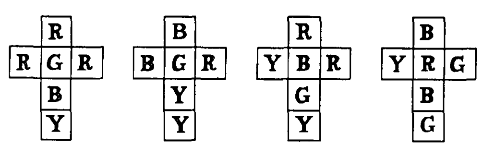
A calendar

You will have to think quite hard about how you will arrange the numbers so that you can show all the dates from 1 to 31!

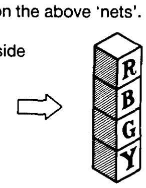
Can you make the stand as well?



A puzzle



Make four cubes and colour them as shown on the above 'nets'.
(R=red, G=Green, B=Blue, Y=Yellow.)
Can you stack them into a tower so that each side of the tower shows four different colours?



Stage 3 Making your own original

Introduction

In this stage, students brainstorm ideas for their own products, then use the techniques that have been developed to design and make prototypes. This will involve some combination of calculation and trial and error, where ideas are successively refined until a satisfactory result is achieved. Some students may wish to develop one of the ideas they have worked on in Stage 2, while others may prefer to begin afresh. All students should aim to produce an original product to the highest possible standards of accuracy, using mathematical techniques wherever appropriate.

Total time needed

About 2 to 4 hours, depending on the complexity of the product.

Organisation and equipment required

Students should work in groups for the initial brainstorming session. They may then either work individually on different ideas, or continue to work together on the development of a more ambitious product. In this latter case, it is important that the project is divided into subtasks which can be shared among the group members.

Each student will need

- a 'Brainstorming' sheet,
- a few sheets of thin white card.

In addition students may find it helpful to have access to the items listed on page T14.

Related assessment criteria

Stage 3 offers students the opportunity to show that they can

(ix) generate possibilities for a design with original features,

(x) draw a design to an acceptable degree of accuracy,

(xi) construct a prototype with original features.

Stage 3 Making your own original

In this stage you will

- brainstorm to find a good idea for a pop-up card, box etc, of your own,
- make a rough version of your own idea in paper,
- make a 'best' version of your idea in card.

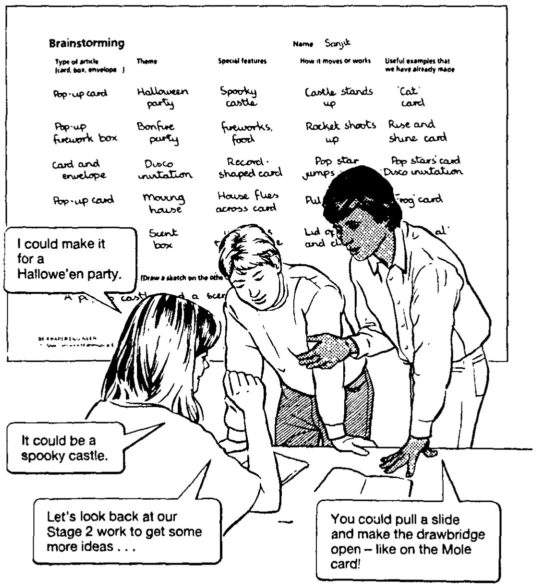
15

STAGE 3

Brainstorming

You will each need a Brainstorming sheet. [1]

In your group, write down some original ideas for pop-up cards, boxes etc, which *you* could make.

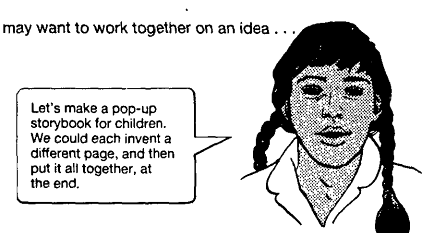


16

STAGE 3

Choosing an idea to work on

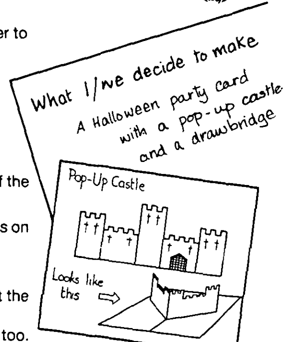
You may want to work together on an idea . . . [2]



. . . or maybe you would prefer to work on your own.
Whichever you decide,

- choose what you want to make,
- describe it at the bottom of the Brainstorming sheet, [3]
- draw sketches of your ideas on the back of the sheet.

If you are working in a group, decide how you will share out the work.
Write that down on the sheet too.



Are you making a box or envelope? If so, turn to page 22.
Otherwise, turn over to page 18. [4]

17

Purpose

To give students the opportunity to consider a range of alternatives before deciding on ideas for further development.

Presentation

Individual work followed by group discussion.

Suggestions and comments

[1] Issue each student with a 'Brainstorming' sheet (M71). The notion of brainstorming may be unfamiliar to some students. Emphasise that the aim of this activity is to list as many imaginative, creative ideas as possible. You could start off by collecting a few ideas from students and listing these on the board. It is helpful to suggest that students note a few ideas on their own before taking it in turns to share their ideas with the rest of their group. This ensures that everyone has something to contribute and that the group discussions are not dominated by the more forceful members.

[2] Each student should now choose an idea to develop. Make sure that, if students do decide to collaborate to make a group product, that there are sufficient tasks to go round. (A single pop-up card is not a suitable group product, for example.)

Examples of products that have been produced by groups are:

- a 'pop-up' dolls' house with furniture
- a model fairground with moving rides
- a model theatre with moveable actors.

[3] Encourage students to describe some of the mechanisms they will need by comparing them with products from Stages 1 and 2. For example, 'We could make it work like the Flying Kite!'

[4] From this point on, the Student's Booklet offers two routes, one for pop-up or moving cards (pages 18–21) and one for boxes and envelopes (pages 22–25).

The general processes involved are similar in both cases, but we have highlighted different aspects – the gradual accumulation of ideas and details in the cards, and the development of an efficient 'net' for the boxes.

All students should work through one route, although some may need to refer to both.

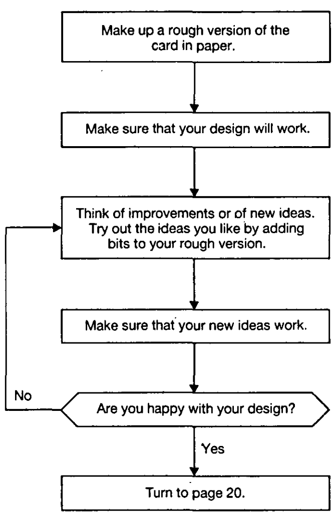
STAGE 3

Developing your idea for a pop-up or moving card

Planning your design in rough

So, you have an idea for a card.

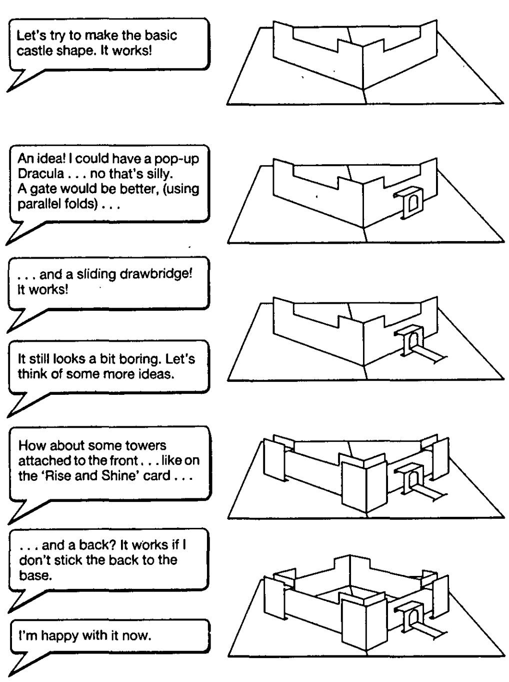
Here is one way to develop it.



18

STAGE 3

Here's how one girl developed her idea . . .



19

Purpose

To enable students to refine their ideas for a pop-up or moving card and then produce

- a rough working version in paper
- an accurate plan which may be used as a copy-master in Stage 4
- an accurate prototype in thin card.

Presentation

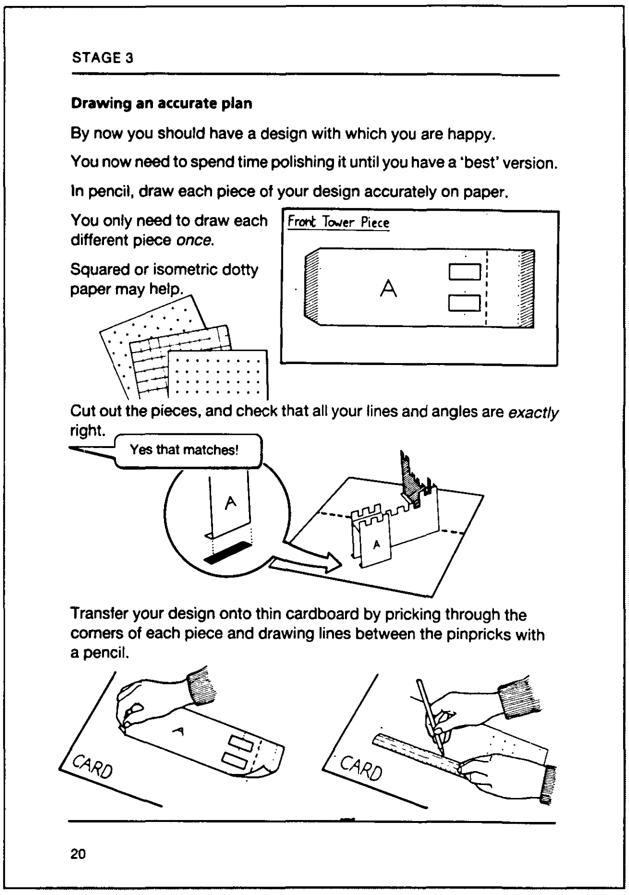
Individual work.

Suggestions and comments

The flow chart shows a process of successive refinements as improvements and new ideas are incorporated into the design. Page 19 illustrates this process with the example of someone designing a pop-up castle. Encourage students to relate the development of this example to the flow chart. The cards produced by students do not need to be as elaborate as this. In fact, simpler ones are sometimes the most effective.

You may prefer to illustrate this process with a simpler model that has been worked through by yourself or by a student from another class. (Perhaps a wall chart could be made showing photographs of attempts including failures.)

Students should aim to produce rough but working versions of their cards, going through cycles of modification and improvement where necessary.



STAGE 3

Drawing an accurate plan

By now you should have a design with which you are happy.

You now need to spend time polishing it until you have a 'best' version.

In pencil, draw each piece of your design accurately on paper.

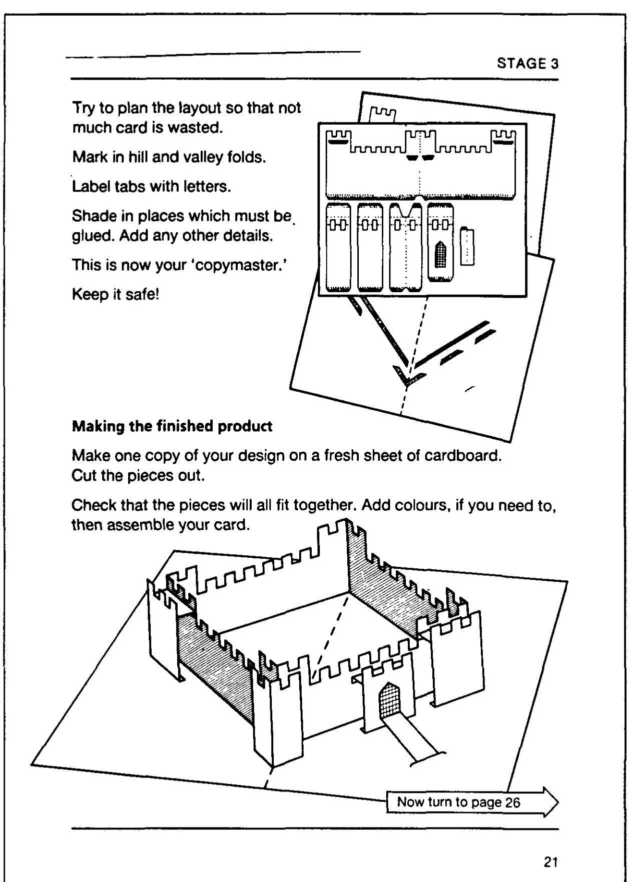
You only need to draw each different piece *once*.

Squared or isometric dotty paper may help.

Cut out the pieces, and check that all your lines and angles are *exactly* right.

Transfer your design onto thin cardboard by pricking through the corners of each piece and drawing lines between the pinpricks with a pencil.

20



STAGE 3

Try to plan the layout so that not much card is wasted.

Mark in hill and valley folds.

Label tabs with letters.

Shade in places which must be glued. Add any other details.

This is now your 'copymaster.'

Keep it safe!

Making the finished product

Make one copy of your design on a fresh sheet of cardboard.
Cut the pieces out.

Check that the pieces will all fit together. Add colours, if you need to, then assemble your card.

Now turn to page 26

21

Students are now asked to draw accurate plans which may be used as copymasters, so that their products are ready for mass-production in Stage 4.

To begin with, they are asked to draw each piece of their designs accurately on paper.

Encourage them to use pencils (so that corrections may be made) and appropriate drawing instruments.

These paper designs will be used as templates, so students do not need to draw each design more than once.

If they wish, students may use lined or dotted, squared or isometric paper to assist with the drawing. (See M67–70.)

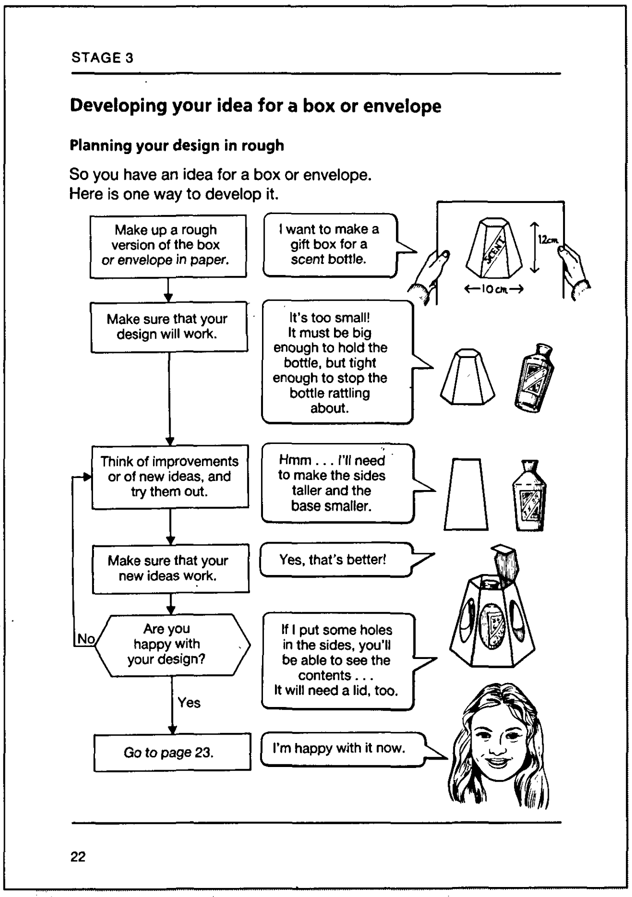
The pieces are then cut out to facilitate the 'checking'. Often, at this point, students will become aware of errors and redraw their designs.

The paper designs are now transferred onto thin cardboard to form the copymasters. Encourage students to set out the designs in an efficient, yet attractive way. They should try to reduce the number of tabs that are needed by combining pieces, if this is possible.

When students are satisfied that their cards are accurately drawn, they may need to go over the pencil lines with fine black drawing ink so that the plans will duplicate clearly.

The designs are now tested by making up prototypes from photocopies of the masters. If a photocopier is unavailable then students can make single copies by pricking through onto fresh sheets of cardboard. Make sure that students do not cut up their copymasters, as these will be required for Stage 4.

If students find mistakes in their designs, then they should amend their copymasters accordingly.



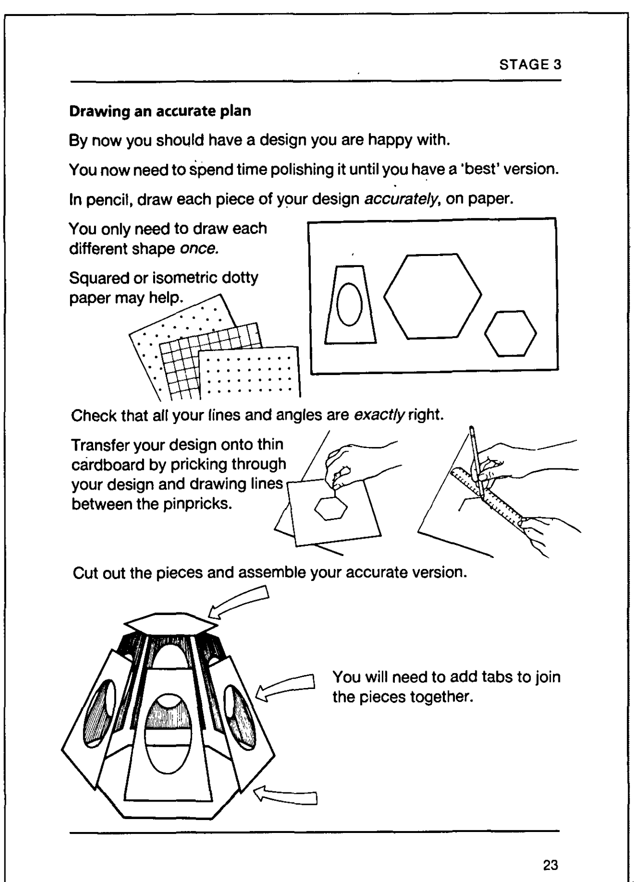
STAGE 3

Developing your idea for a box or envelope

Planning your design in rough

So you have an idea for a box or envelope.
Here is one way to develop it.

22



STAGE 3

Drawing an accurate plan

By now you should have a design you are happy with.

You now need to spend time polishing it until you have a 'best' version.

In pencil, draw each piece of your design *accurately*, on paper.

You only need to draw each different shape *once*.

Squared or isometric dotty paper may help.

Check that all your lines and angles are *exactly* right.

Transfer your design onto thin cardboard by pricking through your design and drawing lines between the pinpricks.

Cut out the pieces and assemble your accurate version.

23

Purpose

To enable students to refine their ideas for a box or envelope and then produce

- a rough version in paper
- an accurate plan which may be used as a copymaster in Stage 4
- an accurate prototype in thin card.

Presentation

Individual work.

Suggestions and comments

The processes outlined here correspond exactly to those for the pop-up or moving card designs.

Again, you may prefer to illustrate these processes using a model made by yourself or by a student from another class.

The method for drawing an accurate plan again corresponds to that for the card designs. Here, however, we suggest that a quick cardboard version is made before the copymaster is produced. This is to encourage the student to think about designing a net which minimises the use of tabs.

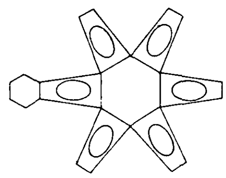
STAGE 3

The more tabs you use, the more difficult your box or envelope will be to make.

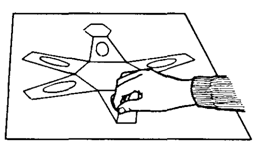
Try to make yours from a single piece of card or paper.

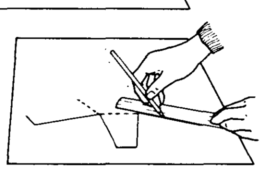
Here is one way to do this.

Cut along the folds of the tabs one by one, until the design can open out flat. This is called a *net*.



Now transfer your net onto a fresh sheet of thin card by pricking through with a pin and drawing lines between pin pricks.





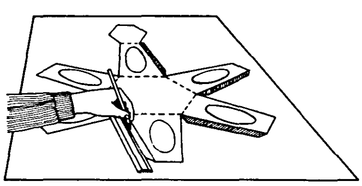
Check that all the measurements are still correct.

24

STAGE 3

Decide where you'll need tabs and draw them in.

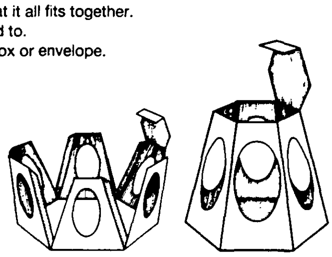
(Don't make them too narrow or your box will fall apart!)



Mark in hill and valley folds.
Label tabs so that it is clear where they go.
Shade in places that must be glued.
Add decoration and any other details.
This is now your 'copymaster'. Keep it safe!

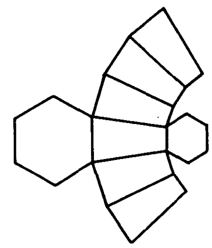
Making the finished product

Make one copy of your design on a fresh sheet of thin cardboard.
Cut the design out.
Fold it up to check that it all fits together.
Add colour if you need to.
Now assemble your box or envelope.



25

The box or envelope may be flattened out in several ways. You may like to ask students if they can think of a better way than the one illustrated here. For example, the following net can be made out of a smaller sheet of cardboard:



When the most efficient arrangement has been decided upon, the designs are then transferred onto thin cardboard and copymasters are made.

As before, students may need to reinforce their pencil lines with black ink to facilitate duplication.

Each student now makes a prototype from a photocopy of the master. Make sure that the copymasters are not cut up, as these will be required for Stage 4.

If students discover any mistakes in their designs, they should amend their copymasters accordingly.

Stage 4 Going into production

Introduction

In this stage, students make up 'kits' which include plans and instructions so that other people can recreate their paper products.

Total time needed

About 1 or 2 hours, but this depends on the complexity of the product being produced.

Organisation and equipment required

You will need to find a way of duplicating a few copies of each kit so that several people can reconstruct each product. If at all possible, arrange for students to visit the reprographics department to learn about various duplicating methods. Perhaps this may be done in small groups as students reach the point when they are ready to duplicate their kits.

Each student will need a few large A4 envelopes in which their kits will be assembled. Other useful items include the cartoon sheet (M72), a camera (a 'Polaroid' is ideal), a typewriter, a word processor, some rub down lettering (e.g. 'Letraset') or a lettering stencil and a tape recorder. None of these is essential, but they do help students to produce products of which they can be proud.

Related assessment criterion

This stage offers students the opportunity to show that they can

(xii) devise instructions to enable someone else to make the product.

Stage 4 Going into production

In this stage, your aim is to produce a few complete kits, so that other people can make your paper product.

Each kit will contain

- a copy of your design
- a set of clear instructions for making your product.

The stage is in 5 sections:

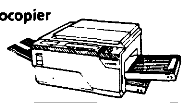
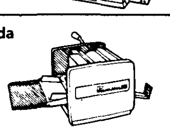
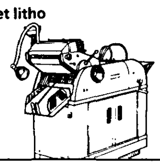
- Finding out about copying ('reprographics')
- Preparing your instructions
- Testing your instructions
- Putting your kits together
- Using your kits.

26

STAGE 4

Finding out about reprographics

Your school probably has several different methods of printing copies from paper originals. 1

Photocopier 	A bright light shines on an original, producing a picture of it on the copy paper.
Banda 	You make an original with special coloured carbon sheets. The machine wets the original and prints copies on paper or card.
Offset litho 	A trained operator makes a special printing 'Plate' – like your original but back-to-front. The plate rotates on a drum at high speed to make copies very quickly.

Find out about each method.

- Will it print in colour?
- Will it print on card?
- Will it print on coloured paper?

Your teacher will help you to decide on which method to use.

Check that your copymasters will produce good copies. 2
You may need to go over some lines with ink.

27

Purpose

To introduce the idea of a kit which will enable someone else to make up the product, and to help students understand the alternative methods of production that they may have at their disposal.

Presentation

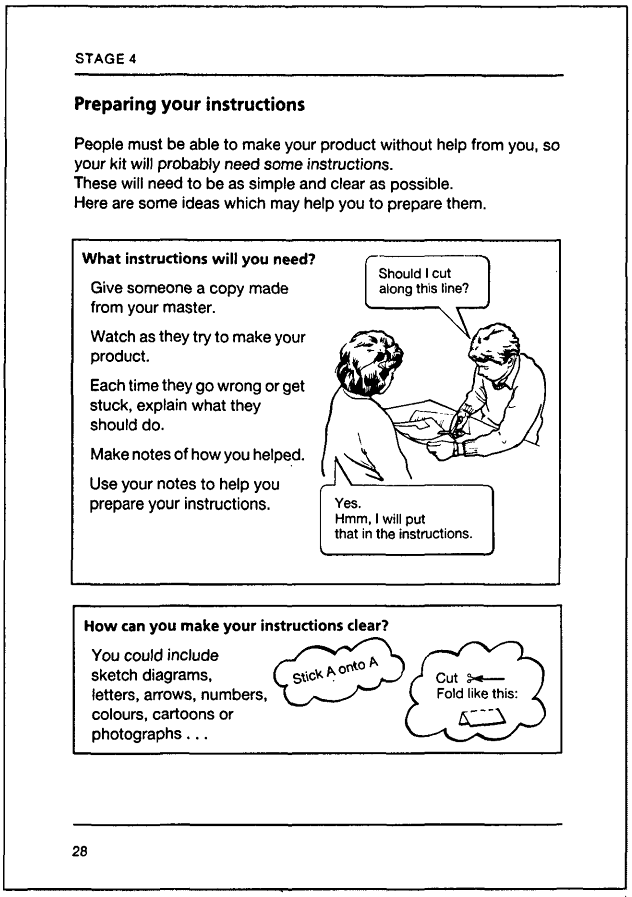
Mainly individual work with the support of the group and incorporating class discussion at various points.

Suggestions and comments

Some of the activities in this stage may be too ambitious for some students. In this case you could ask them to concentrate on devising and testing their instructions for making the products (pages 28 onwards) while you run off a few copies of their design masters, retained from Stage 3.

1 Have at hand some samples of copies made using the various methods available at your school. Where possible, arrange for students to see the different methods in operation.

2 There may only be one possible choice, in which case you should make this clear and show the students how to prepare their masters.



STAGE 4

Preparing your instructions

People must be able to make your product without help from you, so your kit will probably need some instructions.
These will need to be as simple and clear as possible.
Here are some ideas which may help you to prepare them.

What instructions will you need?

Give someone a copy made from your master.

Watch as they try to make your product.

Each time they go wrong or get stuck, explain what they should do.

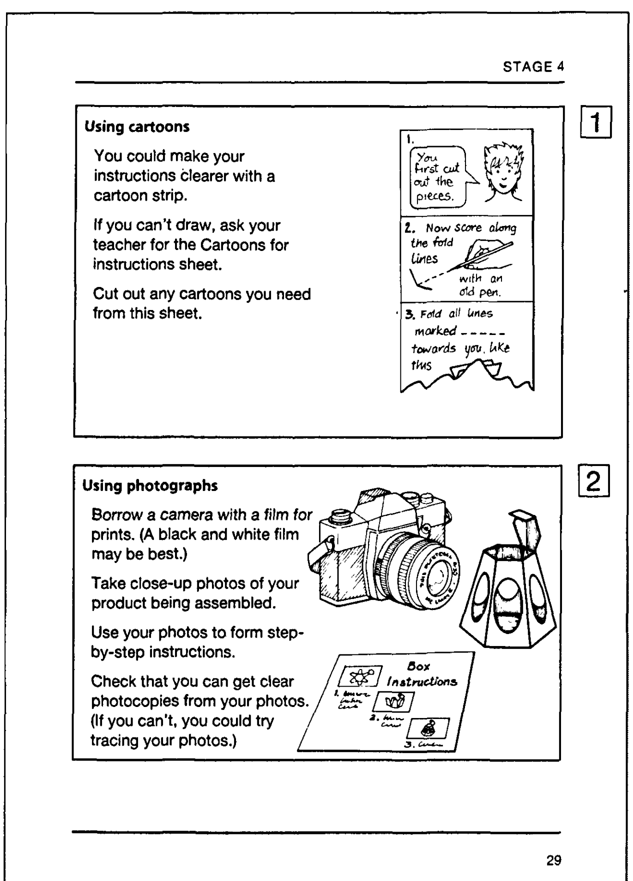
Make notes of how you helped.

Use your notes to help you prepare your instructions.

How can you make your instructions clear?

You could include sketch diagrams, letters, arrows, numbers, colours, cartoons or photographs . . .

28



STAGE 4

Using cartoons

You could make your instructions clearer with a cartoon strip.

If you can't draw, ask your teacher for the Cartoons for instructions sheet.

Cut out any cartoons you need from this sheet.

[1]

Using photographs

Borrow a camera with a film for prints. (A black and white film may be best.)

Take close-up photos of your product being assembled.

Use your photos to form step-by-step instructions.

Check that you can get clear photocopies from your photos. (If you can't, you could try tracing your photos.)

[2]

29

Purpose

To enable students to prepare and test a set of instructions for making their products.

Presentation

Mainly individual, but with some work in pairs.

Suggestions and comments

Students may find it difficult to prepare instructions. Sometimes, for example, they make the instructions over-complicated. Page 28 suggests that they watch as someone else tries to assemble the products without any instructions, recording all the interventions that are necessary. This process may need to be repeated several times with different people.

If a tape recorder is available it may be used to help students translate their oral advice into written instructions – a process which some may find difficult.

[1] The 'Cartoons for instructions' sheet may be duplicated from M72.

[2] The use of a camera may delay the production of the instructions for a week perhaps, but we strongly urge you to consider this option seriously.

STAGE 4

Testing your instructions

Make a copy from your master.
Find someone else who hasn't seen your product before.
This person will be your *tester*.

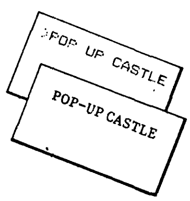
Watch as the tester makes your product using your instructions, but don't say anything!

Check that your tester doesn't need any help – apart from your instructions.

Improve your instructions and test them again if you need to.

When your instructions are 'perfect', make a 'best' copy of them.

Make the writing as clear and as neat as possible.
Use a typewriter or computer printer if you can.



Finally, stick on any cartoons or photos.

30

This could well be carried out at home or informally outside the classroom, ideally using someone who is unfamiliar with the activity. Make it clear that the tester's job is to test the instructions, so the 'originator' should only explain what to do if the instructions do not work. Whenever this happens, the instructions should be modified, then the revised instructions tested again until they can be used independently.

One class of low ability 5th years took their kits into a 2nd year class of similar ability and watched them make the products. This was extremely successful for both classes: it boosted the confidence of the 5th years and provided the 2nd years with some exciting activities.

When the instructions are completed, the kits may be duplicated and assembled in envelopes. We have suggested that the students also design attractive cover sheets.

STAGE 4

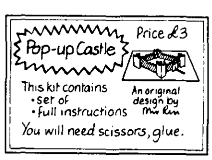
Putting your kits together

You will need large envelopes to put your kits in.

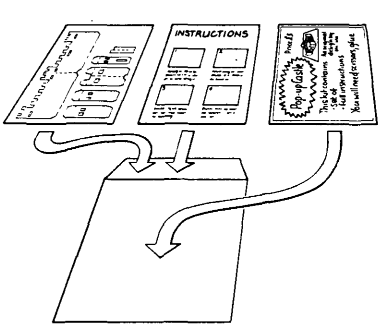
Design a master for a label to stick on your kit envelopes.

The label will need to show

- what the kit will make,
- what is in the kit,
- anything else that is needed.



Make copies of all your masters (designs, instructions, labels).



Put your kits together.

31

The final products may be used in many ways. For example, the cards could be sold at a school fete or at a local community volunteer shop.

STAGE 4

Using your kits

Think of ways in which you could *use* your final products.
Perhaps you could sell them in aid of some worthy cause . . .

32

Some possible further developments

The work of this module may be further developed to illustrate its usefulness and application to real life situations.

- Students may like to design and make a broader range of paper or card products: for example, lampshades, mobiles, model buildings.
- Events that take place during the school year may provide further ideas for packaging problems. (One class packaged mince pies for a Christmas fair, for example.)
- A small business enterprise could be based on the products. This could become a springboard for the consideration of all kinds of manufacturing and marketing issues such as material and duplicating costs, cash flow, production lines, advertising, profit and loss and so on.

2 Developing the mathematics

Developing the mathematics

Introduction

While students are working on this module, *their* main objective is to design and produce an attractive paper product, not to develop particular mathematical techniques. The mathematics is used as a tool to facilitate the design process, and is not seen as an end in itself. You may, however, wish to use the many opportunities provided by the module to motivate the learning of mathematics in a more explicit way. This chapter offers a few ideas on how this may be achieved without destroying the essential flow of activities contained in the module.

Which skills may be developed?

The table below illustrates a few of the mathematical topics related to the context of this module. This list is not intended to be exhaustive because different products will require different techniques. The design of a pop-up dolls' house will make different mathematical demands on students than, say, a simple pop-up greetings card.

Some mathematical topics	**Examples in the context of the module**
Space	
■ Drawing and measuring geometrical figures accurately using appropriate drawing instruments.	Using rulers and protractors to construct paper products in Stages 2 and 3.
■ Understanding and using geometrical concepts. For example,	
symmetry,	Folding and then cutting to produce desired shapes and other effects. For example, Frankenstein card (M7).
angles and parallel lines,	Exploring angled and parallel cuts and folds in Stage 2.
properties of polygons,	Exploring how the pop-up cards work. This leads to many properties of parallelograms, kites and other quadrilaterals.
loci.	Exploring pop-up cards which involve linkages and levers, for example the Frog (M19) and the Flying Kite (M9).
■ Drawing two-dimensional representations of three-dimensional objects.	Drawing nets and plans to create three-dimensional objects in Stages 2, 3 and 4. Drawing isometric or perspective drawings when describing how original products should be made in Stage 4.
■ Making three-dimensional objects from two-dimensional representations.	Creating three-dimensional products from nets, plans and perspective drawings in Stages 1 and 2.
Logic	
■ Following and devising instructions.	Following the instructions in Stage 1, and devising them in Stage 4.
■ Classifying.	Grouping the products made in Stage 1 according to the mechanisms used in their design. 'Why is the Rolls Royce like the Pop Star?'
■ Following and devising flow charts.	Using and inventing classification flowcharts for Stage 1, and using the design process flowcharts in Stage 3.
■ Making conjectures and constructing proofs.	'What are the necessary and sufficient conditions for ensuring that a particular type of pop-up card will work? For example, do the fold lines need to be concurrent in the 'Angled House'? (See M57.) What else needs to be true?'

Symbolisation ■ Devising and using simple algebraic notations to describe geometric results.	Using letters to denote lengths or angles when explaining the conditions under which a particular pop-up card will work.
Trigonometry ■ Finding lengths and angles using appropriate trigonometric ratios.	Calculating how large the base of a pop-up 'wall' card needs to be so that the wall does not protrude beyond the edges of the base when the card is closed. (See M60.)
Ratio ■ Enlargement of drawings.	Making larger versions of pop-up cards and boxes in Stage 2. For example, making a similar box to hold double the quantity.
■ Drawing to scale.	Making a dolls' house including furniture.

How and when may they be introduced?

Mathematical activity may be initiated either by the student or by the teacher. For example:

- a student may become aware of the need to acquire a particular skill in order to make a design work. 'How can I be sure that my pop-up wall won't stick out when the card is closed? Do I have to make the card first or is there a way of calculating it?'
- you may wish to use some ideas from the module to support a more intensive piece of work on a particular topic. 'Today we're going to look at some wooden polyhedra and try to work out how we would make similar models from thin cardboard. Can we devise nets for each of them?'

The first type of situation can lead to an invaluable learning experience because the student wants to know something. Such opportunities occur rather unpredictably, however, and it is inadvisable to spend a great deal of time helping one person if you have a large class to supervise. One possible solution is to ask the student to describe the problem to the whole class and invite help and advice from other students.

Do not expect students to use, autonomously, mathematics that they have only recently been taught. There is a gap, typically of several years, between first 'learning' a skill and being able to use it with flexibility and fluency. Students will tend only to use skills that they have mastered. (Narrowing this gap requires a more 'rounded' approach to learning, with a variety of applications and non-routine problem solving to supplement and give meaning to technical exercises.)

Teacher-initiated work on mathematical techniques relating to the theme of paper engineering may occur before, during, or after working on the module.

Before: 'I'll give them some practice at using protractors now, so that they will be more inclined to use them later on, when they begin work on the module'.

This timing has the advantage that the student will, if all goes well, have techniques polished and ready to be used, but it may seem artificial to learn a new technique before seeing a need for it. Students may tend to assume that the module is merely a vehicle for practising some specific techniques, rather than to develop their autonomy in problem solving.

During: 'They seem to be having difficulty in producing accurate drawings. We'll take a break from the module for a few lessons, and do some exercises which involve using drawing instruments'.

'I'll prepare some problem sheets for my class so that I can keep them together. If one group finishes a stage early, they can do problem sheets until the rest of the class have caught up'.

This timing enables you to respond to needs as they arise, but if students always expect you to produce the method or solution when the going gets difficult, you may reinforce dependence and undermine autonomy. If this is done too often on a class basis then the work on the module may tend to drag on over many weeks and become boring.

After: 'When we finish the module, we will look at the techniques we have used in more depth'.

The experiences of working on the module may motivate students and enable them to perceive the value of techniques when they are taught. Students may still not be able to use techniques autonomously unless they are given further opportunities to apply them in other real problem solving contexts.

Whatever you decide, it is important to be vigilant about preserving the students' strategic control of their work on the module: it is too easy to allow them to revert to the imitative role that the traditional curriculum encourages.

On the following pages we offer a few ideas for developing mathematical tasks on the paper engineering theme. In their present form, the ideas may be too open or too difficult for some students and may need some further development before you can use them. We hope that they stimulate some more ideas of your own.

Some sample ideas

Conjecture and proof

Discovering design principles

It is helpful to know if a design for a card will work, before you start to cut it out and assemble it. This can save a lot of time-consuming trial and error. This worksheet will help you to discover some design 'principles' for pop-up cards.

The diagrams below each show a different 'type' of card.

For each type, try to find out what *must* be true about

- distances between fold lines/gluing lines
- angles between fold lines/gluing lines,

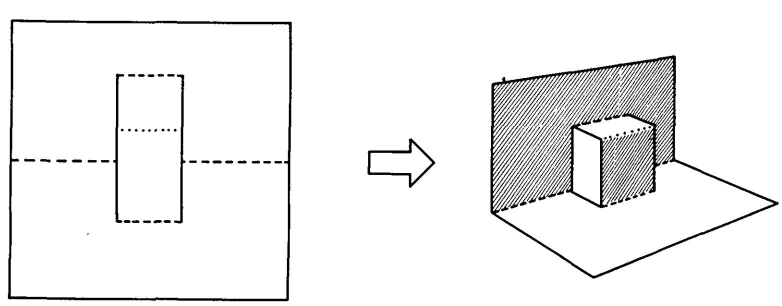
so that

- the cards will open and close through 180° without tearing or creasing in the wrong place,
- no parts will stick out beyond the base of the card when it is closed.

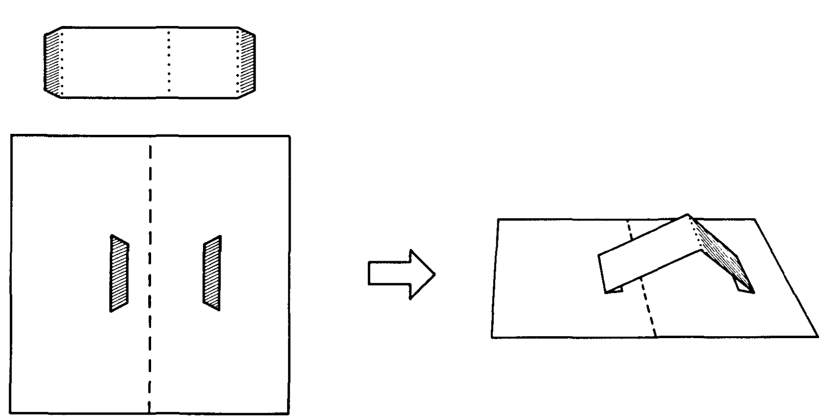
Check that your ideas work by making some models.

Try to prove that your ideas work.

Type 1. Parallel folds – no glue.



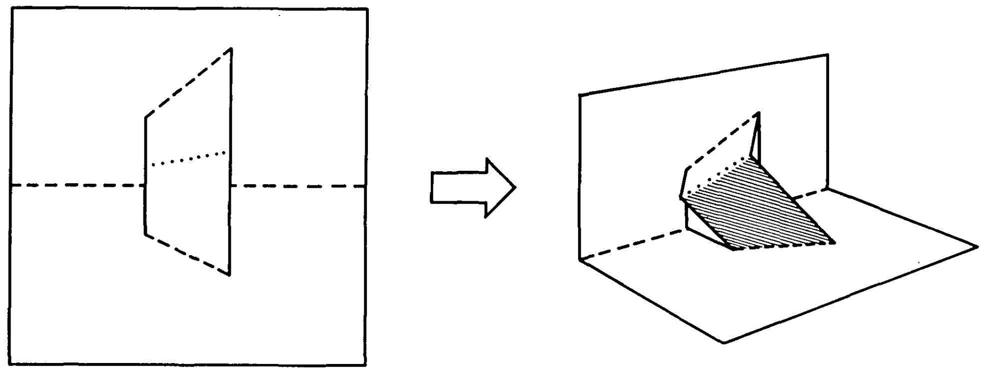
Type 2. Parallel folds – with a piece stuck on.



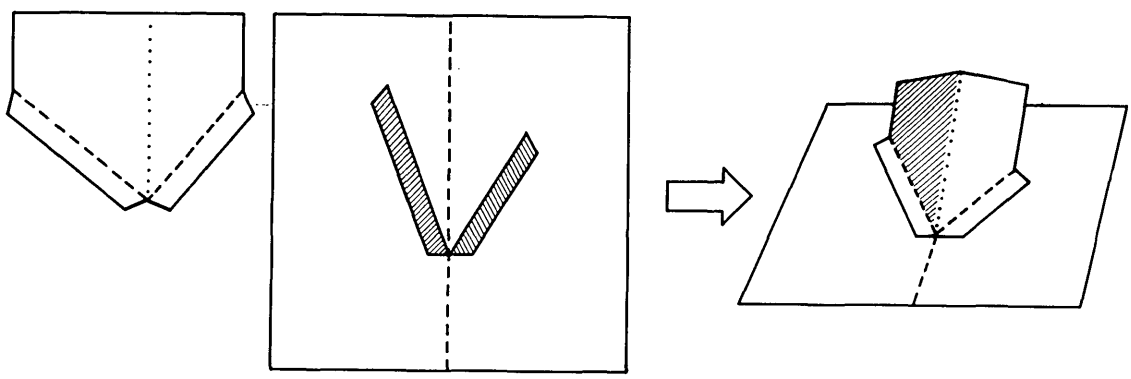
M73

Discovering design principles (continued)

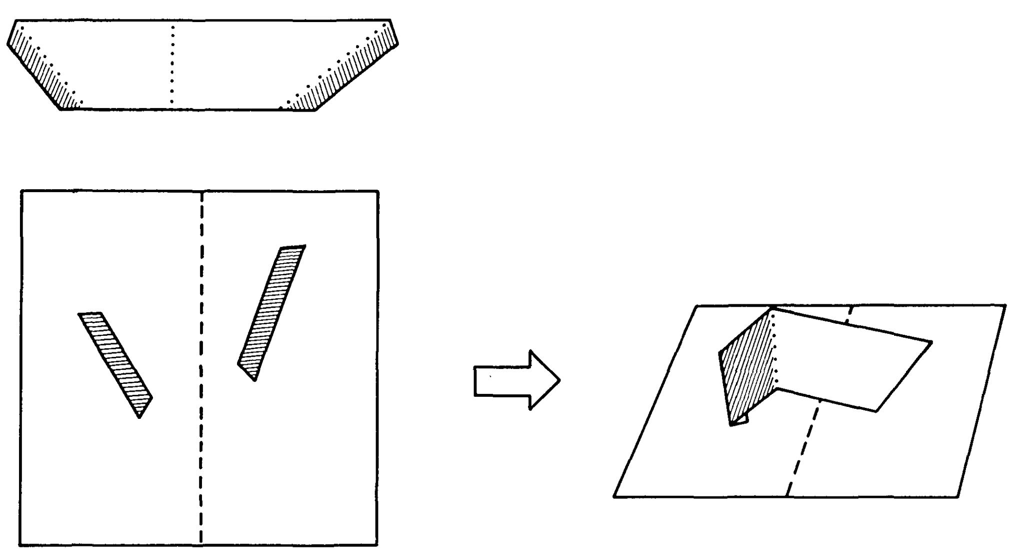
Type 3. Angled folds – no glue.



Type 4. Angled folds – with a 'wall' stuck on.

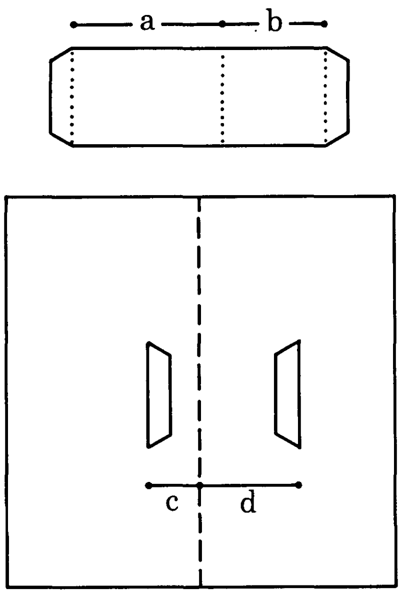


Type 5. Angled folds – with a piece stuck on.



These examples form a progression towards increasing generality. Below, we illustrate some of the results that may be found, but before reading them we strongly urge you to make a few models and try to discover some results for yourself.

Type 2. Parallel folds – with a piece stuck on



For the card to shut properly,

$$a - b = d - c$$

For the card to open to 180°,

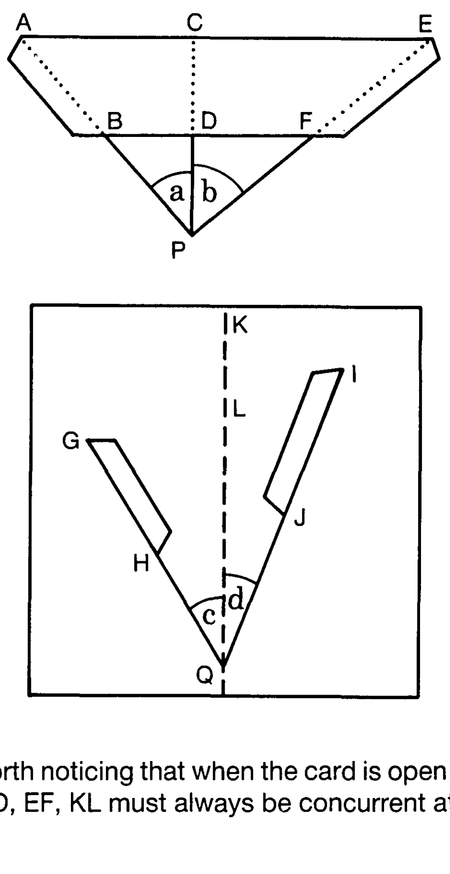
$$a + b \geqslant c + d$$

The piece will not 'stick out' when the card is shut if the base is longer than

$$2(a + c) \quad (= 2(b + d))$$

Note: type 1 is a special case of this card, where $a + b = c + d$, and so in this case, $a = d$, $b = c$.

Type 5. Angled folds – with a piece stuck on



For the card to open and close correctly,

- the 3 fold lines AB, CD and EF when produced must be concurrent at a point, say P.
- the 2 gluing lines GH and IJ when produced must be concurrent with the centre fold KL on the base of the card at Q, say.
- HQ = BP and JQ = FP.
- The angles between the fold lines must satisfy the following conditions

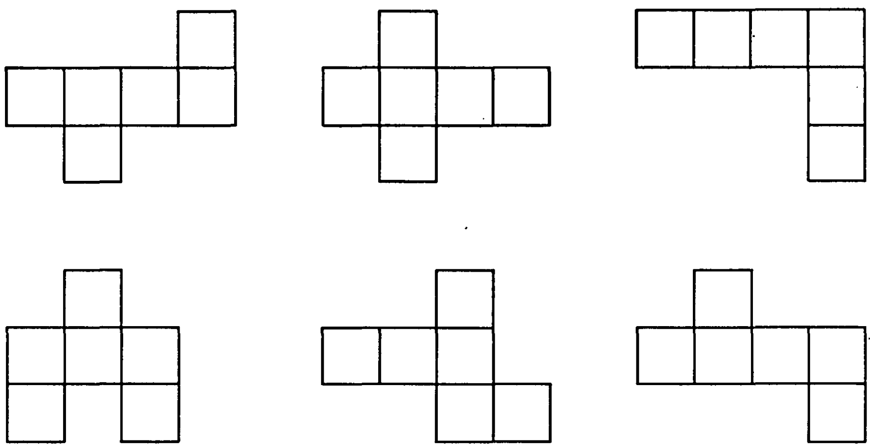
$$a - b = d - c$$
$$a + b \geqslant c + d$$

It is worth noticing that when the card is open at *any* angle, the four fold lines AB, CD, EF, KL must always be concurrent at Q.

Combinatorial geometry

Designing nets for cubes

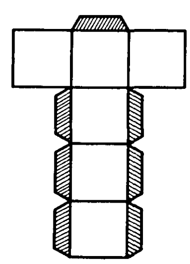
Here are six ways of joining six squares together, edge to edge:



- Which of these arrangements would you say are 'the same' as each other?
- Which *can* be folded up to form a cube?
  How can you tell, without having to make them?
- Which *cannot* be folded up to form a cube? Again, how can you tell?
- How many *different* nets of a cube are there?
  Draw them on dotty paper. How do you *know* that you have found them all?

If you want to make a cube from a net, you really need tabs as well.

- Choose any net.
  How many tabs do you need?
  Show how the tabs may be arranged.
  Now find some different ways of arranging them.
  How many different ways are there?
- Does the number of tabs needed depend on the particular net or arrangement you choose?
  Why?

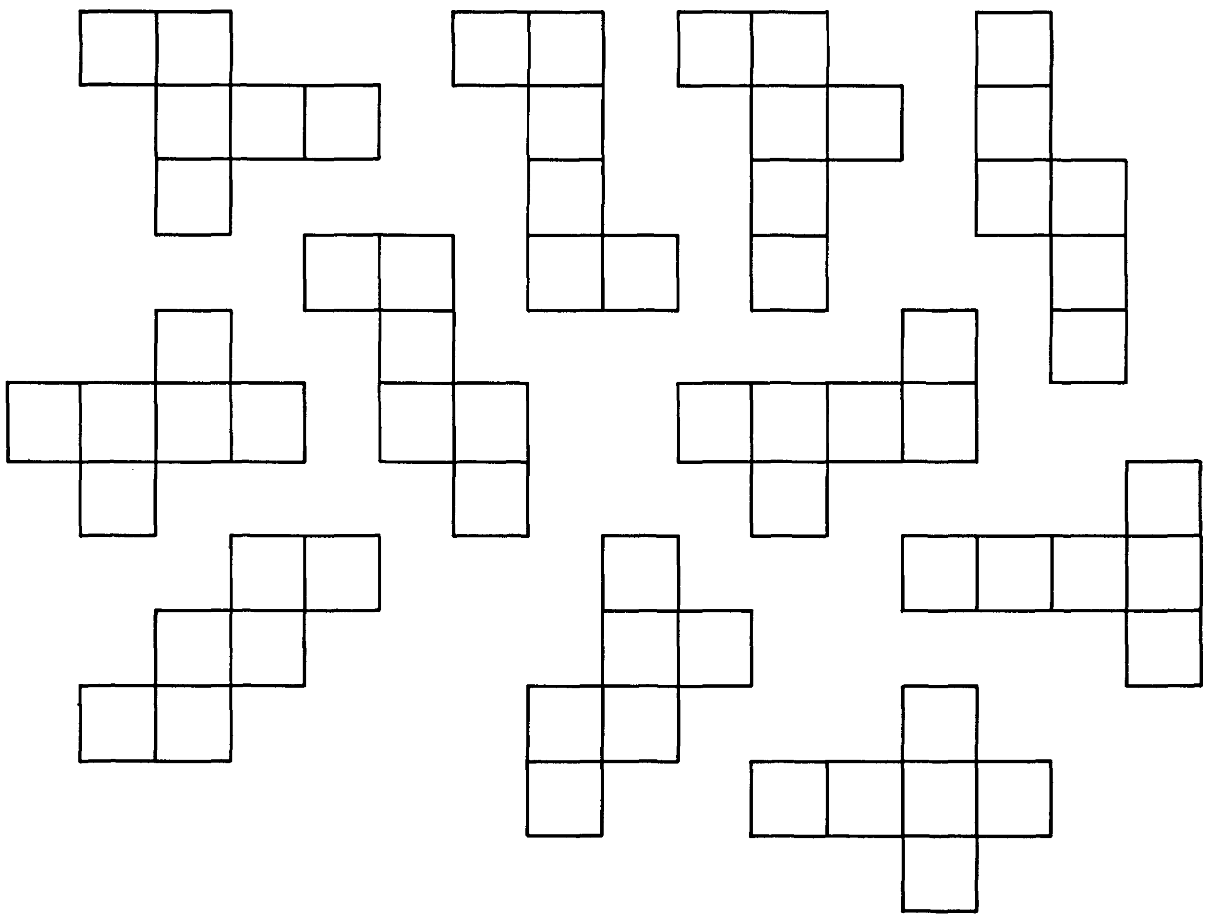


M75

Some students may be surprised to discover that there are many different ways of completing a net for a box (see M64).

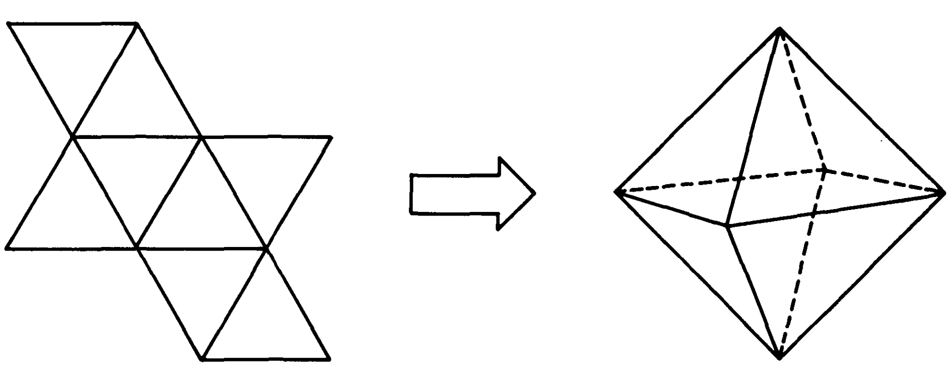
In this activity, the students have to enumerate the possibilities in a systematic way, or they may miss particular arrangements. If there is any doubt, students should cut out copies of the designs and try to make the cubes.

In all, there are 35 'different' ways of joining 6 squares together, edge to edge and, of these, 11 may be made up into cubes. 7 tabs are always required.



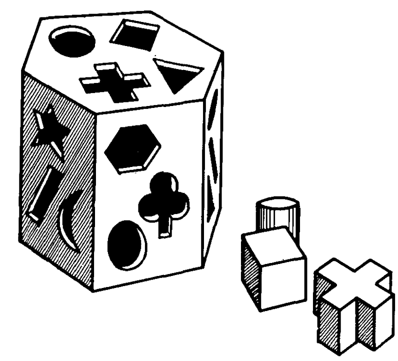
Students will need to reach agreement over the interpretation of the word 'the same'. The above solution assumes that two arrangements are 'the same' if one can be obtained from the other by rotation or reflection. This is an arbitrary definition, however, and students should be allowed to work with a different definition if they so choose.

Of course, this kind of investigation may also be applied to other polyhedra . . . .



Properties of polygons and nets*

Design a shape sorter



Young children are often given 'shape sorters' to help them develop eye and hand co-ordination.

What do you think makes a 'good' shape sorter?

Write down a list of desirable features.

Design your own shapes and shape sorter and make them all out of cardboard.

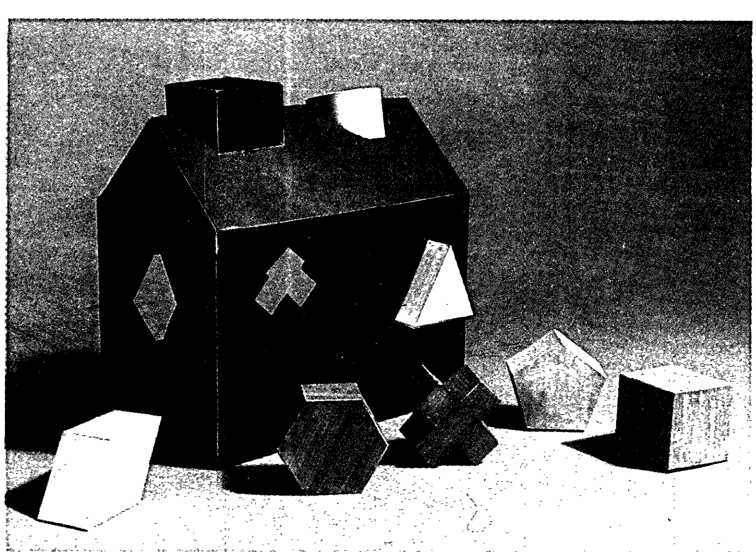
Write about how you designed it, and why you designed it like you did.

Give your shape sorter to young children and watch them play with it. Which shapes do they find easy to put in? Why? Which shapes are harder? Why?

A 'good' shape sorter will ensure that,

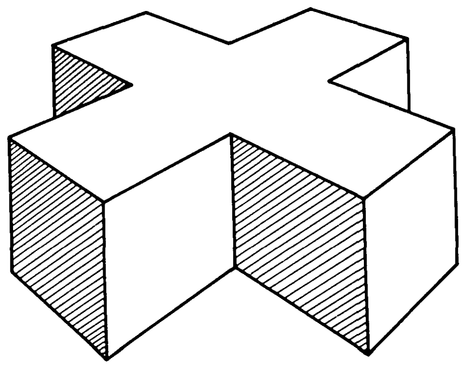
- there is a simple way of retrieving the shapes after they have been inserted,
- each shape can only pass through its own slot,
- some shapes are easy to insert, because they can pass through their slots in several ways, while others are more difficult to insert.

Students often put a great deal of effort into the design of an attractive sorter. One student, for example, produced a 'house' to accommodate her shapes:

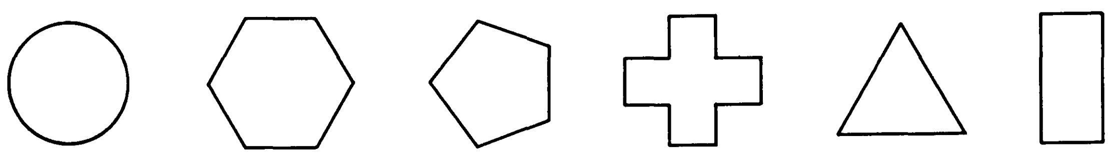


*This activity has been developed as part of a MEG/Shell Centre project which aims to provide support for GCSE coursework.

The design of a net for each shape will also provide an interesting mathematical challenge. For example, can the shape below be constructed from a single piece of card?



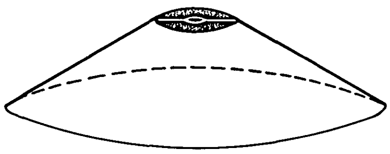
It is possible to predict which shapes a young child will find most difficult to insert into the sorter, by considering the rotational symmetries involved. For example, we could conjecture that the following shapes are in order of difficulty:



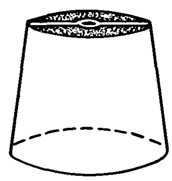
This order may be tested by experiment, and statistics obtained to refute or support the conjecture.

Design a lampshade

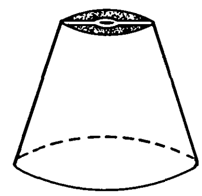
Lampshades come in all shapes and sizes, for example:



Coolie shape



Drum shape

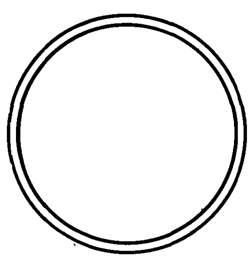


Empire shape

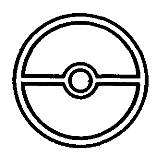
They are usually made from fabric, attached to wire frames.

The frames are designed so that the shades may either be hung from the ceiling or supported by a lampstand. Among the simplest lampshades to make are those with stiff-backed fabrics. All that is then needed for the frame are two rings, one for the top and one for the bottom. For example, you could have two rings like these:

bottom ring



top ring



- If you can, obtain two rings from a shop or from an old lamp.
- Design and make a lampshade to fit these rings, from thin card.
- When you are happy with your design, buy some stiff-backed fabric and make the shade.

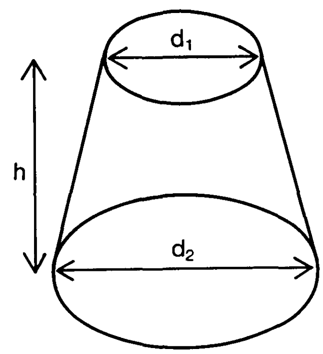
(When making your final design, prepare the frame by covering it with sticky-backed cotton tape. The fabric can be attached to the frame with clothes pegs, until it is positioned correctly, and then glued or sewn into place. A trimming may be added to make the edges neater.)

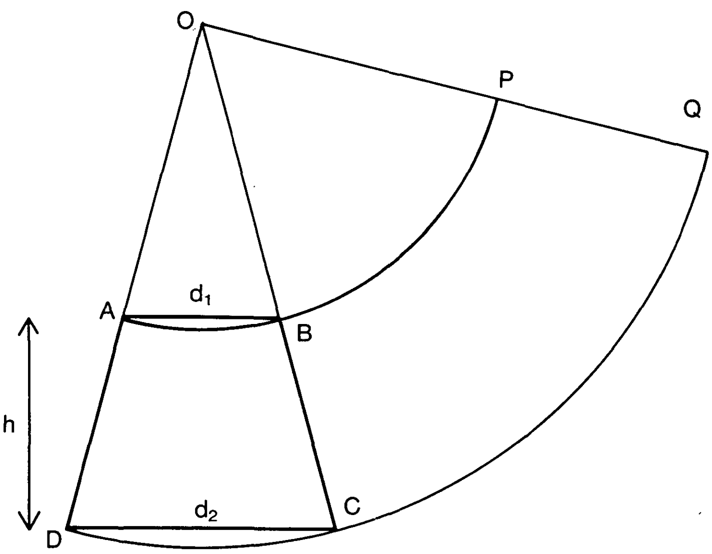
M77

Some students may enjoy this kind of practical challenge. If there is an insufficient number of rings available, then two cardboard circles may be used as a temporary substitute while the students are exploring the design principles.

One possible construction for the design is described below, although it is of course better to allow students to work out their own solutions.

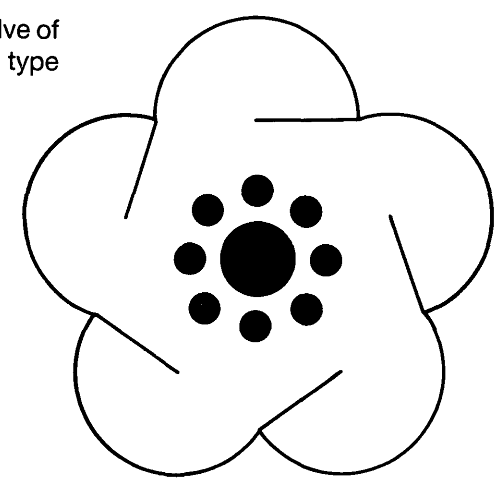
1. Measure the diameters of the two rings, d_1 and d_2, and decide how tall the lampshade will be, h.





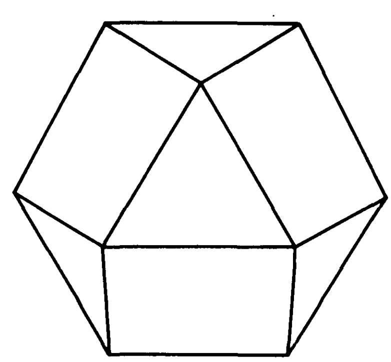
2. Draw the trapezium ABCD on a large sheet of card. Extend DA and CB until they meet at O, say.
3. With OA and OD as radii, draw arcs ABP, DCQ. These arcs should be πd_1 and πd_2 in length respectively. APQD is now the required shape for the lampshade – although it is wise to add a bit extra for overlaps etc.

There are many other ways of designing lampshades. For example, twelve of these shapes will slot together to make a very effective 'dodecahedron' type of lampshade.



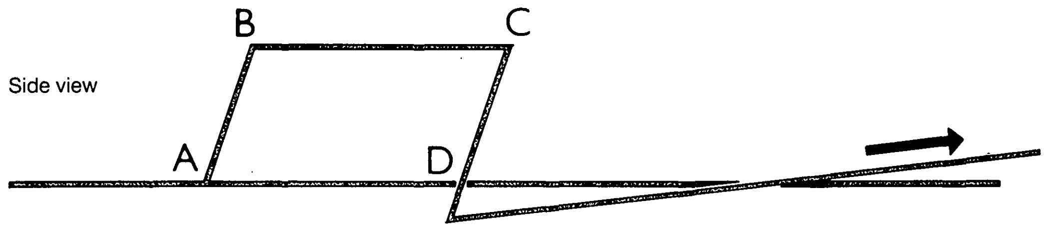
(If the black circles are cut out, they will allow ventilation and more light to escape.)

Similar lampshades could be based on other polyhedra; the cuboctahedron, for example.



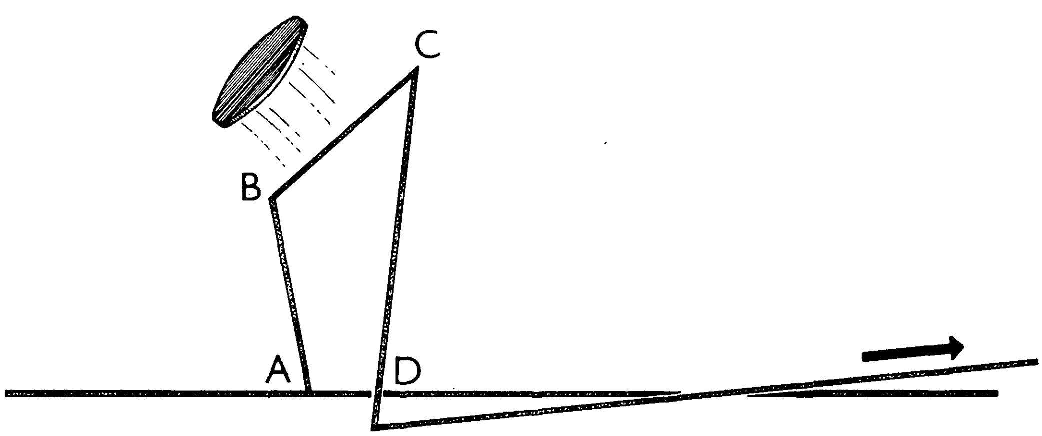
Linkages

Investigating the Frog linkage



The Frog card uses a parallelogram linkage, where AB = CD and BC = AD.

- Investigate what happens if AB≠CD.
- Can you still make the card go flat when the lever is fully pushed in?
  Can you still make the card go flat when the lever is fully pulled out?
  What conditions must be satisfied?
- A student is designing a 'tiddlywinks' game which uses a linkage to toss the counters.



Can you make this linkage, which must satisfy the following conditions?

(i) BC lies flat on the card when the lever is fully pushed in.
(This ensures that it can be stored without breaking.)

(ii) When the lever is fully pulled out and B touches the plane of the card, BC is vertical.

M78

This worksheet illustrates the kind of design problem that a student could be faced with if he or she wants to achieve a specific desired effect.

For the card to go flat when the lever is fully pushed in,

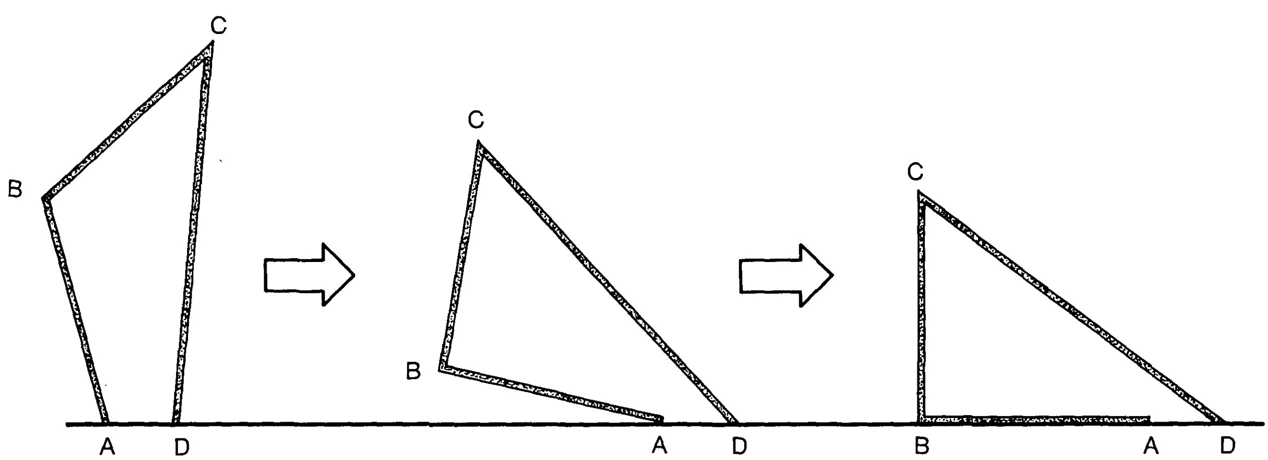
$$AB + BC = AD + CD \quad (1)$$

For the card to go flat when the level is fully pulled out,

$$AB + AD = BC + CD \quad (2)$$

(Equations (1) and (2) can only be true simultaneously if the linkage is a parallelogram.)

The movement of the 'tiddlywinks' linkage is illustrated below:



This mechanism requires that $AB < CD$ (3)

When the lever is fully out $(AB + AD)^2 + BC^2 = CD^2$ (4)

The design problem requires a solution to be found to (1), (3) and (4).

If we specify, for example, AB and AD, then

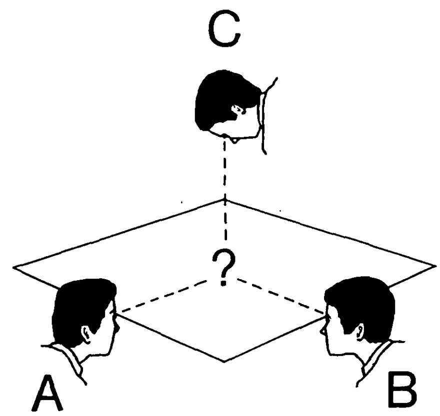
$$BC = \frac{2\ AB.AD}{AB - AD}, \qquad CD = \frac{AB^2 + AD^2}{AB - AD}$$

A sample solution is where AB = 3, AD = 1, BC = 3, CD = 5

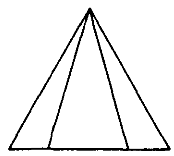
Further design problems may involve the consideration of loci.
(For example, the locus of card 9, The Flying Kite, is particularly interesting.)

Plans and elevations

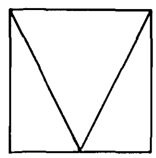
Mystery box



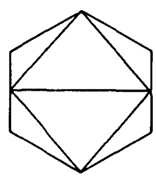
When the box is viewed from A, it looks like an equilateral triangle.



When the box is viewed from B, it looks like a square.



When the box is viewed from C, it looks like a regular hexagon.



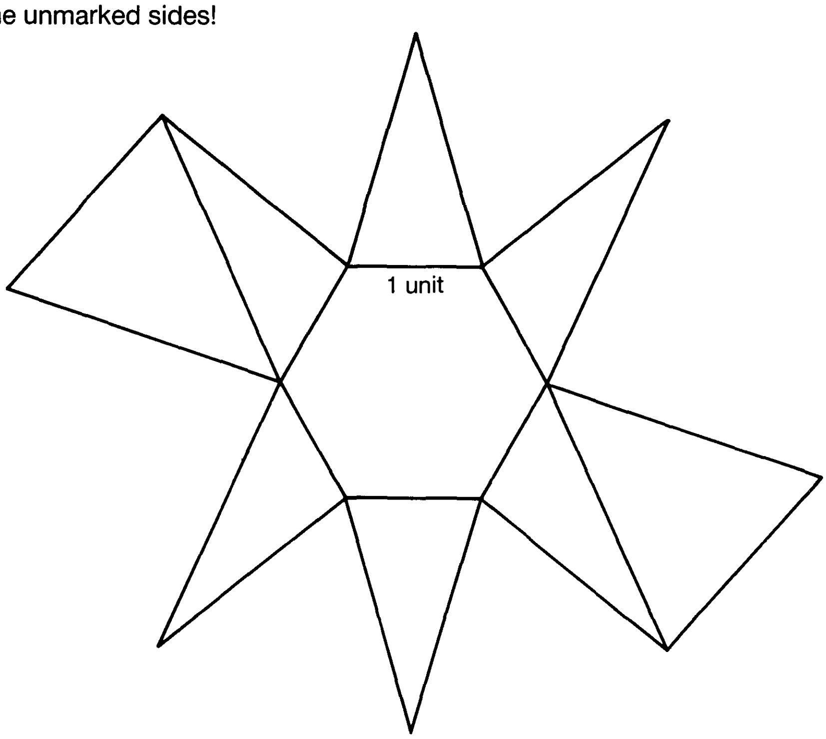
Make the box!

Now *you* write a worksheet like this for a box you have made.

See if someone else can make the box from your worksheet.

M76

Although this particular problem is challenging, it can be solved using a knowledge of Pythagoras' theorem, some drawing skills and patient trial and error. A net for the completed box is shown below. You may like to work out the lengths of the unmarked sides!



Students may like to construct similar problems, using their own box designs.

Perspective drawing

A standard approach to drawing 2-dimensional pictures of 3-dimensional objects is to use isometric grid paper as a basis. It soon becomes clear that the resulting pictures are actually distortions of what you see. Look out of a window, or even round a room, or especially down a corridor, and it is clear that sets of parallel lines receding from you appear to converge towards distant points. These are known as "vanishing points".

You or your students can investigate this directly in many different ways. Below are some suggestions for starting points.

Drawing in perspective

Looking through windows

Look out of any window at nearby buildings.
Look for sets of parallel lines going away from you – roof lines, window frames, etc.

Imagine extending these lines into the distance. What do you notice?

Fix a transparent overhead projector film onto the window. Keeping your head as steady as possible shut one eye and draw what you see onto the film.

Remove the film and trace your picture. Extend sets of parallel lines.

The points where each set meets are called *vanishing points.*

Each set of receding lines has its own vanishing point. How many vanishing points can you find?

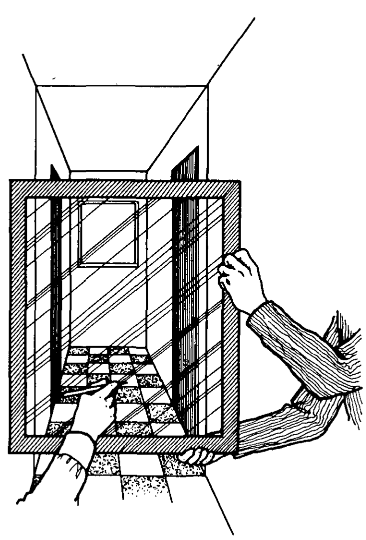
Draw additional lines through the vanishing points. Use them to insert new buildings or features into your picture.

Repeat the 'window' experiment, but this time get a friend to hold a piece of film vertically in front of you as you look through it down a corridor.

Keep your head steady and draw what you see onto the film.

Try again, with a different head position.

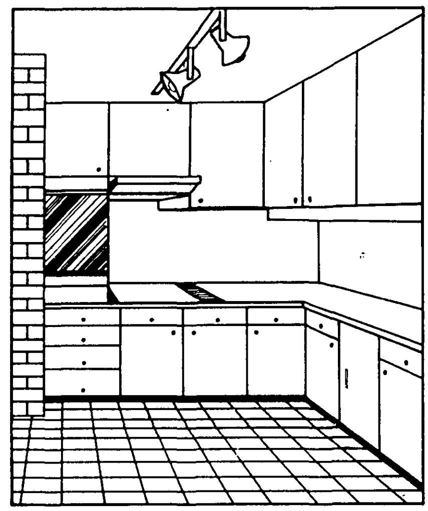
Investigate what happens when the film is not vertical.



M79

Drawing in perspective (continued)

Looking at photographs

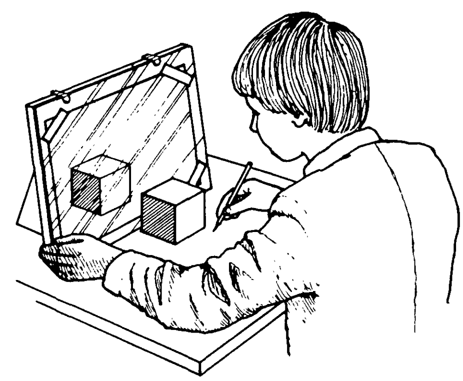


Look through some old magazines or colour supplements.

Find some pictures which contain lots of straight lines. Photographs of luxury kitchens are excellent – especially if they have tiled floors.

Extend all the lines and see what you find.

Looking into mirrors



Fix a piece of transparent overhead projector film onto a mirror.

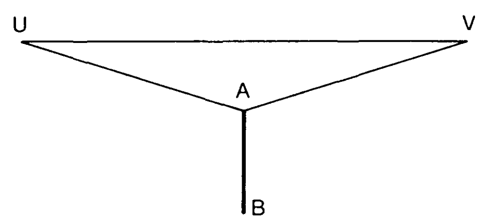
Put a box or any other object with parallel edges in front of the mirror.

Keeping your head steady, shut one eye and mark dots on the film to show the reflected corners of the box. (This is easier than trying to draw edges directly.) Remove the film and join up the dots with straight lines.

Extend all the edges in your picture. What do you find? Repeat for different mirror and head positions.

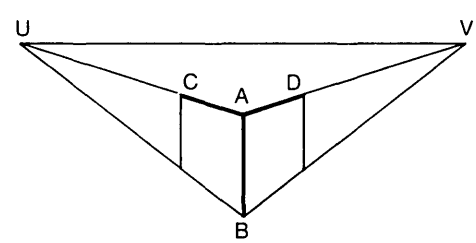
What happens to the vanishing points as you look down on something, horizontally at it, or up at it?

Drawing a cube in perspective

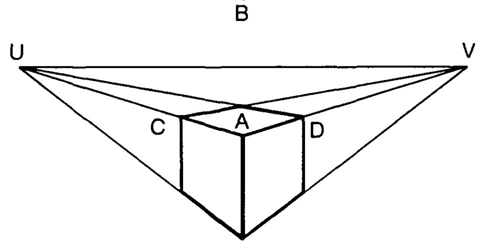


Imagine you are looking diagonally down on a cube on a table. In the diagram, AB is the front vertical edge of the cube. U and V are vanishing points for the horizontal edges.

Join up AU, AV, BU, BV. Mark in two other corners C and D as shown and draw vertical lines through C and D.



Joint up CV and DU to complete the cube.

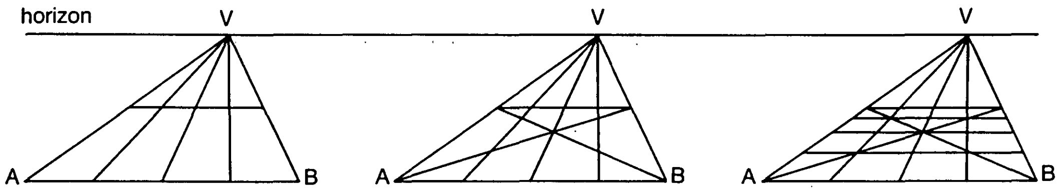


Repeat the drawing with AB in different positions – above UV, nearer U than V, at an angle. See what happens.

Repeat with U and V further apart and then – closer together.

Drawing in perspective (continued)

Try drawing a flat, square grid in perspective, using just one vanishing point:



Divide AB into equal parts.

Join diagonals.

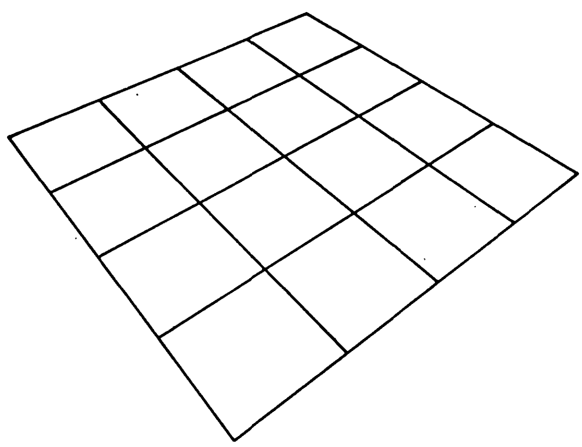
Draw lines parallel to AB through intersections.

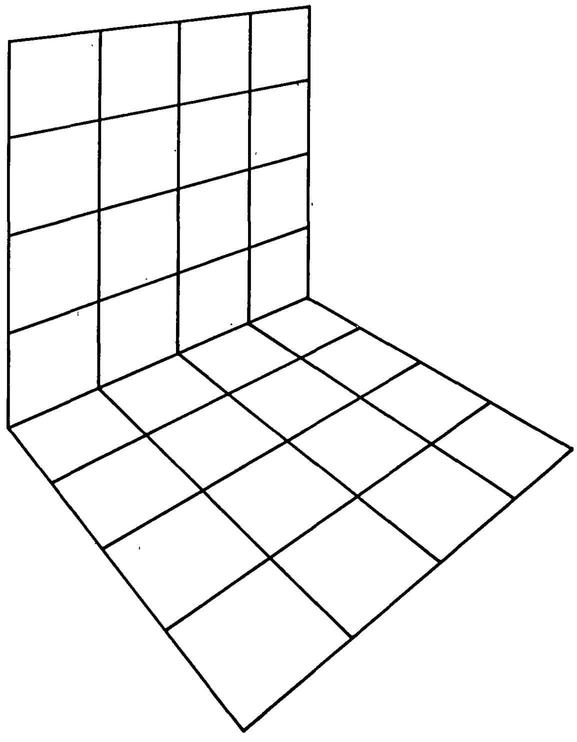
Draw in some other diagonals on your grid, and extend them.

See where they intersect.

Now try drawing a grid using two vanishing points.

Draw in diagonals and check that sets of parallel lines always intersect at vanishing points on the horizon.





Try adding a third dimension. (In the diagram, the vertical lines are all parallel, and there are still two vanishing points.)

Try drawing a perspective picture of a pop-up card or box using one or two vanishing points.

For further study on the topic of perspective drawing, we suggest you refer to:

Perspective Drawing Worksheets (photocopy masters and suggestions for teachers) by R L Lindsay – Shell Centre, University of Nottingham, 1978.

Royal Institution Mathematics Masterclass – video and worksheets on Geometry and Perspective by Professor E. C. Zeeman, 1987.
This is likely to be available on loan from your LEA.

Shadow and Depth by Aad Goddijn – Association of Teachers of Mathematics, 7 Shaftesbury Street, Derby, DE3 8YB.

as well as the drawings of M C Escher (to pose questions) and the pictures of Dürer, Goya, Brunelleschi and others (to provide some partial answers).

The role of assessment

The role of assessment

Introduction

Why assess?

You may wish to assess your students' work for a variety of reasons. The style of assessment you use should reflect the purpose it has to fulfil. For example, you may be using the module in the context of an assessment scheme (for example, the JMB Certificate of Numeracy through Problem Solving, GCSE coursework or Records of Achievement), or you may simply want to assess school work, to enable you and your students to recognise their progress.

The purpose will affect the aspects of your students' work that you decide to assess. These include (with examples)

- strategic skills (the ability to plan and design),
- technical skills (the ability to measure accurately),
- social skills (the ability to discuss and listen).

Each of these poses a distinct assessment challenge.

In this chapter, we will focus on ways of assessing and recording the strategic and technical skills deployed, as referred to in the introduction (pages T5 and T6), and also on how this type of work may be assessed in the context of GCSE coursework.

We include illustrations of students' work and examples of two examination papers for students of different ability levels. These papers may be used to help assess your students' retention of skills and their ability to transfer them to unfamiliar situations within the same context. In designing these assessment tasks we have sought to ensure their curriculum validity (i.e. that each task should also represent a valuable learning activity).

Ways of assessing

Although most of the technical work is done by individual students, it is done within the context of group problem solving. There are opportunities for students to discuss, listen and co-operate with colleagues, and emphasising these aspects will often result in individual students designing more imaginative products than they would have done in isolation. Your observation of the groups at work will enable you to note some of the social skills a student displays. Such skills include

- persistence and attention to detail,
- the ability to co-ordinate the work of the group,
- initiative,
- the ability to work well in a team,
- the ability to communicate.

These skills are valuable and worth recording whenever you are aware that they are being displayed by a student. It is difficult, however, to assess social skills reliably in this context because students' performances will be affected by the nature of the input from other group members. A student may take an active role in one group, but be overshadowed by a dominant personality in another. Merely observing the work of the group as a whole may not give a clear picture of an individual's contribution, acquisition of skills, or understanding of the work of the group.

Because students are working on their own products, it should be possible to assess each individual student's work reliably using a combination of the following methods.

- Observation of the students' complete work.

  For students who are able to keep a written record of the work that they do, this is likely to be the major method of your assessment. It has the advantage that it can be carried out after lessons thus leaving you free to help students informally or carry out assessments of the few students who are unable to produce adequate written records.

- Observation of individual students during lessons.

  In many cases you are likely to see a student doing something that convinces you that he or she has satisfied a criterion. If you are not sure that the student has completed enough work, or done sufficient work unaided, you may ask students to complete an additional task while you observe. For example, in Stage 1 the student could make up another plan.

- One-to-one discussion with an individual student, possibly using a checklist of some kind.

  This method is time-consuming, particularly with larger classes, and it is difficult to carry out an interview if you are simultaneously trying to supervise the rest of the class.

- Students carrying out self-assessments.

  Self-assessments may consist of written notes, the completion of a simple checklist, or verbal comments.

Assessing with reference to criteria

What do the criteria mean?

The criteria listed below are restatements of the general strategic and technical skills in the context of this module, and have been referred to throughout the classroom materials. As students work through the module they are given the opportunity to show that they can, in the context of paper engineering,

(i) follow instructions,
(ii) cut, fold and glue accurately to assemble a 3-dimensional product,
(iii) recognise structural features of a design,
(iv) make a 3-dimensional object from a 2-dimensional representation,
(v) draw a 2-dimensional representation of a 3-dimensional product,
(vi) give a reasoned explanation for design features,
(vii) identify and correct design faults,
(viii) develop an existing idea for a paper product,
(ix) generate possibilities for a design with original features,
(x) draw a design to an acceptable degree of accuracy,
(xi) construct a prototype with original features,
(xii) devise instructions to enable someone else to make the product.

Such criteria assist in providing a useful profile of relative strengths and weaknesses. However, to say that a student has satisfied a criterion, such as 'can identify and correct design faults', has little absolute meaning without specifying

- **the context.** Correcting design faults in a paper product is very different from correcting faults in the design for a bridge.
- **the complexity of the context.** Correcting design faults in a simple 'pop-up' card is much easier than in a working paper clock.
- **the frequency of success.** Has the student satisfied the criterion on many occasions or only once? What proportion of attempts resulted in success?
- **the amount of help given.** How much help did the student receive from the teacher or other group members in understanding and completing the task?
- **the occasion.** When did the student attempt the task? How recently had the student experienced a similar situation?
- **the distance of transfer.** How closely did the task match the student's previous experience?
- **the mode of response.** How did the student respond to the task – in writing or orally?

Thus, if you are basing an assessment on work undertaken on this module alone, you will need to qualify any statement about a student's performance with some additional remarks like 'these abilities have been demonstrated while the student took part in designing and making a simple paper product'.

If, however, students work through a range of extended activities, perhaps corresponding to the range of modules in this series, they are likely to develop the ability to demonstrate similar strategic skills in a variety of different contexts. This will then enable you to make more general statements about a student's ability to satisfy particular criteria.

The table overleaf amplifies the criteria and points to places where students may satisfy them.

Criterion	How it is satisfied	When it may be satisfied (Page numbers refer to the Student's Booklet)
(i) Follow instructions.	The student can comprehend and follow instructions for making up nets and plans.	Stage 1 Making a collection of cards, boxes and envelopes (page 6). Stage 4 Testing your instructions (page 30), if the instructions being followed are clear and complete.
(ii) Cut, fold and glue accurately to assemble a 3-dimensional product.	The products that the student makes work properly, for example, cards pop-up correctly, box lids fit well.	Stage 1 Making a collection of cards, boxes and envelopes (page 6). Stage 3 Making the finished product (pages 21 and 25). Stage 4 Testing your instructions (page 30), if the net or plan has been drawn accurately.
(iii) Recognise structural features of a design.	The student can distinguish between structural (for example, the mechanism) and peripheral (for example, the picture) features of a design.	Stage 1 Looking at things you have made (page 7). Classifying things you have made (page 8). Stage 2 The 'Exploring techniques' sheets, when a student recognises that the structure of a Stage 1 product can be used to make something else. Stage 3 Brainstorming (page 16), (the entries in the final column of the 'Brainstorming' sheet). Planning your design in rough (pages 18 and 22).
(iv) Make a 3-dimensional object from a 2-dimensional representation.	The student can look at either a perspective drawing or a sketch of a plan or net and create a model of it by drawing the pieces and then making it.	Stage 2 The 'Exploring techiques' sheets. Stage 3 Planning your design in rough (pages 18 and 22), if a student draws a sketch of a product before making it.
(v) Draw a 2-dimensional representation of a 3-dimensional product.	The student can create an accurate net or plan which may be made up into a 3-dimensional product.	Stage 2 The 'Exploring techniques' sheets, if a student draws an accurately measured net or plan to make up one of the challenges, this criterion is satisfied as well as criterion (iv). Stage 3 Drawing an accurate plan (pages 20 and 23).
(vi) Give a reasoned explanation for design features.	The student can give reasons for the features he or she is incorporating in a design.	Stage 2 Keeping a record (page 14). An oral explanation is also acceptable.
(vii) Identify and correct design faults.	The student understands why a product fails to work and can overcome this by modifying the design.	Stage 2 The 'Exploring techniques' sheets, where several attempts were made before the product worked successfully. Stage 3 Planning your design in rough (pages 18 and 22).
(viii) Develop an existing idea for a paper product.	The student is able to take an idea for a paper product and develop it in some way (for example, by adding to it, or making it work differently).	Stage 2 The 'Exploring techniques' sheets, where a student develops a Stage 1 product. Stage 3 Planning your design in rough (pages 18 and 22).

Criterion	How it is satisfied	When it may be satisfied (Page numbers refer to the Student's Booklet)
(ix) Generate possibilities for a design with original features.	The student is able to write down different ideas for products and suggest how they could be made.	Stage 3 Brainstorming (page 16).
(x) Draw a design to an acceptable degree of accuracy.	The student can make appropriate use of mathematical equipment to create an accurate plan or net.	Stage 2 The 'Exploring techniques' sheets, if a student draws a net or plan accurately. Stage 3 Drawing an accurate plan (pages 20 and 23).
(xi) Construct a prototype with original features.	The product that the student produces must be accurately made, and with some originality, however small.	Stage 3 Making the finished product (pages 21 and 25).
(xii) Devise instructions to enable someone else to make the product.	A suitably independent person is able to use the instructions to assemble the product without intervention from the student.	Stage 4 Testing your instructions (page 30).

Recording students' achievements

You may find it helpful to record your students' successes on a grid like the one shown opposite. The criteria associated with this particular module have been listed across the top, and there are spaces for students' names to be filled in down the side. The criteria are grouped together under the stage at which they are most likely to be satisfied, although they may, of course, be satisfied at any time during the work.

There are many ways of filling in the cells in the this grid. You may wish to record only that a student has satisfied or 'passed' a criterion, or you may wish to qualify this by recording, for example, the amount of help that was given and the frequency of success. Thus,

P may mean 'pass with no help'

Ph may mean 'pass with a little help'

PH may mean 'pass with a lot of help'

PP may mean that a student has 'passed' the criterion on more than one occasion.

You may also wish to record whether a criterion was satisfied by a written or by an oral response. If a student has not yet shown that he or she can satisfy a criterion, we suggest that the cell should be left blank, and the student be given a further opportunity at a later date so that only positive achievement is recorded.

An alternative would be to use a numerical grading system on a 3- or 5-point scale, but there is always a temptation to add such numbers so that a single score is obtained for each student. Although such a result is convenient, it is meaningless and misses the whole point of criterion-referenced assessment.

You may wish to extend your record sheet to allow space for comments concerning social skills that your students have displayed, together with any particularly noteworthy achievements.

We include below a selection of students' work which we have related to the module criteria. The samples have been chosen to illustrate the level of performance which we believe can be reached or exceeded by a substantial majority of the school population. You may wish to set more demanding standards for more able students.

Record sheet

Criteria satisfied

A student has shown that he or she can:

	Stage 1			Stage 2					Stage 3			Stage 4
Name	(i) follow instructions	(ii) cut, fold and glue accurately to assemble a 3-dimensional product	(iii) recognise structural features of a design	(iv) make a 3-dimensional object from a 2-dimensional representation	(v) draw a 2-dimensional representation of a 3-dimensional product	(vi) give a reasoned explanation for design features	(vii) identify and correct design faults	(viii) develop an existing idea for a paper product	(ix) generate possibilities for a design with original features	(x) draw a design to an acceptable degree of accuracy	(xi) construct a prototype with original features	(xii) devise instructions to enable someone else to make the product

M82

Samples of students' work

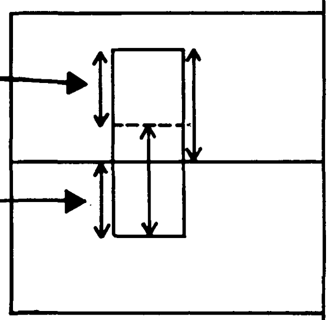
Below, Alison, Heidi and Caroline have been working on cards with parallel folds (M51 and M52) and have demonstrated that they can

(vi) give a reasoned explanation for design features

Alison

The cards which will shut properly
are: A+B

The reason why I can tell is that the length of this



must be the same length as this

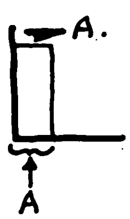
or the orange arrow must be the same length as the other arrow

Alison has given a clear, analytical explanation of how cards with parallel folds work. She has not, however, considered the question of whether or not the picture will stick out when the card is closed. Judging by the clear understanding that she has demonstrated, she should have no difficulty in doing this if prompted.

Heidi

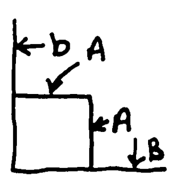
Card A does not work because 2cm of the 'cut out bit' sticks out at the top of it. This is because the part marked A. is too long.

1.



Part A has to be the same length as part b (dia.2) or it sticks out of the top.

2.



This would work.

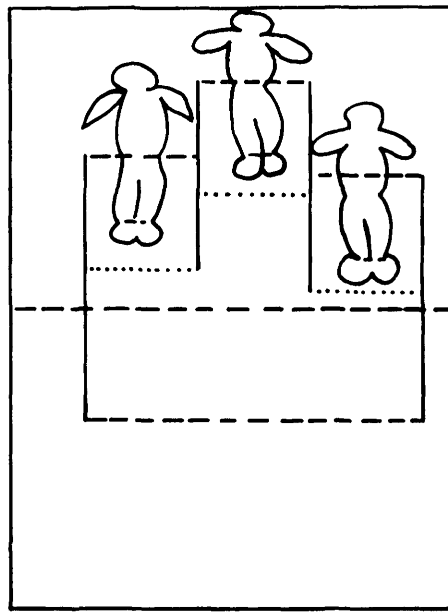
A = B.

Heidi, on the other hand, has explained the conditions for the picture to be hidden in the card when it is closed. As well as this written explanation she has also made up the cards, where she explains why card C does not work by indicating lengths which must be the same. Her work is therefore more complete than Alison's, but not so clearly explained.

Caroline

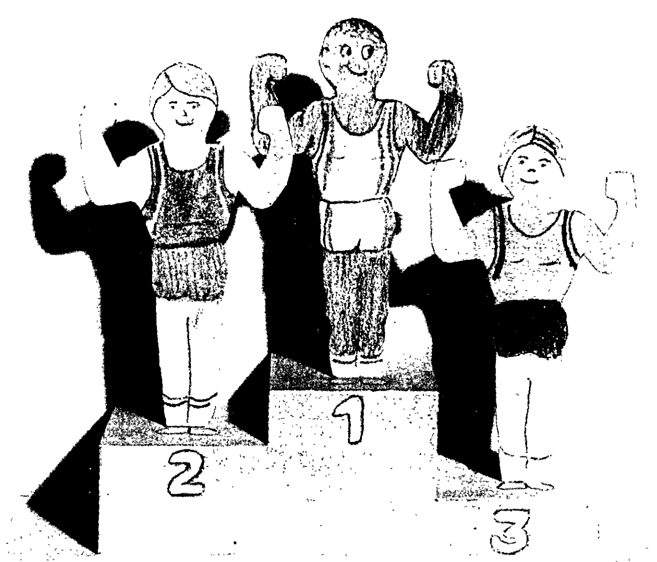
This card Works at 90°

With this card you need to draw the centre line first
but not fold it. you will
then need to draw the shape
of the podium a little way
under the centre line and going
over to the top as shown on
the right you then measure
the amount under the centre
line and draw that amount on
top of each stage in a square.
onto these squares you draw
the atheletes but be careful
not to cut their feet out.
make folds as shown in the
diagram.



Caroline understands well which lengths must be equal for the card to pop-up and close. She, like Alison, has not made sure that the picture is concealed when the card is closed, nor has she made the figures pop-up properly.

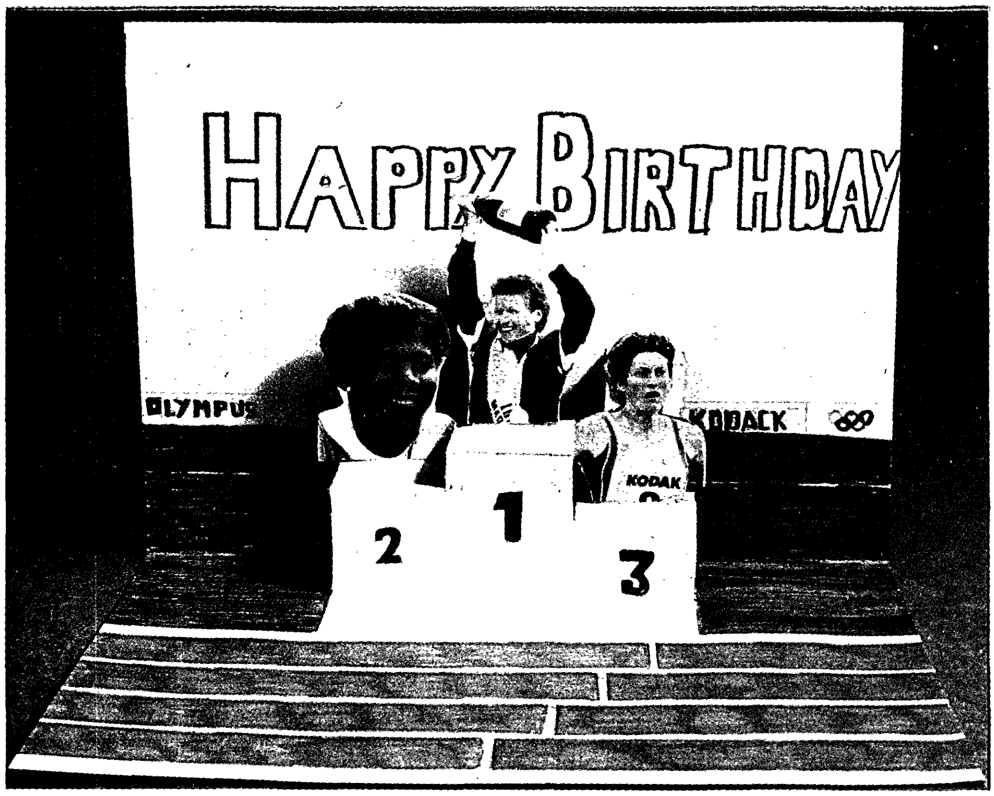
Alison

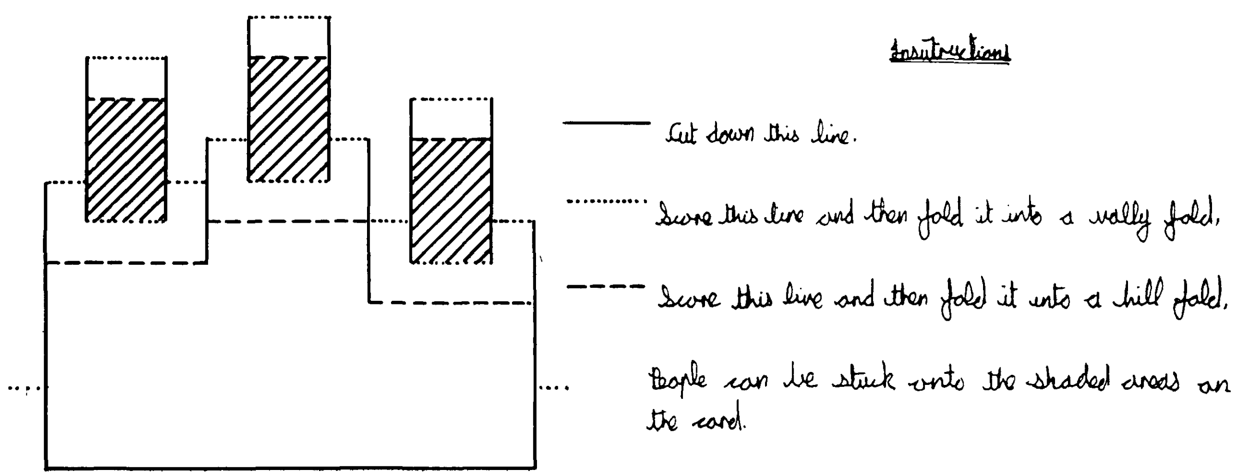


Alison has succeeded in making the podium from the diagram on the sheet and has extended it to include athletes. She has demonstrated that she can

(iv) make a 3-dimensional object from a 2-dimensional representation,
(viii) develop an existing idea for a paper product,
(x) draw a design to an acceptable degree of accuracy.

Justin





Justin has developed the podium idea to create a birthday card. His kit (part of which is shown) is complete and accurate. He has shown that he can

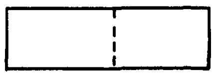
(iv) make a 3-dimensional object from a 2-dimensional representation,
(viii) develop an existing idea for a paper product,
(x) draw a design to an acceptable degree of accuracy,
(xi) construct a prototype with original features,
(xii) devise instructions to enable someone else to make the product.

Curtis and Alison have been working on 'cards with angled stick-ons' (M59 and M60).

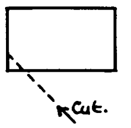
Curtis

How to make:

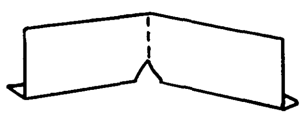
1. Fold a piece of paper in half. Then produce a smaller piece and ...



2.

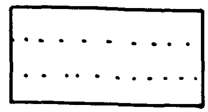


3. Turn flaps back when piece is opened up.



4. Cut out figures and glue to piece. Then add other detail.

5.



Produce two equal pieces of paper, same length to one half of background scene. Hill folds are needed to turn it into a realistic desk. But caution is needed for you must ~~not~~ not overlap the desk with the centre crease.

Curtis succeeded in making the exam card and has written out some clear instructions, including drawings of a 3-dimensional product. An extract is shown above.

He has demonstrated that he can

(v) draw a 2-dimensional representation of a 3-dimensional product,
(xii) devise instructions to enable someone else to make the product.

If he can explain clearly the reasons for instruction 5, he will also demonstrate that he can

(vi) give a reasoned explanation for design features.

Alison

C. Cards with angled stick-ons

To make a wall stand up vertically, the base of the wall must be straight before sticking it on the paper.

To make the wall lean backwards the base of the wall must be cut into a 'Λ' and the centre of the wall must be a valley fold.

To make the wall lean forwards the base of the wall must be cut into a 'V' and the centre of the wall must be a ~~valley~~ hill fold.

Alison has given a clear analysis of this problem, but has not considered all the variations. She also produced some very nice models. If she continues with her investigations she is likely to show that she can

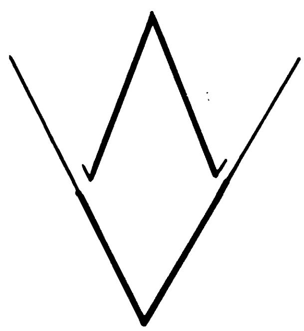
(vi) give a reasoned explanation for design features.

The extracts below from Vicki's and Denise's work on 'Cards with parallel stick-ons' (M54) show that they are working towards demonstrating that they can

(iii) recognise structural features of a design,
(vii) identify and correct design faults.

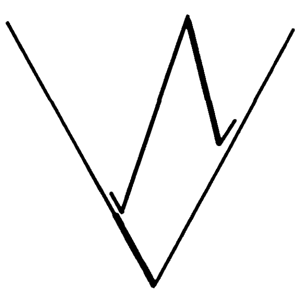
Vicki

Card D



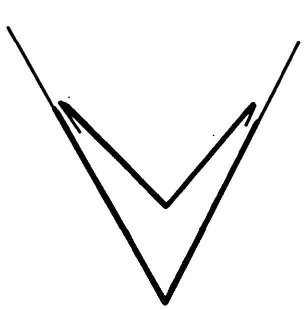
1. This card can be closed because the lines (in red) are the same length.
2. When the card is opened flat on a table it won't tear, because of the same reason.

Card E.



1. When this card is closed it will crease because the measurements (in red) are different lengths.
2. It will tear when opened flat because of the different length.

Card F.

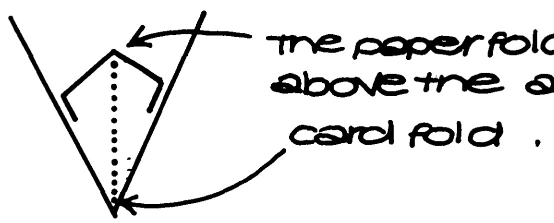


1. When this card is closed the lines ~~[illegible]~~ crease because the ~~[illegible]~~ red lines aren't the same length.
2. When you open the card out flat it will tear, because it has been glued too high up.

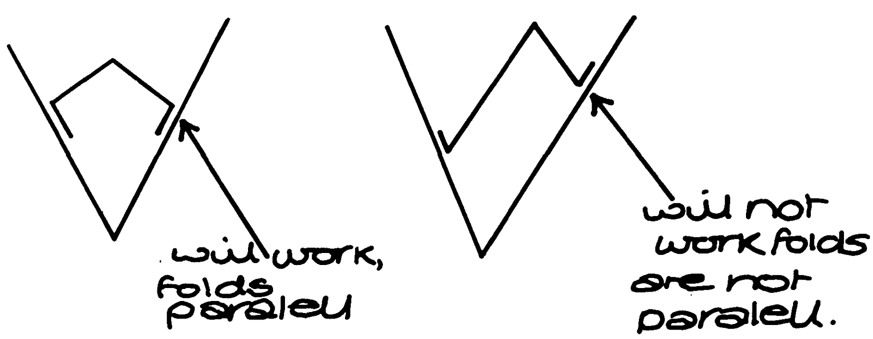
Vicki has done the initial part of this activity very thoroughly, but not all of her statements are complete or correct. She needs to try to make these cards to check her statements before she can satisfy the criteria.

Denise

If you can imagine the cards actually folding then you can tell if it is going to fold.
For example
Card B is definatly going to fold as the card when folded will compress and push together the folded inside. Because the folds in the cards cannot be disthorted in any way then you have to be able to recognise the position of the fold to enable the card to work. the folds, from the examples above should be facing upwards or downwards directly in the centre of the card.



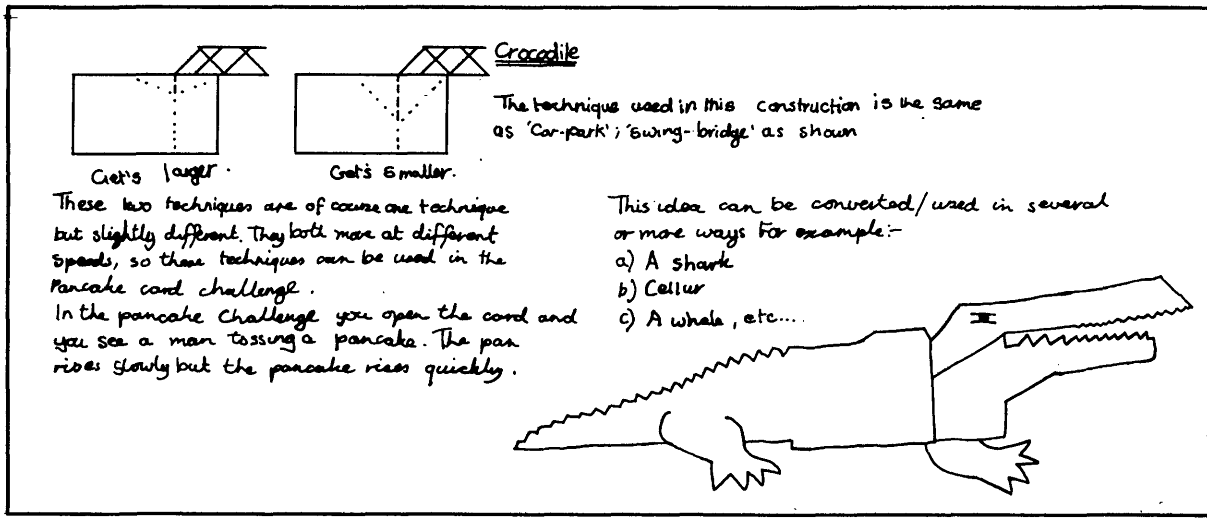
Another thing would be the position at what you stick the paper in. If the two gluing flaps are situated paralell to each other the card will work if not it wont.



Denise has developed a theory that cards fold correctly if the 'paper fold is directly above the card fold'. She needs to make up some examples and then she may realise that some of the cards that she thinks *will* work, will actually tear as they open. She may then be able to correct her errors (criterion (vii)) and then be able to understand the structural conditions that must be satisfied (criterion (iii)).

Curtis and Mark have been working on 'Cards with angled folds' (M57 and 58).

Curtis



Curtis demonstrates that he can

(iii) recognise structural features in a design,

and then, by producing a crocodile (which closes its jaws as the book opens) he has also demonstrated that he can

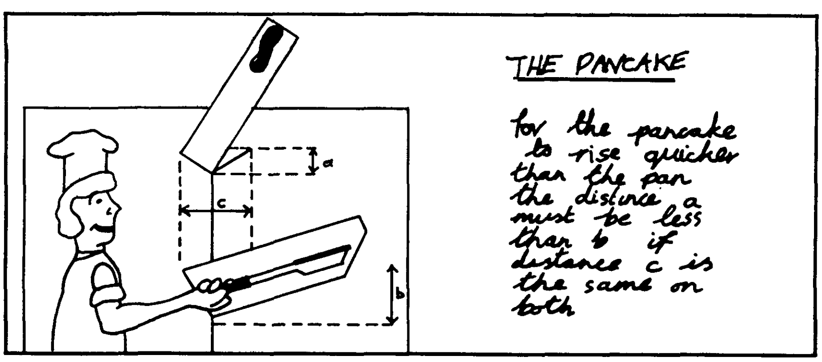
(viii) develop an existing idea for a paper product,
(x) draw a design to an acceptable degree of accuracy,
(xi) construct a prototype with original features.

He is also on the way to demonstrating that he can

(vi) give a reasoned explanation for design features.

He has not made it clear whether the larger or the smaller angle makes the arm move the quicker. If he can do this, or make the pancake card to demonstrate it, then he will satisfy the criterion.

Mark



Mark has shown a very clear understanding of how the speed of movement depends on the angle of fold, and has been able to apply it accurately.

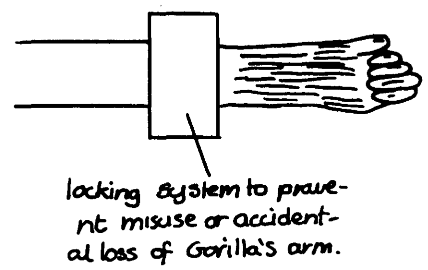
This working model shows that he can

(iii) recognise structural features of a design,
(iv) make a 3-dimensional object from a 2-dimensional representation,
(vi) give a reasoned explanation for design features.

Curtis and Mark have been working on 'Cards with slides and pivots' (M61 and M62).

Curtis

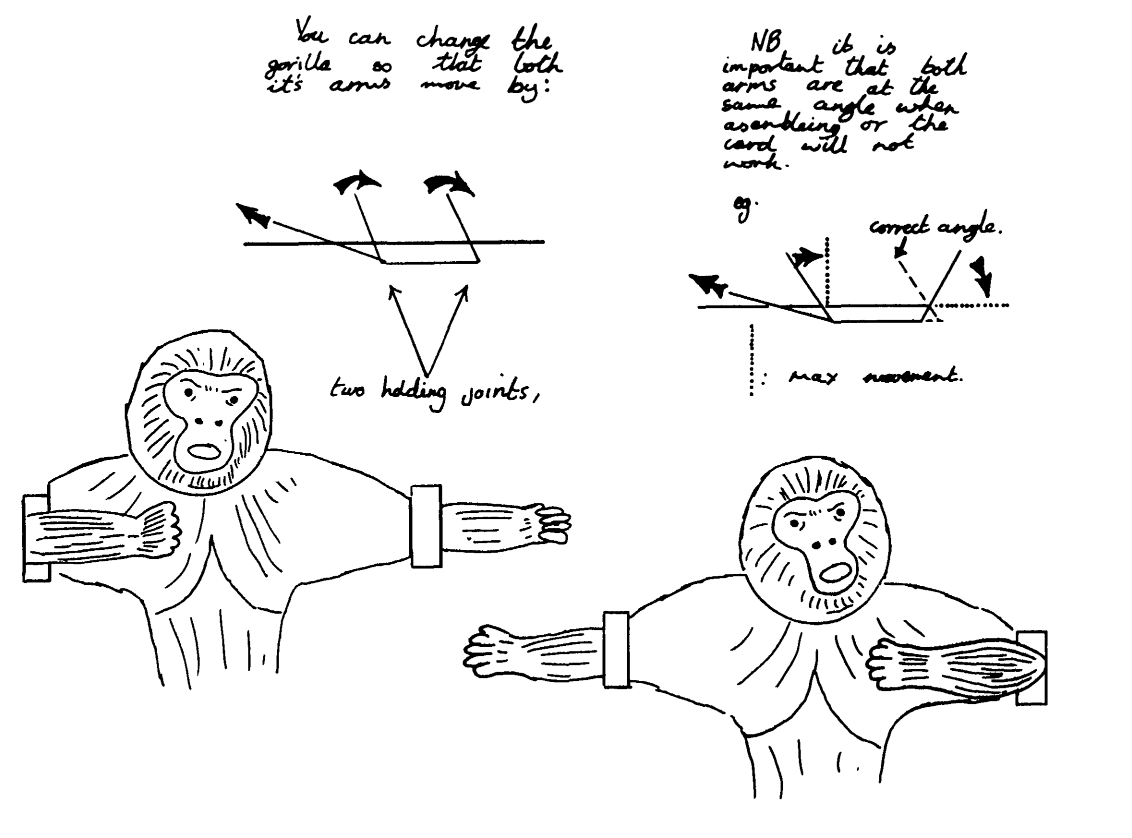
There is however a slight mistake in this construction. There isnt a very good locking system on the slide which if you pull the slide too far the arm disapears behind the ape itself.



Curtis made his own version of the gorilla with only one arm moving. He has noticed a fault in his design and was able to suggest, and implement, a way of overcoming it. He thus shows that he can

(vii) identify and correct a design fault.

Mark

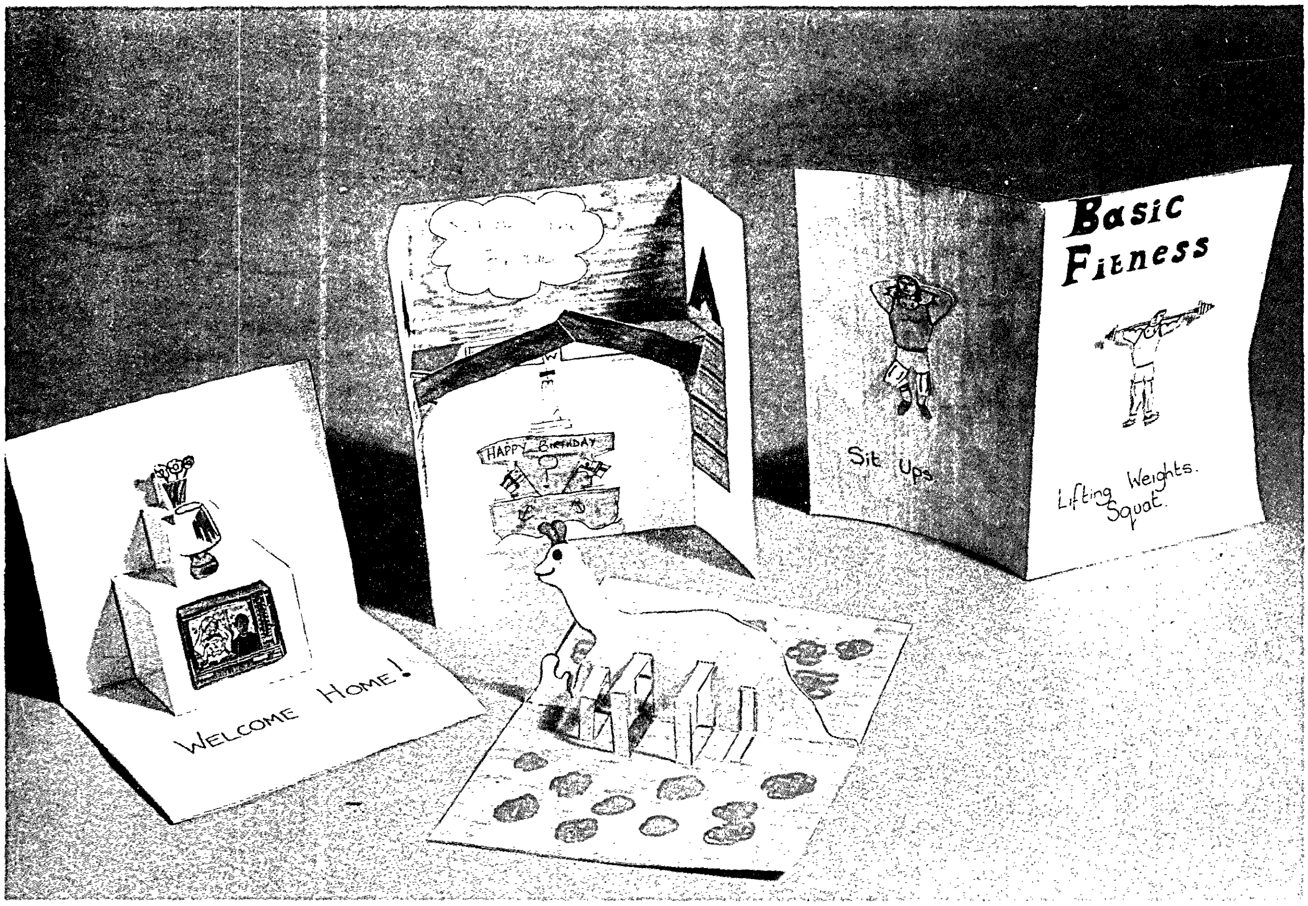


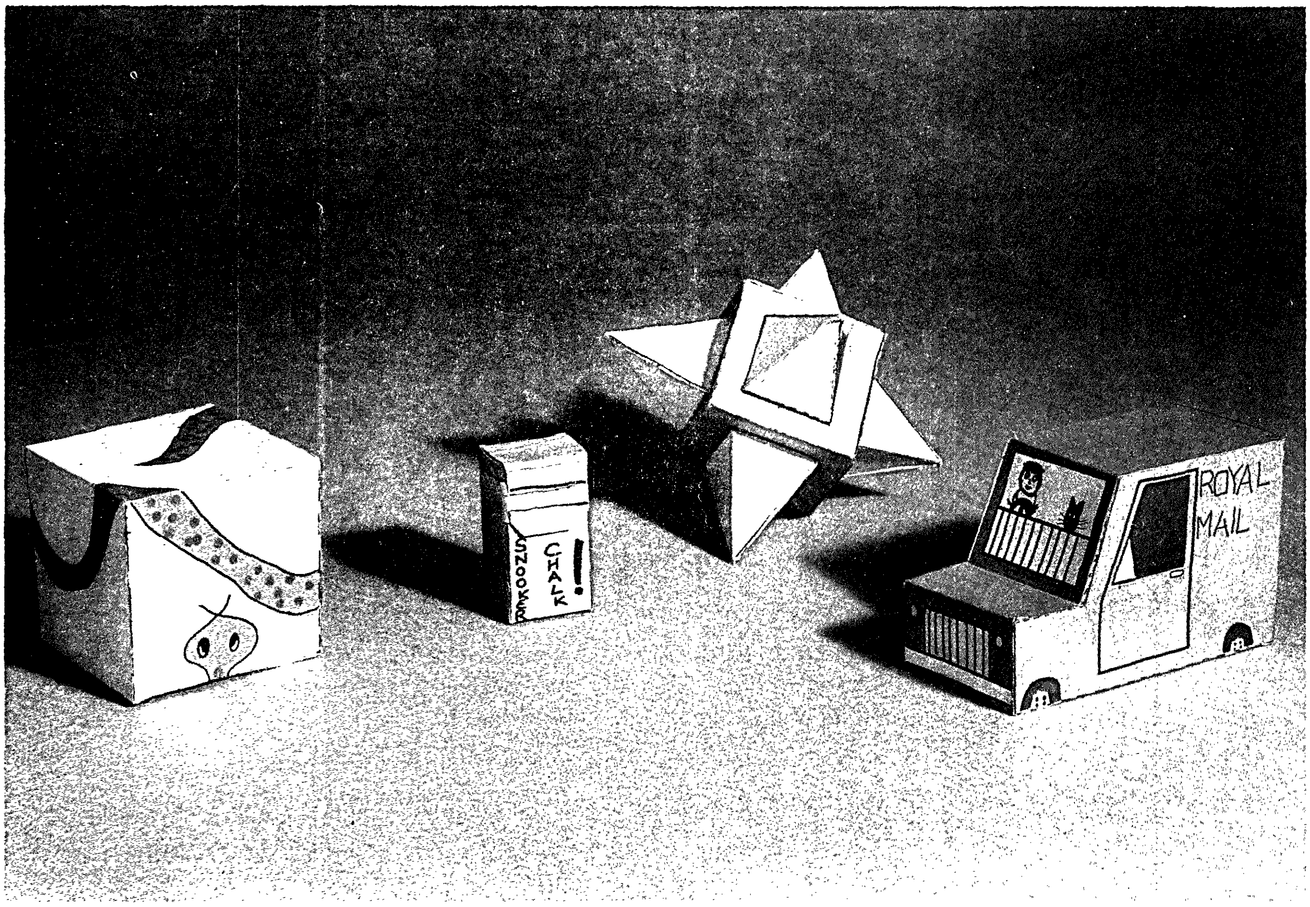
Mark gradually developed his gorilla. His first one pops up as on the sheet and both arms move. His description and diagram, showing the way he developed the mechanism to do this, show that he can

(vi) give a reasoned explanation for design features,

(viii) develop an existing idea for a paper product.

He then made a new model making the head roll from side to side as well. The two extreme positions are shown here.


Basic Fitness
Sit Ups
Lifting Weights.
Squat.
HAPPY BIRTHDAY
WELCOME HOME!


SNOOKER CHALK!
ROYAL MAIL

The two photographs show products from one low ability fifth year class. In the process of making them, the students all satisfied most of the criteria. Their teacher felt very positive about their achievement and describes some of the aspects of the work and effect on the students.

'I decided to let them try out their final designs on a younger class to give their work credibility.
I organised the children into groups – mixing the fifth years in with the second years. The second years were allowed to keep the models and cards which they made.

My fifth year pupils grew in self-confidence when the second years were making up their designs. Their behaviour, which for four years had been rather suspect, was excellent: they were well mannered, spoke clearly and precisely and were very helpful. This arrangement with fifth years helping me continued, at the request of the second year pupils, until they left school. For the first time in the whole of their school career my fifth years actually felt that they were of worth and, even more importantly, brighter than others in the school.

I feel very strongly that if we build up the self-confidence of these youngsters, their whole approach to working improves and they are more inclined to produce something of worth. This was certainly the case with this group of fifth years. They were the bottom set not capable of entering GCSE exams and yet they all passed Paper Engineering at Standard Level.* These were probably the only certificates they were likely to be given in school. To have these certificates signed by the JMB and the headmaster, with specific comments from me, gave them status amongst their peers, parents and other school children.

This group took part in the trial scheme for
Design a Board Game
Produce a Quiz Show and
Be a Paper Engineer.

Their results in each module improved.
Attendance at maths lessons improved.
Attitude to work improved.
Being able to organise themselves improved, so that instead of asking for equipment, they went and got what they wanted straight away.
Presentation of work improved.
Links with other subject areas improved, in particular with CDT and craft subjects.
They became so absorbed in their work they didn't even notice when visitors arrived.'

*The examination referred to was included in the trials of the Joint Matriculation Board's Certificate of Numeracy through Problem Solving.

Assessing process and product

The strategic and technical skills described above have much in common with the Mathematics aims in the GCSE National Criteria. These aims state that

"all courses should enable pupils to:

2.5 solve problems, present the solutions clearly, check and interpret results;

2.7 recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve a problem;

2.12 produce and appreciate imaginative and creative work arising from mathematical ideas."

Many of the GCSE Mathematics assessment objectives can be met through an assessment of the activities arising out of this module, from the more general objectives such as

"3.7 estimate, approximate and work to degrees of accuracy appropriate to the context,

3.8 use mathematical and other instruments to measure,

3.17 carry out practical and investigational work and undertake extended pieces of work,"

to those which apply more particularly to this module, such as

"3.11 recognise and use spatial relationships in two and three dimensions, particularly in solving problems."

The module activities thus provide a good starting point for GCSE mathematics, particularly coursework, though for GCSE there is likely to be a need to enhance the 'mathematical' aspects of the work.

The various GCSE Examining Groups have their individual approaches to coursework in their Mode 1 syllabuses. For example, one Group specifies broad areas for each of the candidate's four pieces of coursework, namely 'Practical Geometry', 'Statistics and Probability', 'Everyday Applications of Mathematics' and 'An Investigation'. This requires coursework to be assessed under the five headings of 'Overall design and strategy', 'Mathematical content', 'Accuracy', 'Clarity of argument' and 'Presentation' together with a controlled element, with marks awarded under each heading.

For another group the candidate has to submit three units of coursework, one of which is an extended piece of work. Oral assessment is also included. Although specific areas of mathematics are not laid down for each of the pieces of coursework, the units together should include examples of drawings, patterns, statistical surveys, sampling, measurement and constructions. In this case the group expects teachers to assess coursework under the four headings of 'comprehension of the task', 'planning', 'carrying out the task' and 'communication'. Here there is less emphasis on specific mathematical content within a given unit, while more emphasis is given to general problem solving skills such as 'considering a range of methods, looking at patterns, testing conjectures, generalising, justifying and evaluating.'

Whatever coursework syllabus is followed – and these two examples give some idea of the range available within Mode 1 – you will need to provide guidance so that the syllabus requirements are met and at the same time encourage the candidate to develop individual ideas in as open a way as possible. You may therefore have to adapt the approach suggested in the later stages of the Student's Booklet to give a greater opportunity for a student to develop mathematical ideas individually, and to fulfil other syllabus requirements. It may also be sensible to limit the range of initial module activities to allow more time for an individual to develop his or her own ideas. Chapter 2, 'Developing the Mathematics' offers a selection of suggestions for areas of mathematics which a student might explore within a coursework unit.

An example of GCSE coursework

On the following pages, we include an excerpt from the later stages of a unit of GCSE Mathematics extended coursework. The student has produced three designs, each with an accompanying discussion, and has then used these discussions as a basis for recommending 'the most suitable design'.

Despite similarities with the approach in the Student's Booklet, there is a greater emphasis on the mathematical content of the coursework and less on the finished product. This reflects the differences in the objectives between a typical GCSE Mathematics syllabus and those of the 'Numeracy through Problem Solving' scheme.

Part of the coursework instructions devised by the school

> During this Project you will be looking at a variety of types of packaging and using the ideas you see, together with your own, to design and make a container to package an item of your own choice.
>
> Choose an item to package.
>
> Design three different containers to package this item. In each case, draw an accurate net for the design, with clear instructions on how to assemble it.
>
> Discuss your designs and select which one you think would be the most suitable, giving your reasons.

Part of a student's response

Lai

Stage 3

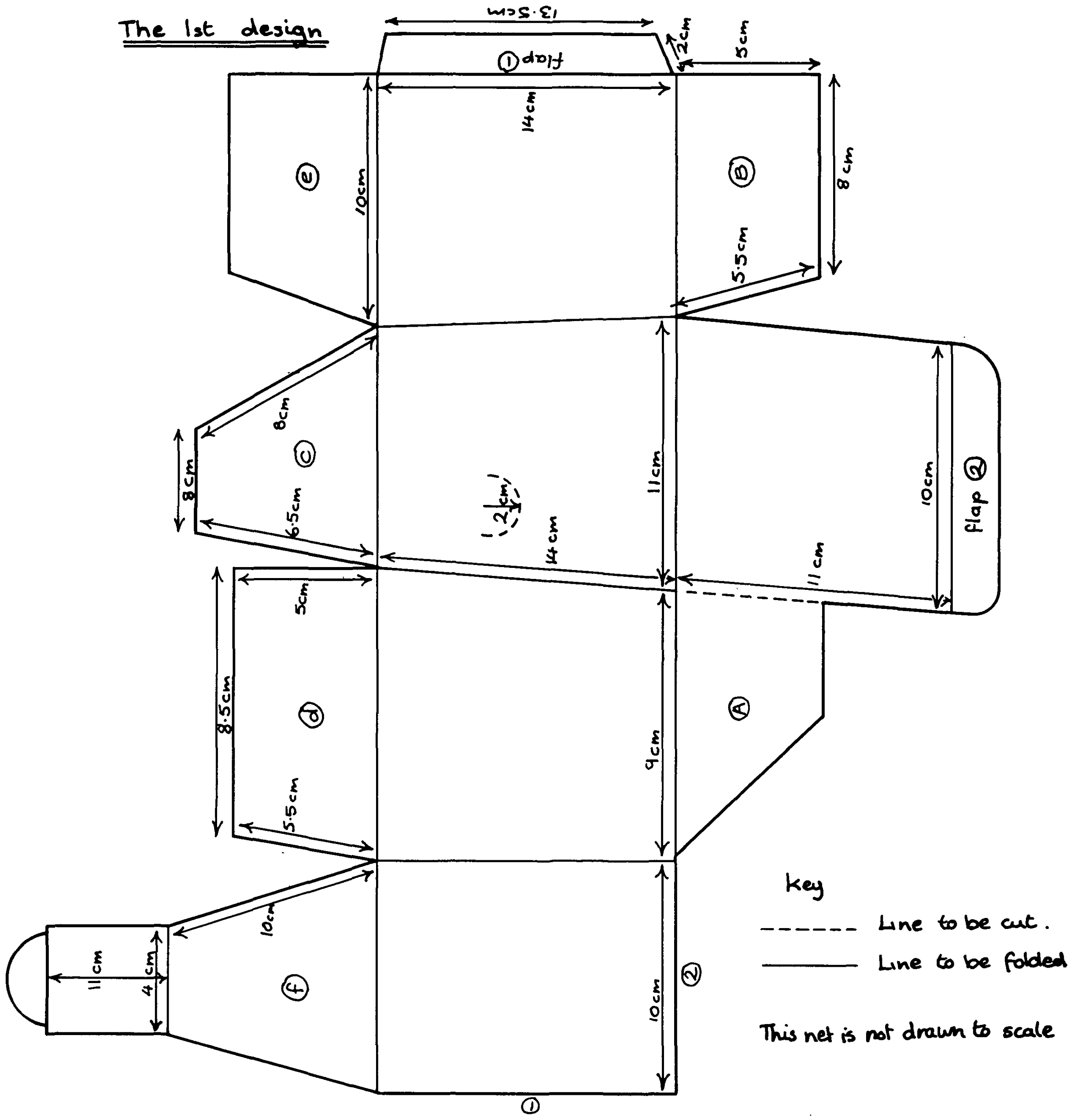
Designing 3 different containers to package a soft toy

The item which I have chose to package is a tall narrow toy (soft)

It's height is approximately 12 cm

It's width is approximately 7cm

The 1st design



We include Lai's assembly instructions and her discussion only for the first design. The assembly instructions indicate that the designs have been made up to test their effectiveness. No doubt the teacher will also have discussed the work with Lai during its progress to gain further indications of the mathematical processes being used.

How to assemble the net for the 1st design?

① Draw the net onto card with the measurements given.
② Score all the lines to make it easier to fold.
③ fold the scored lines inwards.
④ Assemble it to form a solid.
⑤ Stick flap ① onto the other end marked ①.
⑥ stick flap ② onto where into where flap Ⓐ and Ⓑ folds, so the flap will be attached to where the line is marked with ②.
⑦ Before assembling the lid, make sure the sticking of the flaps are firmally secured.
⑧ Fold © inwards with ⓓ and ⓔ ontop and the lid ⓕ covering it.
⑨ The thin rounded flap has to be pushed into the semi circle slot to close the container.

Discussing the 1st Design

The box is very easy to open having to lift the flap. when closing it you just have to push this flap into the slit. It provides reasonable protection.
I think the design is very attractive because it is a cuboid pushed to one side. The container is quite easy to produced once knowing how to assemble it to form a solid. The cutting of the slit caused the greatest error having to cut it to an exact curve.

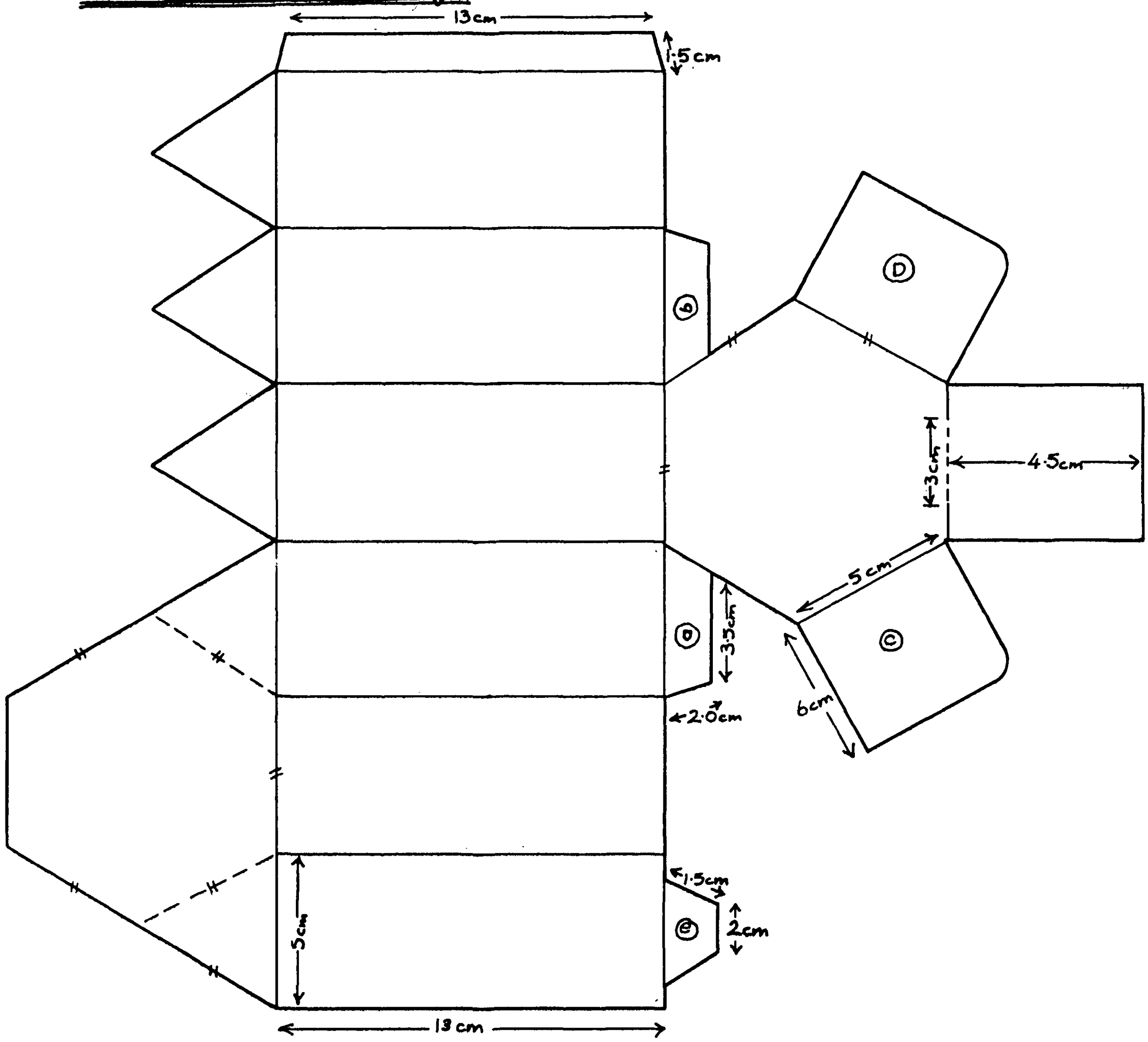
Volume of the box
= area of the end
× length of the solid.

area of the end = Length × width
= 10cm × 10cm
= 100 cm^2

the length of the solid = 14cm

volume = 100 × 14 = 1400 cm^3

The net for the ~~3rd~~ 2nd design



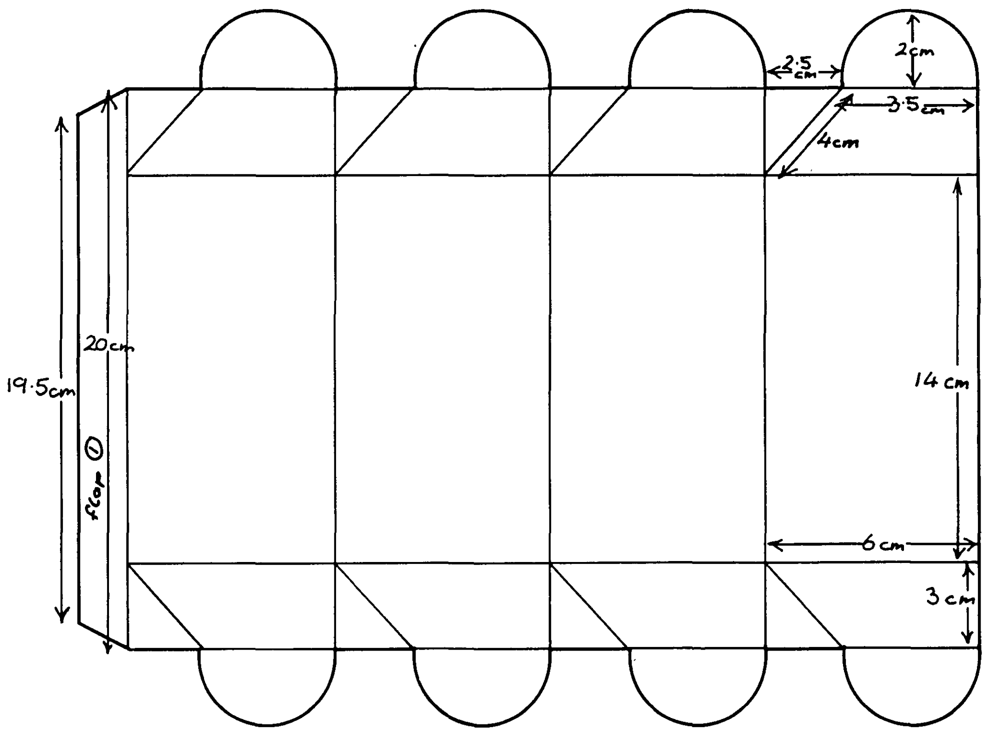
This net is not drawn to scale

Key

———————— folding lines

- - - - - - - cutting lines

The net for the ~~2nd~~ 3rd design



This net is not drawn to scale.

The most suitable design.

The design which I've selected and would be most suitable is design 1, the solid which is a cuboid pushed to one side.

The reason why I have selected this are:

1) The box is easily opened.
2) It will be used as a storage container. In this case storing a cuddly toy.
3) It provides reasonable protection with a firmally secured base
4) The design is very attractive because of it's shape.
5) It is easily produced.
6) The net is not complicated to assemble.
7) The volume is the greatest so the box gives a much larger space to hold the content.
8) The area of the base is $100 cm^2$ This figure tells us that the area is very high allowing the base to stand easily.

Written examination papers

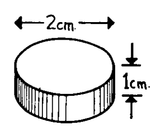
The two examination papers which follow may serve as a useful resource in assessing a student's ability to retain and transfer the skills to fresh situations in the same context.

Paper 1 is intended for nearly all students.
Paper 2 is intended for approximately the top 30% of students.

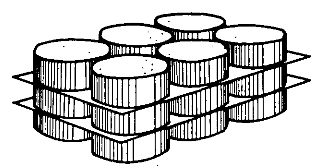
PAPER 1

Designing a box

Steve and Ann have made a large number of round peppermint sweets. Each sweet is 2cm diameter and 1cm thick.



They plan to sell them in boxes. Each box will hold 18 sweets, in layers of 6.

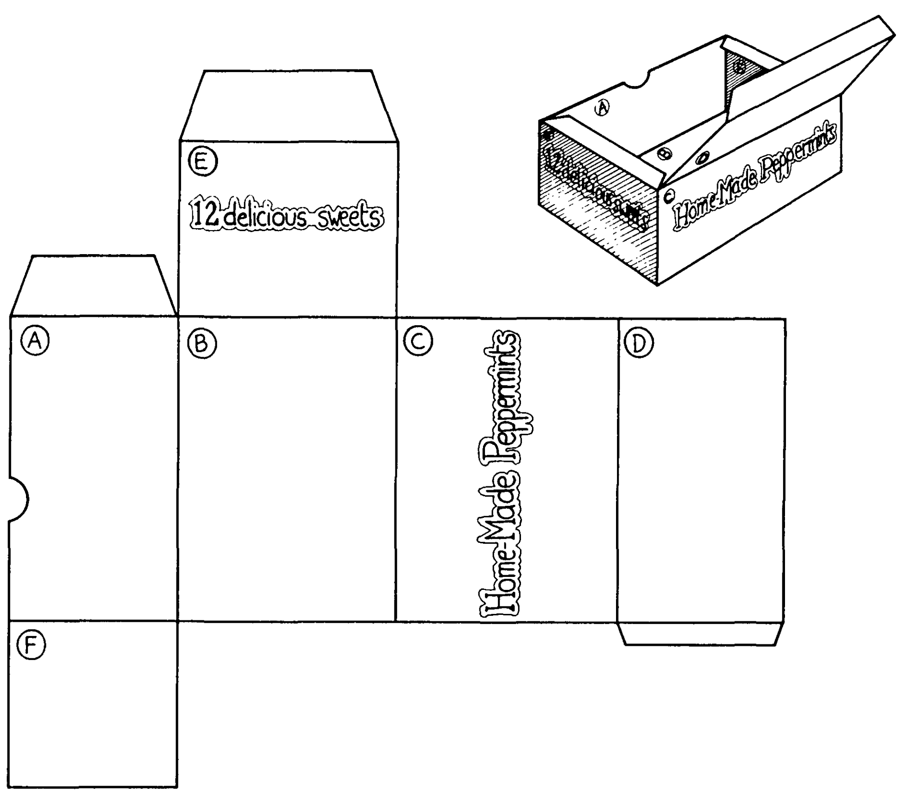


Below is their first design for a box. But . . . .

It's no good!
This won't even make a box!

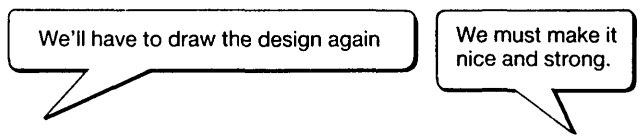
1. Using a copy of the design, cut out and try to make the box.

   List as many mistakes in the design as you can. You can show them on the diagram below if you wish.

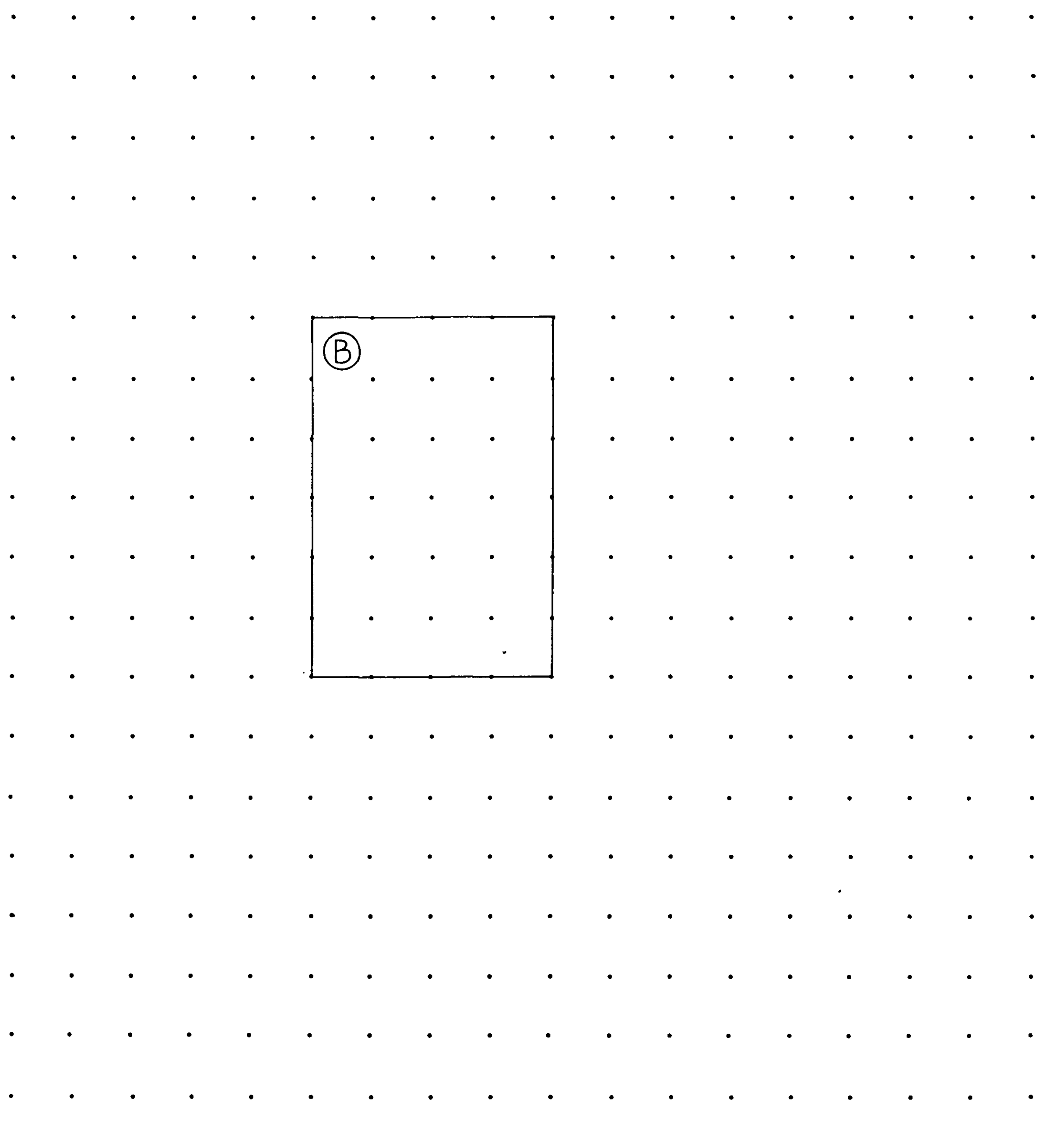


PAPER 1

Designing a box (continued)

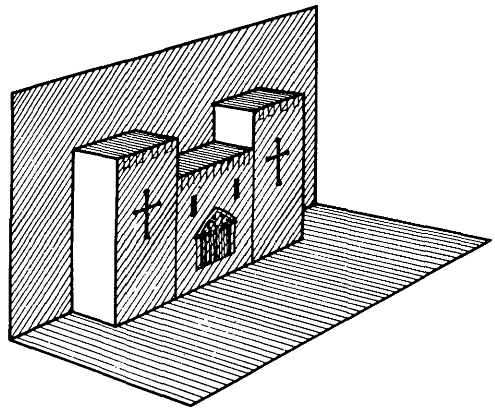


2. Complete the correct design as accurately as you can.



PAPER 1

A Pop-up castle

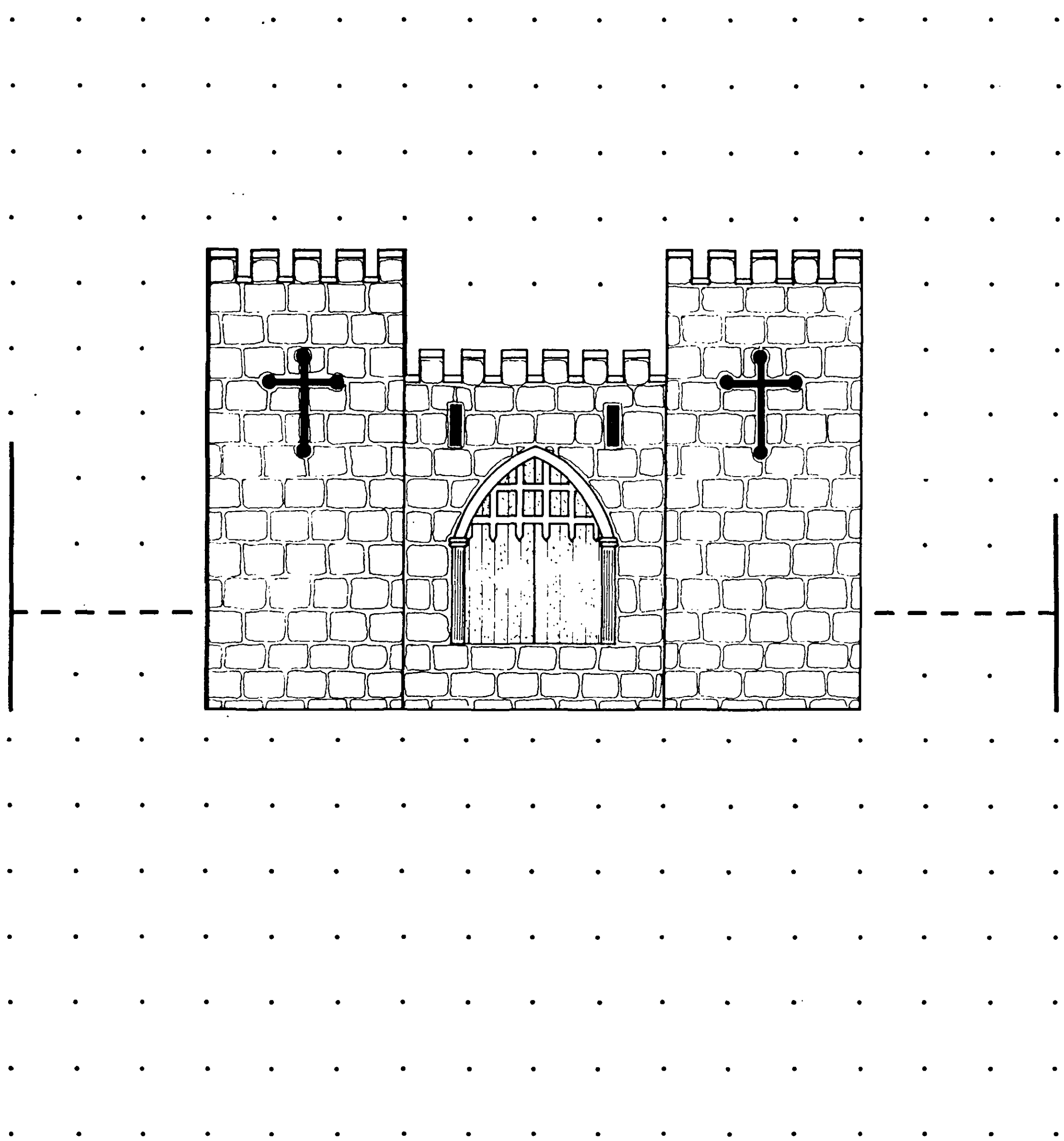


The picture shows a Castle pop-up card which Ann and Steve are making.

When the card is closed,

- the castle must not stick out beyond the edges of the card,
- the card must fit into an 8½cm by 16½cm envelope.

1. Using a copy of the design, cut out and make the card. Now complete the design for the card shown below as accurately as you can. (Use ——— for cut lines, · · · · · · · for hill folds, — — — — — — — for valley folds.)



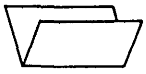
3

PAPER 1

A Pop-up castle (continued)

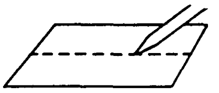
2. Here are four instructions (A, B, C, D) to help someone else make the card from the design:

A Fold along these lines ------------ to make "valleys"

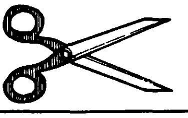


B Fold along these lines .... to make "hills"

C Score along the fold lines



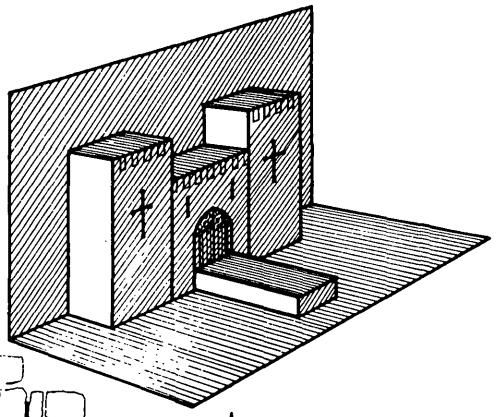
D Cut along these thick lines ______



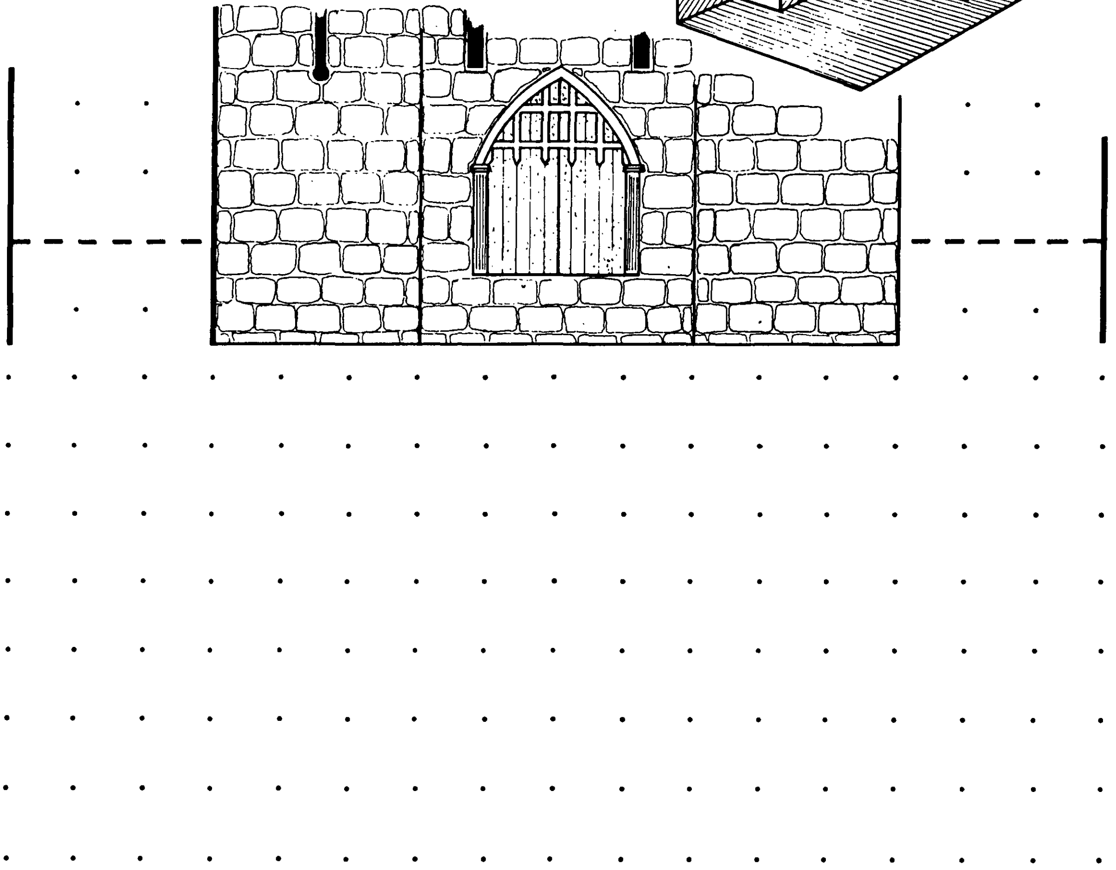
In what order should the instructions be?

3. Anne and Steve decide to change their design by making a bridge which leads to the door.

   Add this bridge to the card you have already made.



Now show below, as accurately as you can, the extra cuts and folds that are needed on the design.



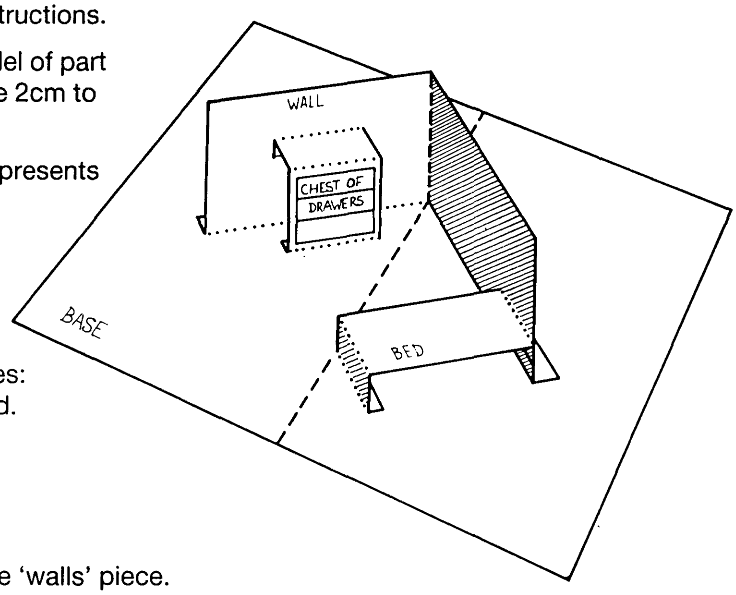
PAPER 2

A Pop-up bedroom card

Ann and Steve plan to produce an accurate design for the card, with assembly instructions.

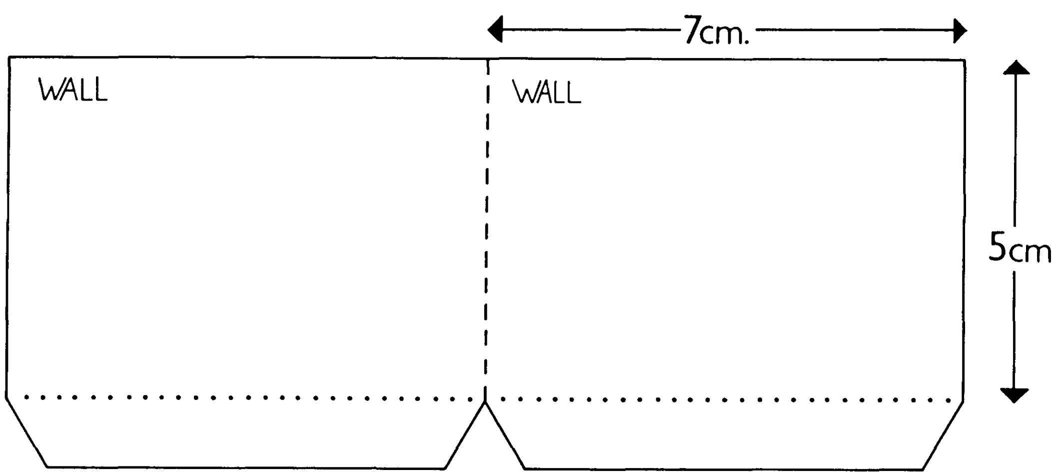
They want the card to be a scale model of part of a real bedroom. They decide to use 2cm to represent 1m.

So, for example, 10cm on the card represents 5m in the bedroom.



The card will be made from four pieces: Base, walls, chest of drawers and bed.

1. Here is an accurate diagram for the 'walls' piece.
   What are the height and length of the wall in the real bedroom?



2. Cut out the 'walls' piece from a copy of this sheet.
   Use it and some rough paper to make up the walls and base of the card.

3. The chest of drawers is 1.5m wide by 0.7m deep by 0.9m high. On the card the pop-up chest will be fixed by glue to the wall and floor.

   Draw an accurate plan for this piece, including tabs for gluing.

1

A Pop-up bedroom card (continued)

4. The card also includes a bed which is 4cm long and 1cm high. What is the greatest width the bed can have, so that the card will open and close properly? (Make sure it doesn't extend beyond the wall!)

   Draw an accurate plan for this piece, including tabs for gluing.

5. When the card is closed, the pop-up parts should all be completely hidden.

   At the same time, Ann and Steve want the base of the card to be as small as possible.

   Draw an accurate plan for the base, showing clearly any fold lines and the exact positions where any tabs should be glued.

6. Give full instructions which explain clearly how to assembly an accurate version of the card, starting with the four pieces.

M88

Checklist for the teacher

Real problem solving demands a different balance of teaching styles and strategies from the traditional curriculum. Students need to learn how to

- assume more control of, and responsibility for, their work
- share and develop ideas with each other
- give and accept constructive criticism
- turn their own ideas into actions, and to learn from the consequences of their own decisions.

To effect this change of roles, it is helpful if the teacher can:

Frequently

- praise achievement

  > Those seem to be really good ideas.

- encourage students to think further and deeper

  > Look back to the products in Stage 1. Which could help you to make *your* design?

- ask clarifying questions

  > Can you explain that to me again?
  > I didn't quite understand.

- agree targets

  > Do you think you could finish a rough design by Monday?

- encourage students who lack confidence.

  > Try making something simple to start with. You can always develop it later on.

Occasionally

- make a suggestion if a student is running out of ideas

  > Why not design something that could be used to publicise the school fete?

- divert a student from a particularly unhelpful idea

  > Don't colour in your rough models. Add colour and detail *after* you have made sure that your design will work.

Avoid

- taking over a problem

  > Why don't you do it like this . . .

- making negative comments

  > You're being much too ambitious.

- determining whose view is accepted.

  > Sarinda's idea seems to be the best.