The Language of Functions and Graphs

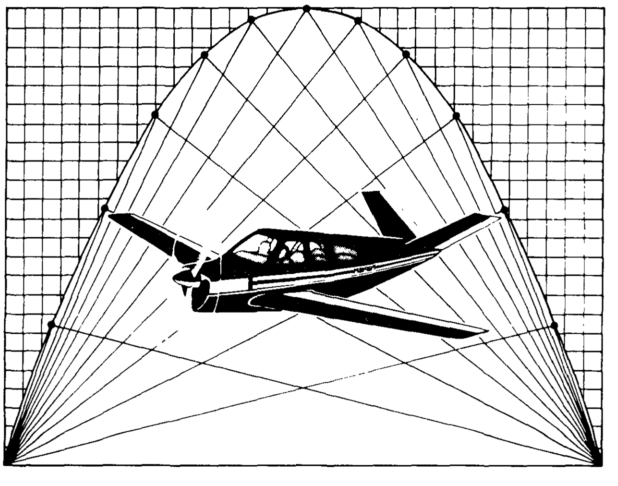
Masters for Photocopying

CONTENTS

* The numbers in brackets refer to the corresponding pages in the Module books.

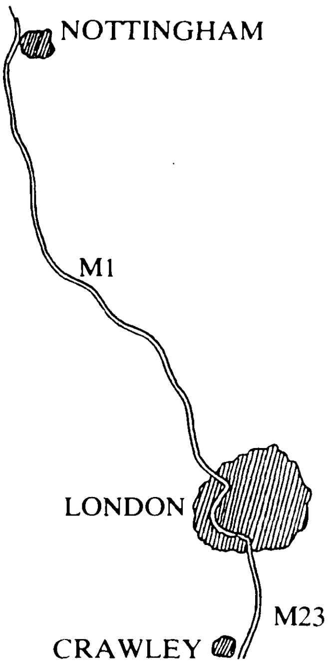
† The masters for materials prefixed with an A, B or T should be used to form four paged booklets, by photocopying back to back and folding in half.

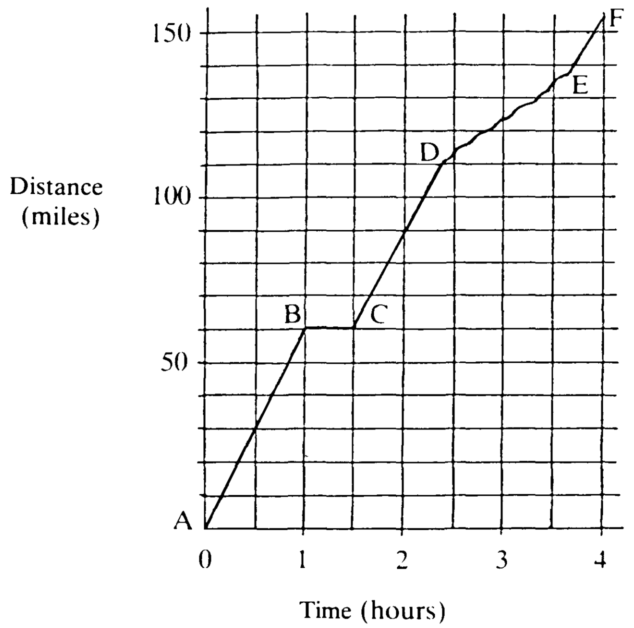
Specimen Examination Questions



THE JOURNEY

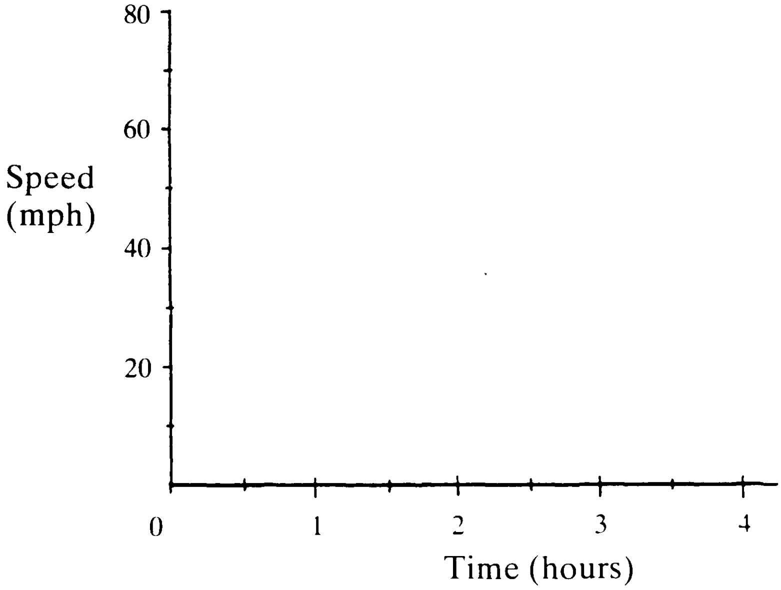
The map and the graph below describe a car journey from Nottingham to Crawley using the M1 and M23 motorways.





(i) Describe each stage of the journey, making use of the graph *and* the map. In particular describe and explain what is happening from A to B; B to C; C to D; D to E and E to F.

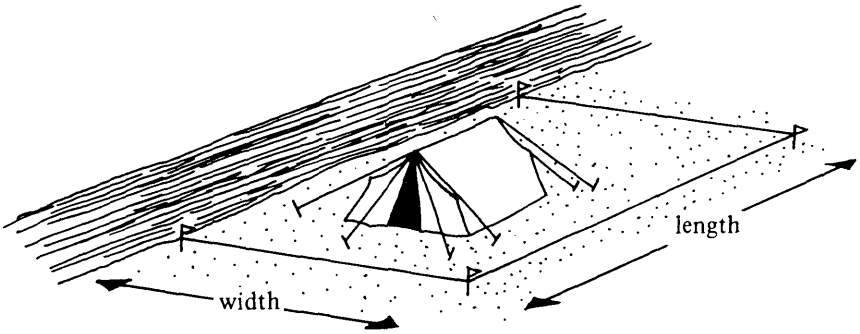
(ii) Using the information given above, sketch a graph to show how the speed of the car varies during the journey.



CAMPING

On their arrival at a campsite, a group of campers are given a piece of string 50 metres long and four flag poles with which they have to mark out a rectangular boundary for their tent.

They decide to pitch their tent next to a river as shown below. This means that the string has to be used for only three sides of the boundary.

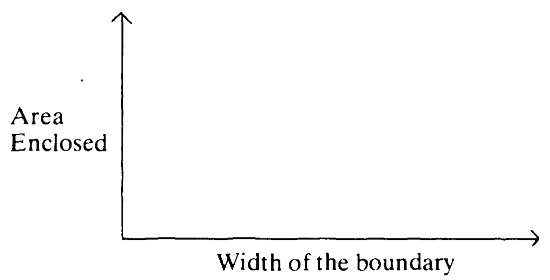


(i) If they decide to make the width of the boundary 20 metres, what will the length of the boundary be?

(ii) Describe in words, as fully as possible, how the length of the boundary changes as the width increases through all possible values. (Consider both small and large values of the width.)

(iii) Find the area enclosed by the boundary for a width of 20 metres and for some other different widths.

(iv) Draw a *sketch* graph to show how the area enclosed changes as the width of the boundary increases through all possible values. (Consider both small and large values of the width.)

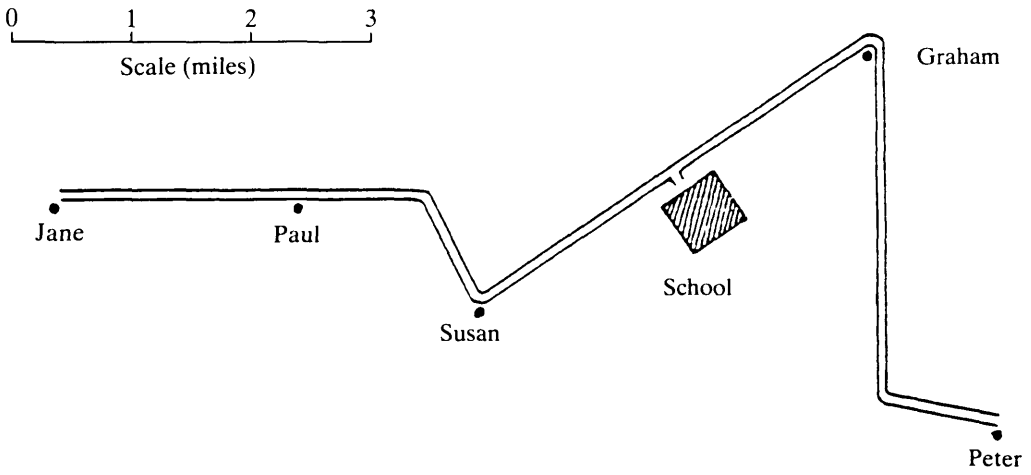


The campers are interested in finding out what the length and the width of the boundary should be to obtain the greatest possible area.

(v) Describe, in words, a method by which you could find this length and width.

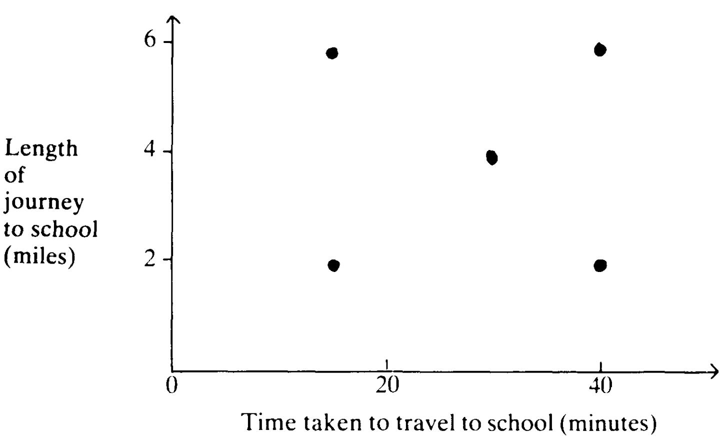
(vi) Use the method you have described in part (v) to find this length and width.

GOING TO SCHOOL



Jane, Graham, Susan, Paul and Peter all travel to school along the same country road every morning. Peter goes in his dad's car, Jane cycles and Susan walks. The other two children vary how they travel from day to day. The map above shows where each person lives.

The following graph describes each pupil's journey to school last Monday.



i) Label each point on the graph with the name of the person it represents.

ii) How did Paul and Graham travel to school on Monday? ______________

____

iii) Describe how you arrived at your answer to part (ii) ______________

____

____

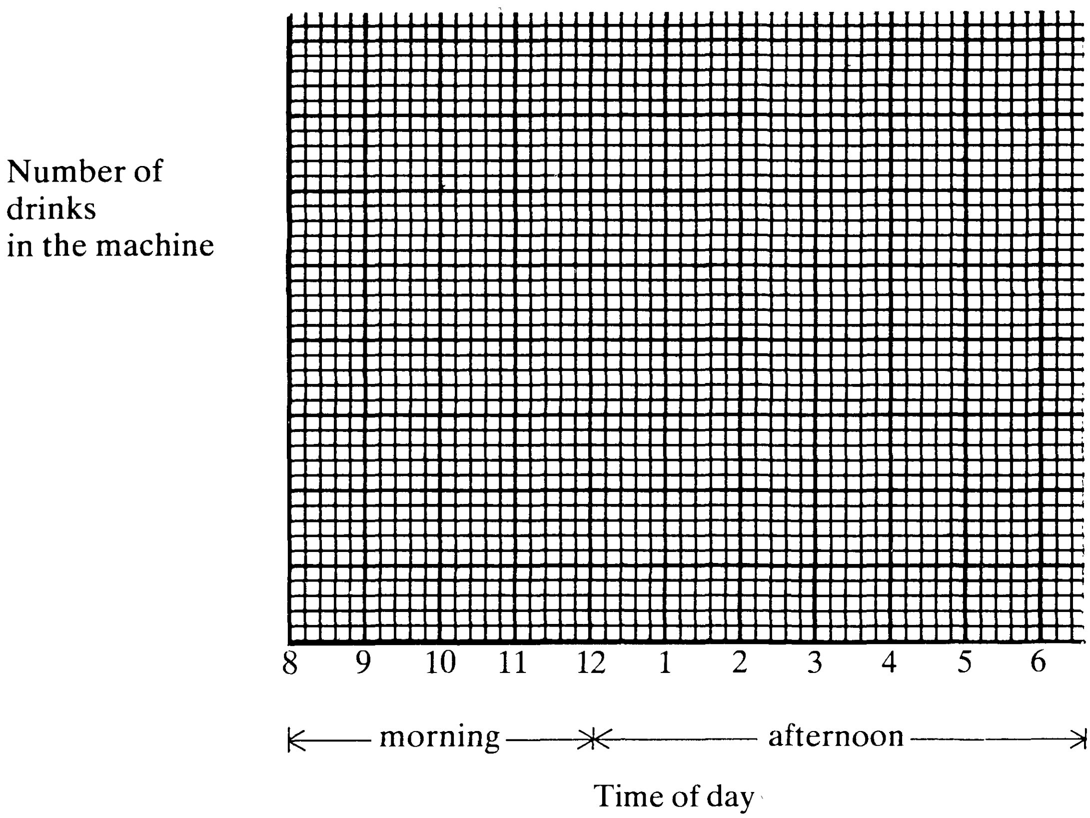
(continued)

THE VENDING MACHINE

A factory cafeteria contains a vending machine which sells drinks.
On a typical day:

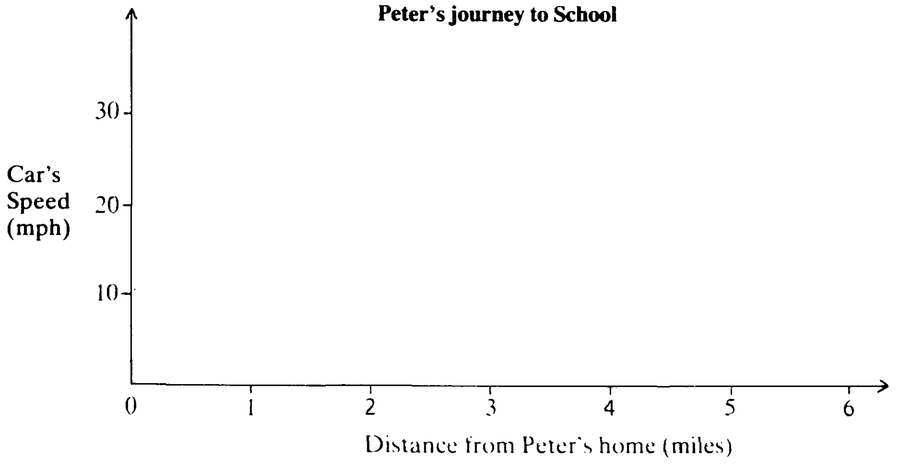
* the machine starts half full.
* no drinks are sold before 9 am or after 5 pm.
* drinks are sold at a slow rate throughout the day, except during the morning and lunch breaks (10.30-11 am and 1-2 pm) when there is greater demand.
* the machine is filled up just before the lunch break. (It takes about 10 minutes to fill).

Sketch a graph to show how the number of drinks in the machine might vary from 8 am to 6 pm.

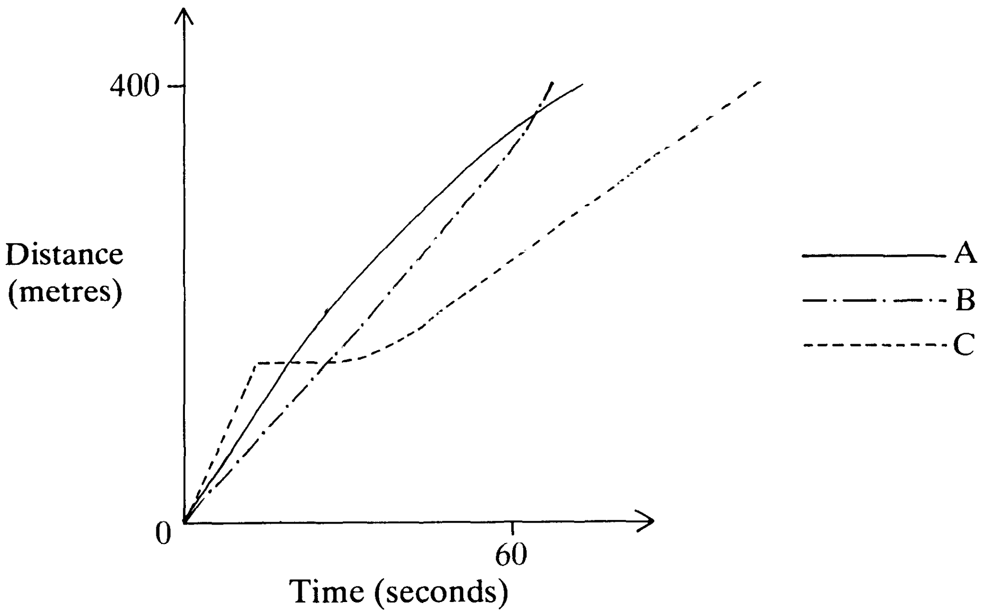


GOING TO SCHOOL (continued)

iv) Peter's father is able to drive at 30 mph on the straight sections of the road, but he has to slow down for the corners. *Sketch* a graph on the axes below to show how the car's speed varies along the route.



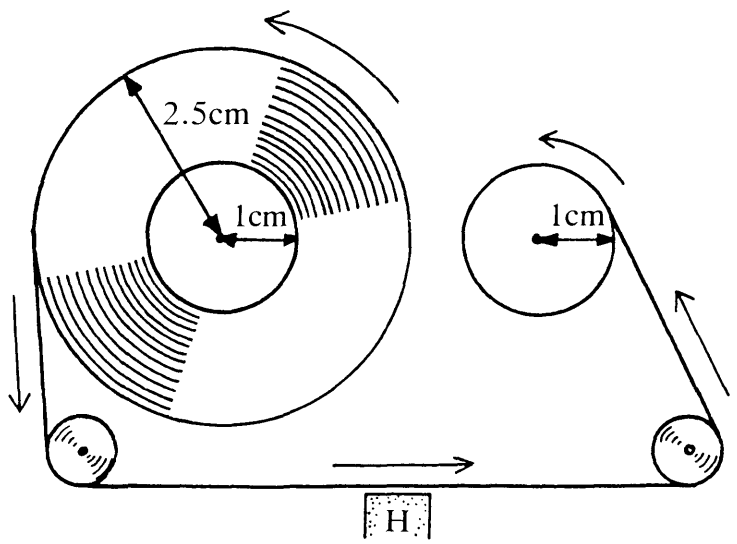
THE HURDLES RACE



The rough sketch graph shown above describes what happens when 3 athletes A, B and C enter a 400 metres hurdles race.

Imagine that you are the race commentator. Describe what is happening as carefully as you can. You do not need to measure anything accurately.

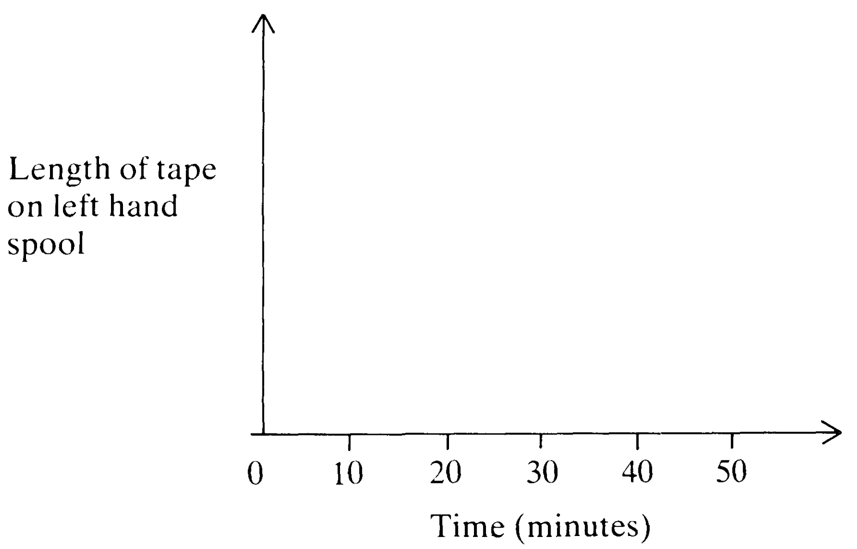
THE CASSETTE TAPE



This diagram represents a cassette recorder just as it is beginning to play a tape. The tape passes the "head" (Labelled H) at a constant speed and the tape is wound from the left hand spool on to the right hand spool.

At the beginning, the radius of the tape on the left hand spool is 2.5 cm. The tape lasts 45 minutes.

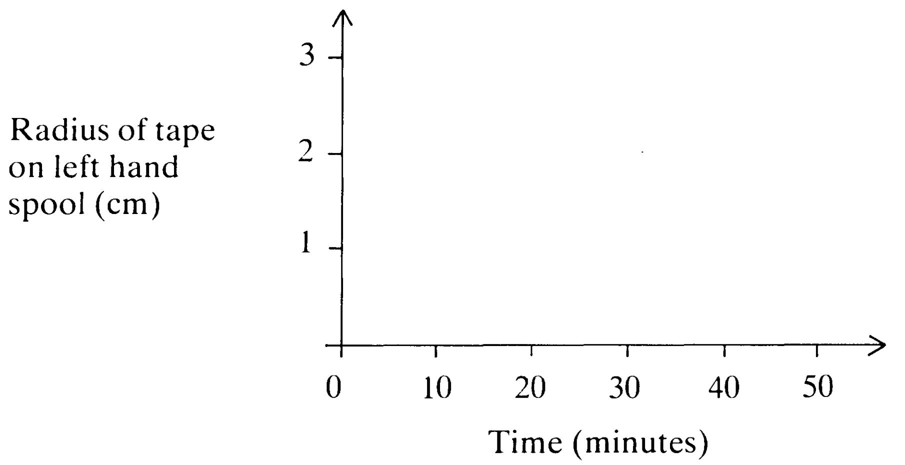
(i) Sketch a graph to show how the *length* of the tape on the left hand spool changes with time.



(continued)

THE CASSETTE TAPE (continued)

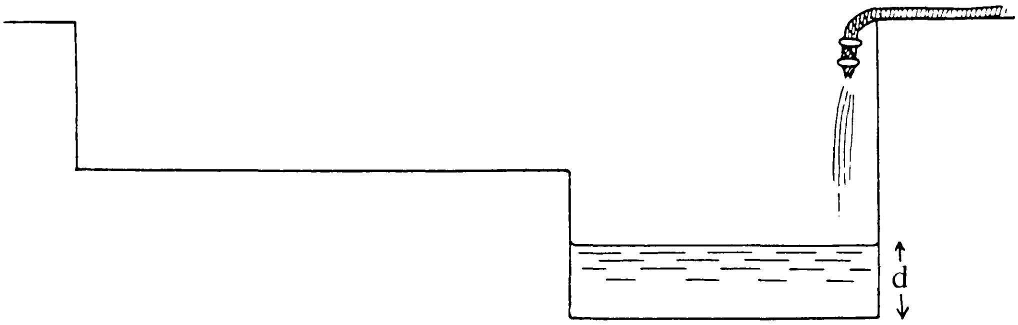
(ii) Sketch a graph to show how the *radius* of the tape on the left hand spool changes with time.



(iii) Describe and explain how the radius of the tape on the *right-hand* spool changes with time.

FILLING A SWIMMING POOL

(i) A rectangular swimming pool is being filled using a hosepipe which delivers water at a constant rate. A cross section of the pool is shown below.



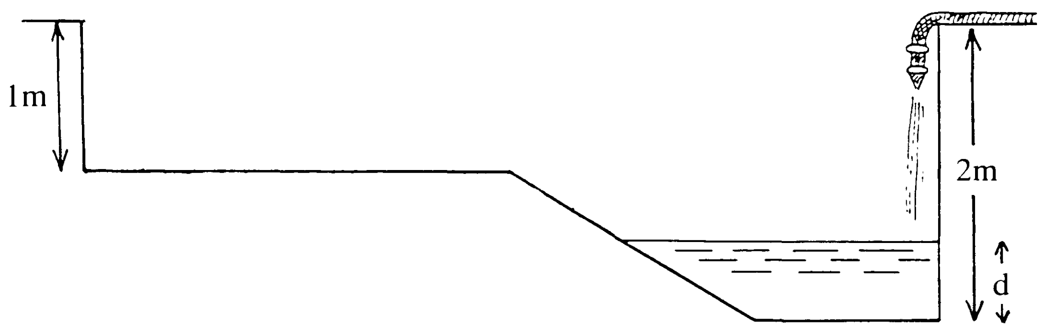
Describe fully, in words, how the depth (d) of water in the deep end of the pool varies with time, from the moment that the empty pool begins to fill.

____

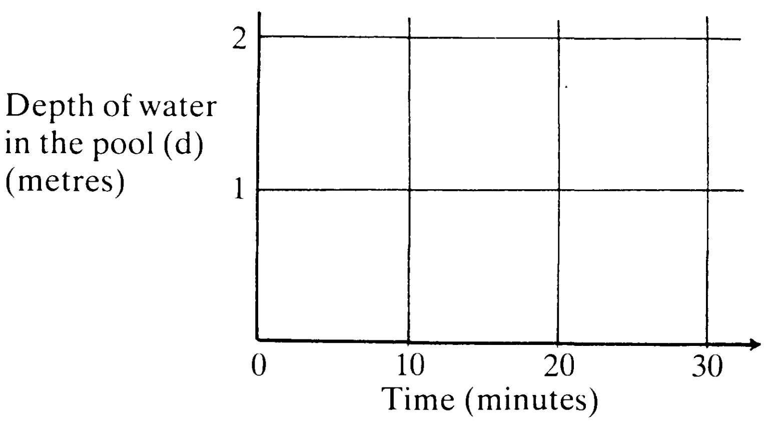
____

____

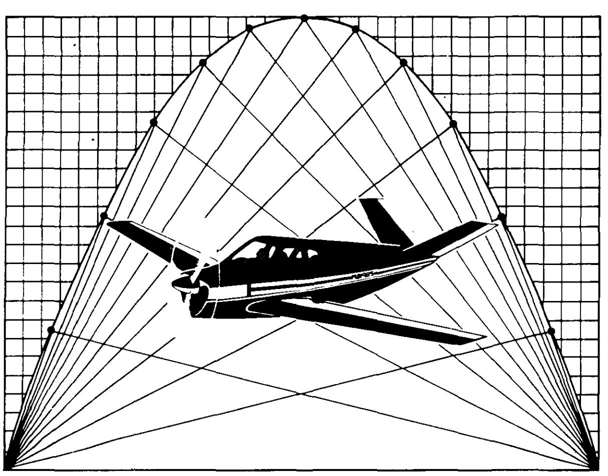
(ii) A different rectangular pool is being filled in a similar way.



Sketch a graph to show how the depth (d) of water in the deep end of the pool varies with time, from the moment that the empty pool begins to fill. Assume that the pool takes thirty minutes to fill to the brim.



Classroom Materials

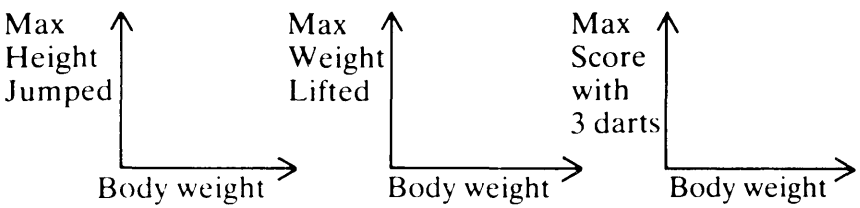


4. Sport

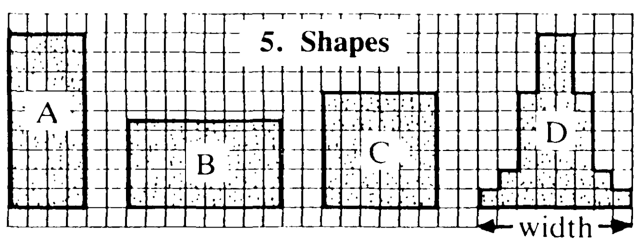
Suppose you were to choose, at random, 100 people and measure how heavy they are. You then ask them to perform in 3 sports;

High Jumping, Weight Lifting and Darts.

Sketch scattergraphs to show how you would expect the results to appear, and explain each graph, underneath. Clearly state any assumptions you make . . .

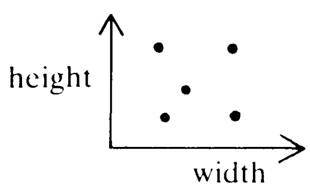


5. Shapes



These four shapes each have an area of 36 square units.

* Label four points on the graph below, with the letters A, B, C and D.
* Can you draw a fifth shape, with an area of 36 square units, to correspond to the other point? Explain.
* Draw a scattergraph to show *every rectangle* with an area of 36 square units.
* Finally, what happens if you include *all* shapes, with the same area, on your graph?

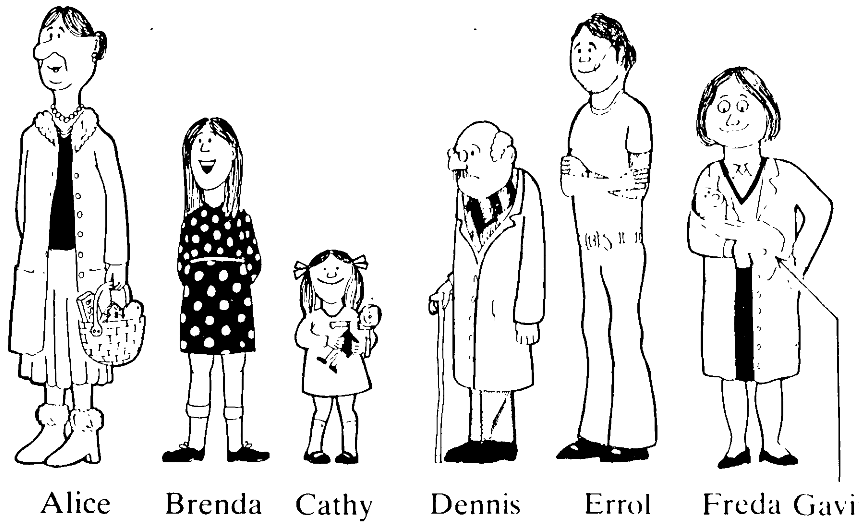


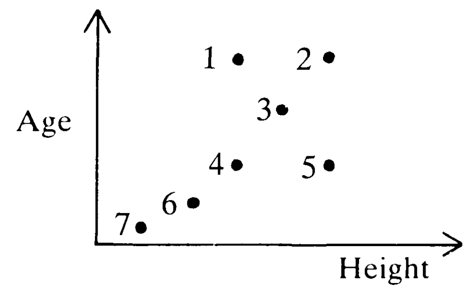
A1 INTERPRETING POINTS

As you work through this booklet, discuss your answers with your neighbours and try to come to some agreement.

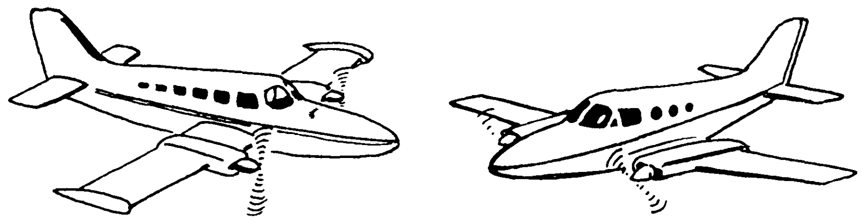
1. The Bus Stop Queue

Who is represented by each point on the scattergraph, below?

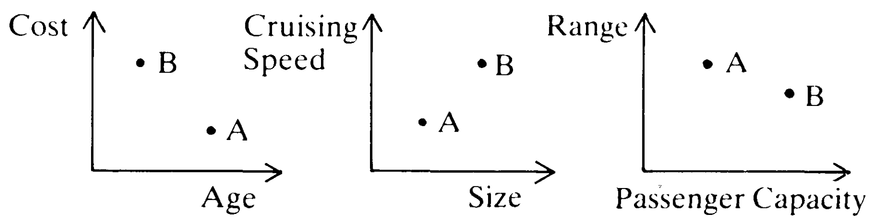




2. Two Aircraft



The following quick sketch graphs describe two light aircraft, A and B: (note: the graphs have *not* been drawn accurately)



The first graph shows that aircraft B is more expensive than aircraft A. What else does it say?

* Are the following statements true or false?

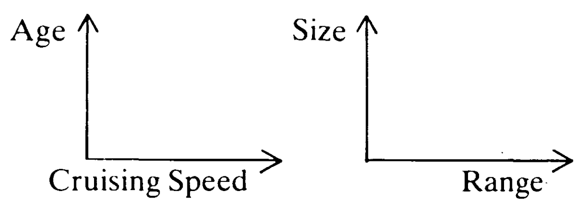
  "The older aircraft is cheaper"?

  "The faster aircraft is smaller"?

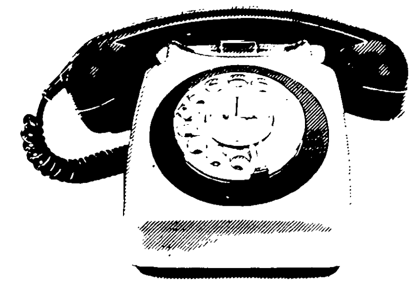
  "The larger aircraft is older"?

  "The cheaper aircraft carries fewer passengers"?

* Copy the graphs below. On each graph, mark and label two points to represent A and B.

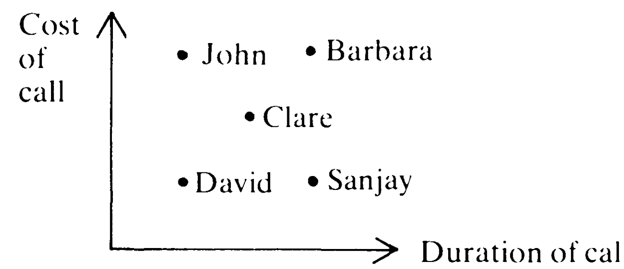


3. Telephone Calls



One weekend,
Five people made telephone calls to various part of the country.

They recorded both the cost of their calls, and the length of time they were on the telephone, on the graph below:

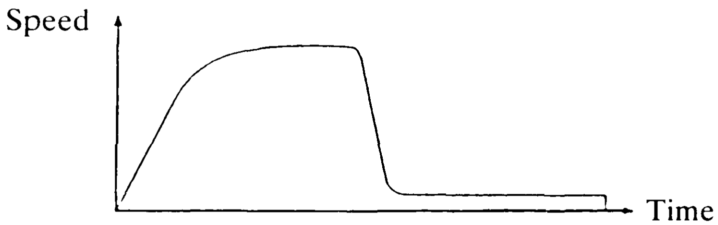


* Who was ringing long-distance? Explain your reasoning carefully.
* Who was making a local call? Again, explain.
* Which people were dialling roughly the same distance? Explain.
* Copy the graph and mark other points which show people making local calls of different durations.
* If you made a similar graph showing every phone call made in Britain during one particular week-end, what would it look like? Draw a sketch, and clearly state any assumptions you make.

Finally, discuss and write about this problem:

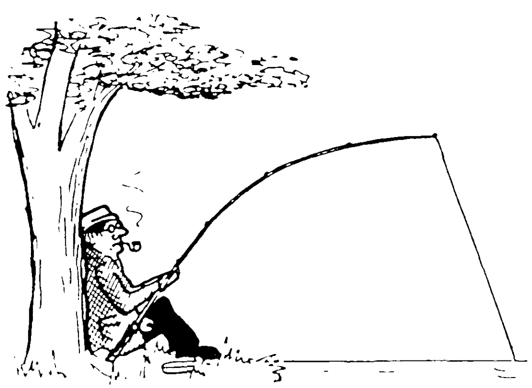
Which Sport?

Which sport will produce a graph like this?



Choose the best answer from the following and explain exactly how it fits the graph.

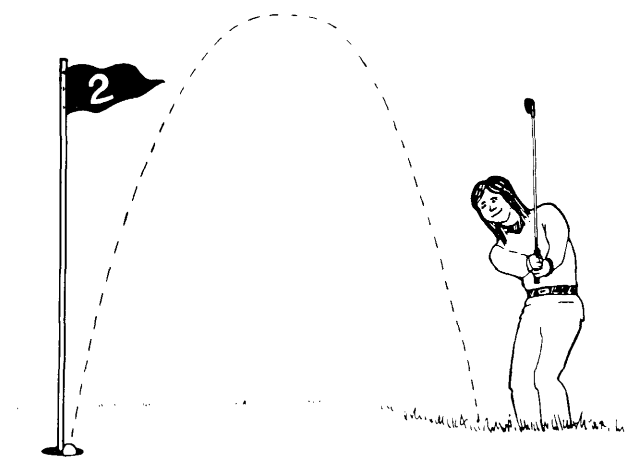
Write down reasons why you reject alternatives.



Fishing
Pole Vaulting
100 metre Sprint
Sky Diving
Golf
Archery
Javelin Throwing
High Jumping
High Diving
Snooker
Drag Racing
Water Skiing

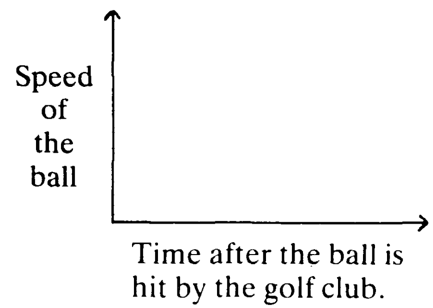
A2 ARE GRAPHS JUST PICTURES?

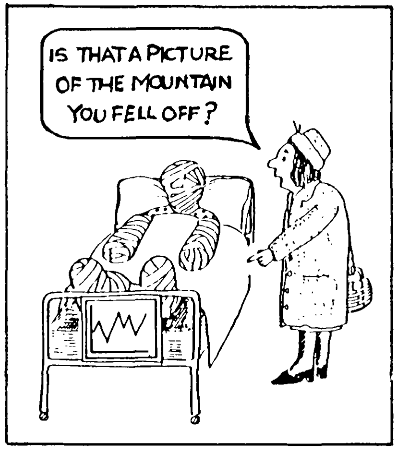
Golf Shot



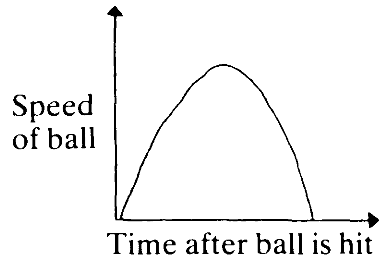
How does the speed of the ball change as it flies through the air in this amazing golf shot?

* Discuss this situation with your neighbour, and write down a clear description stating how you both think the speed of the golf ball changes.
* Now sketch a rough graph to illustrate your description:



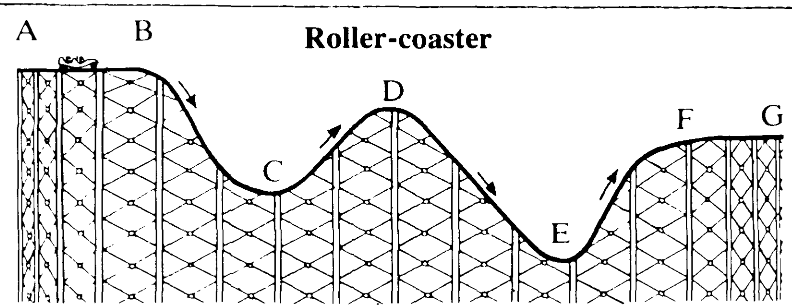


Peter attempted the golf question and produced a graph like this:



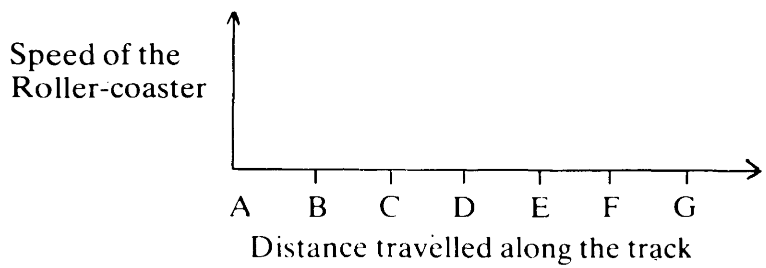
* Comment on it.
* Can you suggest *why* Peter drew his graph like this?
* Can you see any connection between Peter's attempt and the cartoon on page 1?

Now try the problem below:



The picture above shows the track of a roller-coaster, which is travelling at a slow constant speed between A and B. How will the speed of this roller-coaster vary as it travels along the track from A to G?

Describe your answer both in words and by sketching a graph in your book.



This next activity will help you to see how well you have drawn your sketch graph.

Fold this booklet so that you cannot see the picture of the roller-coaster track.

Try to answer the following questions using *only your sketch graph*.

* Along which parts of the track was the roller-coaster travelling quickly? slowly?
* Was the roller-coaster travelling faster at B or D? D or F? C or E?
* Where was the roller-coaster accelerating (speeding up)? decelerating (slowing down)?

Check your answers to these questions by looking back at the picture of the roller-coaster track. If you find any mistakes, *redraw* your sketch graph. (It is better to use a fresh diagram than to try and correct your first attempt.)

* Now invent some roller-coaster tracks of your own. Sketch a graph for each one, on a separate sheet of paper. Pass *only the sketch graphs* to your neighbour.

  Can she reconstruct the shape of the original roller-coaster tracks?

* Do you notice any connection between the shape of a roller-coaster track, and the shape of its graph? If so write down an explanation.

  Are there any exceptions?

Sketch graphs to illustrate the following statements. Label your axes with the variables shown in brackets. For the last statement you are asked to sketch two graphs on the same axes.

"In the spring, my lawn grew very quickly and it needed cutting every week, but since we have had this hot dry spell it needs cutting less frequently."

(length of grass/time)

"When doing a jigsaw puzzle, I usually spend the first half an hour or so just sorting out the edge pieces. When I have collected together all the ones I can find, I construct a border around the edge of the table. Then I start to fill in the border with the centre pieces. At first this is very slow going but the more pieces you put in, the less you have to sort through and so the faster you get."

(number of pieces put in jigsaw/time).

"The Australian cottony cushion scale insect was accidentally introduced into America in 1868 and increased in number until it seemed about to destroy the Californian citrus orchards where it lived. Its natural predator, a ladybird, was artificially introduced in 1889 and this quickly reduced the scale insect population. Later, DDT was used to try to cut down the scale insect population still further. However, the net result was to *increase* their numbers as, unfortunately, the ladybird was far more susceptible to DDT than the scale insect! For the first time in fifty years the scale insect again became a serious problem."
Use the same axes . . .
(scale insect population/time); (ladybird population/time).

A3 SKETCHING GRAPHS FROM WORDS

Picking Strawberries

* **Using axes like the ones below, sketch a graph to illustrate this situation.**

Total time it will take to finish the job

Number of people picking strawberries

* **Compare your graph with those drawn by your neighbours. Try to come to some agreement over a correct version.**
* **Write down an explanation of how you arrived at your answer. In particular, answer the following three questions.**
  - **should the graph 'slope upwards' or 'slope downwards'? Why?**
  - **should the graph be a straight line? Why?**
  - **should the graph meet the axes? If so, where? If not, why not?**

Choose the best graph to fit each of the ten situations described below. (Particular graphs may fit more than one situation.) Copy the graph, label your axes and explain your choice, stating any assumptions you make. If you cannot find the graph you want, draw your own version.

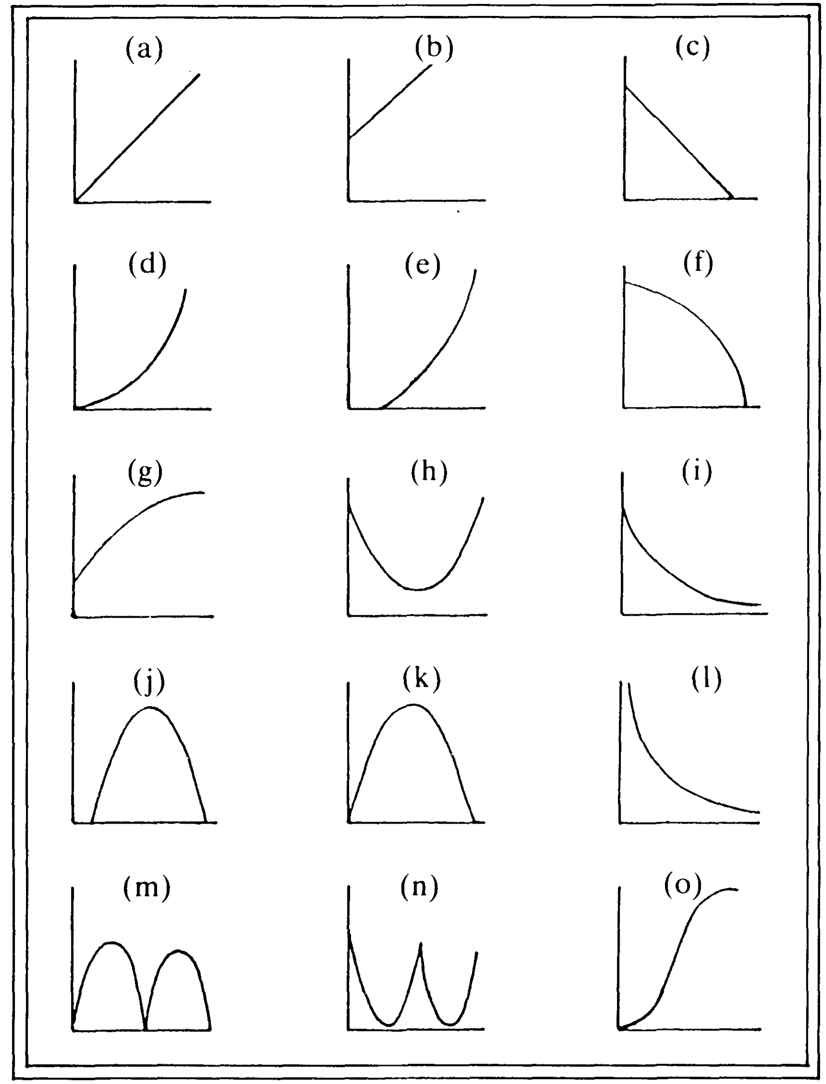
1. "Prices are now rising more slowly than at any time during the last five years."
2. "I quite enjoy cold milk or hot milk, but I loathe lukewarm milk!"
3. "The smaller the boxes are, then the more boxes we can load into the van."
4. "After the concert there was a stunned silence. Then one person in the audience began to clap. Gradually, those around her joined in and soon everyone was applauding and cheering."
5. "If cinema admission charges are too low, then the owners will lose money. On the other hand, if they are too high then few people will attend and again they will lose. A cinema must therefore charge a moderate price in order to stay profitable."

In the following situations, *you* have to decide what happens. Explain them carefully in words, and choose the best graph, as before.

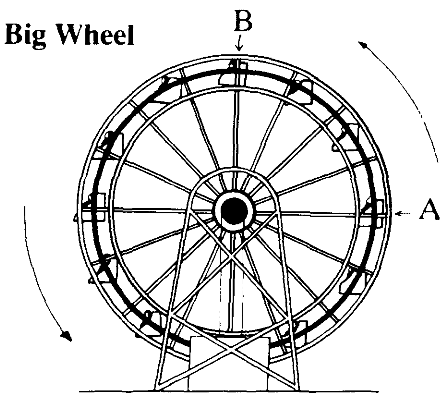
How does . . .

6. the cost of a bag of potatoes depend on its weight?
7. the diameter of a balloon vary as air is slowly released from it?
8. the time for running a race depend upon the length of the race?

9. the speed of a girl vary on a swing?
10. the speed of a ball vary as it bounces along?



The Big Wheel

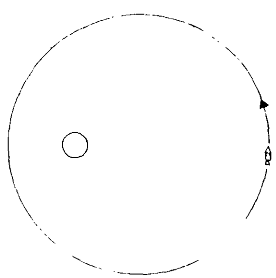


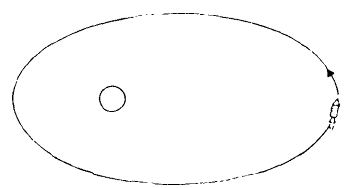
The Big Wheel in the diagram turns round once every 20 seconds. On the *same* pair of axes, sketch two graphs to show how both the height of car A and the height of car B will vary during a minute.

Describe how your graphs will change if the wheel turns more quickly.

Orbits

Each of the diagrams below shows a spacecraft orbiting a planet at a constant speed.
Sketch two graphs to show how the distance of the spacecraft from the planet will vary with time.



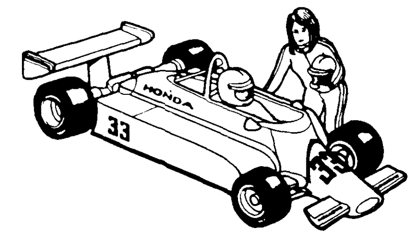


Using a dotted line on the same axes, show how your graphs will change if the speed of the spacecraft increases as it gets nearer to the planet.

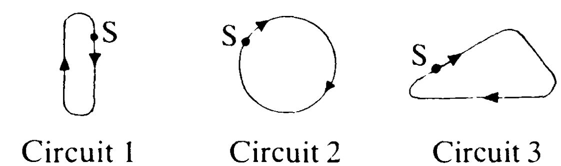
Now invent your own orbits and sketch their graphs, *on a separate sheet of paper*. Give *only* your graphs to your neighbour. Can she reconstruct the orbits from the graphs alone?

A4 SKETCHING GRAPHS FROM PICTURES

Motor Racing



How do you think the speed of a racing car will vary as it travels on the *second lap* around each of the three circuits drawn below? (S = starting point)



Explain your answer in each case both in words and with a sketch graph. State clearly any assumptions that you make.

Compare your graphs with those produced by your neighbours. Try to produce three graphs which you all agree are correct.

Look again at the graph you drew for the third circuit. In order to discover how good your sketch is, answer the following questions looking *only* at your sketch graph. When you have done this, check your answer by looking back at the picture of the circuit. If you find any mistakes *redraw* your sketch graph.

— Is the car on the first or second lap?

— How many bends are there on the circuit?

— Which bend is the most dangerous?

— Which "straight" portion of the circuit is the longest? Which is the shortest?

— Does the car begin the third lap with the same speed as it began the second? Should it?

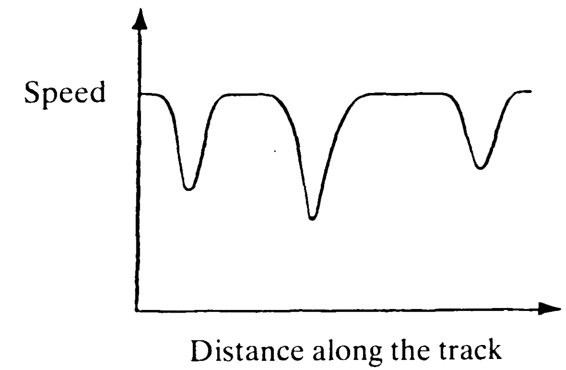
Now invent a racing circuit of your own with, at most, four bends.

Sketch a graph *on a separate sheet of paper* to show how the speed of a car will vary as it goes around your circuit.

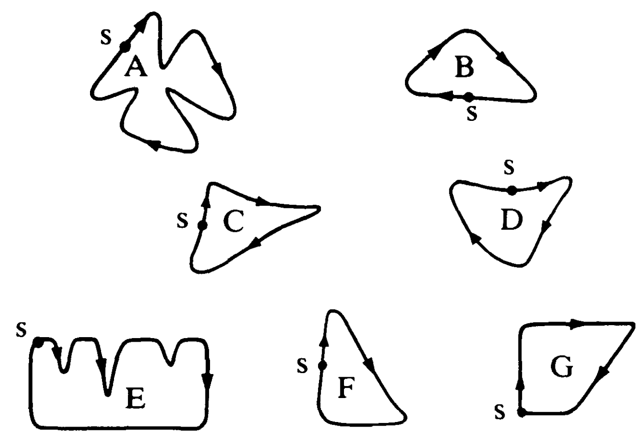
Pass *only* your graph to your neighbour.

Can she reconstruct the shape of the original racing circuit?

The graph below shows how the speed of a racing car varies during the second lap of a race.

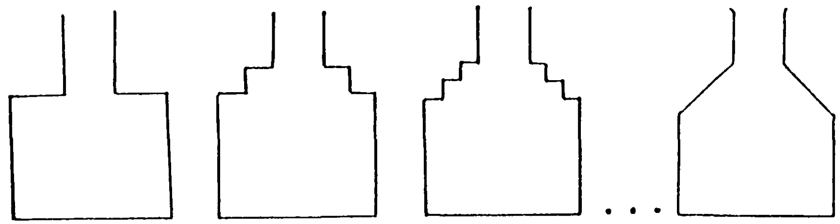


Which of these circuits was it going round?



Discuss this problem with your neighbours.
Write down your reasons each time you reject a circuit.

* Draw sketch graphs for the following sequence of bottles.



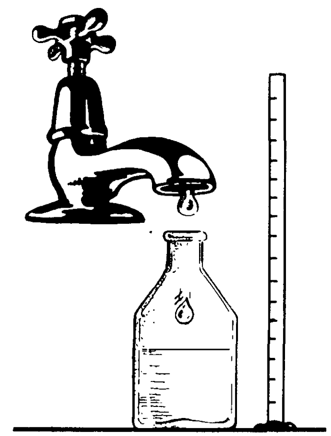
* Using your sketches explain why a bottle with straight sloping sides does not give a straight line graph (ie: explain why the ink bottle does *not* correspond to graph g).

* Invent your own bottles and sketch their graphs on a separate sheet of paper.
  Pass only *the graphs* to your neighbour.
  Can he reconstruct the shape of the original bottles using only your graphs?
  If not, try to discover what errors are being made.

* Is it possible to draw two different bottles which give the same height-volume graph?
  Try to draw some examples.

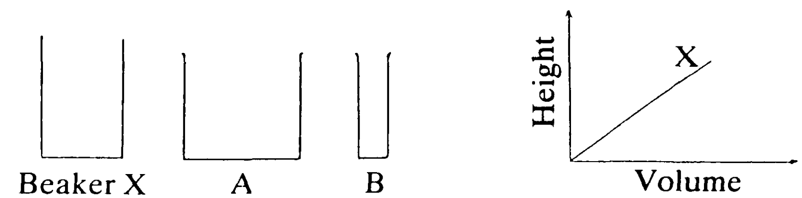
A5 LOOKING AT GRADIENTS

Filling Bottles

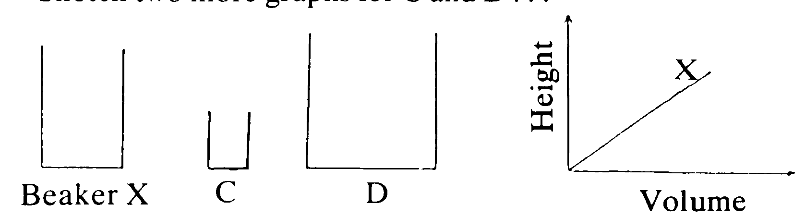


In order to callibrate a bottle so that it may be used to measure liquids, it is necessary to know how the height of the liquid depends upon the volume in the bottle.

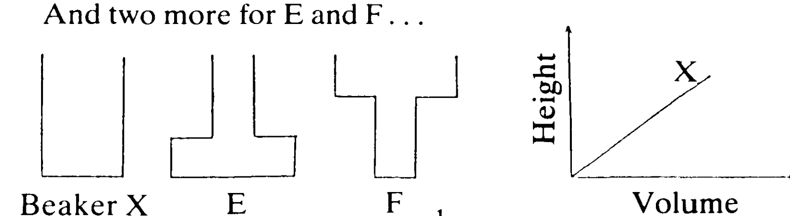
The graph below shows how the height of liquid in beaker X varies as water is steadily dripped into it. Copy the graph, and *on the same diagram*, show the height-volume relationship for beakers A and B.



Sketch two more graphs for C and D . . .



And two more for E and F . . .

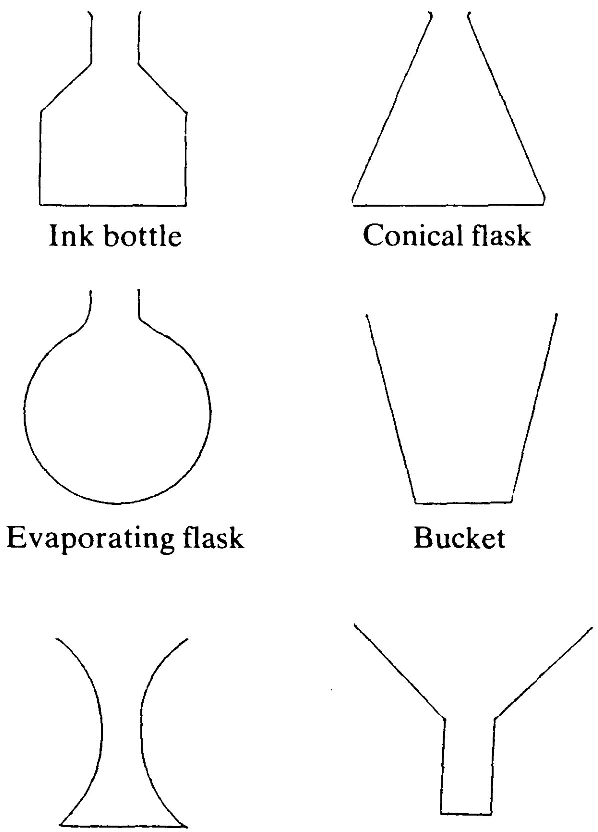


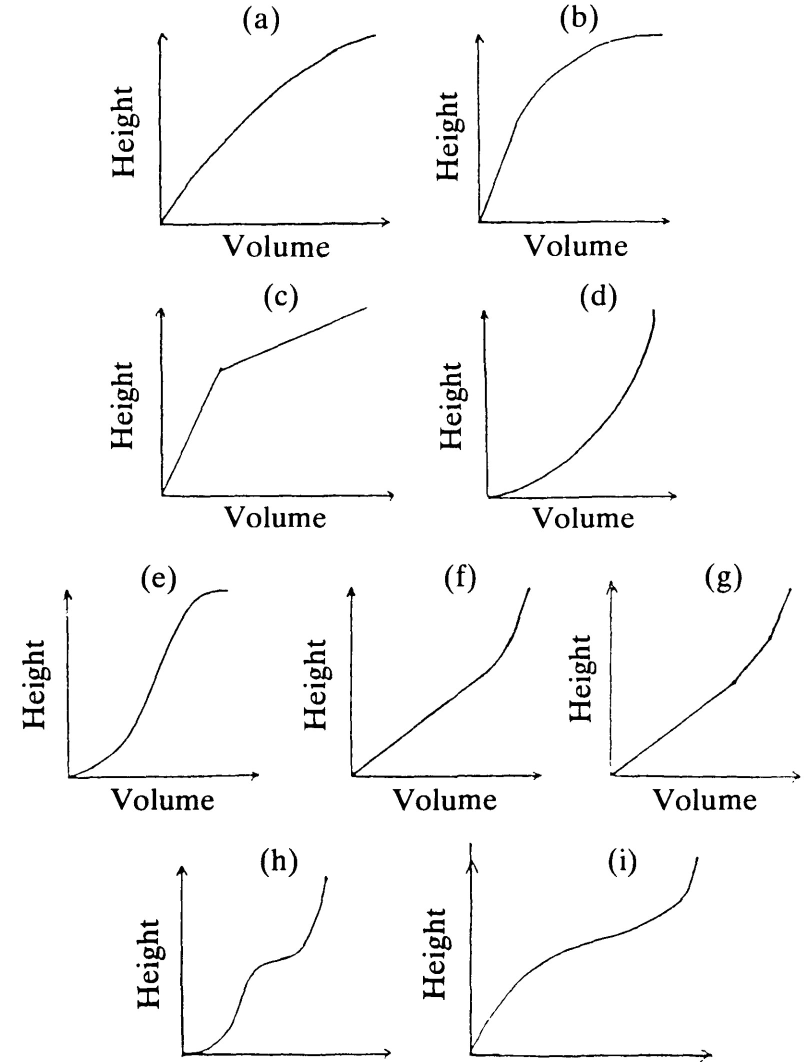
Here are 6 bottles and 9 graphs.

Choose the correct graph for each bottle.

Explain your reasoning clearly.

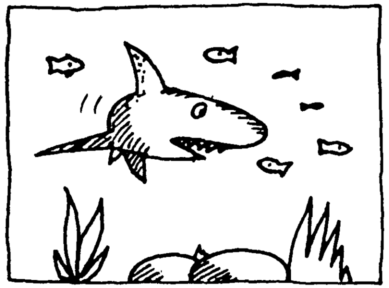
For the remaining 3 graphs, sketch what the bottles should look like.



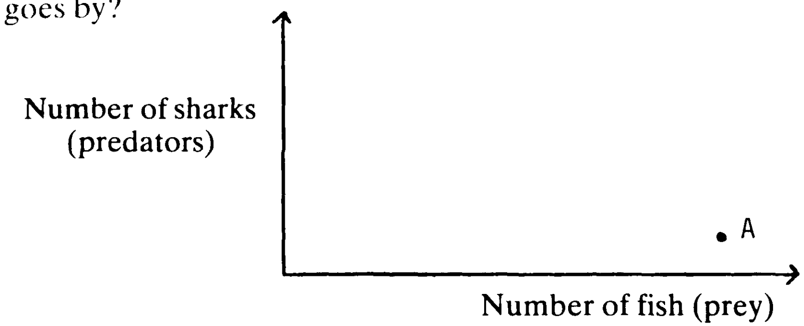


4. Sharks and Fish

Below is a simplified description of what can happen when two species interact. The sharks are the predators and the fish are the prey. The situation in statement A has been represented on the graph by a point.



How does this point move as time goes by?



(A) Due to the absence of many sharks, there is an abundant supply of fish in the area . . .

(B) Sensing a plentiful supply of fish for food, sharks enter the area.

(C) The sharks eat many of the fish until . . .

(D) . . .the fish population is insufficient to support all the sharks. Many sharks therefore decide to leave.

(E) With few sharks around, the fish population increases once again.

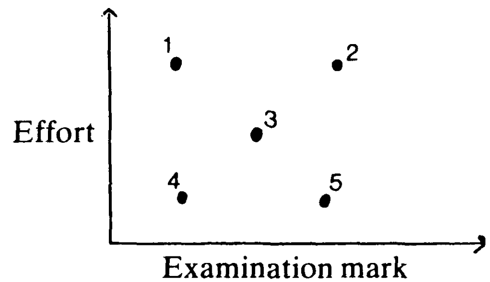
(F) The area now contains enough food to support more sharks, so they return . . .

(G) and begin to eat the fish . . .until . . .

INTERPRETING POINTS

1. School Reports

Each school report is represented by one of the points in the graph below. Label four points with the names Alex, Suzy, Catherine and David. Make up a school report for the remaining point.

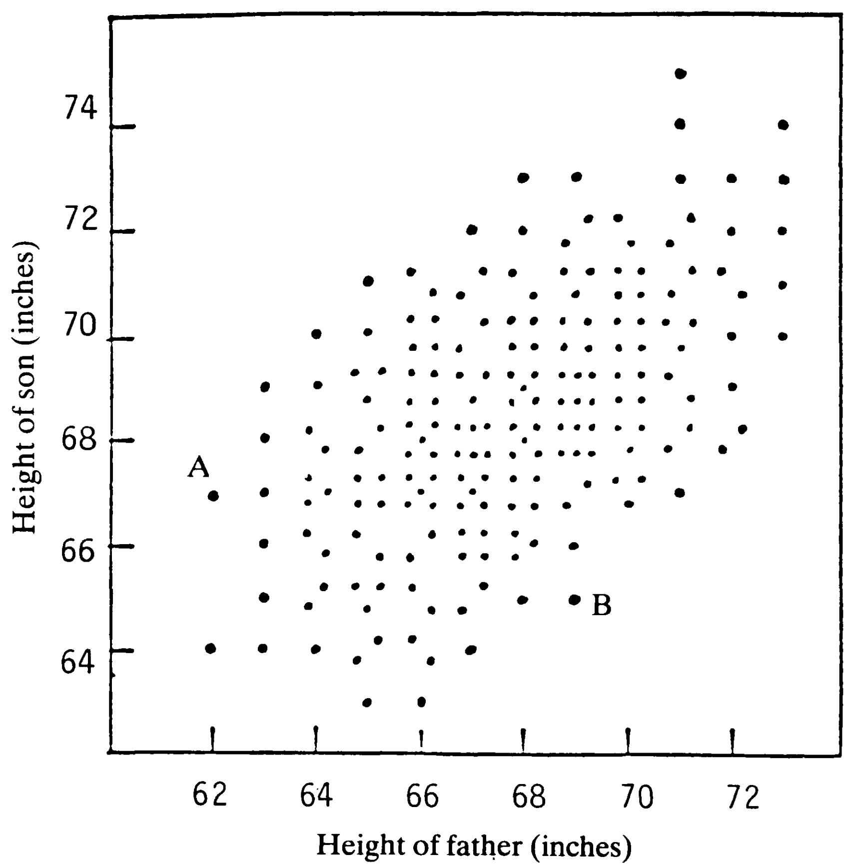


2. Is Height Hereditary?

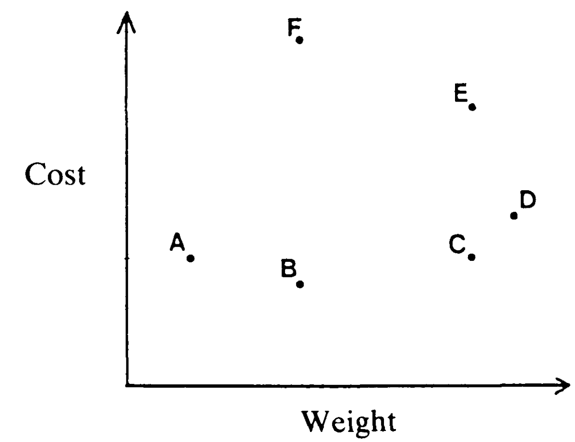
In an experiment, 192 fathers and sons were measured.
(The sons were measured when they had attained their full adult height.)

* What can you say about points A and B?

* What conclusions can be drawn from this graph?



3. Bags of Sugar



Each point on this graph represents a bag of sugar.

(a) Which bag is the heaviest?
(b) Which bag is the cheapest?
(c) Which bags are the same weight?
(d) Which bags are the same price?
(e) Which of F or C would give better value for money? How can you tell?
(f) Which of B or C would give better value for money? How can you tell?
(g) Which two bags would give the same value for money? How can you tell?

Sketch graphs to illustrate the following situations.
You have to decide on the variables and the relationships involved. Label your axes carefully, and explain your graphs in words underneath.

How does . . .

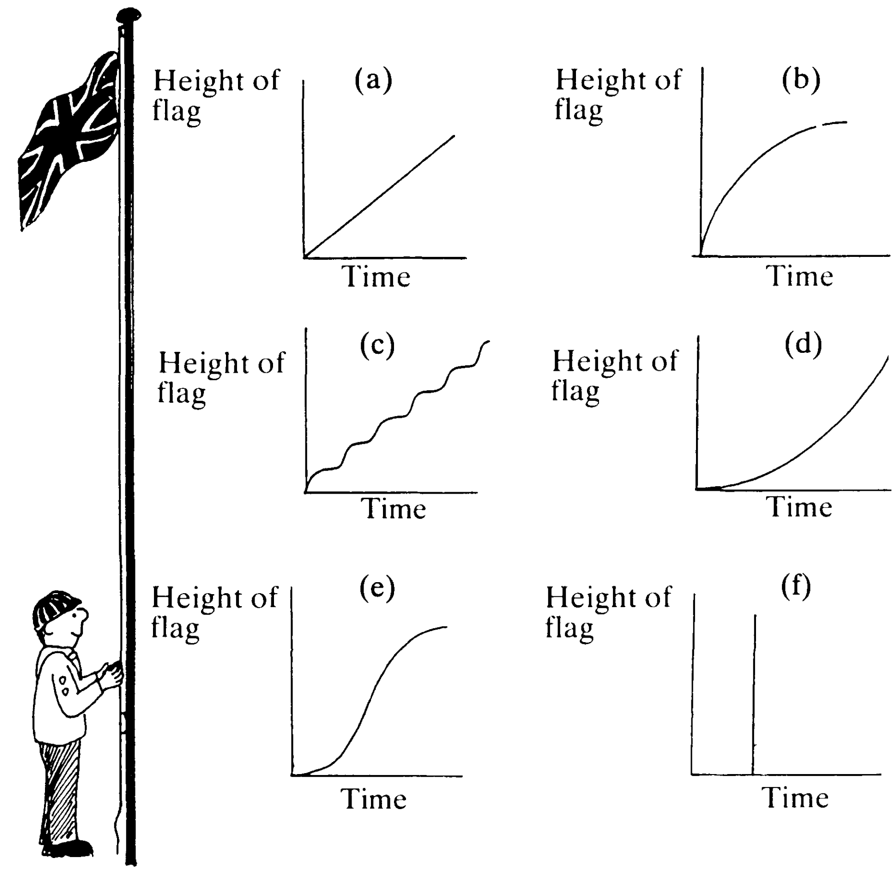
1 Your height vary with age?
2 The amount of dough needed to make a pizza depend upon its diameter?
3 The amount of daylight we get depend upon the time of year?
4 The number of people in a supermarket vary during a typical Saturday?
5 The speed of a pole-vaulter vary during a typical jump?
6 The water level in your bathtub vary, before, during and after you take a bath?

SKETCHING GRAPHS FROM WORDS

Hoisting the flag

Every morning, on the summer camp, the youngest boy scout has to hoist a flag to the top of the flagpole.

(i) Explain in words what each of the graphs below would mean.
(ii) Which graph shows this situation most realistically? Explain.
(iii) Which graph is the least realistic? Explain.



Choose the best graph to describe each of the situations listed below. Copy the graph and label the axes clearly with the variables shown in brackets. If you cannot find the graph you want, then draw your own version and explain it fully.

1) The weightlifter held the bar over his head for a few unsteady seconds, and then with a violent crash he dropped it. (height of bar/time)

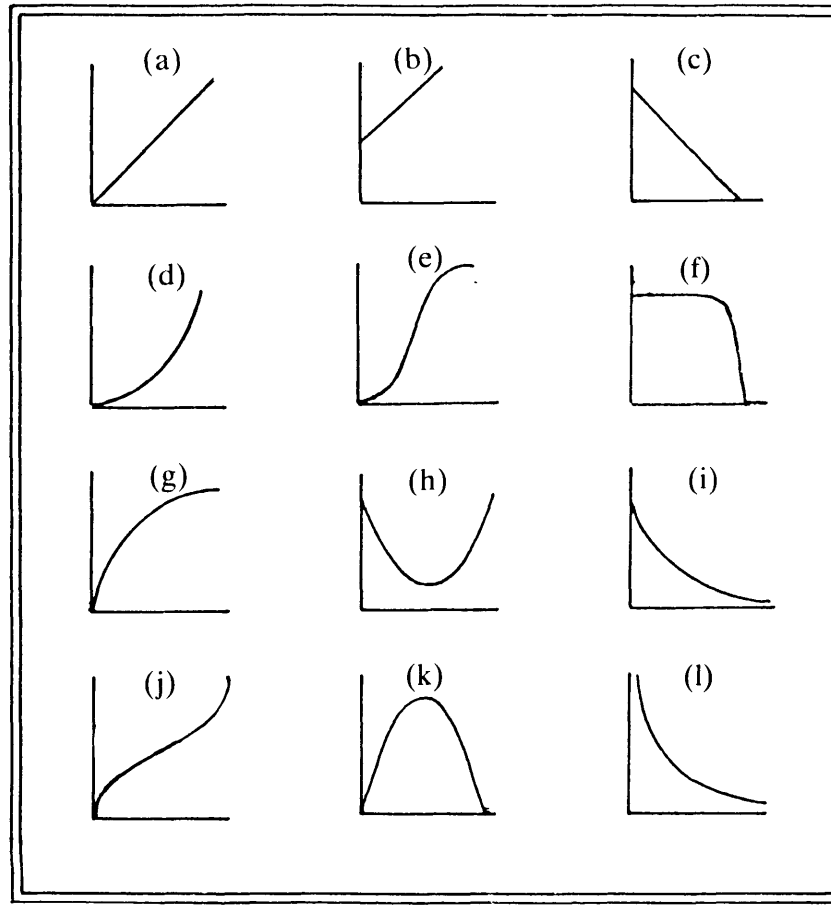
2) When I started to learn the guitar, I initially made very rapid progress. But I have found that the better you get, the more difficult it is to improve still further. (proficiency/amount of practice)

3) If schoolwork is too easy, you don't learn anything from doing it. On the other hand, if it is so difficult that you cannot understand it, again you don't learn. That is why it is so important to pitch work at the right level of difficulty. (educational value/difficulty of work)

4) When jogging, I try to start off slowly, build up to a comfortable speed and then slow down gradually as I near the end of a session. (distance/time)

5) "In general, larger animals live longer than smaller animals and their hearts beat slower. With twenty-five million heartbeats per life as a rule of thumb, we find that the rat lives for only three years, the rabbit seven and the elephant and whale even longer. As respiration is coupled with heartbeat—usually one breath is taken every four heartbeats—the rate of breathing also decreases with increasing size. (heart rate/life span)

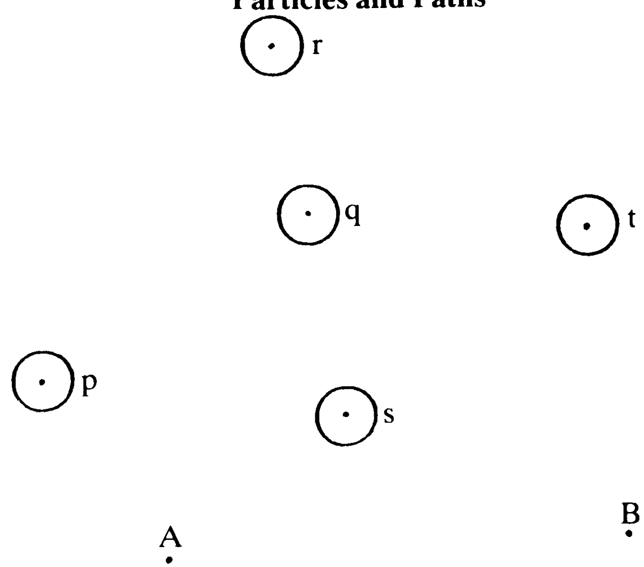
6) As for 5, except the variables are (heart rate/breathing rate)



Now make up three stories of your own to accompany three of the remaining graphs. Pass your stories to your neighbour. Can they choose the correct graphs to go with the stories?

SKETCHING GRAPHS FROM PICTURES

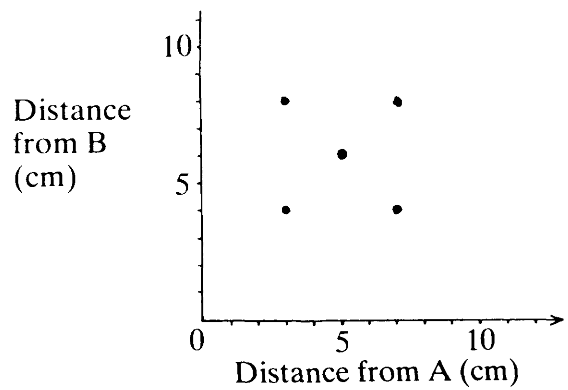
Particles and Paths



In the diagram above, there are 5 particles labelled p, q, r, s and t.

* Without measuring, can you label each point on the graph below with the correct letter?

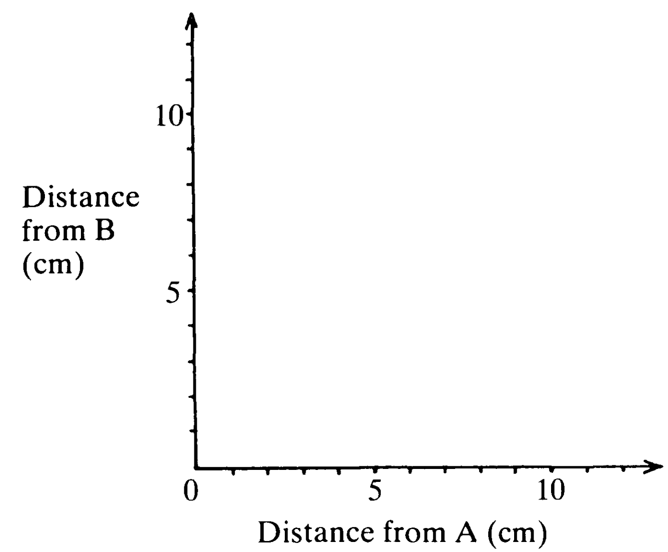
  Now *check* your answer by measurement (A and B are 6 cm apart)



In the accompanying booklet, particles are moving along a number of different paths.

For each situation:

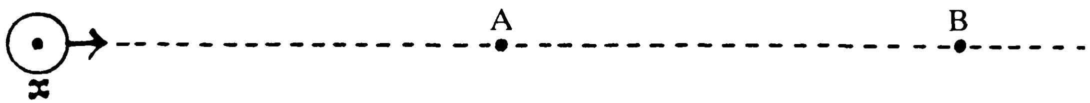
* Sketch a rough graph to show how the distance from B will vary with the distance from A.



* *Check* your answer by measuring various positions, recording your answers in a table and by *plotting* a few points accurately.
* Try to find a formula which describes the relationship between the two distances.

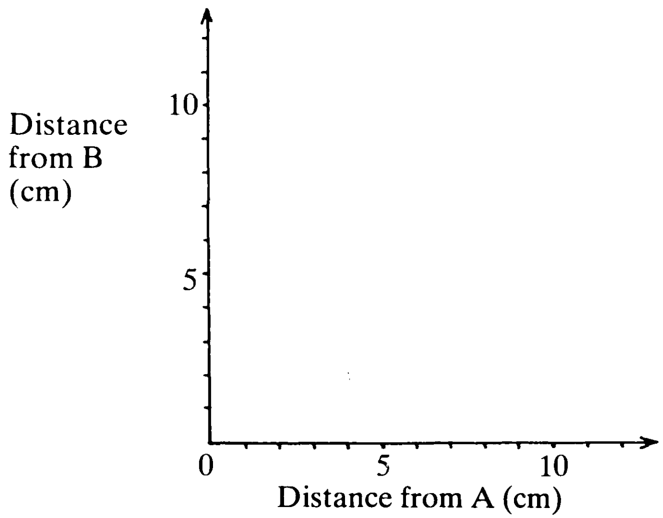
Continue exploring other paths and their graphs.

Write up all your findings.



In this diagram, particle x is moving slowly along the path shown by the dotted line, from left to right.

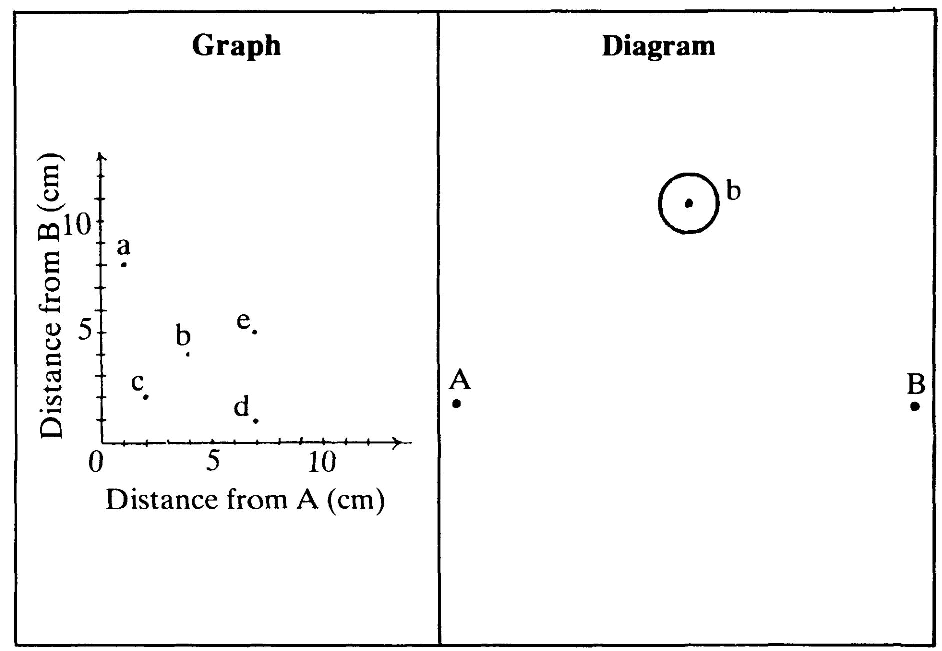
* Sketch a graph to show how the distance from B relates to the distance from A during this motion.



* Check your answer by measuring various positions and recording them in the table:

Distance from A (cm)	6	5	4	3	2	1	0	1	2
Distance from B (cm)	12								

Write down any formulae that you can find which fit your graph.



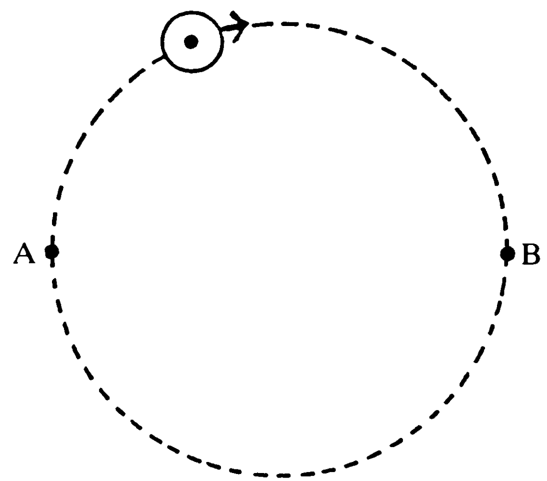
Try to mark the positions of the five particles a, b, c, d and e on the right hand diagram (b has been done for you).

* Which positions are impossible to mark? Why is this? Try to mark other points on the graph which would give impossible positions on the diagram. Shade in these forbidden regions on the graph.

* One position of particle b has been shown. Is this the only position which is 4 cm from both A and B? Mark in any other possible positions for particle b.

* Which points on the graph give *only one* possible position on the diagram?

(vi)



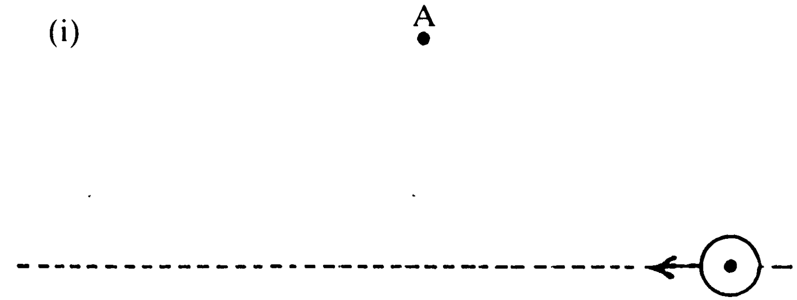
Now continue exploring other paths and their graphs.

Write up all your findings.

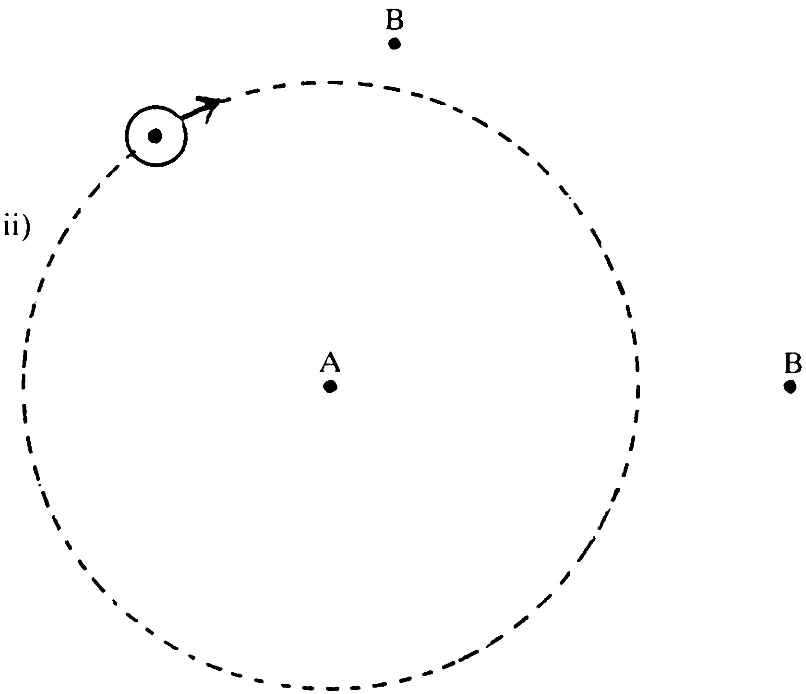
SKETCHING GRAPHS FROM PICTURES (contd)

Particles and Paths

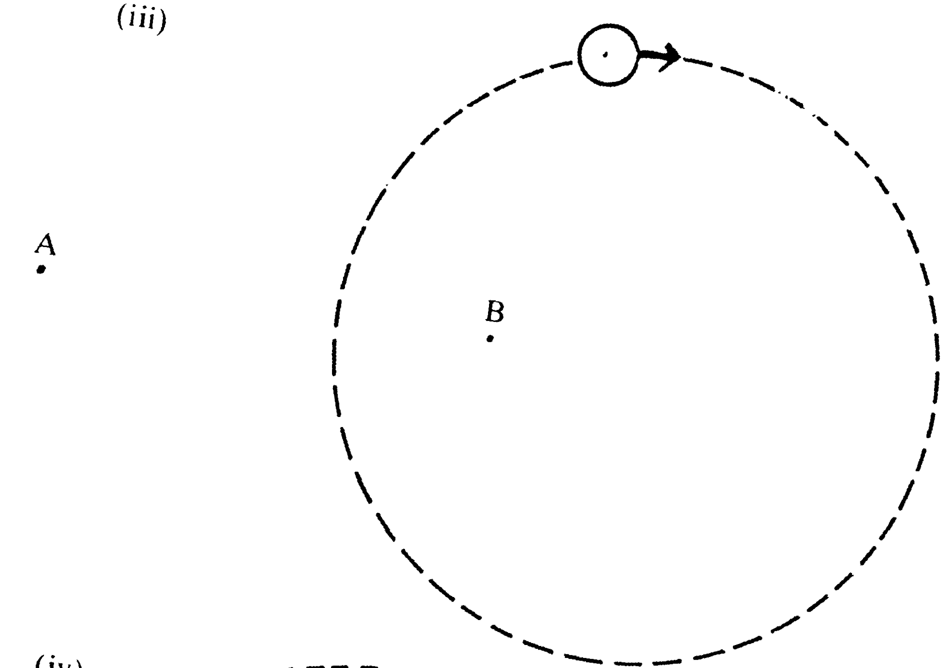
(i)



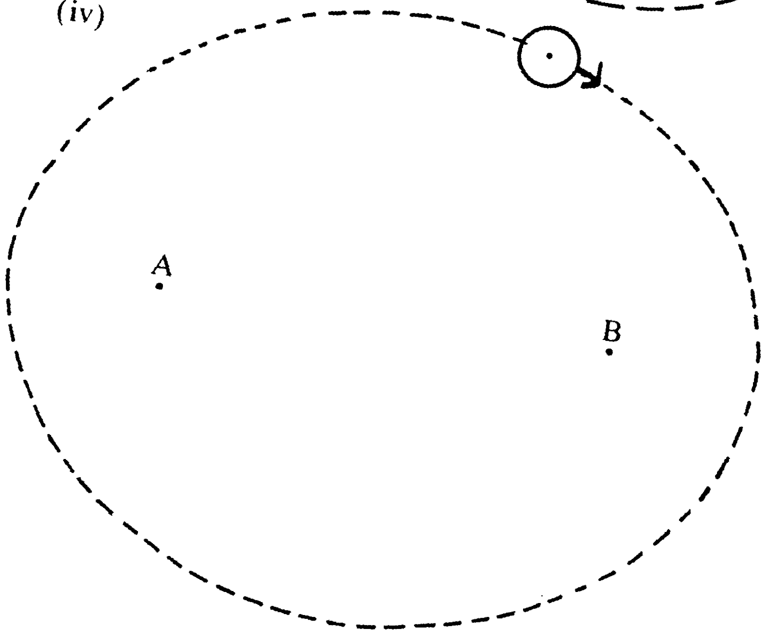
(ii)



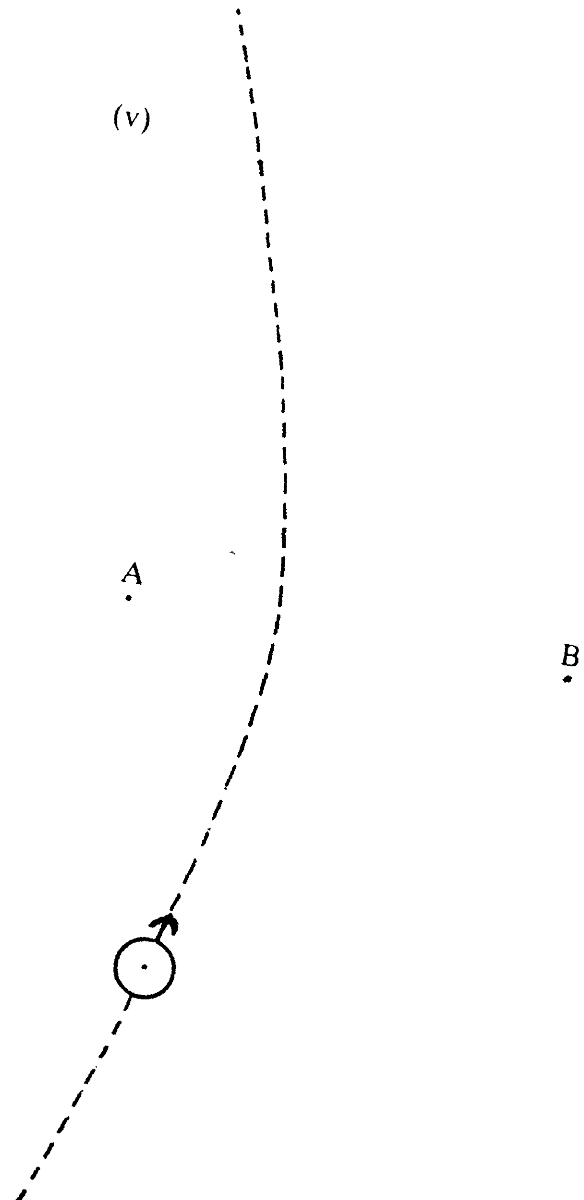
(iii)



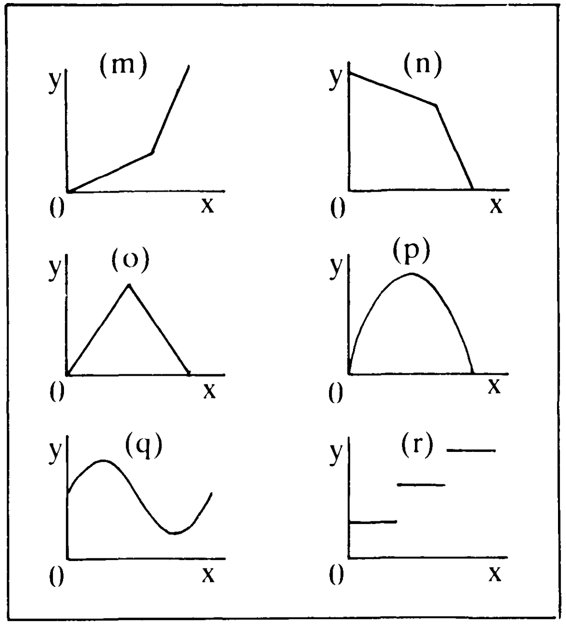
(iv)



(v)



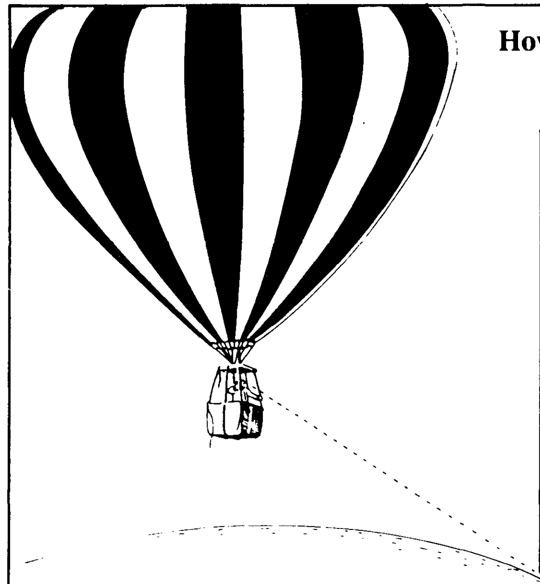
Try to *make up* tables of numbers which will correspond to the following six graphs: (They do not need to represent real situations).



Now make up some tables of your own, and sketch the corresponding graphs on a separate sheet of paper. (Again they do not need to represent real situations). Pass only *the tables* to your neighbour.
She must now try to sketch graphs from your tables. Compare her solutions with yours.

B1 SKETCHING GRAPHS FROM TABLES

In this booklet, you will be asked to explore several tables of data, and attempt to discover any patterns or trends that they contain.

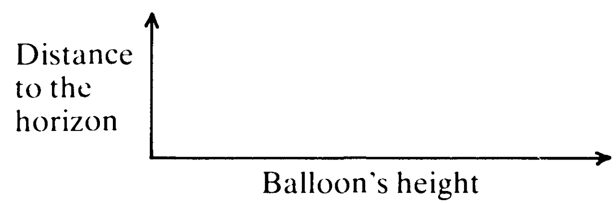


How far can you see?

Balloon's height (m)	Distance to the horizon (km)
5	8
10	11
20	16
30	20
40	23
50	25
100	36
500	80
1000	112

Look carefully at the table shown above.

* Without accurately plotting the points, try to sketch a rough graph to describe the relationship between the balloon's height, and the distance to the horizon.



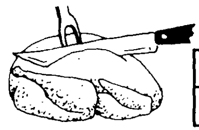
Explain your method for doing this.

Without plotting, choose the best sketch graph (from the selection on page 3) to fit each of the tables shown below. Particular graphs may fit more than one table. Copy the most suitable graph, name the axes clearly, and explain your choice. If you cannot find the graph you want, draw your own version.

1. Cooling Coffee

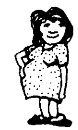
Time (minutes)	0	5	10	15	20	25	30
Temperature (C°)	90	79	70	62	55	49	44

2. Cooking Times for Turkey



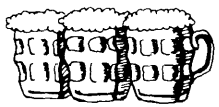
Weight (lb)	6	8	10	12	14	16	18	20
Time (hours)	2½	3	3½	4	4½	5	5½	6

3. How a Baby Grew Before Birth



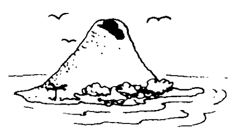
Age (months)	2	3	4	5	6	7	8	9
Length (cm)	4	9	16	24	30	34	38	42

4. After Three Pints of Beer . . .



Time (hours)	1	2	3	4	5	6	7
Alcohol in the blood (mg/100ml)	90	75	60	45	30	15	0

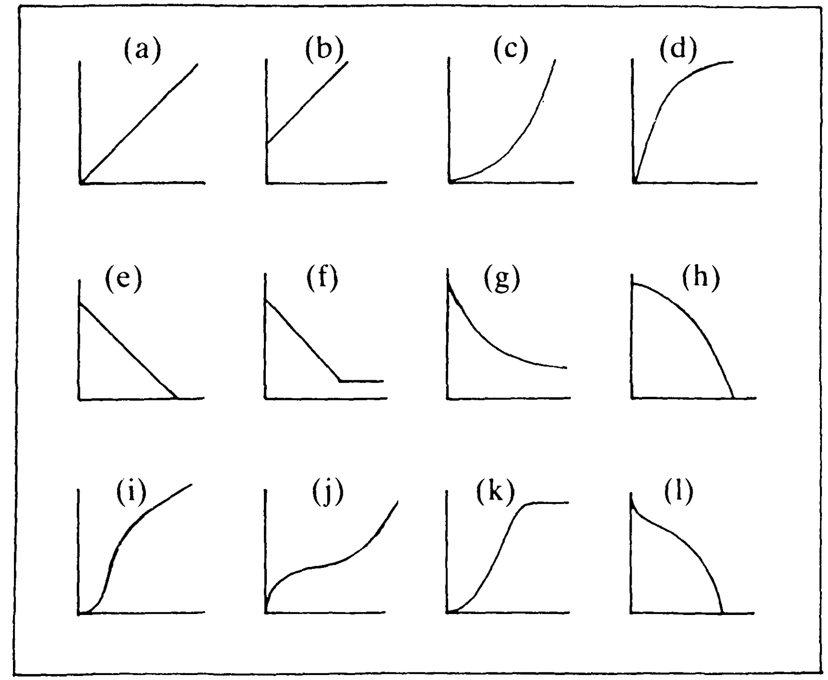
5. Number of Bird Species on a Volcanic Island



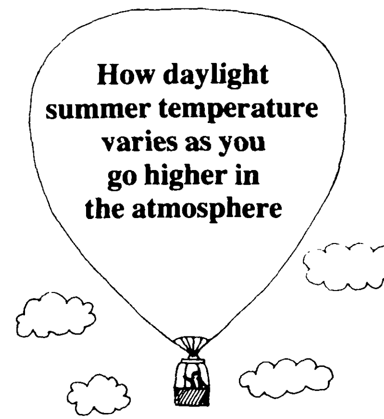
Year	1880	1890	1900	1910	1920	1930	1940
Number of Species	0	1	5	17	30	30	30

6. Life Expectancy

Age (years)	Number of Survivors	Age (years)	Number of Survivors
0	1000	50	913
5	979	60	808
10	978	70	579
20	972	80	248
30	963	90	32
40	950	100	1



Without plotting, try and sketch a graph to illustrate the following table:



Altitude (km)	Temperature (°C)	Altitude (km)	Temperature (°C)
0	20	60	−12
10	−48	70	−56
20	−50	80	−80
30	−38	90	−90
40	−18	100	−75
50	6	110	−20

B1. (contd) SOME HINTS ON SKETCHING GRAPHS FROM TABLES

Look again at the balloon problem, "How far can you see?"

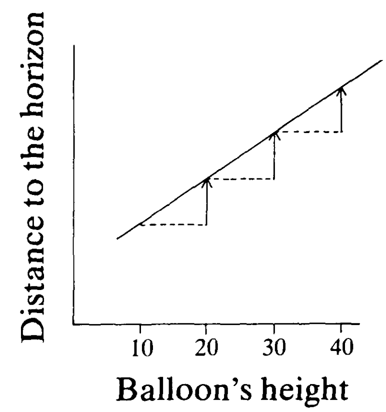
The following discussion should help you to see how you can go about sketching quick graphs from tables without having to spend a long time plotting points.

* As the balloon's height increases by *equal* amounts, what happens to the 'distance to the horizon'? Does it increase or decrease?

Balloon's height (m)	5	10	20	30	40	50	100	500	1000
Distance to horizon (km)	8	11	16	20	23	25	36	80	112

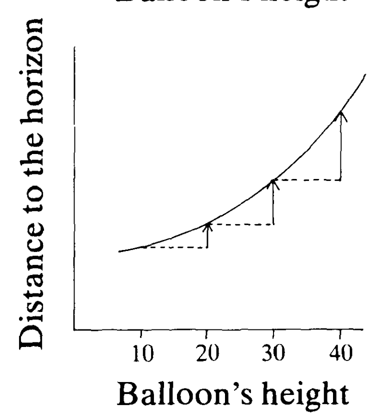
Does this distance increase by equal amounts?...

⇨



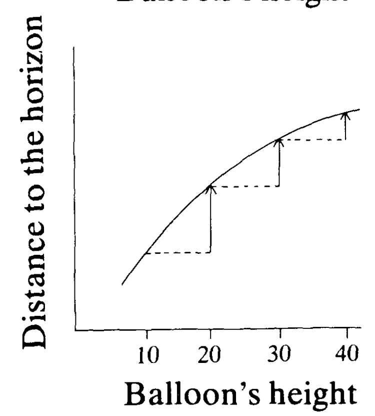
...or increase by greater and greater amounts?...

⇨



...or increase by smaller and smaller amounts?

⇨

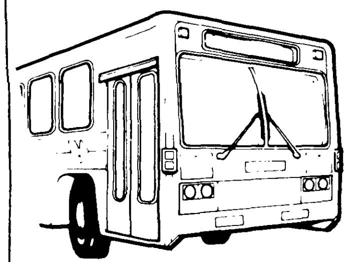


Now ask yourself:
- do the other numbers in the table fit in with this overall trend?
- will the graph cross the axes? If so, where?

For each of the two situations which follow,

(i) Describe your answer by *sketching* a rough graph.
(ii) Explain the shape of your graph in words.
(iii) Check your graph by constructing a table of values, and redraw it if necessary.
(iv) Try to find an algebraic formula.

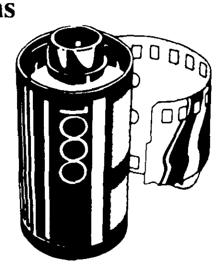
The Outing



A coach hire firm offers to loan a luxury coach for £120 per day. The organiser of the trip decides to charge every member of the party an equal amount for the ride.
How will the size of each person's contribution depend upon the size of the party?

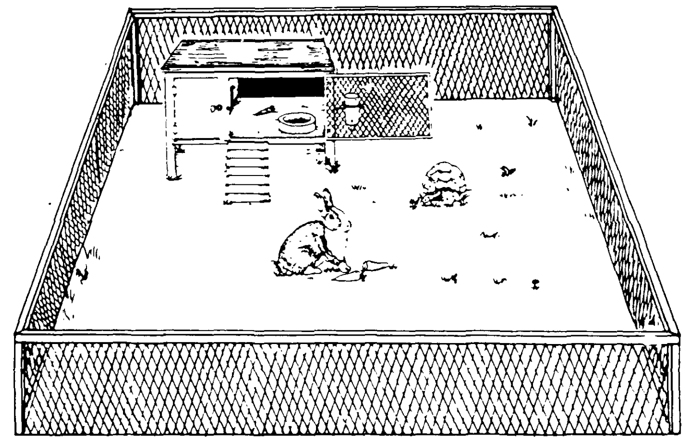
Developing Photographs

"Happy Snaps" photographic service offer to develop your film for £1 (a fixed price for processing) plus 10p for each print. How does the cost of developing a film vary with the number of prints you want developed?



B2 FINDING FUNCTIONS IN SITUATIONS

The Rabbit Run



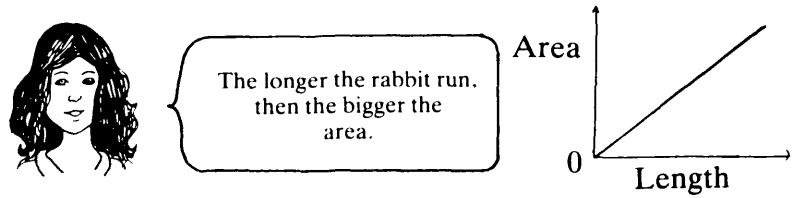
Length

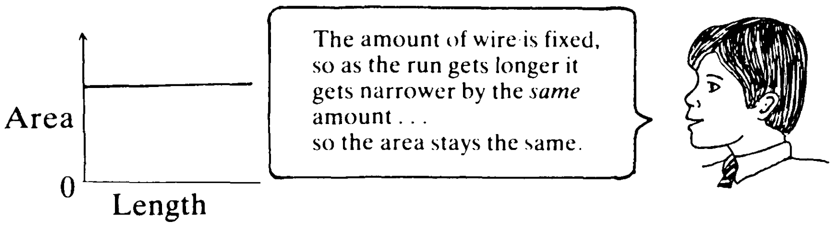
A rectangular rabbit run is to be made from 22 metres of wire fencing. The owner is interested in knowing how the area enclosed by the fence will depend upon the length of the run.

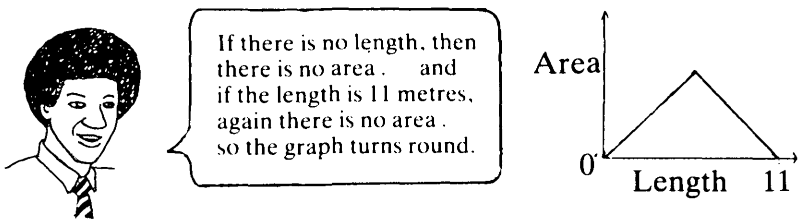
Think carefully about this situation, and discuss it with your neighbour.

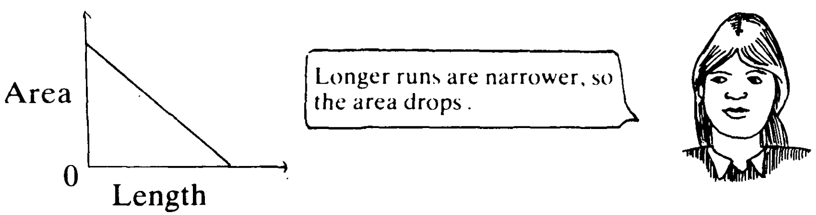
* Describe, in writing, how the enclosed area will change as the length increases through *all* possible values.
* Illustrate your answer using a sketch graph:

The pupils shown below have all attempted this problem. Comment on their answers, and try to explain their mistakes.









* In order to see how good your sketch is, construct a table of values:

Length of run (metres)												
Area (square metres)												

* Do you notice any patterns in this table?
  Write down what they are and try to explain *why* they occur.

* Now, redraw your sketch using the patterns you have observed. (This does not need to be done accurately).

* Using your sketch and your table of values, find out what the dimensions of the boundary should be to obtain the greatest possible space for the rabbit to move around in.

* Finally, try to find an algebraic formula which fits this situation.

* Check your sketch graphs by plotting *a few* points accurately on graph paper. Share this work out with your neighbour so that it doesn't take too long.

* Do just *one* of the two investigations shown below:

> Draw an accurate graph to show how the effect of Triazolam wears off.
>
> After how many hours has the amount of drug in the blood halved?
>
> How does this "Half life" depend on the size of the initial dose?
>
> Write down and explain your findings.

> Investigate the effect of taking a $4\mu g$ dose of Methohexitone every hour.
>
> Draw an accurate graph and write about its implications.

B3 LOOKING AT EXPONENTIAL FUNCTIONS

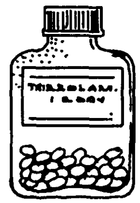
Hypnotic Drugs

Sometimes, doctors prescribe 'hypnotic drugs' (e.g. sleeping pills) to patients who, either through physical pain or emotional tension, find that they cannot sleep. (Others are used as mild sedatives or for anaesthetics during operations). There are many different kinds of drugs which can be prescribed. One important requirement is that the effect of the drug should wear off by the following morning, otherwise the patient will find himself drowsy all through the next day. This could be dangerous if, for example, he has to drive to work! Of course, for someone confined to a hospital bed this wouldn't matter so much.

Imagine that a doctor prescribed a drug called Triazolam. (Halcion®).

After taking some pills, the drug eventually reaches a level* of 4µg/l in the blood plasma.

How quickly will the drug wear off?



Look at the table shown below:

Drug name (and Brand name)	Approximate formula
Triazolam (Halcion®)	$y = A \times (0.84)^x$
Nitrazepam (Mogadon®)	$y = A \times (0.97)^x$
Pentobombitone (Sonitan®)	$y = A \times (1.15)^x$
Methohexitone (Brietal®)	$y = A \times (0.5)^x$

KEY A = size of the initial dose in the blood
y = amount of drug in the blood
x = time in hours after the drug reaches the blood.

For Triazolam, the formula is $y = A \times (0.84)^x$

In our problem the initial dose is 4 µg/l, so this becomes

$$y = 4 \times (0.84)^x$$

---

* Please note that in this worksheet, doses and blood concentrations are not the same as those used in clinical practice, and the formulae may vary considerably owing to physiological differences between patients.

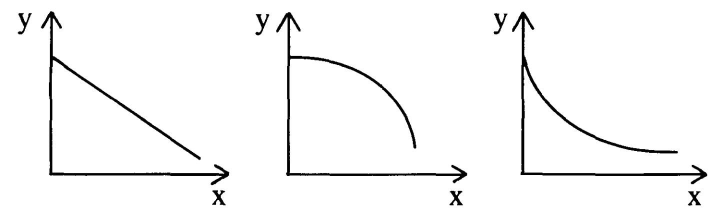
* Continue the table below, using a calculator, to show how the drug wears off during the first 10 hours.

You do not need to plot a graph.

Time (hours)	Amount of drug left in the blood
x	y
0	4
1	3.36 (= 4 × 0.84)
2	2.82 (= 3.36 × 0.84)
.	.
.	.
.	.

* Which of the following graphs best describes your data?

Explain how you can tell *without plotting*



* *On the same pair of axes,* sketch four graphs to compare how a 4µg dose* of each of the drugs will wear off.
(*Guess* the graphs—*do not* draw them accurately)

* Only three of the drugs are real. The other was intended as a joke! Which is it? Explain how you can tell.

What would happen if you took this drug?

At the moment, we have 3 variables; length, breadth, and thickness. If we keep two of these variables *fixed*, then we may be able to discover a relationship between the third variable and the weight the plank will support.

So . . .

* Collect together all the data which relates to a plank with breadth 30 cm and thickness 2 cm, and make a table:

Length of plank (l metres)				
Maximum weight supported (w kg wt)				

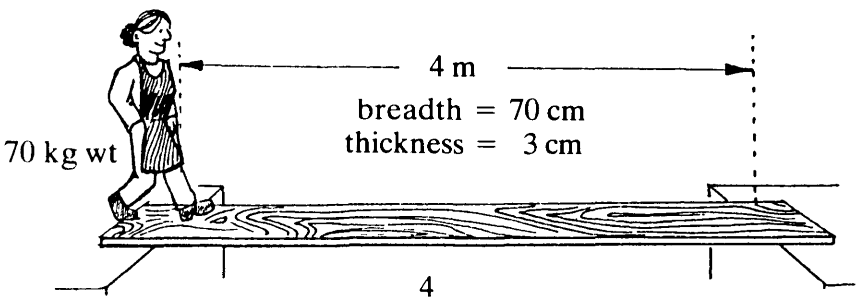
Describe any patterns or rules that you spot. (Can you predict, for example, the value of w when $l = 6$?)
Does your sketch graph agree with this table?
Try to write down a formula to fit this data.

* Now look at all bridges with a fixed length and breadth, and try to find a connection between the thickness and the maximum weight it will support.
Describe what you discover, as before.

* Now look at all planks with a fixed length and thickness.

For geniuses only! Can you combine *all* your results to obtain a formula which can be used to predict the strength of a bridge with *any* dimensions?

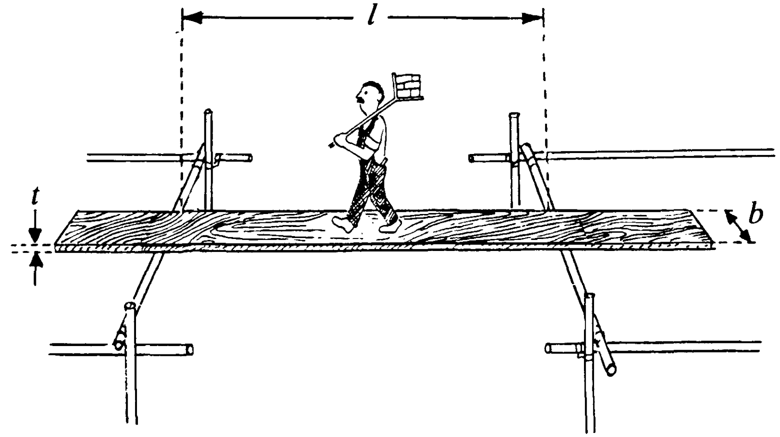
* Finally, what will happen in this situation?



B4 A FUNCTION WITH SEVERAL VARIABLES

In this booklet you will be considering the following problem:

Bridges

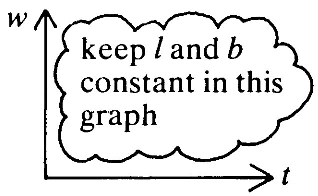


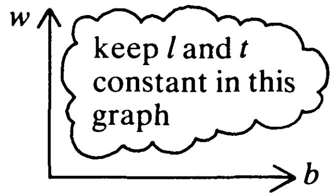
How can you predict whether a plank bridge will collapse under the weight of the person crossing it?

* Imagine the distance between the bridge supports (l) being slowly changed. How will this affect the maximum weight (w) that can safely go across?

Sketch a graph to show how w will vary with l.

* Now imagine that, in turn, the thickness (t) and the breadth (b) of the bridge are changed. Sketch two graphs to show the effect on w.





* Compare your graphs with those drawn by your neighbour. Try to convince her that your graphs are correct. It does not matter too much if you cannot agree at this stage.
* Write down an explanation for the shape of each of your graphs.

The table on the next page shows the maximum weights that can cross bridges with different dimensions. The results are written in order, from the strongest bridge to the weakest.

* Try to discover patterns or rules by which the strength of a bridge can be *predicted* from its dimensions.

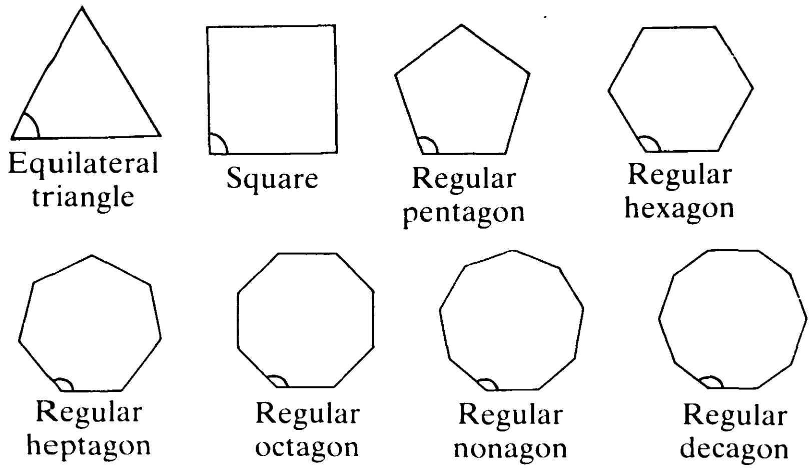
Some Hints: Try reorganising this table, so that l, b and t vary in a systematic way.

Try keeping b and t fixed, and look at how w depends on l . . .

If you are still stuck, then there are more hints on page 4.

Distance between supports l(m)	Breadth b(cm)	Thickness t(cm)	Maximum supportable weight w(kg wt)
2	40	5	250
1	20	5	250
2	50	4	200
2	40	4	160
1	20	4	160
2	20	5	125
2	30	4	120
1	20	3	90
2	20	4	80
1	30	2	60
4	40	3	45
1	20	2	40
2	10	4	40
2	30	2	30
3	30	2	20
3	10	3	15
4	30	2	15
5	30	2	12
1	20	1	10
4	40	1	5

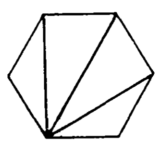
Regular Polygons



How does the size of one of the interior angles depend upon the number of sides of the polygon?

* Describe your answer in words and by means of a rough sketch graph.
* Draw up a table of values, and check your sketch.

  (If you find this difficult, it may help if you first calculate the total sum of *all* the angles inside each polygon by subdividing it into triangles, for example:



sum of angles
$= 4 \times 180°$
$= 720°$
so each angle is . . .)

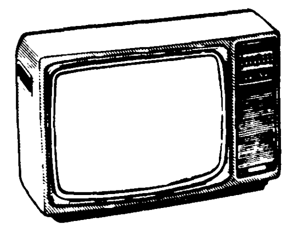
* Explain, in words, how you would calculate the size of an interior angle for a regular n sided polygon.
  Can you write this as a formula?

FINDING FUNCTIONS IN SITUATIONS

For each of the four situations which follow,

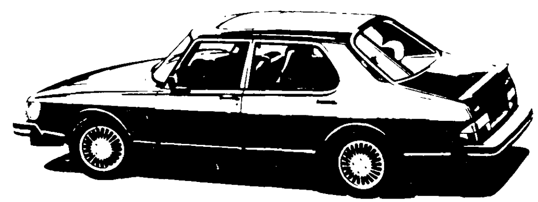
(i) Describe your answer by *sketching* a rough graph.
(ii) Explain the shape of your graph in words.
(iii) Check your graph by constructing a table of values, and redraw it if necessary.
(iv) If you can, try to find an algebraic formula, but do not worry too much if this proves difficult.

1 Renting a Television



A TV rental company charge £10 per month for a colour set. An introductory offer allows you to have the set rent-free for the first month. How will the total cost change as the rental period increases?

2 The Depreciating Car



When it was new, my car cost me £3,000. Its value is depreciating at a rate of 20% per year. This means that after one year its value was

£3,000 × 0.8 = £2,400

and after two years, its value was

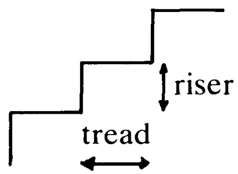
£2,400 × 0.8 = £1,920 and so on.

How does its value continue to change?

The instructions on what to do for these two questions are at the top of page 1.

3 Staircases

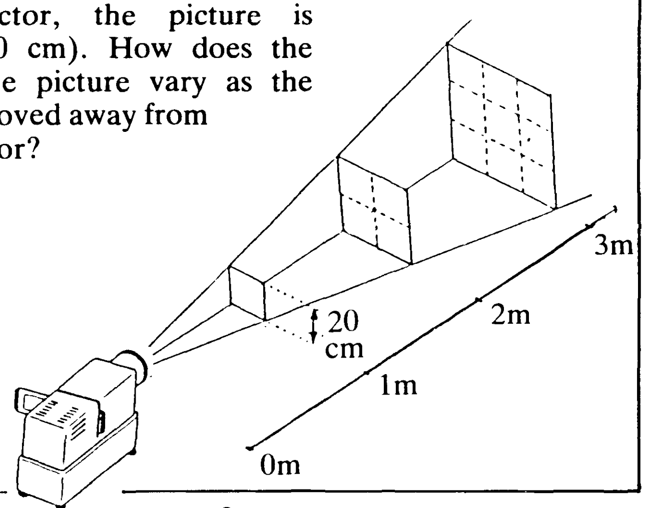
"The normal pace length is 60 cm. This must be decreased by 2 cm for every 1 cm that the foot is raised when climbing stairs."



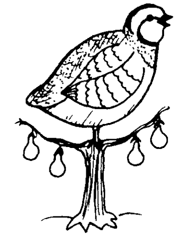
If stairs are designed according to this principal, how should the "tread length" (see diagram) depend upon the height of each "riser"?

4 The Film Show

When a square colour slide is projected onto a screen, the area of the picture depends upon the distance of the projector from the screen as illustrated below. (When the screen is 1 metre from the projector, the picture is 20 cm × 20 cm). How does the *area* of the picture vary as the screen is moved away from the projector?



The Twelve Days of Christmas



"On the first day of Christmas my true love sent to me:
A partridge in a pear tree.
On the second day of Christmas my true love sent to me:
Two turtle doves and a partridge in a pear tree.
On the third . . .

On the *twelfth* day of Christmas my true love sent to me: 12 drummers drumming, 11 pipers piping, 10 lords a-leaping, 9 ladies dancing, 8 maids a-milking, 7 swans a-swimming, 6 geese a-laying, 5 gold rings, 4 calling birds, 3 french hens, 2 turtle doves, and a partridge in a pear tree."

After twelve days, the lady counts up all her gifts.

* How many turtle doves did she receive *altogether*? (No, not two).
* If we call 'a partridge in a pear tree' the first kind of gift, a 'turtle dove' the second kind of gift . . .etc, then how many gifts of the n th kind were received during the twelve days? Draw up a table to show your results.
* Sketch a rough graph to illustrate your data. (You do *not* need to do this accurately).
* Which gift did she receive the most of?
* Try to find a formula to fit your data.

1. Speed conversion chart

Miles per hour	10	20	30	40	50	60	70	80
Kilometres per hour	16.1	32.2	48.3	64.4	[illegible]	96.6	112.7	128.7

2. Radio frequencies and wavelengths

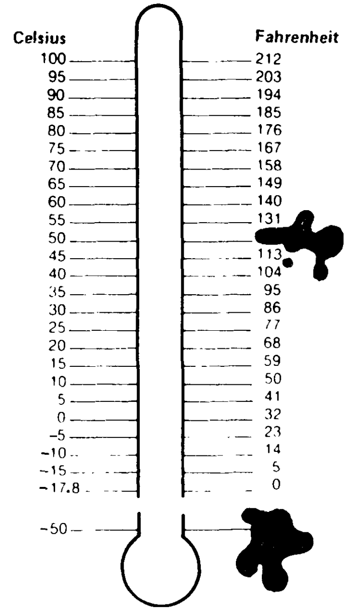
		Radio 4						
Frequency (KHz)	100	200	300	400	500	600	700	800
Wavelength (m)	3000	1500	1000	750	600	500	429	375

		Radio 2		Radio 1			Radio 3
Frequency (KHz)	900	909	1000	1089	1100	1200	1215
Wavelength (m)	333	[illegible]	300	[illegible]	273	250	[illegible]

3. A Pendulum Clock

Length of pendulum (cm)	Time for 100 swings (seconds)
0	0
5	45
10	63
15	77
20	89
25	100
30	110
35	118
40	126
45	134
50	141
60	[illegible]

4. Temperature conversion

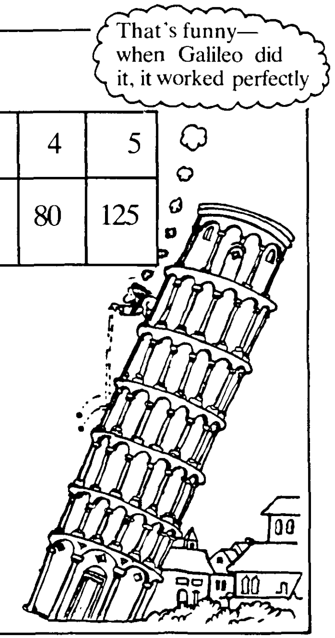


FINDING FUNCTIONS IN TABLES OF DATA

Try the following problem. When you have finished, or when you get stuck, *read on.*

Dropping a stone

Time (seconds)	0	1	2	3	4	5
Distance fallen (metres)	0	5	20	45	80	125



* Sketch a rough graph to illustrate this data.
* Can you see any rules or patterns in this table? Describe them in words and, if possible, by formulae.
* A stone is dropped from an aircraft. How far will it fall in 10 seconds?

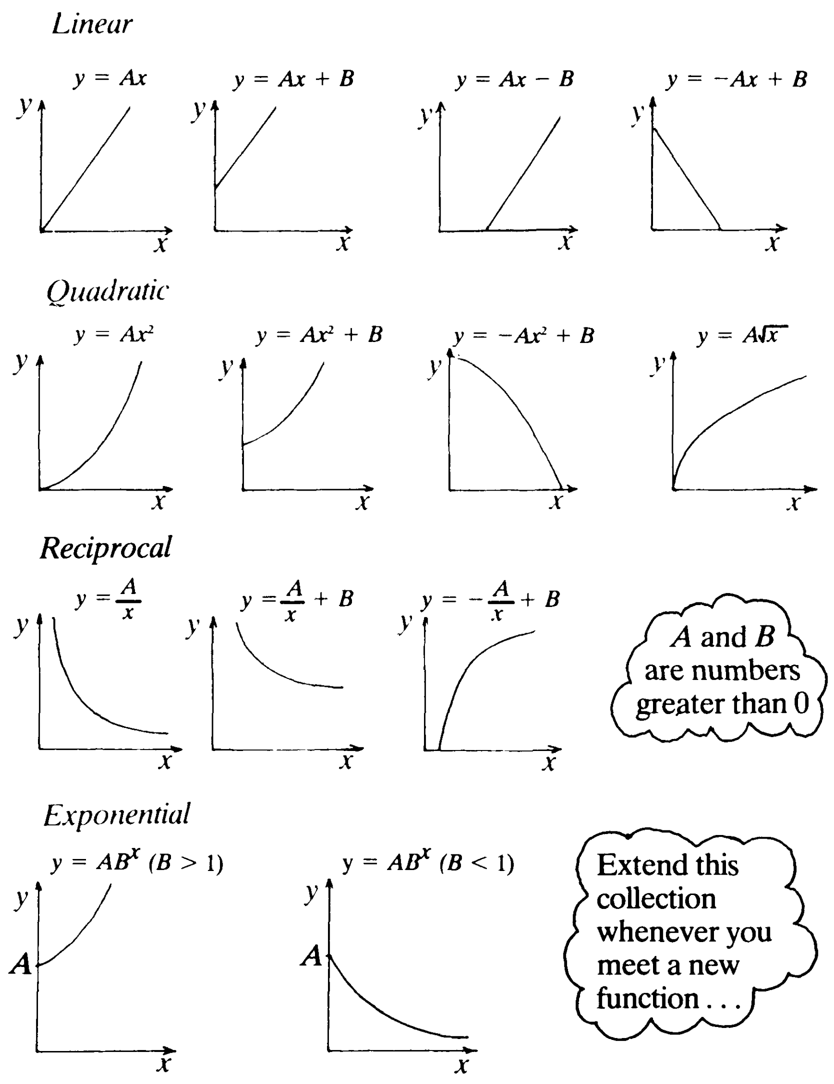
Tables of data often conceal a simple mathematical rule or 'function' which, when known, can be used to predict unknown values.

This function can be very difficult to find, especially if the table contains rounded numbers or experimental errors.

It helps a great deal if you can recognise a function from the shape of its graph. On the next page is a 'rogue's gallery' of some of the most important functions.

* Which graph looks most like your sketch for the 'dropping a stone' problem?

"Rogue's Gallery"



Fitting a formula to the data

By now; you have probably realised that the graph labelled $y = Ax^2$ is the only one which fits the 'dropping a stone' data.

In our case

y = distance fallen (metres)
x = time (seconds)
and A is fixed positive number.

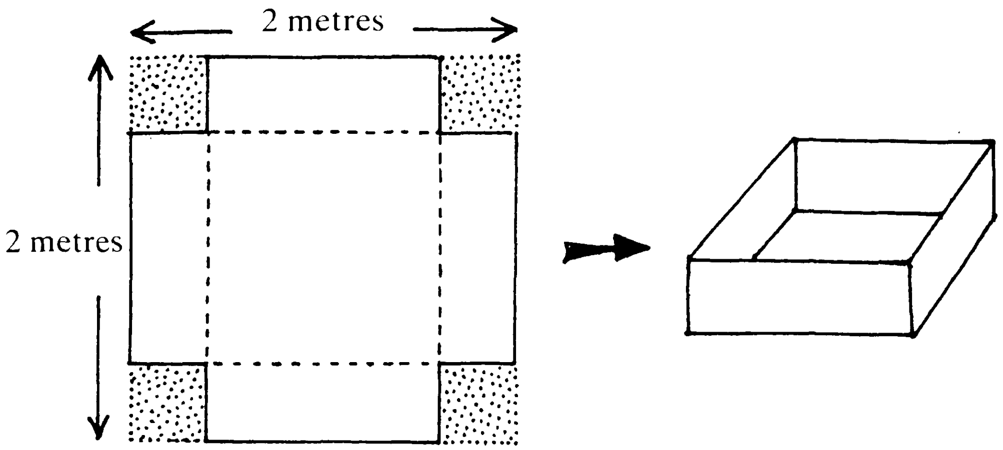
* Try to find the value of A that makes the function fit the data either by trial and error or by substituting for values of x and y and solving the resulting equation.

* Use your resulting formula to find out how far the stone will fall in ten seconds..

Now look at the tables on the next page

* Sketch a ***rough*** graph to illustrate the type of function shown in each table. (You do *not* need to plot points accurately).
* Try to find patterns or rules in the tables and write about them.
* Use the "Rogue's gallery" to try to fit a function to the data in each table.
* Some of the entries in the tables have been hidden by ink blots. Try to find out what these entries should be.

DESIGNING A WATER TANK



A square metal sheet (2 metres by 2 metres) is to be made into an open-topped water tank by cutting squares from the four corners of the sheet, and bending the four remaining rectangular pieces up, to form the sides of the tank. These edges will then be welded together.

* How will the final volume of the tank depend upon the size of the squares cut from the corners?

Describe your answer by:

a) Sketching a rough graph

b) Explaining the shape of your graph in words

c) Trying to find an algebraic formula

* How large should the four corners be cut, so that the resulting volume of the tank is as large as possible?

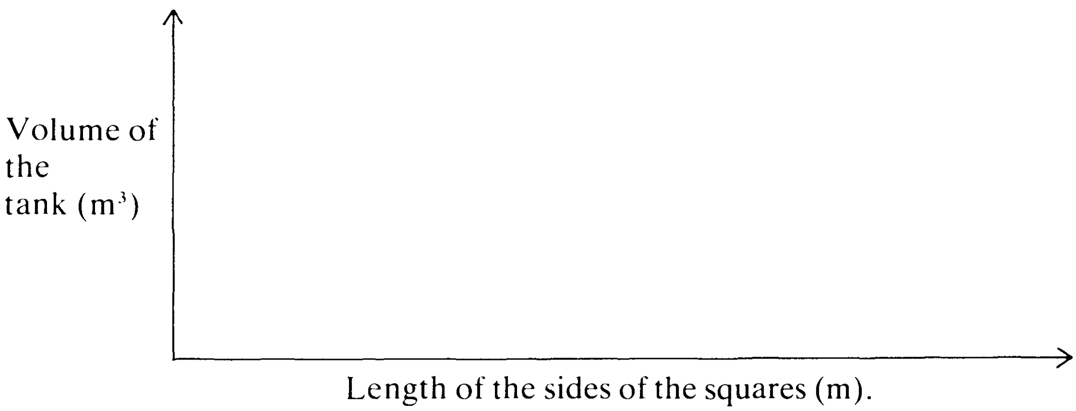
DESIGNING A WATER TANK . . . SOME HINTS

* Imagine cutting very small squares from the corners of the metal sheet. In your mind, fold the sheet up. Will the resulting volume be large or small? Why?

Now imagine cutting out larger and larger squares . . . .

What are the largest squares you can cut? What will the resulting volume be?

* Sketch a rough graph to describe your thoughts and explain it fully in words underneath:



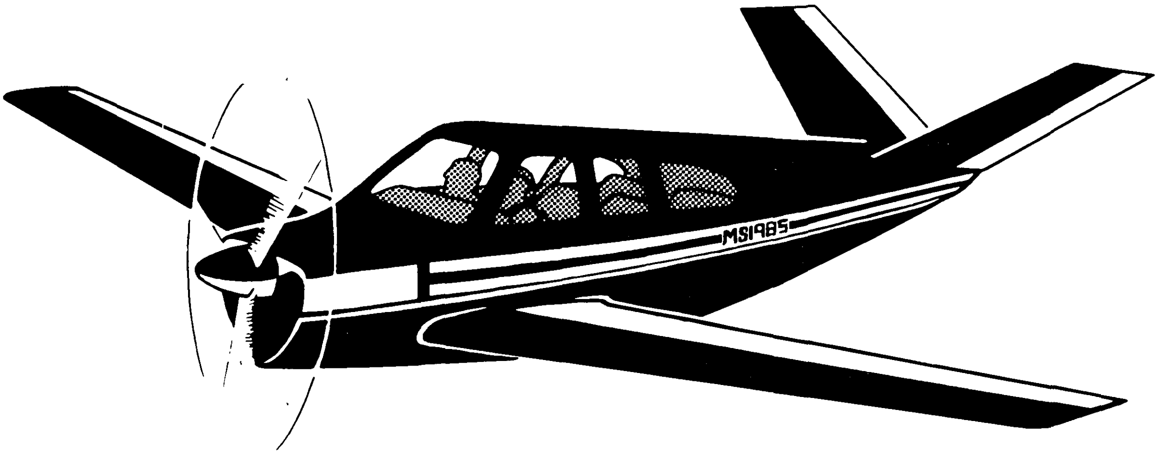
* In order to find a formula, imagine cutting a square x metres by x metres from each corner of the sheet. Find an expression for the resulting volume.

* Now try plotting an accurate graph.
(A suitable scale is 1 cm represents 0.1 metres on the horizontal axis, and 1 cm represents 0.1 cubic metres on the vertical axis).

How good was your sketch?

* *Use your graph* to find out how large the four corner squares should be cut, so that the resulting volume is maximised.

THE POINT OF NO RETURN



Imagine that you are the pilot of the light aircraft in the picture, which is capable of cruising at a steady speed of 300 km/h in still air. You have enough fuel on board to last four hours.

You take off from the airfield and, on the outward journey, are helped along by a 50 km/h wind which increases your cruising speed relative to the ground to 350 km/h.

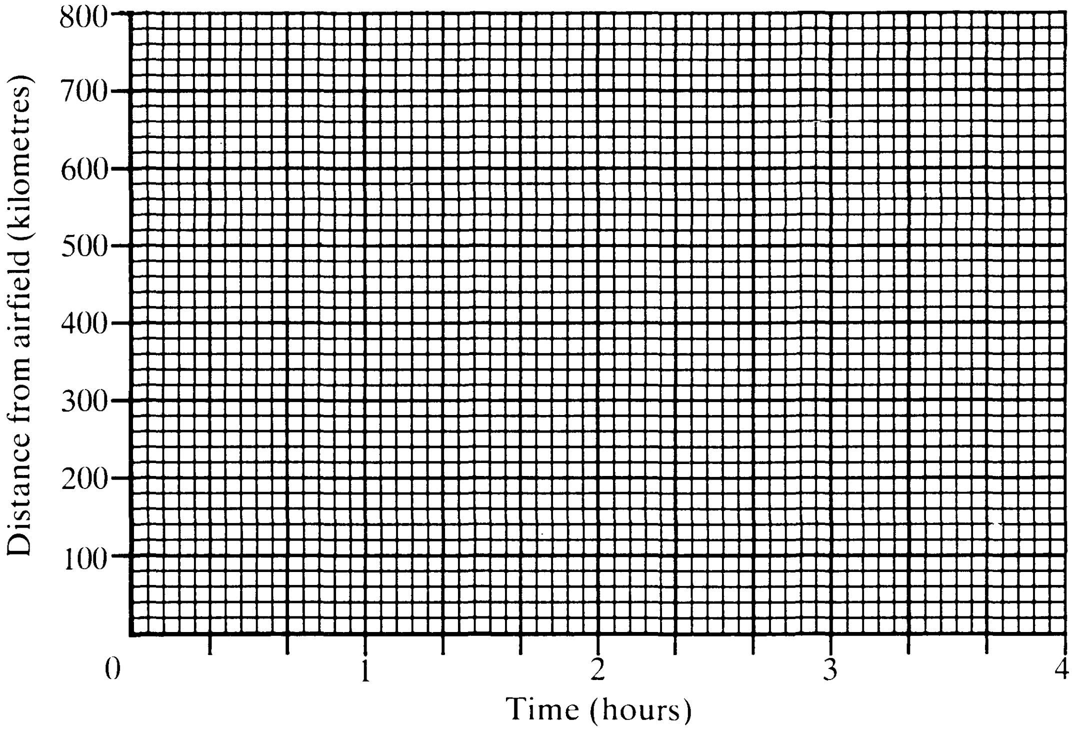
Suddenly you realise that on your return journey you will be flying *into* the wind and will therefore slow down to 250 km/h.

* What is the maximum distance that you can travel from the airfield, and still be sure that you have enough fuel left to make a safe return journey?

* Investigate these 'points of no return' for different wind speeds.

THE POINT OF NO RETURN . . . SOME HINTS

* Draw a graph to show how your distance from the airfield will vary with time.
How can you show an outward speed of 350 km/h?
How can you show a return speed of 250 km/h?



* Use your graph to find the maximum distance you can travel from the airfield, and the time at which you should turn round.

* On the same graph, investigate the 'points of no return' for different wind speeds. What kind of pattern do these points make on the graph paper? Can you explain why?

* Suppose the windspeed is w km/h,
the 'point of no return' is d km from the airfield
and the time at which you should turn round is t hours.

Write down two expressions for the outward speed of the aircraft, one involving w and one involving d and t.

Write down two expressions for the homeward speed of the aircraft, one involving w and one involving d and t.

Try to express d in terms of only t, by eliminating w from the two resulting equations.

Does this explain the pattern made by your 'points of no return'?

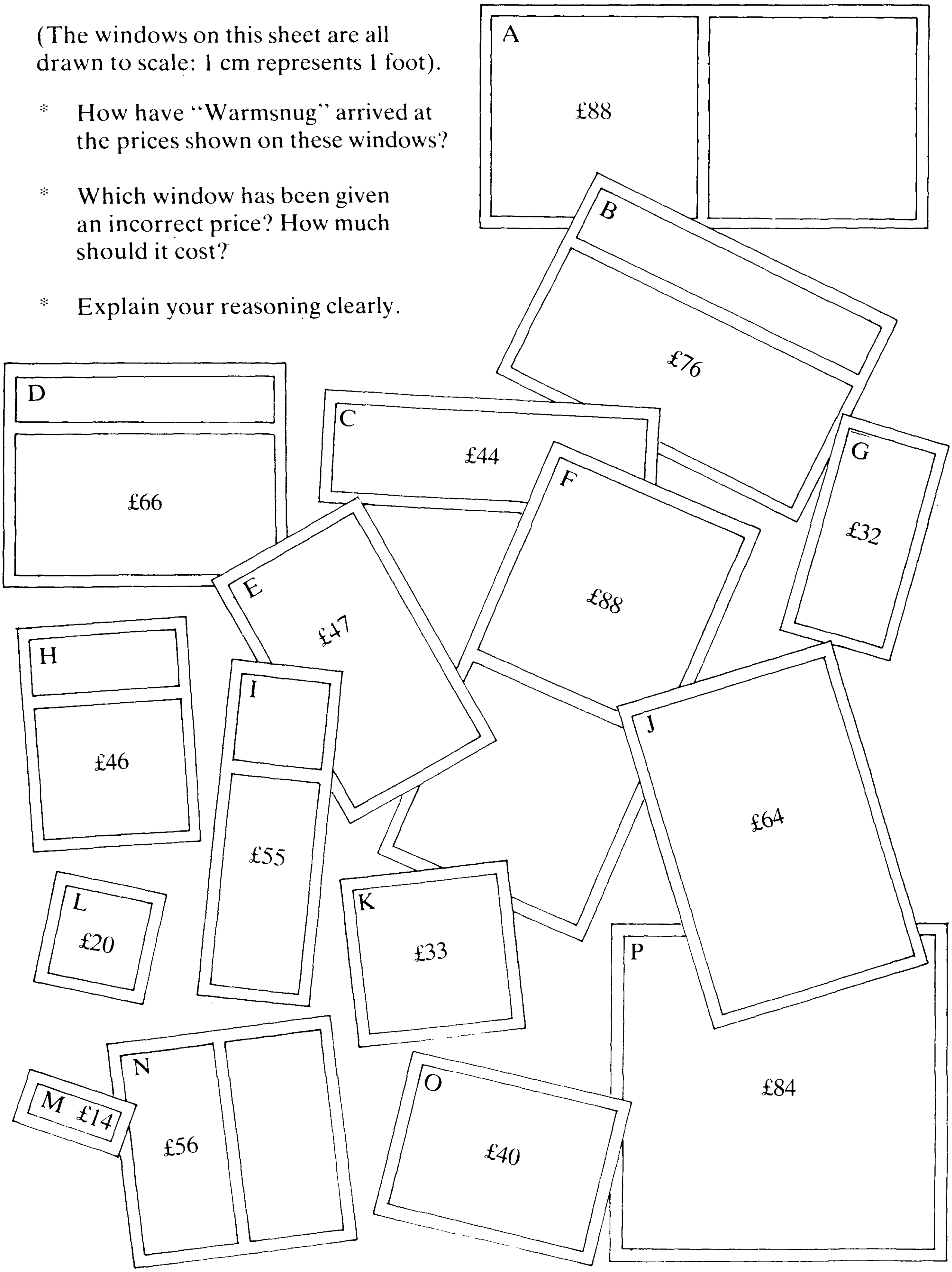
"WARMSNUG DOUBLE GLAZING"

(The windows on this sheet are all drawn to scale: 1 cm represents 1 foot).

* How have "Warmsnug" arrived at the prices shown on these windows?

* Which window has been given an incorrect price? How much should it cost?

* Explain your reasoning clearly.



"WARMSNUG" DOUBLE GLAZING ... SOME HINTS

* Write down a list of factors which *may* affect the price that "Warmsnug" ask for any particular window:

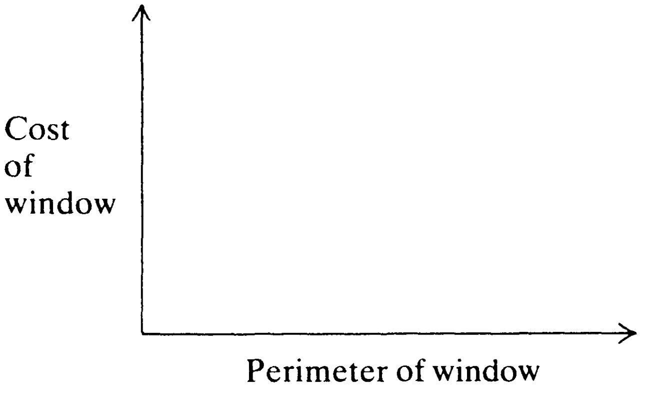
  e.g. Perimeter,
  Area of glass needed,
  ....
  ....

* Using your list, examine the pictures of the windows in a systematic manner.

* Draw up a table, showing all the data which you think may be relevant. (Can you share this work out among other members of your group?)

* Which factors or combinations of factors is the most important in determining the price?

  Draw scattergraphs to test your ideas. For example, if you think that the perimeter is the most important factor, you could draw a graph showing:



* Does your graph confirm your ideas? If not, you may have to look at some other factors.

* Try to find a point which does not follow the general trend on your graph. Has this window been incorrectly priced?

* Try to find a formula which fits your graph, and which can be used to predict the price of *any* window from its dimensions.

PRODUCING A MAGAZINE

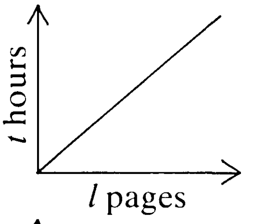
A group of bored, penniless teenagers want to make some money by producing and selling their own home-made magazine. A sympathetic teacher offers to supply duplicating facilities and paper free of charge, at least for the first few issues.

1 a) Make a list of all the important decisions they must make.
Here are three to start you off:

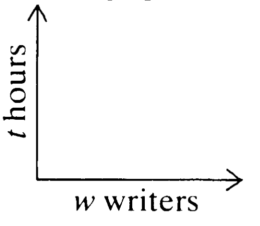
How long should the magazine be?	(l pages)
How many writers will be needed?	(w writers)
How long will it take to write?	(t hours)

b) Many items in your list will depend on other items.
For example,

For a fixed number of people involved, the longer the magazine, the longer it will take to write.



For a fixed length of magazine, the more writers there are, . . .



Complete the statement, and sketch a graph to illustrate it.

Write down other relationships you can find, and sketch graphs in each case.

2 The group eventually decides to find out how many potential customers there are within the school, by producing a sample magazine and conducting a survey of 100 pupils, asking them "Up to how much would you be prepared to pay for this magazine?" Their data is shown below:

Selling price (s pence)	Nothing	10	20	30	40
Number prepared to pay this price (n people)	100	82	58	40	18

How much should they sell the magazine for in order to maximise their profit?

3 After a few issues, the teacher decides that he will have to charge the pupils 10p per magazine for paper and duplicating.

How much should they sell the magazine for now?

PRODUCING A MAGAZINE... SOME HINTS

1 Here is a more complete list of the important factors that must be taken into account:

Who is the magazine for?	(schoolfriends?)
What should it be about?	(news, sport, puzzles, jokes..?)
How long should it be?	(*l* pages)
How many writers will it need?	(*w* writers)
How long will it take to write?	(*t* hours)
How many people will buy it?	(*n* people)
What should we fix the selling price at?	(*s* pence)
How much profit will we make altogether?	(*p* pence)
How much should we spend on advertising?	(*a* pence)

* Can you think of any important factors that are *still* missing?

* Sketch graphs to show how: t depends on w; w depends on l; n depends on s; p depends on s; n depends on a.

* Explain the shape of each of your graphs in words.

2 * Draw a graph of the information given in the table of data.

* Explain the shape of the graph.

* What kind of relationship is this?
(Can you find an approximate formula which relates n to s?)

* From this data, draw up a table of values and a graph to show how the *profit* (p pence) depends on the selling price (s pence).
(Can you find a formula which relates p and s?)

* Use your graph to find the selling price which maximises the profit made.

3 Each magazine costs 10p to produce.

* Suppose we fix the selling price at 20p.

How many people will buy the magazine? How much money will be raised by selling the magazine, (the 'revenue')? How much will these magazines cost to produce? How much actual profit will therefore be made?

* Draw up a table of data which shows how the revenue, production costs and profit all vary with the selling price of the magazine.

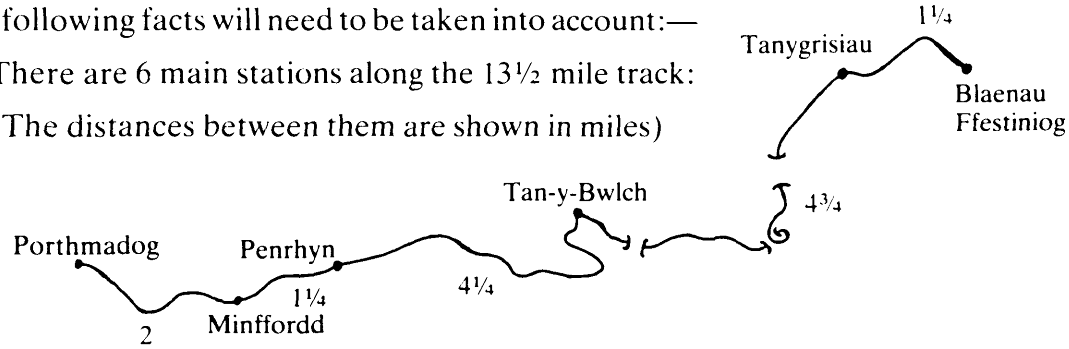
* Draw a graph from your table and use it to decide on the best selling price for the magazine.

THE FFESTINIOG RAILWAY

This railway line is one of the most famous in Wales. Your task will be to devise a workable timetable for running this line during the peak tourist season.

The following facts will need to be taken into account:—

* There are 6 main stations along the 13½ mile track:
(The distances between them are shown in miles)



* Three steam trains are to operate a shuttle service. This means that they will travel back and forth along the line from Porthmadog to Blaenau Ffestiniog with a 10-minute stop at each end. (This should provide enough time for drivers to change etc.)

* The three trains must start and finish each day at Porthmadog.

* The line is single-track. This means that trains cannot pass each other, except at specially designed passing places. (You will need to say where these will be needed. You should try to use as few passing places as possible.)

* Trains should depart from stations at regular intervals if possible.

* The journey from Porthmadog to Blaenau Ffestiniog is 65 minutes (including stops at intermediate stations. These stops are very short and may be neglected in the timetabling).

* The first train of the day will leave Porthmadog at 9.00 a.m.

* The last train must return to Porthmadog by 5.00 p.m. (These times are more restricted than those that do, in fact, operate.)

THE "FFESTINIOG RAILWAY" ... SOME HINTS

Use a copy of the graph paper provided to draw a distance-time graph for the 9.00 a.m. train leaving Porthmadog.

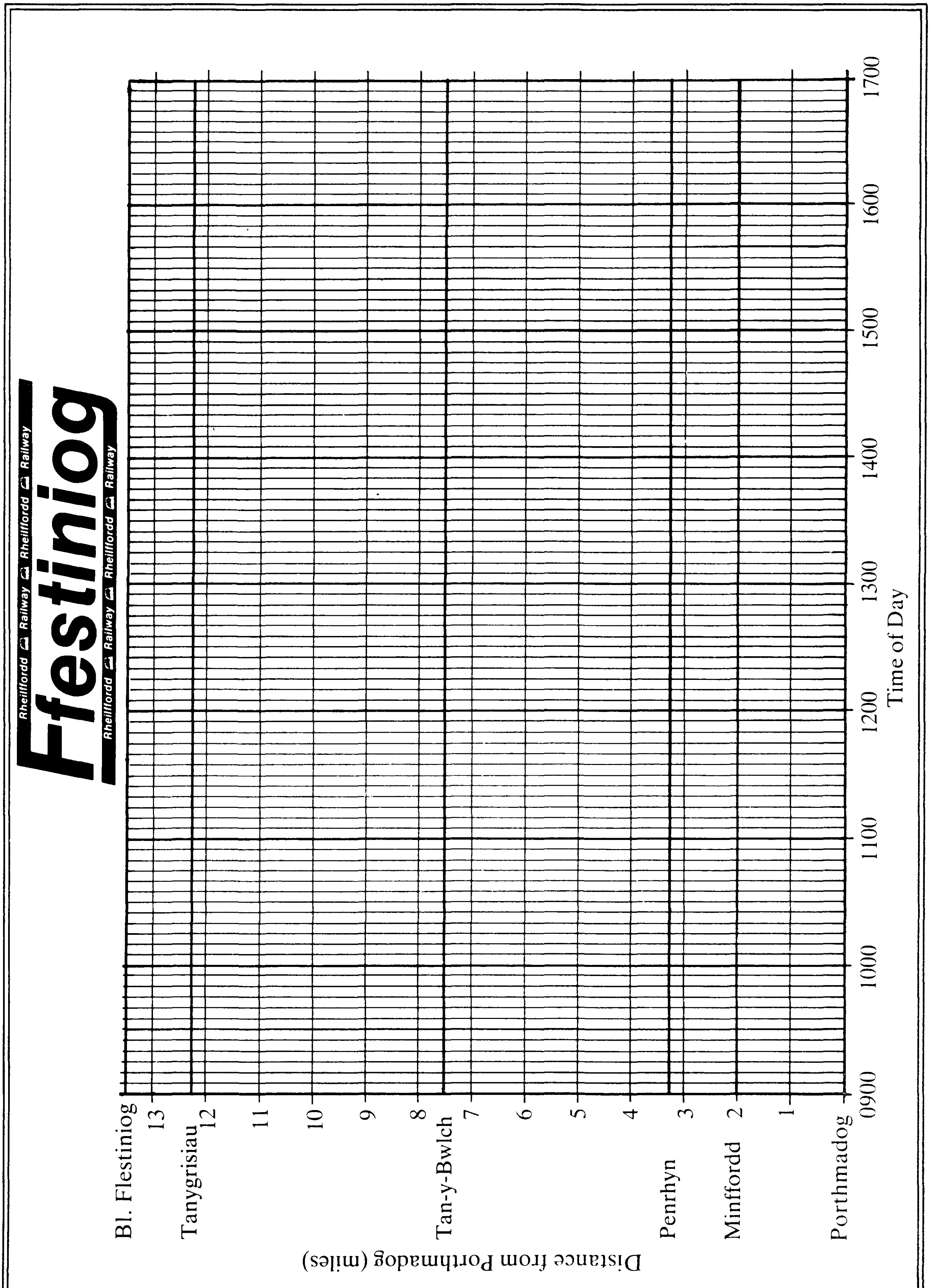
Try to show, accurately:

- The outward journey from Porthmadog to Blaenau Ffestiniog.
- The waiting time at Blaenau Ffestiniog.
- The return journey from Blaenau Ffestiniog to Porthmadog.
- The waiting time at Porthmadog . . . and so on.

What is the interval between departure times from Porthmadog for the above train?
How can we space the two other trains regularly between these departure times?
Draw similar graphs for the other two trains.
How many passing places are needed? Where will these have to be?
From your graph, complete the following timetable:

Miles	Station		Daily Timetable						
0	Porthmadog	d	09.00						
2	Minffordd	d							
3¼	Penrhyn	d							
7½	Tan-y-Bwlch	d							
12¼	Tanygrisiau	d							
13½	Blaenau Ffestiniog	a							
0	Blaenau Ffestiniog	d							
1¼	Tanygrisiau	d							
6	Tan-y-Bwlch	d							
10¼	Penrhyn	d							
11½	Minffordd	d							
13½	Porthmadog	a							

Ask your teacher for a copy of the real timetable, and write about how it compares with your own.


Ffestiniog
Rheilffordd Railway Rheilffordd Railway
Bl. Ffestiniog
Tanygrisiau
Tan-y-Bwlch
Penrhyn
Minffordd
Porthmadog
13
12
11
10
9
8
7
6
5
4
3
2
1
0900
1000
1100
1200
1300
1400
1500
1600
1700
Time of Day
Distance from Porthmadog (miles)

Rheilffordd Railway Rheilffordd Railway

Ffestiniog

Rheilffordd Railway Rheilffordd Railway

MOUNTAINS, LAKES AND COASTLINE

Take the famous Ffestiniog Railway for a memorable journey through the Snowdonia National Park. From the coast at Porthmadog the little train climbs through tranquil pastures and magnificent forests, past lakes and waterfalls, round horseshoe bends and even a complete spiral, sometimes clinging to the side of the mountain and sometimes tunnelling under it. Much of the area is so remote that there are not even any motor roads and the train stops occasionally at isolated cottages whose inhabitants depend entirely on the railway. 13½ miles and one hour's journey time from Porthmadog is Blaenau Ffestiniog, over 700 feet above sea level. Here are the slate mines at Llechwedd and Gloddfa Ganol which are both open to visitors.

To cater for all your requirements there are gift shops at Porthmadog, Tan-y-Bwlch and Blaenau Ffestiniog, a self-service restaurant at Porthmadog and station buffets at Tan-y-Bwlch and Blaenau Ffestiniog.

So sit back, relax and take the journey of a lifetime. Let our stewards wait on you with snacks and drinks from the buffet car or minibar trolley. For the enthusiast, there's even more — many of the trains are pulled by unique and historic steam locomotives some of which have served the line for over a hundred years.

Your complete day out . . .

SPECIAL INCLUSIVE EXCURSIONS FROM PORTHMADOG

STWLAN DAM (Daily 26 May to 13 September)
Depart Porthmadog by most "daytime" trains (see timetable) for bus connection from Tanygrisiau. This is one of the most spectacular 'bus routes in Britain, ascending to a height of 1650 feet. The bus returns to Tanygrisiau station from where you may return to Porthmadog or, if you so wish, continue your rail journey to Blaenau Ffestiniog at no extra charge. Allow 2½ hours for the complete excursion if returning direct from Tanygrisiau or 3 hours 20 minutes if returning via Blaenau Ffestiniog.

LLECHWEDD CAVERNS (Monday to Saturday 30 March to 2 November, also Sundays 26 May to 8 September)
Depart Porthmadog by any train up to 1220 (1310 when operating). Transfer to 'bus at Blaenau Ffestiniog for short trip across town to Llechwedd Caverns. Then take either the battery electric train or the deep mine incline into the heart of a Victorian slate mine. Allow at least 5 hours to do justice to the complete excursion.

FFESTINIOG LINK TOURS (Monday to Friday 27 May to 13 September)
The new joint Ffestiniog/British Rail station in Blaenau Ffestiniog has enabled us to provide easy rail access to the Conwy Valley and North Wales Coast. The journey from Porthmadog to Llandudno offers 44 miles of spectacular mountain and coastal scenery. Depart Porthmadog at 0950 (or 0840 when operating) for this highlight of your holiday. A shorter version of this tour from Porthmadog to Betws-y-Coed or Llanrwst is also available.

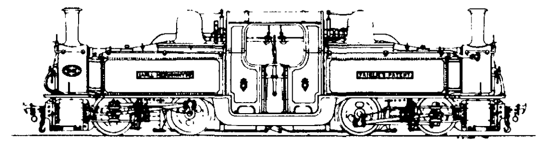
GLODDFA GANOL Slate Mine. Free admission will be granted during the 1985 season to any child whose parent produces a full return Ffestiniog ticket between Porthmadog and Blaenau Ffestiniog. A 'bus service operates between Blaenau Ffestiniog station and Gloddfa Ganol.

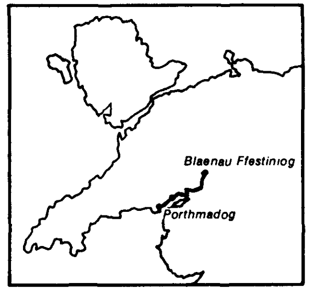
The Great Little Trains of Wales

NARROW GAUGE WANDERER TICKET

GREAT VALUE — 8 days unlimited travel on any of the following lines: FFESTINIOG RAILWAY, TALYLLYN RAILWAY, VALE OF RHEIDOL RAILWAY, BALA LAKE RAILWAY, WELSHPOOL AND LLANFAIR LIGHT RAILWAY, LLANBERIS LAKE RAILWAY, WELSH HIGHLAND RAILWAY, BRECON MOUNTAIN RAILWAY.

Adults: £13 Children aged 5 and under 16: £6.50.





MORE MILES FOR YOUR MONEY

THIRD CLASS (First class available at supplementary charge)

Principal Fares (available in either direction)	ORDINARY SINGLE	ORDINARY RETURN	ECONOMY★ RETURN
Porthmadog to Blaenau Ffestiniog	£2.80	£5.60	£4.60
Porthmadog to Tan-y-Bwlch	£1.70	£3.40	—
Tan-y-Bwlch to Blaenau Ffestiniog	£1.70	£3.40	—
Porthmadog to Penrhyn	90p	£1.80	—

★ Travel out by diesel service shown black on timetable. Return by ANY train.

Reductions for Children and Senior Citizens as follows:

Children under 5 free. ONE CHILD UNDER 16 TRAVELS FREE IN THIRD CLASS FOR EACH ADULT PAYING THIRD CLASS ORDINARY OR ECONOMY FARES. Additional children aged 5 and under 16 travel at half fare.
Senior Citizens travel at half fare on return fares only.

PLEASE NOTE FREE CHILD FACILITY

Family Fares up to 22% cheaper than three years ago!

Fares correct at time of going to press but liable to alteration without notice.

Did you know . . .
that the Ffestiniog Railway has a supporters club?
The FESTINIOG RAILWAY SOCIETY
is a voluntary organisation dedicated to supporting the continued existence of the Ffestiniog Railway.

You can join at one of the Railway's shops, or, send £6 (£3 for Juniors under the age of 18) to the Membership Secretary: J. Manisty, 4 Kingsgate Street, Winchester, Hants. SO23 9PD. (Members receive travel privileges and a quarterly magazine.)

If you would like further information about the Ffestiniog Railway and the Society, ask at the booking office for a copy of the leaflet. *An introduction to the Festiniog Railway Society.*

Ffestiniog Railway, Porthmadog, Gwynedd.
Telephone No.: (0766) 2340/2384
Member — Ten Top Attractions of North Wales

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Rheilffordd Railway Rheilffordd Railway

Ffestiniog

Rheilffordd Railway Rheilffordd Railway

Porthmadog to Blaenau Ffestiniog

Time Table 1985

CHILDREN FREE

one passenger (under 16) travels free for each adult paying normal 3rd class fare

HIGH SUMMER

Monday 15 July to Friday 30 August
(Also Spring Holiday Week Sunday 26 May to Thursday 30 May)

MONDAYS TO THURSDAYS

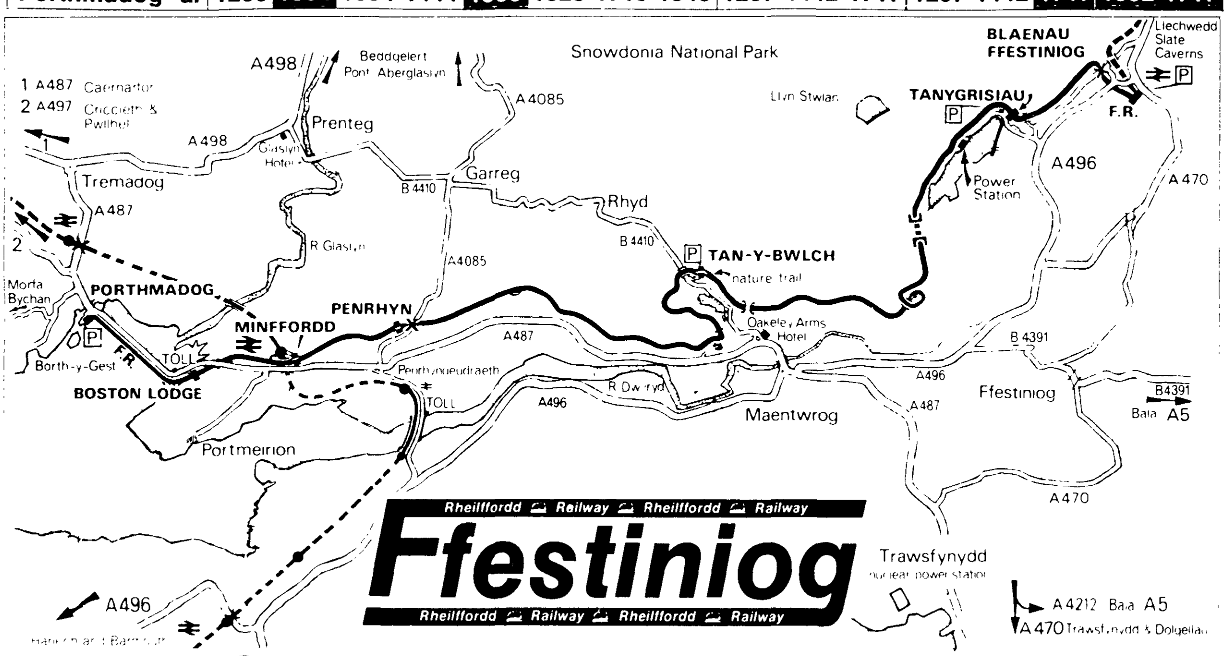
Porthmadog	**d.**	**0840**	0950	1035	1125	1220	1310	1400	1455	1545	1635	**1900**
British Rail Cambrian Coast line (See notes below) NO SUNDAY SERVICE: Pwllheli	d.	0755		0938		1135			1400	1500		1640
Minffordd	a.	0825		1008		1205			1433	1530		1710
Barmouth	d.		0817		0955		1155		1343		1520	1752
Minffordd	a.		0901		1040		1237		1422		1606	1834
Minffordd	**d.**	**0849**	0959	1044	1134	1229	1319	1409	1504	1554	1644	**1909**
Penrhyn	**d.**	**"Early**	1005	1050	1140	1235	1325	1415	1510	1600	1650	**1915**
Tan-y-Bwlch	**d.**	**Bird"**	1025	1112	1205	1255	1345	1435	1530	1620	1710	**1935**
Tanygrisiau	**d.**		1043	1135	1223	1313	1407	1458	1548	1641	1728	**1953**
Bl. Ffestiniog	**a.**	**0940**	1055	1146	1233	1325	1417	1507	1600	1652	1740	**2005**
British Rail Conwy Valley line (See notes below) NO SUNDAY SERVICE: Bl. Ffestiniog	d.	0945	1105			1330	1450		1625		1748	2020
Llandudno Jcn.	a.	1042	1204			1431	1548		1726		1845	2117
Llandudno	a.	1055	1214			1441	1558		1736		1903	2127
Llandudno	d.	0830	0950		1110			1333	1450		1630	1745
Llandudno Jcn.	d.	0840	1000		1121			1343	1501		1640	1803
Bl. Ffestiniog	a.	0940	1100		1224			1445	1609		1741	1903
Bl. Ffestiniog	**d.**	**1015**	1105	1155	1245	1340	1430	1520	1615	1700	1746	**2015**
Tanygrisiau	**d.**	**1021**	1111	1201	1251	1346	1436	1526	1621	1706	1752	**2021**
Tan-y-Bwlch	**d.**	**1045**	1133	1225	1315	1405	1500	1550	1640	1730	1814	**2043**
Penrhyn	**d.**	**1101**	1149	1245	1335	1424	1520	1606	1659	1746	1830	**2059**
Minffordd	**d.**	**1106**	1154	1250	1340	1429	1525	1611	1706	1751	1835	**2104**
British Rail Cambrian Coast line (See notes below) NO SUNDAY SERVICE: Minffordd	d.		1205		1433	1530		1710			1955	
Barmouth	a.		1246		1514	1616		1751			2036	
Minffordd	d.		1237		1422		1606			1834		2212
Pwllheli	a.		1305		1450		1633			1901		2239
Porthmadog	**a.**	**1120**	1209	1304	1354	1444	1539	1629	1719	1805	1848	**2117**

FRIDAYS, SATURDAYS AND SUNDAYS

Porthmadog	**d.**	0950	**1035**	1125	1220	**1310**	1400	1455	163
Pwllheli	d.	0755g	0938		1135			1400	1500
Minffordd	a.	0825g	1008		1205			1433	1530
Barmouth	d.	0817		0955		1155FO	1247SO	1343	1520
Minffordd	a.	0901		1040		1237FO	1325SO	1422	1606
Minffordd	**d.**	0959	**1044**	1134	1229	**1319**	1409	1504	164
Penrhyn	**d.**	1005	**1050**	1140	1235	**1325**	1415	1510	165
Tan-y-Bwlch	**d.**	1025	**1112**	1205	1255	**1345**	1435	1530	171
Tanygrisiau	**d.**	1043	**1135**	1223	1313	**1407**	1458	1548	172
Bl. Ffestiniog	**a.**	1055	**1146**	1233	1325	**1417**	1507	1600	174
Bl. Ffestiniog	d.	1105FO			1330	1450		1625	1748
Llandudno Jcn.	a.	1204FO			1431	1548		1726	1845
Llandudno	a.	1214FO			1441	1558		1736	1903
Llandudno	d.	0830 0950FO		1110			1333	1450	1630
Llandudno Jcn.	d.	0840 1000FO		1121			1343	1501	1640
Bl. Ffestiniog	a.	0940 1100FO		1224			1445	1609	1741
Bl. Ffestiniog	**d.**	1105	**1155**	1245	1340	**1430**	1520	1615	174
Tanygrisiau	**d.**	1111	**1201**	1251	1346	**1436**	1526	1621	175
Tan-y-Bwlch	**d.**	1133	**1225**	1315	1405	**1500**	1550	1640	181
Penrhyn	**d.**	1149	**1245**	1335	1424	**1520**	1606	1659	183
Minffordd	**d.**	1154	**1250**	1340	1429	**1525**	1611	1706	183
Minffordd	d.	1205		1433	1530		1710		1955
Barmouth	a.	1246		1514	1616		1751		2036
Minffordd	d.	1237FO	1325SO	1422		1606		1834	2212
Pwllheli	a.	1305FO	1353SO	1450		1633		1901	2239
Porthmadog	**a.**	1209	**1304**	1354	1444	**1539**	1625	1719	184

AUTUMN AND WINTER

		Saturday 31 August to Sunday 15 September: MONDAYS TO FRIDAYS								SATS. & SUNS.			DAILY Monday 16 Sept. to Sunday 3 Nov.			Daily 26 Dec to 1 Jan, Sats & Suns 15 Feb 1986 to 23 March 1986	
Porthmadog	**d.**	0950	**1035**	1125	1220	**1310**	1400	1455	1635	0950	1220	1455	0950	1220	**1455**	**1115**	**1455**
British Rail Cambrian Coast line (See notes below) NO SUNDAY SERVICE: Pwllheli	d.	0755	0938		1135				1500	0838	1135	1400	0755g	1135		0938	1135
Minffordd	a.	0825	1008		1205				1530	0911	1205	1433	0825g	1205		1008	1205
Barmouth	d.	0817		0955				1343	1520	0817	0955	1343	0817	0955	1343	0955	1343
Minffordd	a.	0901		1040				1422	1606	0901	1040	1422	0901	1040	1422	1040	1422
Minffordd	**d.**	0959	**1044**	1134	1229	**1319**	1409	1504	1644	0959	1229	1504	0959	1229	**1504**	**1124**	**1504**
Penrhyn	**d.**	1005	**1050**	1140	1235	**1325**	1415	1510	1650	1005	1235	1510	1005	1235	**1510**	**1130**	**1510**
Tan-y-Bwlch	**d.**	1025	**1112**	1205	1255	**1345**	1435	1530	1710	1025	1255	1530	1025	1255	**1530**	**1150**	**1530**
Tanygrisiau	**d.**	1043	**1135**	1223	1313	**1407**	1458	1548	1728	1043	1313	1548	RQ	RQ	**RQ**	**RQ**	**RQ**
Bl. Ffestiniog	**a.**	1055	**1146**	1233	1325	**1417**	1507	1600	1740	1055	1325	1600	1055	1325	**1600**	**1220**	**1600**
British Rail Conwy Valley line (See notes below) NO SUNDAY SERVICE: Bl. Ffestiniog	d.	1105			1330	1450		1625	1748		1330	1625		1330	1625	1330	1625
Llandudno Jcn.	a.	1204			1431	1548		1726	1845		1431	1726		1431	1726	1431	1726
Llandudno	a.	1214			1441	1558		1736	1903		1441	1736		1441	1736	1441	1736
Llandudno	d.	0950		1110			1333	1450	1630	0830	1110	1450	0830	1110	1450	1110	1450
Llandudno Jcn.	d.	1000		1121			1343	1501	1640	0840	1121	1501	0840	1121	1501	1121	1501
Bl. Ffestiniog	a.	1100		1224			1445	1609	1741	0940	1224	1609	0940	1224	1609	1224	1609
Bl. Ffestiniog	**d.**	1105	**1155**	1245	1340	**1430**	1520	1615	1746	1105	1340	1615	1105	1340	**1615**	**1230**	**1615**
Tanygrisiau	**d.**	1111	**1201**	1251	1346	**1436**	1526	1621	1752	1111	1346	1621	RQ	RQ	**RQ**	**RQ**	**RQ**
Tan-y-Bwlch	**d.**	1133	**1225**	1315	1405	**1500**	1550	1640	1814	1133	1405	1640	1133	1405	**1640**	**1258**	**1640**
Penrhyn	**d.**	1149	**1245**	1335	1424	**1520**	1606	1659	1830	1149	1424	1659	1149	1424	**1659**	**1314**	**1659**
Minffordd	**d.**	1154	**1250**	1340	1429	**1525**	1611	1706	1835	1154	1429	1704	1154	1429	**1704**	**1319**	**1704**
British Rail Cambrian Coast line (See notes below) NO SUNDAY SERVICE: Minffordd	d.	1205			1530		1710		1955	1205	1530	1710	1205	1530	1710	1530	1710
Barmouth	a.	1246			1616		1751		2033	1246	1616	1751	1246	1616	1751	1616	1751
Minffordd	d.			1422		1606		1834	2212	1325	1606	1834	1422	1606	1834	1422	1834
Pwllheli	a.			1450		1633		1901	2239	1353	1633	1901	1450	1633	1901	1450	1901
Porthmadog	**a.**	1209	**1304**	1354	1444	**1539**	1625	1719	1848	1207	1442	1717	1207	1442	**1717**	**1332**	**1717**



by Narrow Gauge Train through the Mountains of Snowdonia

NOTES

NOTES ON BRITISH RAIL CONNECTING SERVICES

- b — until 12 May
- c — from 13 May
- g — Saturdays 18 May to 28 September, retimed to Pwllheli dep. 0838, Minffordd arr. 0911
- FO — Fridays only
- SO — Saturdays only

British Rail services may be subject to alteration at Bank Holiday periods.

NOTES ON FFESTINIOG RAILWAY SERVICES

1134 1140 1200 1218 1230 denotes DIESEL HAULED TRAINS — 'ECONOMY' FARES AVAILABLE. Considerable reductions for passengers commencing a FULL RETURN JOURNEY by such trains. The return journey may be made by any train.

All other services are normally steam hauled.

RQ Stops on request only. Passengers wishing to alight should inform the guard before boarding. Passengers wishing to join should give a clear handsignal to the driver. All trains other than the 0840 ex. Porthmadog will also call on request at Boston Lodge, Plas and Dduallt.

Every effort possible will be made to ensure running as timetable but the Ffestiniog Railway will not guarantee advertised connections nor the advertised traction in the event of breakdown or other obstruction of services.

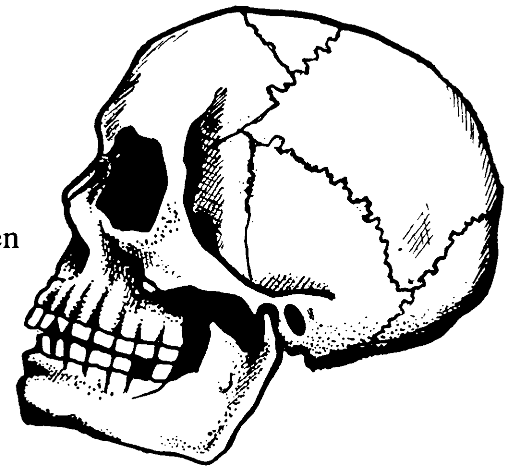
Special parties and private charter by arrangement.

Please apply to:
FFESTINIOG RAILWAY
HARBOUR STATION
PORTHMADOG
GWYNEDD
Telephone:
PORTHMADOG
(0766) 2340/2384

FATHER CHRISTMAS EXCURSIONS

A special service will operate on 21 and 22 December. Details available from 1 October. Father Christmas will meet the trains and distribute presents to the children.
All seats reservable — Advance Booking Essential.

CARBON DATING



Carbon dating is a technique for discovering the age of an ancient object, (such as a bone or a piece of furniture) by measuring the amount of Carbon 14 that it contains.

While plants and animals are alive, their Carbon 14 content remains constant, but when they die it decreases to radioactive decay.

The amount, a, of Carbon 14 in an object t thousand years after it dies is given by the formula:

$$a = 15.3 \times 0.886^{t}$$

(The quantity "a" measures the rate of Carbon 14 atom disintegrations and this is measured in "counts per minute per gram of carbon (cpm)")

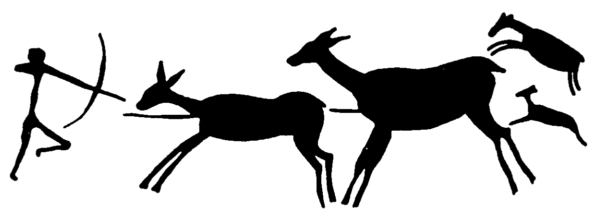
1 Imagine that you have two samples of wood. One was taken from a fresh tree and the other was taken from a charcoal sample found at Stonehenge and is 4000 years old.

How much Carbon 14 does each sample contain? (Answer in cpm's)

How long does it take for the amount of Carbon 14 in each sample to be halved?

These two answers should be the same, (Why?) and this is called the *half-life* of Carbon 14.

2 Charcoal from the famous Lascaux Cave in France gave a count of 2.34 cpm. Estimate the date of formation of the charcoal and give a date to the paintings found in the cave.



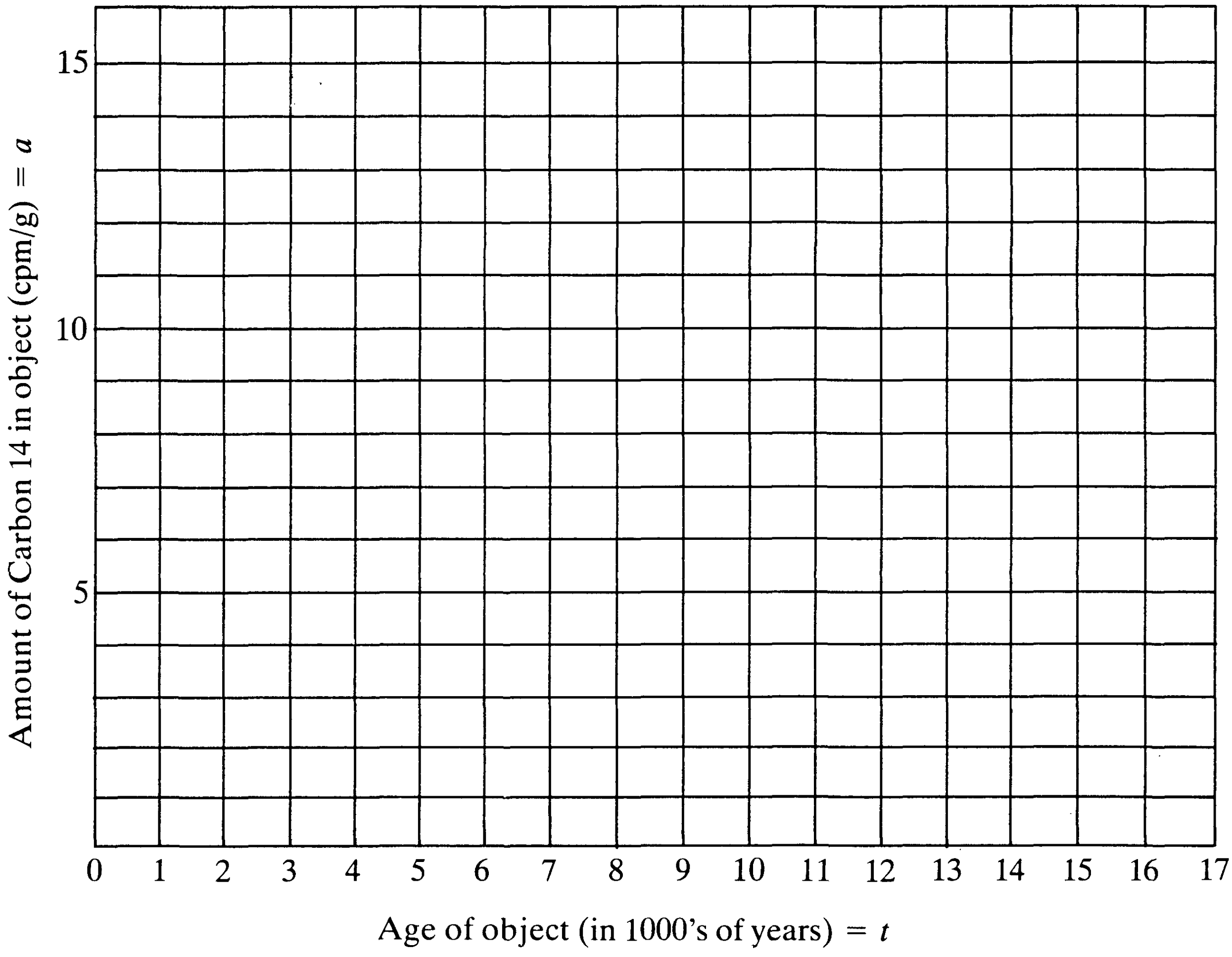
3 Bones A and B are x and y thousand years old respectively. Bone A contains three times as much Carbon 14 as bone B.

What can you say about x and y?

CARBON DATING . . . SOME HINTS

Using a calculator, draw a table of values and plot a graph to show how the amount of Carbon 14 in an object varies with time.

t (1000's of years)	0	1	2	3	4	5	6	7	8	9	10	. . .	17
a (c.p.m)													



Use your graph to read off answers to the questions.

DESIGNING A CAN

A cylindrical can, able to contain half a litre of drink, is to be manufactured from aluminium. The volume of the can must therefore be 500 cm^3.

* Find the radius and height of the can which will use the least aluminium, and therefore be the cheapest to manufacture. (i.e., find out how to minimise the surface area of the can).

  State clearly any assumptions you make.

* What shape is your can? Do you know of any cans that are made with this shape? Can you think of any practical reasons why more cans are not this shape?

DESIGNING A CAN . . . SOME HINTS

* You are told that the volume of the can must be 500 cm^3.

  If you made the can very tall, would it have to be narrow or wide? Why?

  If you made the can very wide, would it have to be tall or short? Why?

  Sketch a *rough* graph to describe how the height and radius of the can have to be related to each other.

* Let the radius of the can be r cm, and the height be h cm.

  Write down algebraic expressions which give

  — the volume of the can

  — the total surface area of the can, in terms of r and h.
  (remember to include the two ends!).

* Using the fact that the volume of the can must be 500 cm^3, you could
  either: – try to find some possible pairs of values for r and h
  (do this systematically if you can).
  – for each of your pairs, find out the corresponding surface area.
  or: – try to write one single expression for the surface area in terms of r, by eliminating h from your equations.

* Now plot a graph to show how the surface area varies as r is increased, and use your graph to find the value of r that minimises this surface area.

* Use your value of r to find the corresponding value of h. What do you notice about your answers? What shape is the can?

MANUFACTURING A COMPUTER

Imagine that you are running a small business which assembles and sells two kinds of computer: Model A and Model B (the cheaper version). You are only able to manufacture up to 360 computers, of either type, in any given week.

The following table shows all the relevant data concerning the employees at your company:

Job Title	Number of people doing this job	Job description	Pay	Hours worked
Assembler	100	This job involves putting the computers together	£100 per week	36 hours per week
Inspector	4	This job involves testing and correcting any faults in the computers before they are sold	£120 per week	35 hours per week

The next table shows all the relevant data concerning the manufacture of the computers.

	Model A	Model B
Total assembly time in man-hours for each computer	12	6
Total inspection and correction time in man-minutes for each computer	10	30
Component costs for each computer	£80	£64
Selling price for each computer	£120	£88

At the moment, you are manufacturing and selling 100 of Model A and 200 of Model B each week.

* What profit are you making at the moment?
* How many of each computer should you make in order to improve this worrying situation?
* Would it help if you were to make some employees redundant?

MANUFACTURING A COMPUTER . . . SOME HINTS

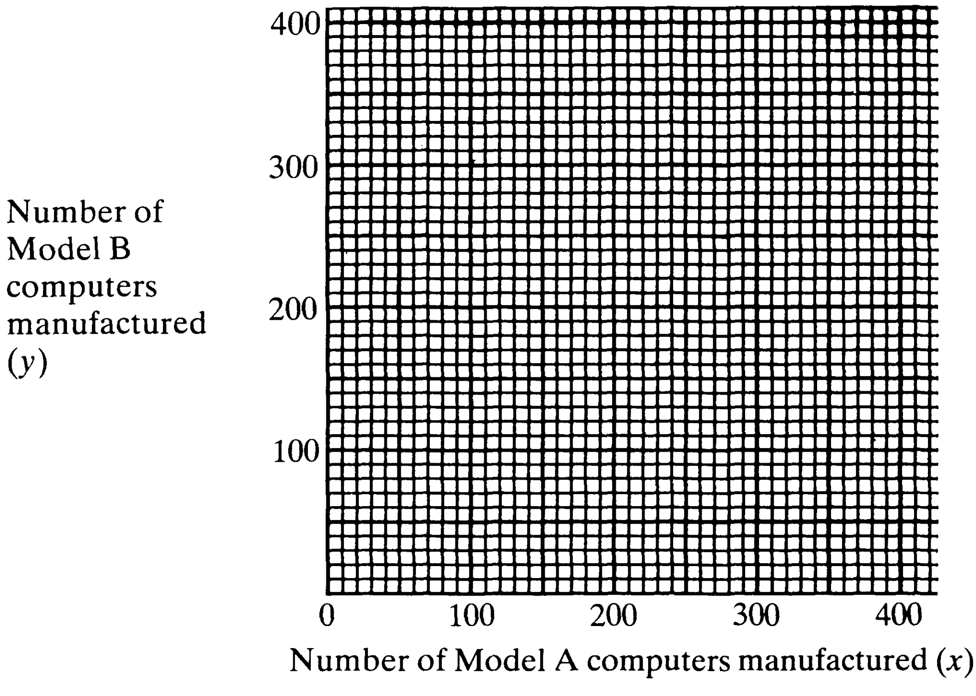
1 Suppose you manufacture 100 Model A's and 200 Model B's in one week:

* How much do you pay in wages?
* How much do you pay for components?
* What is your weekly income?
* What profit do you make?

2 Now suppose that you manufacture x Model A and y Model B computers each week.

* Write down 3 inequalities involving x and y. These will include:
  — considering the time it takes to assemble the computers, and the total time that the assemblers have available.
  — considering the time it takes to inspect and correct faults in the computers, and the total time the inspectors have available.

Draw a graph and show the region satisfied by all 3 inequalities:

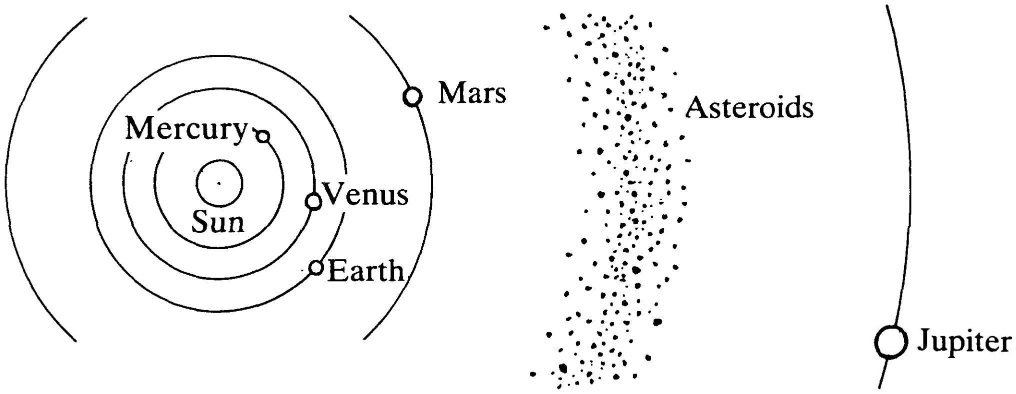


3 Work out an expression which tells you the profit made on x Model A and y Model B computers.

4 Which points on your graph maximise your profit?

THE MISSING PLANET 1.

In our solar system, there are nine major planets, and many other smaller bodies such as comets and meteorites. The five planets nearest to the sun are shown in the diagram below.



Between Mars and Jupiter lies a belt of rock fragments called the 'asteroids'. These are, perhaps, the remains of a tenth planet which disintegrated many years ago. We shall call this, planet 'X'. In these worksheets, you will try to discover everything you can about planet 'X' by looking at patterns which occur in the other nine planets.

How far was planet 'X' from the sun, before it disintegrated?

The table below compares the distances of some planets from the Sun with that of our Earth. (So, for example, Saturn is 10 times as far away from the Sun as the Earth. Scientists usually write this as 10 A.U. or 10 'Astronomical Units').

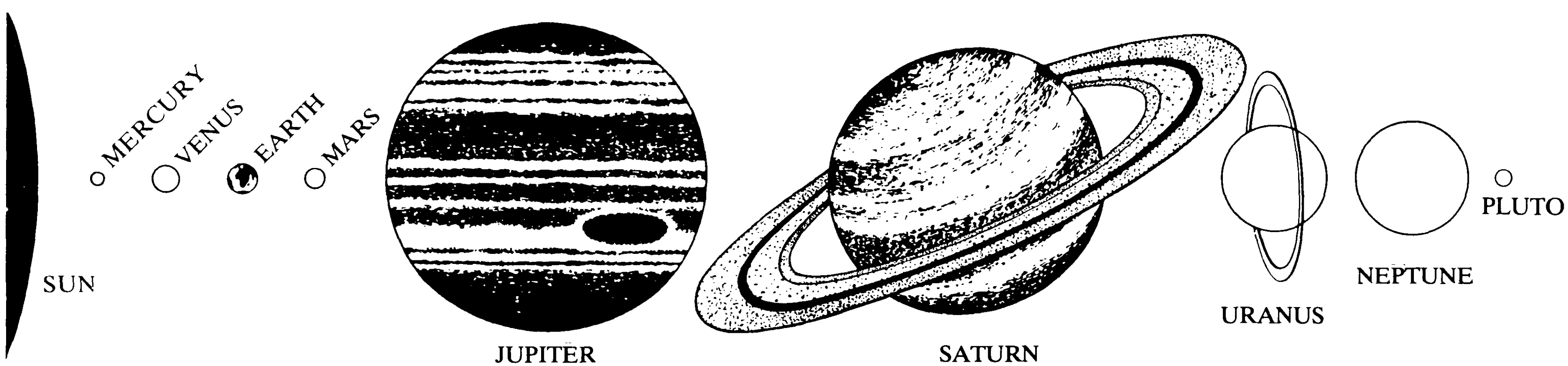
- Can you spot any pattern in the sequence of *approximate* relative distances.
- Can you use this pattern to predict the missing figures?
- So how far away do you think planet 'X' was from the Sun? (The Earth is 93 million miles away)
- Check your completed table with the planetary data sheet.

Where does the pattern seem to break down?

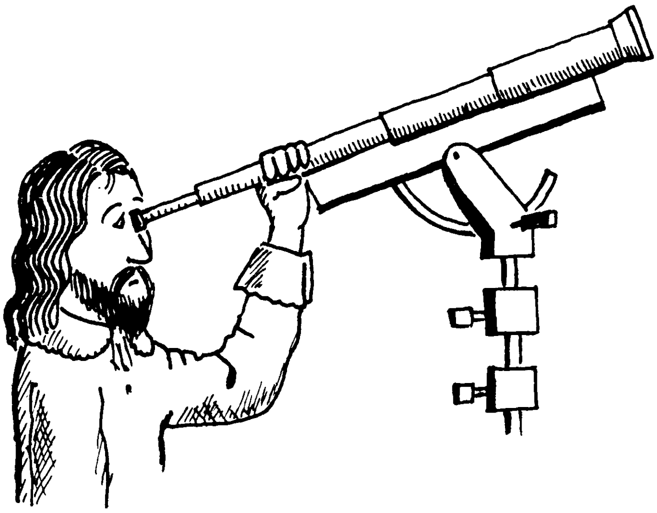
Planet	Relative Distance from Sun, approx (exact figures are shown in brackets)	
Mercury	?	
Venus	0.7	(0.72)
Earth	1	(1)
Mars	1.6	(1.52)
Planet X	?	
Jupiter	5.2	(5.20)
Saturn	10	(9.54)
Uranus	19.6	(19.18)
Neptune	?	
Pluto	?	

PLANETARY DATA SHEET

Planet	Average distance from the Sun. (millions of miles)	Diameter in miles.	Speed at which it flies through Space (mph)	Speed at which a point on the equator spins round (mph)	Time taken to go once round the Sun. (years)	Time taken to spin round once	Number of 'moons'
Earth	93	7 926	66 641	1 040	1	23.9 hours	1
Jupiter	484	88 700	29 216	28 325	11.86	9.9 hours	12
Mars	142	4 217	53 980	538	1.88	24.6 hours	2
Mercury	36	3 032	107 132	7	0.24	58.7 days	0
Neptune	2 794	30 800	12 147	6 039	164.8	15.8 hours	2
Pluto	3 674	3 700	10 604	77	248	6.3 days	0
Saturn	887	74 600	21 565	22 892	29.46	10.2 hours	10
Uranus	1 784	32 200	15 234	9 193	84.02	10.7 hours	5
Venus	67	7 521	78 364	4	0.61	243 days	0



THE MISSING PLANET . . . SOME BACKGROUND INFORMATION



In 1772, when planetary distances were still only known in relative terms, a German astronomer named David Titius discovered the same pattern as the one you have been looking at. This 'law' was published by Johann Bode in 1778 and is now commonly known as "Bode's Law". Bode used the pattern, as you have done, to *predict* the existence of a planet 2.8 AU from the sun. (2.8 times as far away from the Sun as the Earth) and towards the end of the eighteenth century scientists began to search systematically for it. This search was fruitless until New Year's Day 1801, when the Italian astronomer Guiseppe Piazzi discovered a very small asteroid which he named Ceres at a distance 2.76 AU from the Sun—astonishingly close to that predicted by Bode's Law. (Since that time, thousands of other small asteroids have been discovered, at distances between 2.2 and 3.2 AU from the sun.)

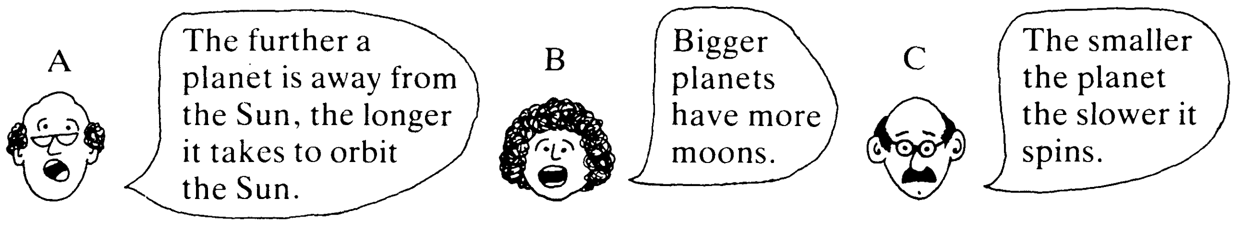
In 1781, Bode's Law was again apparently confirmed, when William Herschel discovered the planet Uranus, orbitting the sun at a distance of 19.2 AU, again startlingly close to 19.6 AU as predicted by Bode's Law. Encouraged by this, other astronomers used the 'law' as a starting point in the search for other distant planets.

However, when Neptune and Pluto were finally discovered, at 30 AU and 39 AU from the Sun, respectively, it was realised that despite its past usefulness, Bode's 'law' does not really govern the design of the solar system.

THE MISSING PLANET 2.

Look at the Planetary data sheet, which contains 7 statistics for each planet.

The following scientists are making hypotheses about the relationship between these statistics:

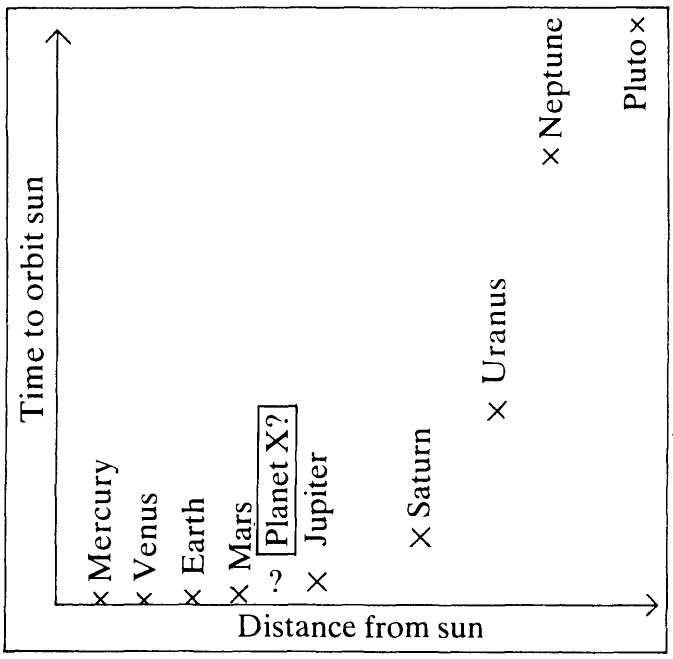


* Do you agree with these hypotheses? How true are they? (Use the data sheet)

* Invent a list of your own hypotheses.
Sketch a graph to illustrate each of them.

One way to test a hypothesis is to draw a scattergraph. This will give you some idea of how strong the relationship is between the two variables.

For example, here is a 'sketch' scattergraph testing the hypothesis of scientist A:



Notice that:
There *does* appear to be a relationship between the distance a planet is from the Sun and the time it takes to orbit once. The hypothesis seems to be confirmed.

We can therefore predict the orbital time for Planet X. It should lie between that of Mars (2 years) and Jupiter (12 years). (A more accurate statement would need a more accurate graph.)

- *Sketch* scattergraphs to test your own hypotheses. What else *can* be found out about Planet X? What *cannot* be found?

THE MISSING PLANET 3.

After many years of observation the famous mathematician Johann Kepler (1571-1630) found that the time taken for a planet to orbit the Sun (T years) and its average distance from the Sun (R miles) are related by the formula

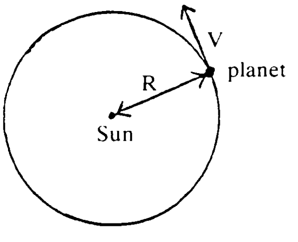
$$\frac{R^3}{T^2} = K$$ where K is a constant value.

* Use a calculator to check this formula from the data sheet, and find the value of K.

Use your value of K to find a more accurate estimate for the orbital time (T) of Planet X. (You found the value of R for Planet X on the first of these sheets).

* We asserted that the orbits of planets are 'nearly circular'. Assuming this is so, can you find another formula which connects

- — The average distance of the planet from the Sun (R miles)
- — The time for one orbit (T years)
- — The speed at which the planet 'flies through space' (V miles per hour)?



(Hint: Find out how far the planet moves during one orbit. You can write this down in two different ways using R, T and V)
(Warning: T is in *years*, V is in miles per *hour*)

Use a calculator to check your formula from the data sheet.
Use your formula, together with what you already know about R and T, to find a more accurate estimate for the speed of Planet X.

* Assuming that the planets are spherical, can you find a relationship connecting

- — The diameter of a planet (d miles)
- — The speed at which a point on the equator spins (v miles per hour)
- — The time the planet takes to spin round once (t hours)?

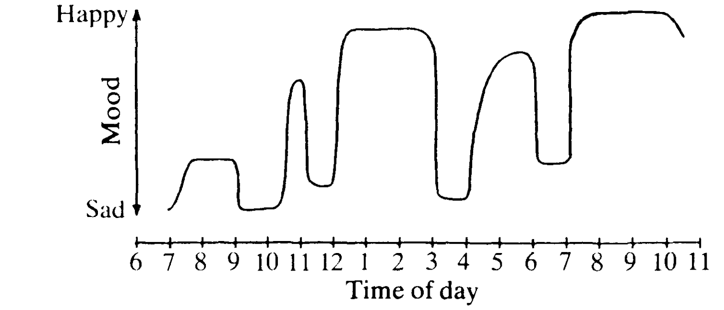
Check your formula from the data sheet.

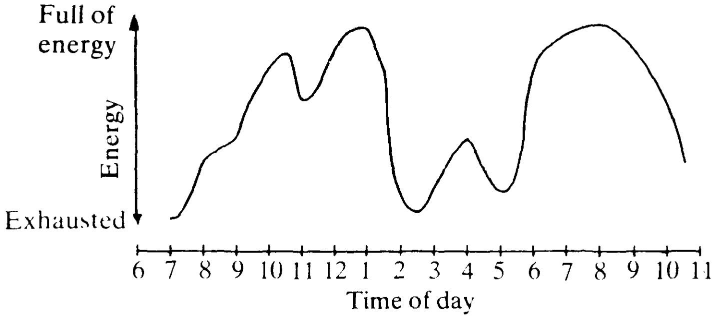
FEELINGS

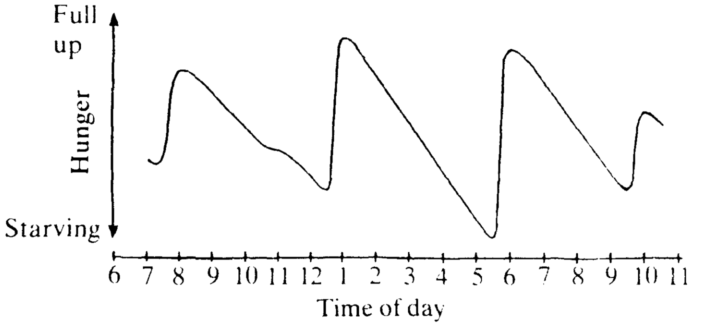
These graphs show how a girl's feelings varied during a typical day.

Her timetable for the day was as follows:

7.00 am	woke up
8.00 am	went to school
9.00 am	Assembly
9.30 am	Science
10.30 am	Break
11.00 am	Maths
12.00 am	Lunchtime
1.30 pm	Games
2.45 pm	Break
3.00 pm	French
4.00 pm	went home
6.00 pm	did homework
7.00 pm	went 10-pin bowling
10.30 pm	went to bed







(a) Try to explain the shape of each graph, as fully as possible.

(b) How many meals did she eat?
Which meal was the biggest?
Did she eat at breaktimes?
How long did she spend eating lunch?
Which lesson did she enjoy the most?
When was she "tired and depressed?" Why was this?
When was she "hungry but happy?" Why was this?

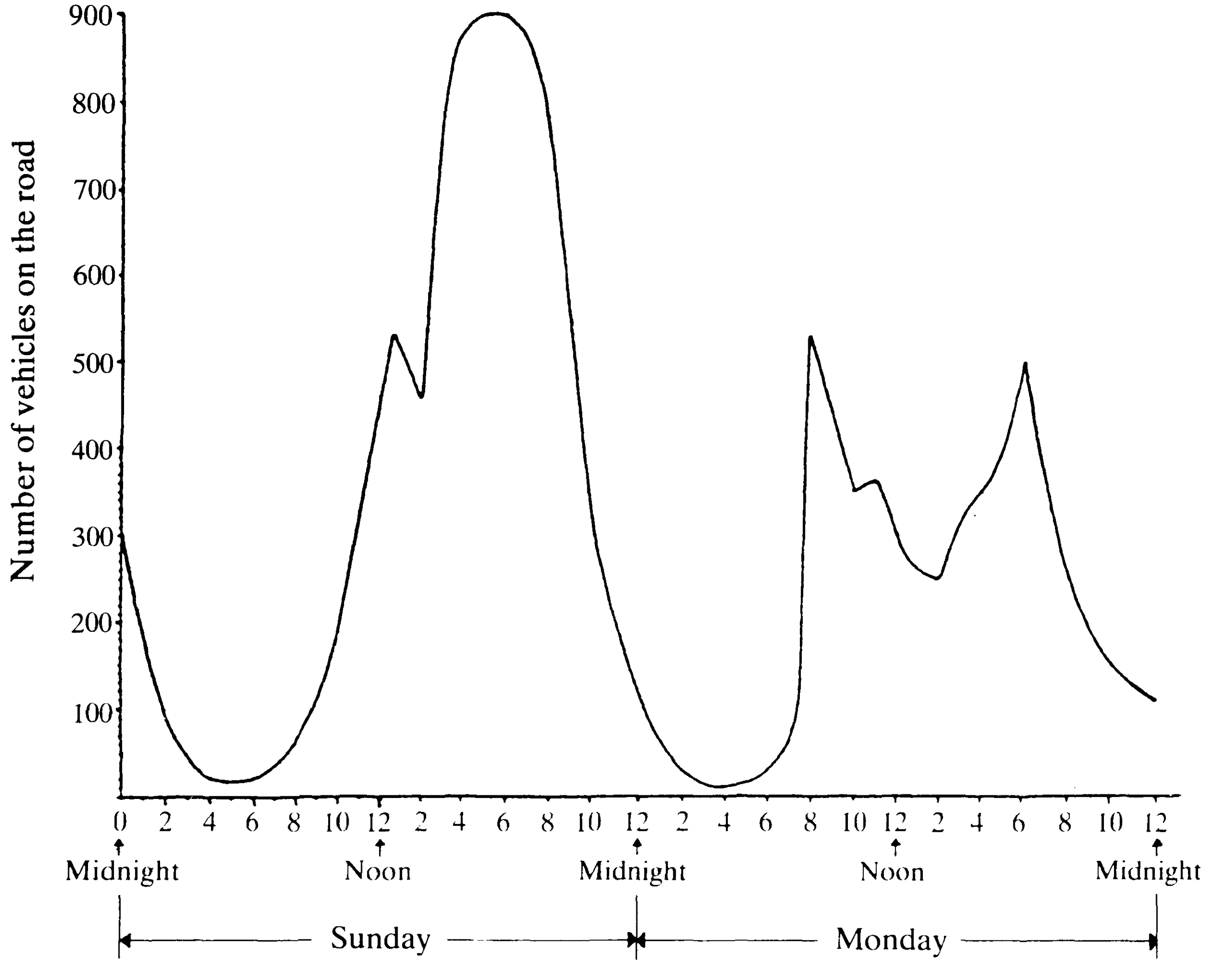
Make up some more questions like these, and give them to your neighbour to solve.

(c) Sketch graphs to show how *your* feelings change during the day. See if your neighbour can interpret them correctly.

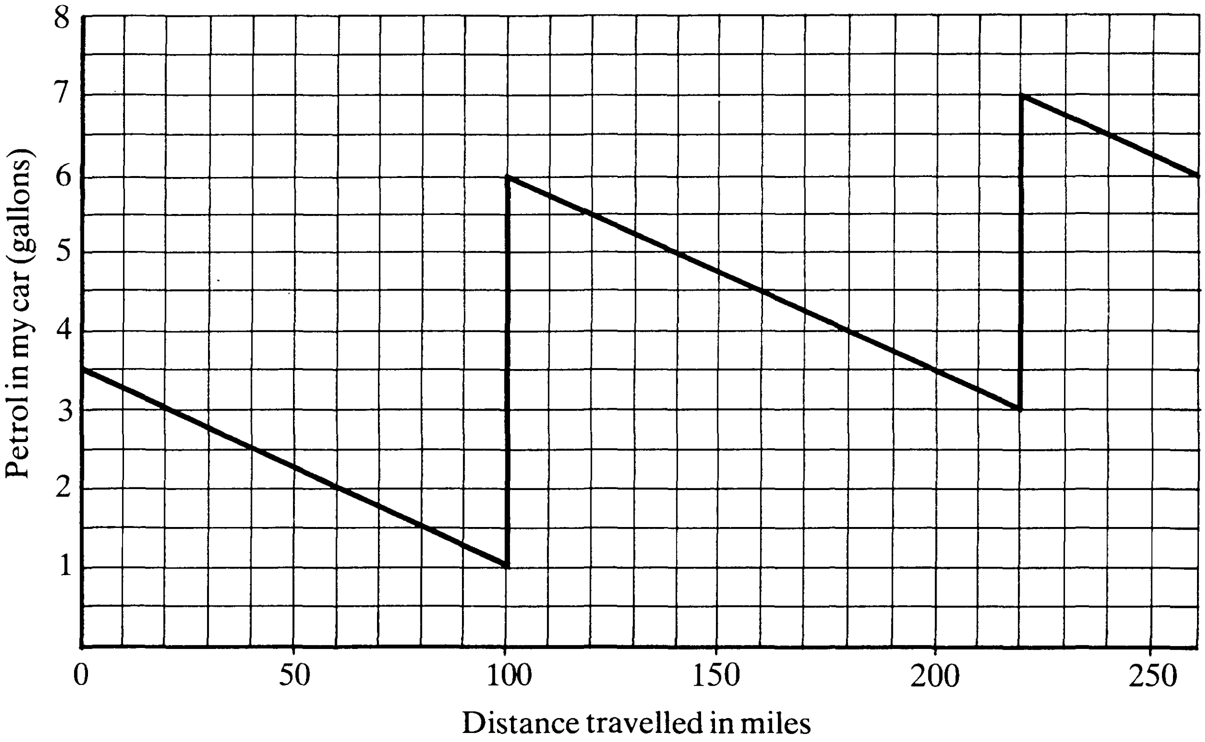
THE TRAFFIC SURVEY

A survey was conducted to discover the volume of traffic using a particular road. The results were published in the form of the graph which shows the number of cars using the road at any specified time during a typical Sunday and Monday in June.

1. Try to explain, as fully as possible, the shape of the graph.
2. Compare Sunday's graph with Monday's. What is suprising?
3. Where do you think this road could be? (Give an example of a road you know of, which may produce such a graph.)



THE MOTORWAY JOURNEY



The above graph shows how the amount of petrol in my car varied during a motorway journey.

Write a paragraph to explain the shape of the graph. In particular answer the following questions:

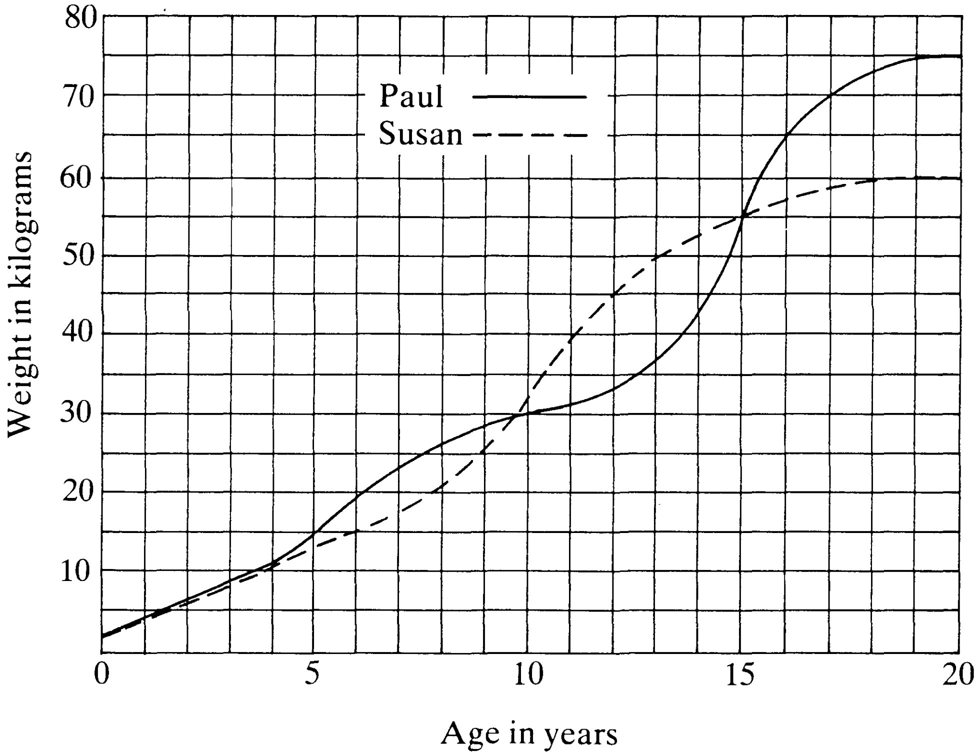
1 How much petrol did I have in my tank after 130 miles?
2 My tank holds about 9 gallons. Where was it more than half full?
3 How many petrol stations did I stop at?
4 At which station did I buy the most petrol? How can you tell?
5 If I had not stopped anywhere, where would I have run out of petrol?
6 If I had only stopped once for petrol, where would I have run out?
7 How much petrol did I use for the first 100 miles?
8 How much petrol did I use over the entire journey?
9 How many miles per gallon (mpg) did my car do on this motorway?

I left the motorway, after 260 miles, I drove along country roads for 40 miles and then 10 miles through a city, where I had to keep stopping and starting. Along country roads, my car does about 30 mpg, but in the city it's more like 20 mpg.

10 Sketch a graph to show the remainder of my journey.

GROWTH CURVES

Paul and Susan are two fairly typical people. The following graphs compare how their weights have changed during their first twenty years.



Write a paragraph comparing the shape of the two graphs. Write down everything you think is important.

Now answer the following:

1 How much weight did each person put on during their "secondary school" years (between the ages of 11 and 18)?

2 When did Paul weigh more than Susan? How can you tell?

3 When did they both weigh the same?

4 When was Susan putting on weight most rapidly?
How can you tell this from the graph?
How fast was she growing at this time? (Answer in kg per year).

5 When was Paul growing most rapidly? How fast was he growing at this time?

6 Who was growing faster at the age of 14? How can you tell?

7 When was Paul growing faster than Susan?

8 Girls tend to have boyfriends older than themselves. Why do you think this is so? What is the connection with the graph?

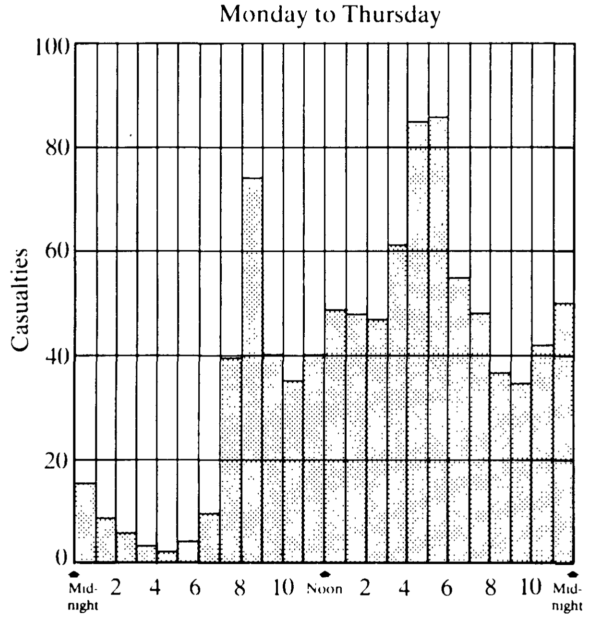
ROAD ACCIDENT STATISTICS

The following four graphs show how the number of road accident casualties per hour varies during a typical week.

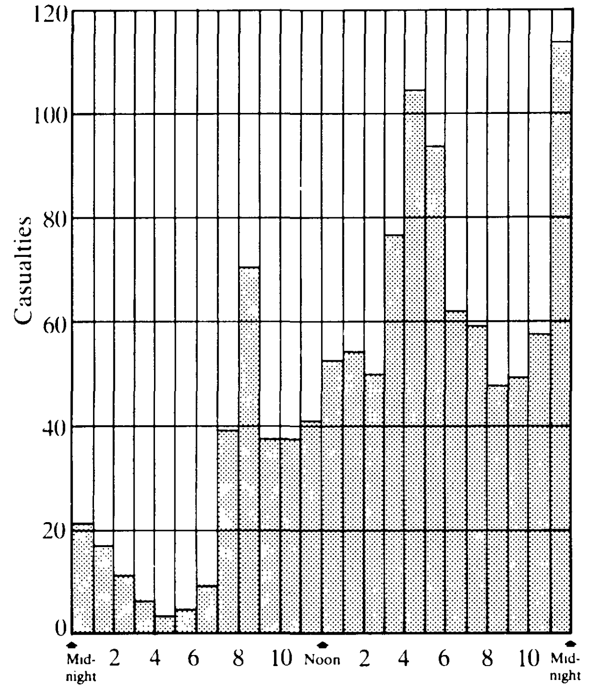
Graph A shows the normal pattern for Monday, Tuesday, Wednesday and Thursday.

* Which graphs correspond to Friday, Saturday and Sunday?
* Explain the reasons for the shape of each graph, as fully as possible.
* What evidence is there to show that alcohol is a major cause of road accidents?

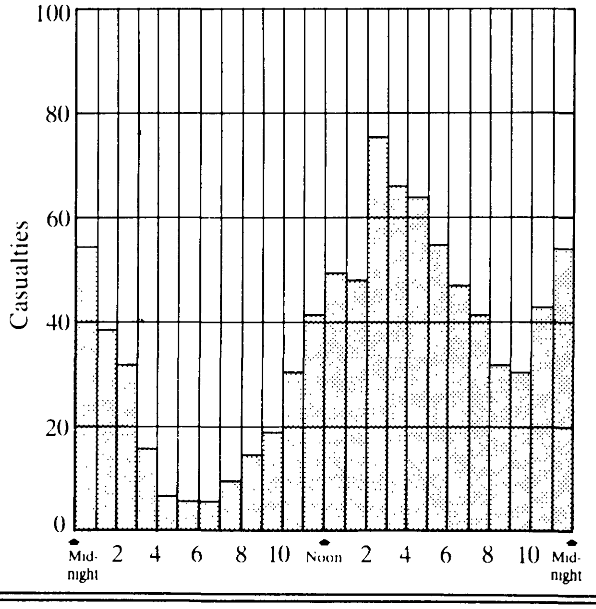
Graph A



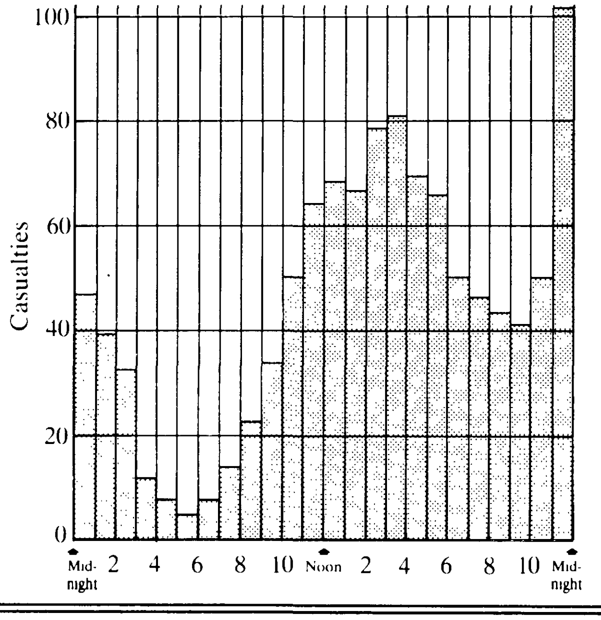
Graph B



Graph C



Graph D



THE HARBOUR TIDE

The graph overleaf, shows how the depth of water in a harbour varies on a particular Wednesday.

1 Write a paragraph which describes in detail what the graph is saying:

When is high/low tide? When is the water level rising/falling?

When is the water level rising/falling most rapidly?

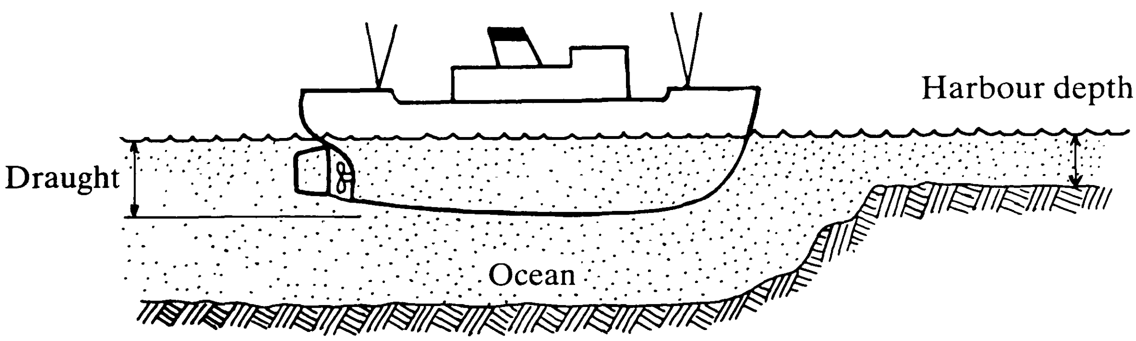
How fast is it rising/falling at this time?

What is the average depth of the water? How much does the depth vary from the average?

2 Ships can only enter the harbour when the water is deep enough. What factors will determine when a particular boat can enter or leave the harbour?

The ship in the diagram below has a draught of 5 metres when loaded with cargo and only 2 metres when unloaded.

Discuss when it can safely enter and leave the harbour.



Make a table showing when boats of different draughts can safely enter and leave the harbour on Wednesday.

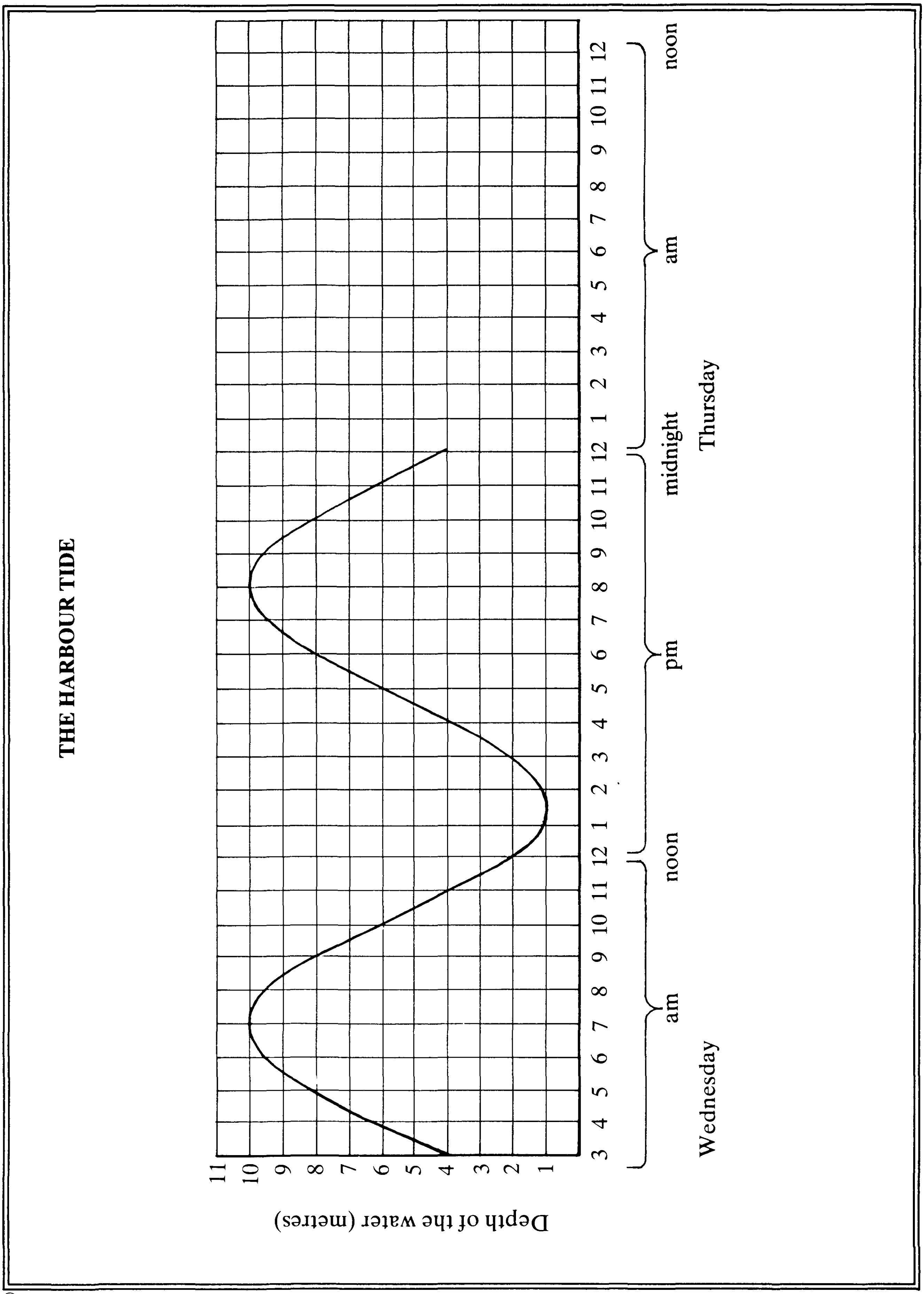
3 Try to complete the graph in order to predict how the tide will vary on Thursday. How will the table you draw up in question 2 need to be adjusted for Thursday? Friday? . . .

4 Assuming that the formula which fits this graph is of the form

$$d = A + B\cos(28t + 166)^\circ$$

(Where d = depth of water in metres
t = time in hours after midnight on Tuesday night)

Can you find out the values of A and B?
How can you do this without substituting in values for t?



THE HARBOUR TIDE
Depth of the water (metres)
1 2 3 4 5 6 7 8 9 10 11
3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
am
noon
pm
midnight
am
noon
Wednesday
Thursday

ALCOHOL

Read through the data sheet carefully, and then try to answer the following questions:

> Using the chart and diagram on page 2, describe and compare the effects of consuming different quantities of different drinks.
> (eg: Compare the effect of drinking a pint of beer with a pint of whisky)
> Note that 20 fl oz = 4 gills = 1 pint.
> Illustrate your answer with a table of some kind.

> An 11 stone man leaves a party at about 2 am after drinking 5 pints of beer. He takes a taxi home and goes to bed. Can he legally drive to work at 7 am the next morning? When would you advise him that he is fit to drive? Explain your reasoning as carefully as possible.

The five questions below will help you to compare and contrast the information presented on the data sheet.

1 *Using only the information presented in words by the "Which?" report,* draw an accurate graph showing the effect of drinking 5 pints of beer at 2 am.

   a) What will the blood alcohol level rise to?

   b) How long will it take to reach this level?

   c) How quickly will this level drop?

   d) What is the legal limit for car drivers? How long will this person remain unfit to drive? Explain your reasoning.

2 *Using only the formula provided,*

   draw another graph to show the effect of drinking 5 pints of beer.

   How does this graph differ from the graph produced above?

   Use your formula to answer 1a) b) c) d) again.

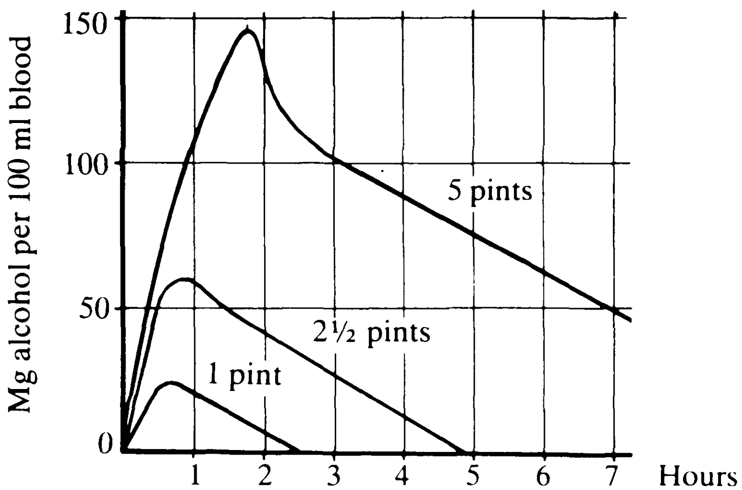
   Compare your answers with those already obtained.

3 *Using only the table of data from the AA book of driving,*

   draw another graph to show the effect of an *11 stone* man drinking 5 pints of beer.

   Compare this graph to those already obtained.

   Answer 1a) b) c) d) from this graph, and compare your answers with those above.



(adapted from a Medical textbook)

If you are above average weight (8-10 stone) subtract an hour; below it, add an hour.

Consumption (mg alcohol/100 ml blood)	After 1 Hour	After 2 Hours	After 3 Hours	After 4 Hours	After 5 Hours	After 6 Hours	After 7 Hours	After 8 Hours
15	0	0	0	0	0	0	0	0
30	15	0	0	0	0	0	0	0
45	45	30	15	0	0	0	0	0
60	60	45	30	15	0	0	0	0
75	75	75	60	45	30	15	0	0
90	90	90	75	60	45	30	15	0
105	105	105	90	75	60	45	30	15
120	120	120	105	90	75	60	45	30
135	135	135	135	120	105	90	75	60
150	150	150	150	135	120	105	90	75
165	165	165	165	165	150	135	120	105
180	180	180	180	180	165	150	135	120
195	195	195	195	195	180	165	150	135
210	210	210	210	210	210	195	180	165

(adapted from the A.A. Book of Driving)

ALCOHOL . . . DATA SHEET

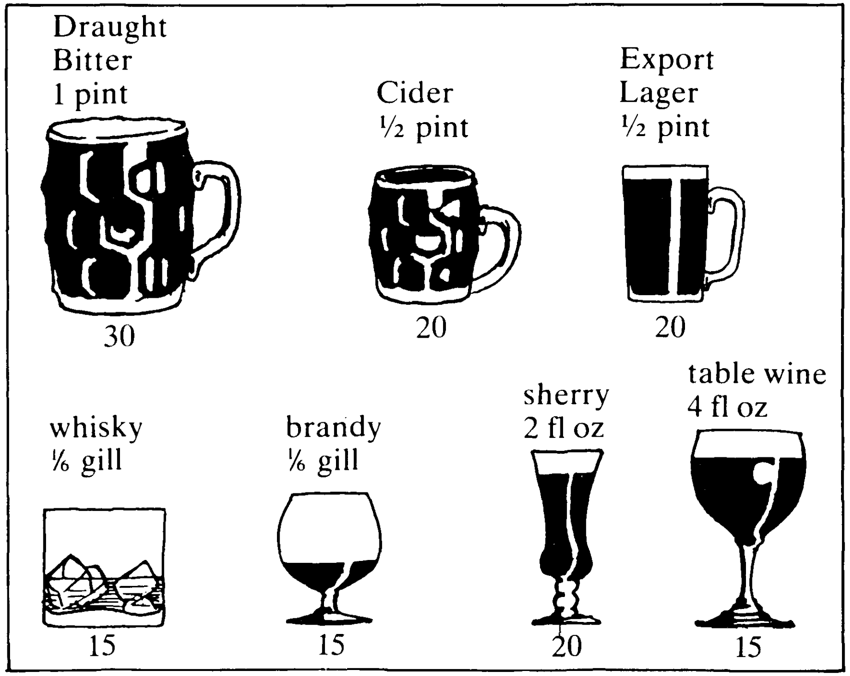
Alcohol is more easily available today, and more is drunk, than at any time over the past 60 years. At parties, restaurants and pubs you will be faced with the decision of how much to drink. Hundreds of thousands of people suffer health and social problems because they drink too much, so we feel you should know some facts.

What happens to alcohol in the body?

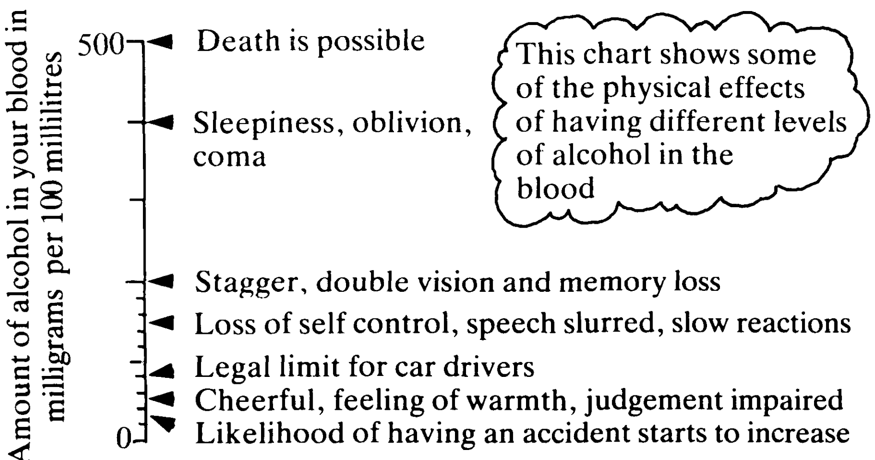
Most of it goes into the bloodstream. The exact amount will depend on how much has been drunk, whether the stomach is empty or not, and the weight of the person. We measure this amount by seeing how much alcohol (in milligrams) is present in 100 millilitres of blood.

How does alcohol affect behaviour?

You cannot predict the effect of alcohol very accurately, since this will depend on how much you drink, and on your personality. Some people become noisy and others sleepy. Alcohol will affect your judgement, self control and skills (like driving a car).



The figures below the glasses show the concentration of alcohol in the blood (in mg per 100 ml) after drinking the measure quoted.



Experts generally agree that a person who regularly drinks more than 4 pints of beer a day (or the equivalent in other forms of drink) is running a high risk of damaging his health. However, smaller amounts than this may still be harmful.

How do the effects of drinking wear off?

The information shown below was taken from four different sources. Do they agree with each other?

Clearly there's an urgent need for more public education about this. Here is a rough guide. An 11 stone man normally raises his blood/alcohol level by about 30mg/100ml with each drink (pint of beer, 2 glasses of wine, or double measure of spirits). So after 2½ such drinks he will probably be just below the legal limit (if he eats a meal at the same time, he may be able to go up to, say, three drinks without going over the limit).

It takes about an hour for the blood/alcohol level to reach a peak. After this time-assuming you've stopped drinking-the blood/alcohol level starts to fall at the rate of about 15mg/100ml (half a drink) per hour. This means that the rate at which you drink is important. For example, your blood/alcohol level will probably be higher after drinking 2½ pints of beer in quick succession than after 4 pints taken over an evening.

Don't look on the 80mg/100ml as a target to aim just short of. Many people (particularly the young) aren't safe to drive at levels well below this, and virtually everyone's reactions are at least slightly slower by the time the blood/alcohol limit approaches 80mg/100ml. For safety's sake you shouldn't drive if your blood/alcohol level is likely to be 50mg/100ml or more. And bear in mind that, after a night's heavy drinking, you may still be unsafe to drive (and over the legal limit) the next morning. Note also that it's an offence to drive or be in charge of a car while 'unfit through drink'-for which you could be convicted even if your blood/alcohol level is below 80mg/100ml

(from a "Which?" report on alcohol).

Let the amount of alcohol in the blood at any time be a mg/100ml.
Let the number of beers drunk be b
Let the number of hours that have passed since the drinking took place be h hours.

Then $a = 30b - 15h + 15$

ALCOHOL (continued)

4 *Compare the graph taken from the Medical textbook* with those drawn for questions 1, 2 and 3. Answer question 1a) b) c) and d) concerning the 11 stone man from this graph.

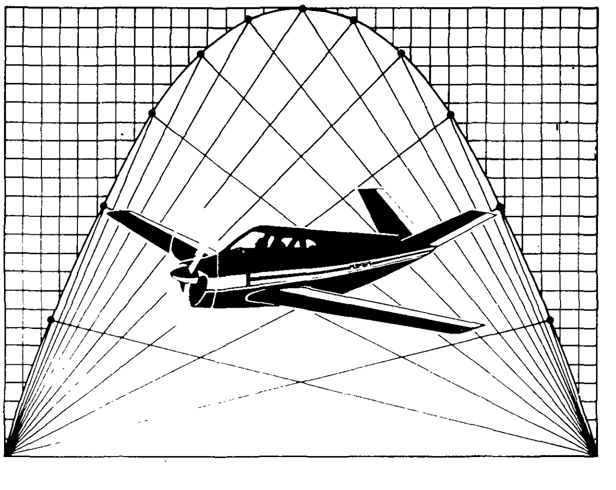
5 Compare the advantages and disadvantages of each mode of representation: words, formula, graph and table, using the following criteria:

Compactness	(does it take up much room?)
Accuracy	(is the information over-simplified?)
Simplicity	(is it easy to understand?)
Versatility	(can it show the effects of drinking different amounts of alcohol easily?)
Reliability	(which set of data do you trust the most? Why? Which set do you trust the least? Why?)

A business woman drinks a glass of sherry, two glasses of table wine and a double brandy during her lunch hour, from 1 pm to 2 pm. Three hours later, she leaves work and joins some friends for a meal, where she drinks two double whiskies.

Draw a graph to show how her blood/alcohol level varied during the entire afternoon (from noon to midnight). When would you have advised her that she was unfit to drive?

Support Materials



A SUGGESTED PROGRAMME OF MEETINGS ON THE MODULE

One way to explore the contents of 'The Language of Functions and Graphs' is to arrange a series of departmental meetings. A possible programme is outlined below.

Meeting 1 What's in the Box?

- Identify the contents of the box and browse through it.
- Consider which classes will use the materials first and arrange that, if possible, two or more colleagues try out worksheets A1 and A2 (pages 64 and 74 of the main module book) so that their experiences may be discussed at the next meeting.
- Arrange for everyone to have access to the materials over the next few days.

Meeting 2 Looking at the Video (issues 1 and 2)

- Compare notes and experiences with Worksheet A1 and Worksheet A2.
- Having used Worksheet A2 with classes it will be of interest to see the beginning of the video tape. This commences with two teachers and their classes working with Worksheet A2 followed by discussion. Join in the discussion at pauses 1 and 2 on the tape.
- Plan to use further materials, including Worksheet A5, in parallel with colleagues.

Meeting 3 Looking at the Video (issues 3 and 4)

- Compare classroom experiences, including sessions using Worksheet A5.
- View the rest of the video tape which shows different approaches to Worksheet A5 and further discussion. Join in discussion pauses 3 and 4.
- Plan some further parallel classroom explorations.

Meeting 4 How Can the Micro Help?

- Compare classroom experiences.
- Explore the microcomputer programs using the supporting booklets, and Chapter 4 (this could well take two lunchtime periods).
- Plan some further parallel classroom explorations, using the micro if possible.

Meeting 5 Tackling a Problem in a Group

- Compare classroom experiences.
- The activity on page 207 of the main module book suggests a problem to tackle together with colleagues. If possible tape record some of the group discussion to analyse in the next meeting.
- Plan some further parallel classroom explorations, using groupwork if possible.

Meeting 6 Ways of Working in the Classroom

- Compare classroom experiences.
- Consider Chapter 3 of the Support Materials on page 218 of the main module book. If you have recorded group discussion from Meeting 5, select 3-5 mins. of it to analyse using the schemes on page 221.
- Discuss ways of managing classroom discussion. Refer to the checklist on the inside back cover of the main module book.
- Plan some further parallel classroom experiences, including whole class discussion, if possible.

Meeting 7 Assessing the Examination Questions

- Compare classroom experiences.
- Chapter 5 of the Support Materials page 234 of the main module book offers a set of activities to clarify the assessment objectives of the materials and gives children's scripts for a 'marking' exercise. These scripts are also provided in the pack of 'Masters for Photocopying'.
- Plan further activities and meetings.

Script A Sharon

Competiter (A) starts off with a good pace
and is getting faster and starts to slow alittle
at the end but not drasticly
Competiter (B) is making a good pace but
he isn't going as fast as (A) about
half way in the race - Right near the
end he decides to quicken up his
pace. He is taking more time to do
the race.
Competitor (C) starts off with a really
fast run but he ~~ff~~ tires himself out
and has to keep himself as the same
pace for awhile. I think he's stopped
he's not making any mileage at all
but he stops running again. But
he has taken the most time.

Script B Sean

In the first seconds of the race C made
the best start followed by A and B bringing up the rear
but after a few seconds C has hit a hurdle and fallen
which leaves A in the lead followed by B. Once C has
got up again he starts once more but cannot catch up.
In the later stages of the race A is beginning to tire
and B is putting on a final burst of acceleration to
reach the tape first followed closely by A
and C came last.

Script C Simon

HURDLES RACE

2 C gets out of the blocks first followed by A then b. Oh tragedy C has fallen at about 120 m. So A is in the lead coming up to the finish followed by b then C. Oh and b is putting up a late challenge and the result is 1st B 2nd A 3rd C

Script D David

They're off All going well As they come up to the hundrened metre mark B leads from A with C behind
Oh no C has hit the hurdle badly but yes he's alright and ~~they're~~ he's up again.
Approaching the 200 metre mark A has overtaken B
C is still lagging behind badley.
At 300 metres its still A from B. C is out of the race because he so far behind.
A is tiring, Yes B has over-taken A At the line its B then A with C still hobbeding round the track.

Script E Jackie

Athlete A on the first 100m is in second place when he has past the 100m mark the time is about 10 seconds His speed stays about the same through the next 100m and as he passes 300m mark the time is about 50 seconds He finishes the race in about 1minute 10 seconds

Athlete B on the first 100m is slower on the first 100m than Athlete A his time after 100m is about 20 seconds. His speed stays about the same through the next 100m and as he passes the 300m mark the time is about 60 seconds.
He finishes the race in about 1minute 5 seconds so he quickened up near the end.

Athlete C is quicker than Athlete A, B in the first 100m at about the 150 metre mark he ~~goes~~ stops gradually but quickens up again on the last 200m but he finishes the race in about 1minute 40 seconds.

Script F Nicola

No C ~~go~~ runs fast at the beginning with A a bit slower + B the slowest of all. A then picks up speed and ~~B~~ B is going almost as fast, but C now slows down quite a lot. A + B are side by side as they near the end of the race but B wins, just by a few seconds. C is third, quite a while after A.

Marking Record Form

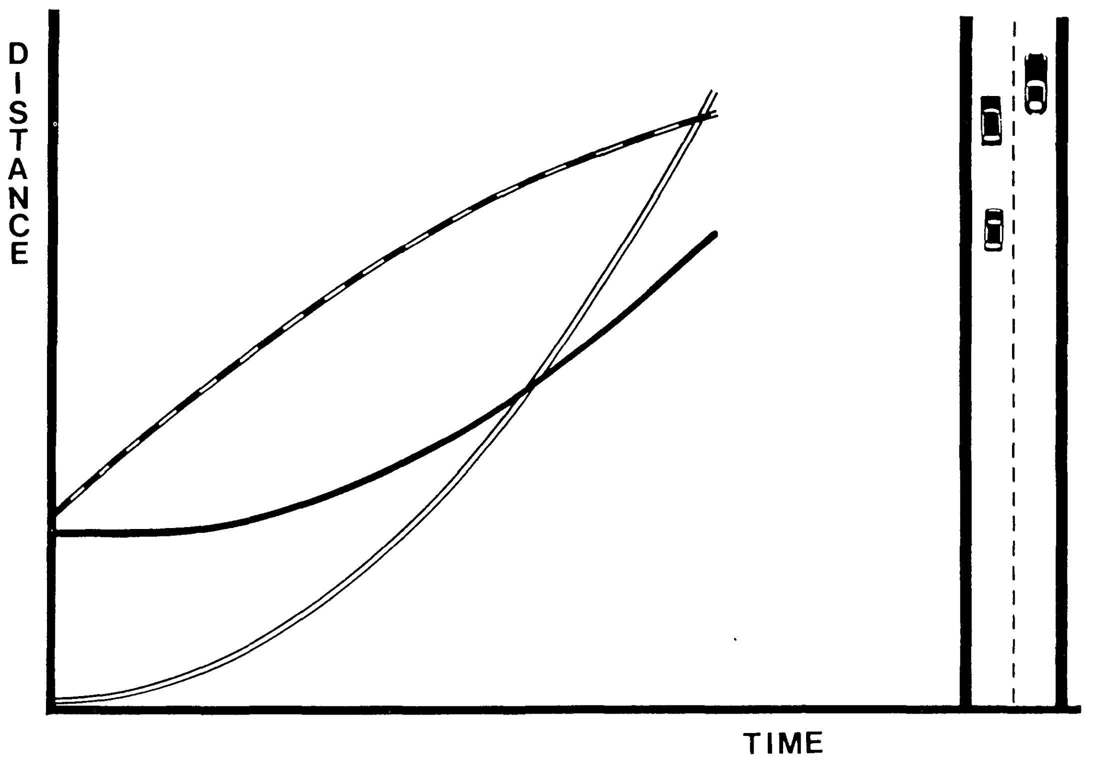
Script	Marker 1				Marker 2				Marker 3				Marker 4			
	R_0	R_1	M_1	M_2	R_0	R_1	M_1	M_2	R_0	R_1	M_1	M_2	R_0	R_1	M_1	M_2
A Sharon																
B Sean																
C Simon																
D David																
E Jackie																
F Nicola																

Key:

Impression rank order	R_0
Raw mark	M_1
Mark rank order	R_1
Revised mark (if any)	M_2

The Language of Functions and Graphs

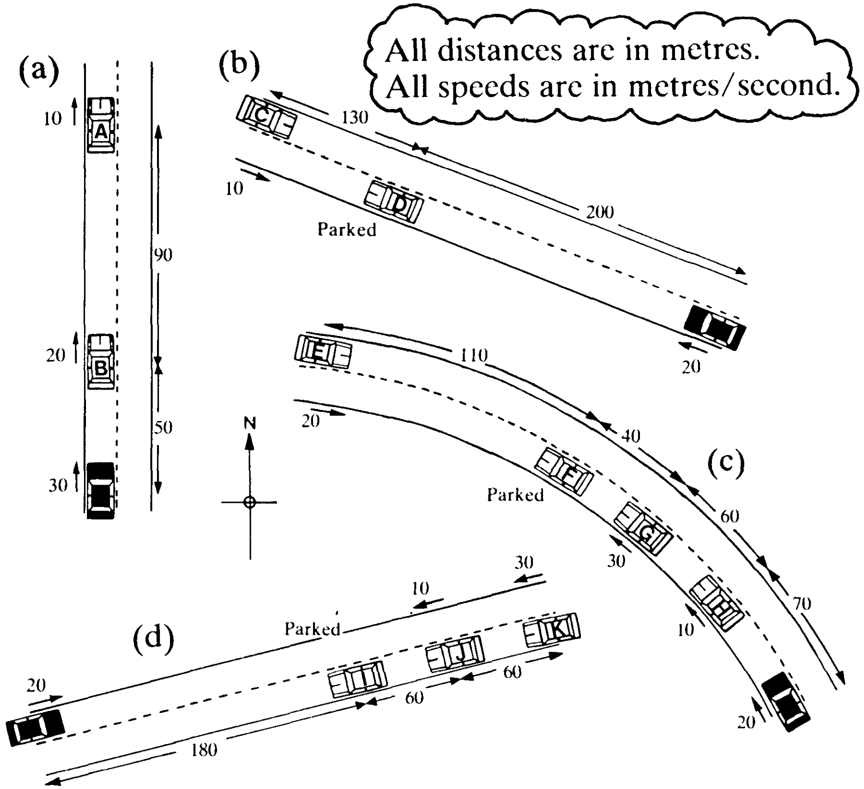
Traffic



An Approach to Distance — Time Graphs

Joint Matriculation Board
Shell Centre for Mathematical Education

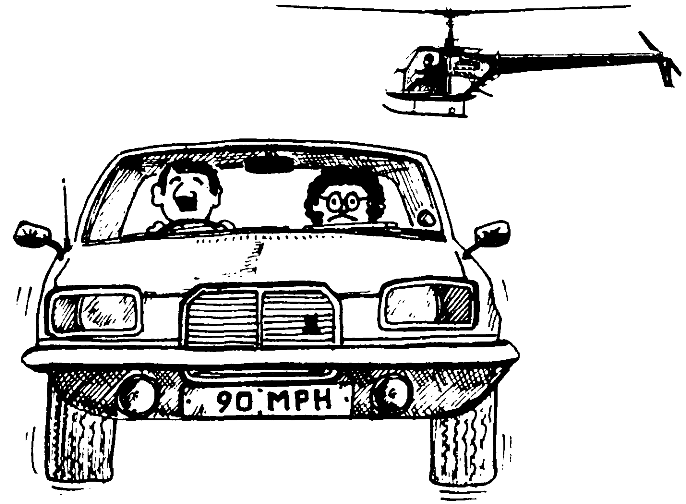
As the helicopter flies Northwards, it spots various other traffic situations on different roads...



Investigate each of these situations, in turn, on a separate "snapshot blank". Be careful to give each car its correct speed and starting position. (The black car always starts at 0 metres along the road.) Write about what happens.

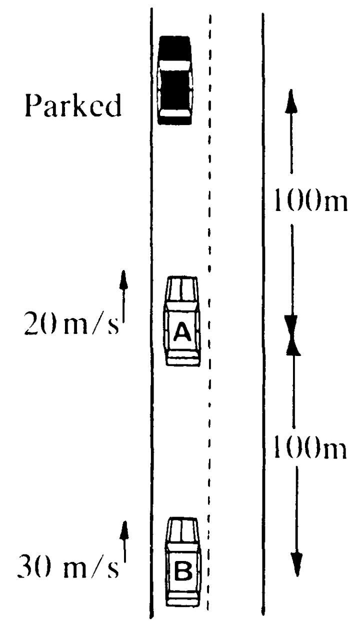
Now make up a traffic situation of your own. Write down what you think will happen, and then draw a 'snapshot' diagram to see if you were right.

T1. TAKING PHOTOGRAPHS FROM A HELICOPTER



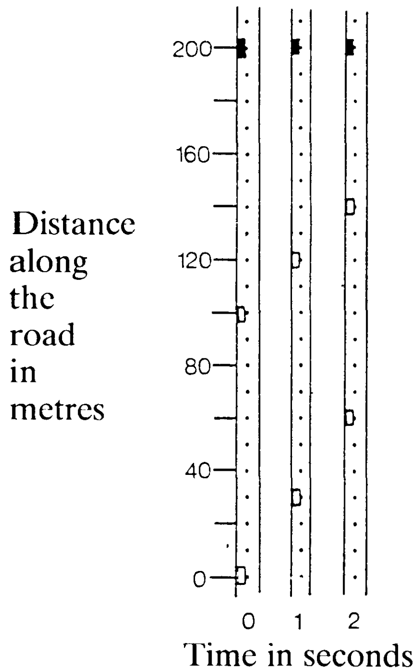
A police helicopter spots three cars travelling due North along a narrow country lane and takes this photograph of the scene.

The lane is just wide enough for two cars to pass each other safely. If the cars continue at the same steady speeds, what do you think might happen in the next few seconds?



Suppose that a police officer in the helicopter takes a photograph of the scene every second.

His first three photographs look like this:

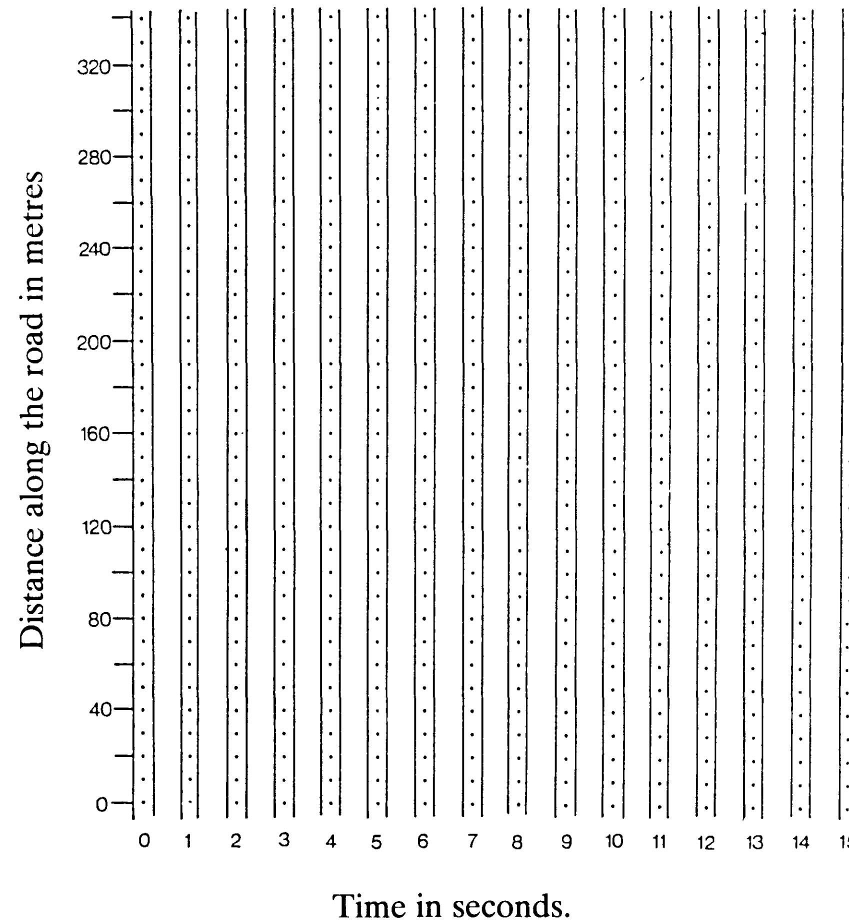


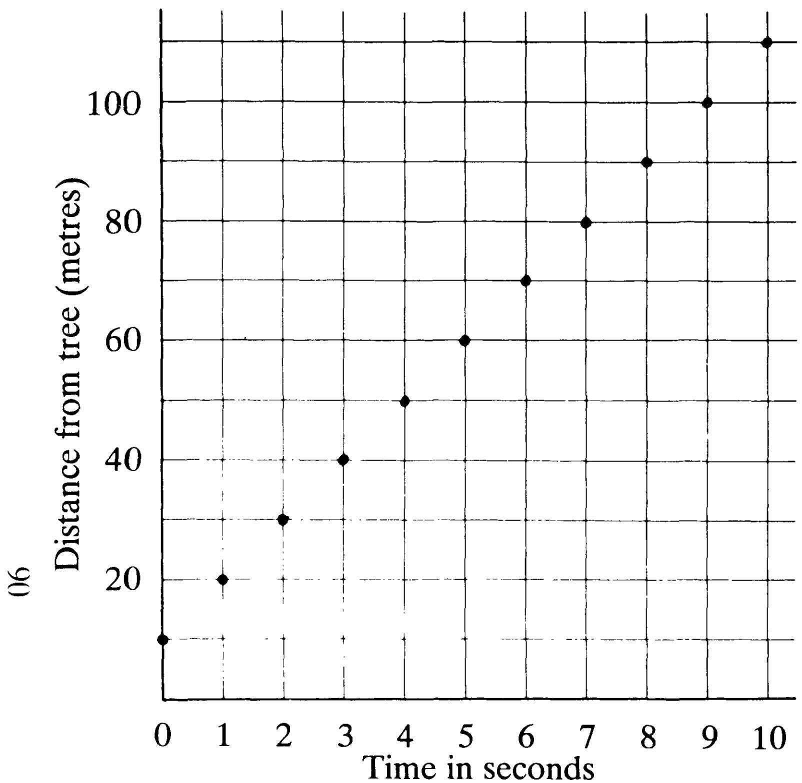
On a sheet of "snapshot blanks", complete a series of photographs taken at one-second intervals.

Write down what happens.

Now suppose that the black car had been travelling at 10 m/s ....

"Snapshot Blanks"





Add extra dots showing the position of the black car at ½ sec intervals, then at $^1/_{10}$ sec intervals, then at $^1/_{100}$ sec intervals . . .

What happens?

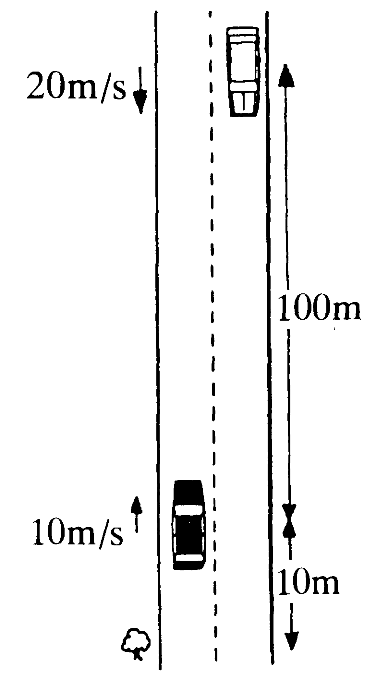
Add the 'cinefilm' pictures for the white car. Find out as accurately as you can where they pass each other.

Make up your own traffic situation to investigate on graph paper. (You may like to have a greater number of cars this time.)

Write down what you think will happen first, and then draw a graph to see if you were right.

T2. FROM PHOTOGRAPHS TO CINE FILM.

Attempt the problem below, before reading on.



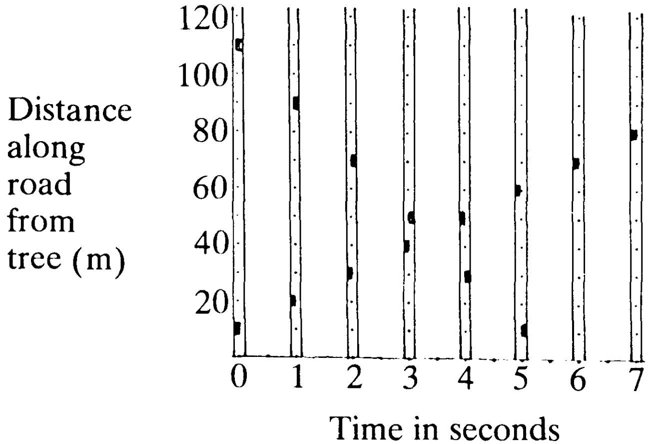
When will these two cars meet each other?

How far will they be from the tree at that time?

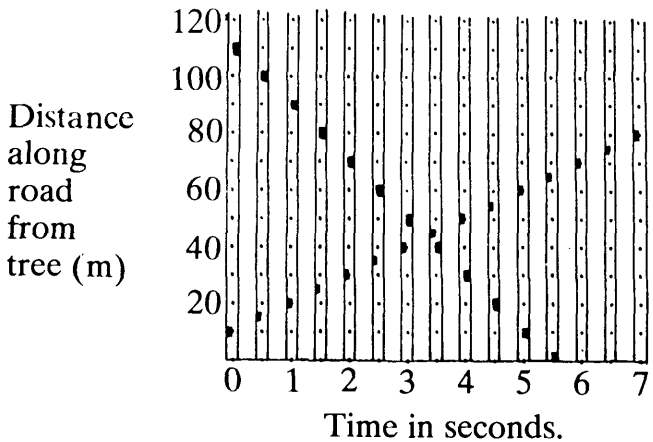
How accurate are your answers?

The police took photographs of this situation with three different cameras.

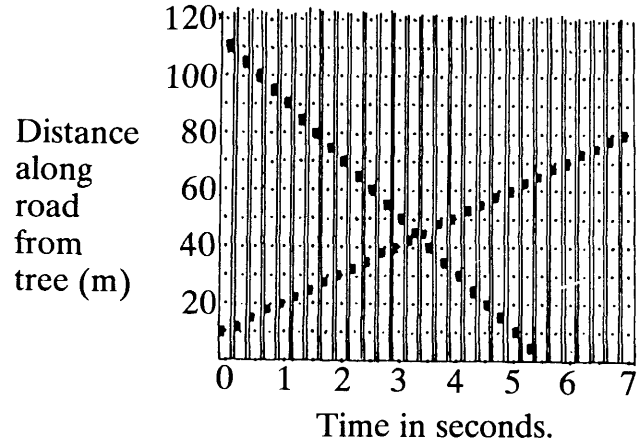
Camera A



Camera B



Camera C



Camera A takes one photograph per second.

Camera B takes ? photographs per second.

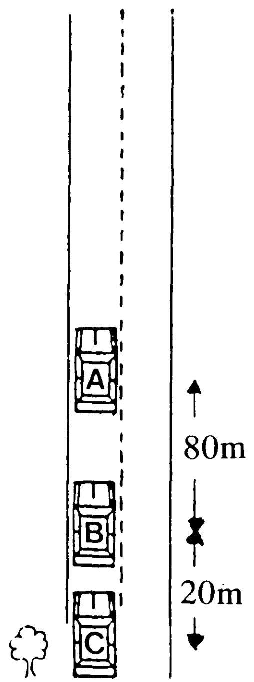
Camera C takes ? photographs per second.

What will the "snapshot" diagram look like if even faster cameras are used?

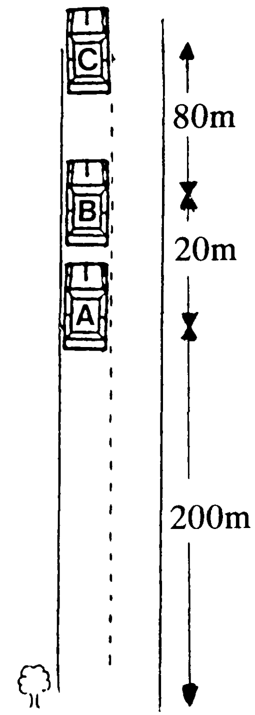
Draw 'photographs' of the black car taken at 1 second intervals, but this time use a sheet of graph paper, as shown on the next page.

Notice that the road is now represented by a single vertical line and the position of the 'car' is shown by a small black dot on it:

Here is a traffic situation



Here is another view of the same road, 10 seconds later



Write an account of what happened in between these two times. (Remember to include times, distances and speeds)

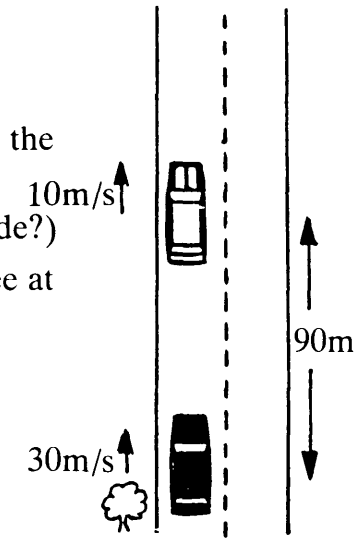
You will need to draw a graph to answer this question.

T3. MORE TRAFFIC PROBLEMS

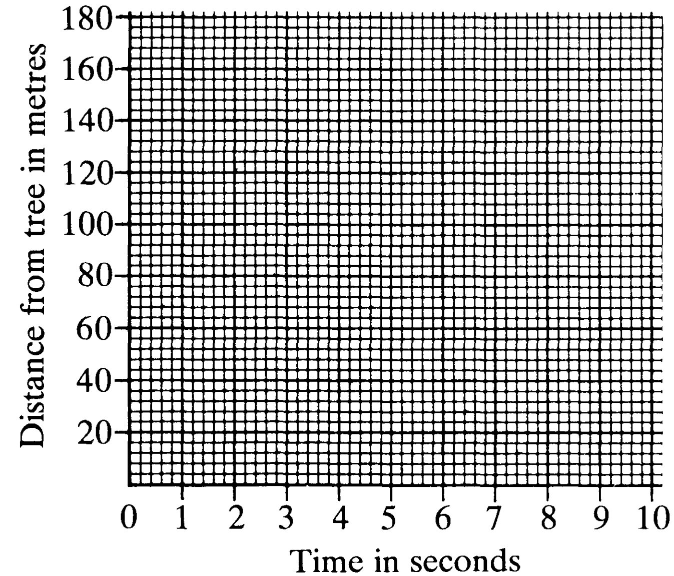
Assuming that both cars continue to travel at the same steady speeds,

- When will the black car overtake the white car?

  (i.e. When will they be side by side?)
- How far will they be from the tree at that time?
- How accurate are your answers?

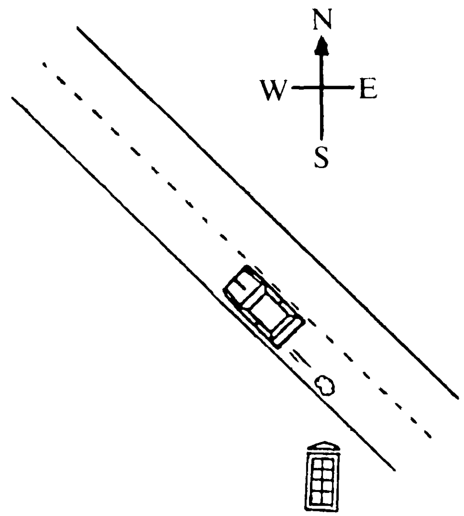


(Draw axes as shown below, onto a sheet of graph paper. Draw a graph and use it to answer this question.)



The main road joining Nettle Village to Little Huntingford runs North-West.

A telephone box stands by the side of the road.



The graph shown opposite was drawn to show the progress of some traffic along the 200 metre stretch of road beyond the telephone box.

All timings were measured from the moment when a green car passed the telephone box.

a) Draw a picture to show the traffic situation after 5 seconds.

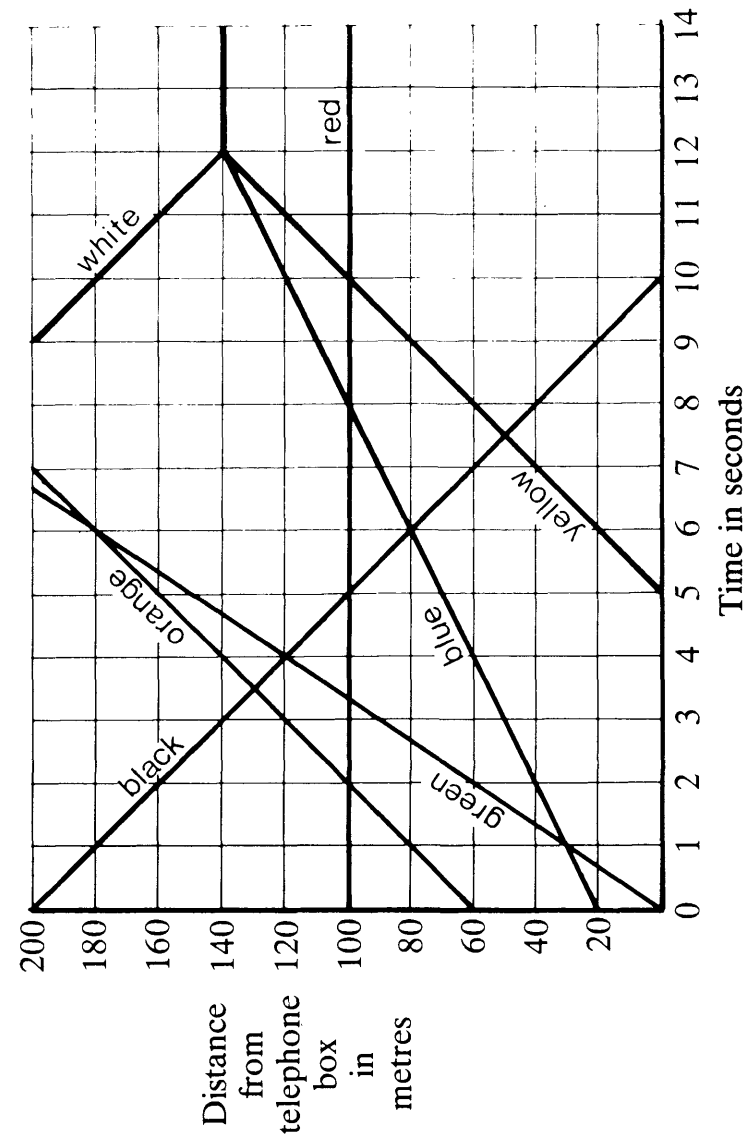
b) Which cars were travelling due Northwest? Southeast? East?

c) Write a short story describing what you would have seen if you had been the pilot of the helicopter.
(Remember to mention speeds, times, distances and directions in your account.)

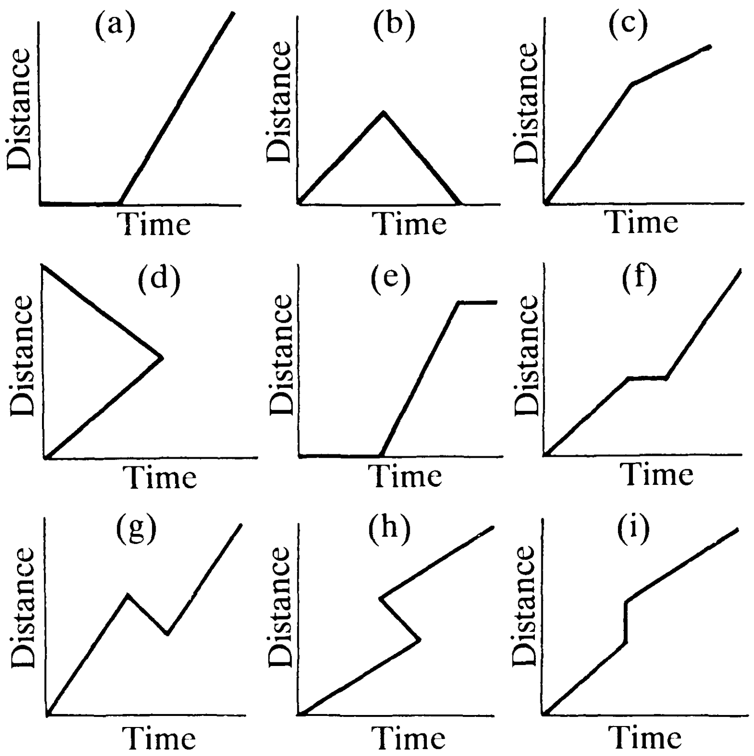
d) Write another eyewitness account of the situation from the point of view of the driver of one of the cars. (State clearly which car you choose.)

Read and discuss your neighbour's accounts.

e) Make up your own traffic situation and give it to your neighbour to describe.



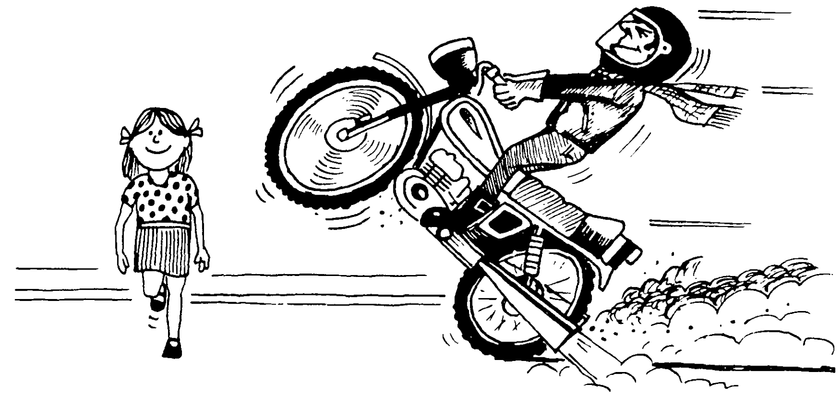
A schoolboy has answered some similar problems and has produced the following graphs for his answers.



i) Some of these graphs *cannot* possibly be correct! Which graphs can never represent the journey of a single vehicle? Why?

ii) For the rest of the graphs, make up situations which could cause these graphs to be drawn. You needn't worry about the exact values of speed, time or distance — just a rough story will do.

T4. MORE TRAFFIC PROBLEMS



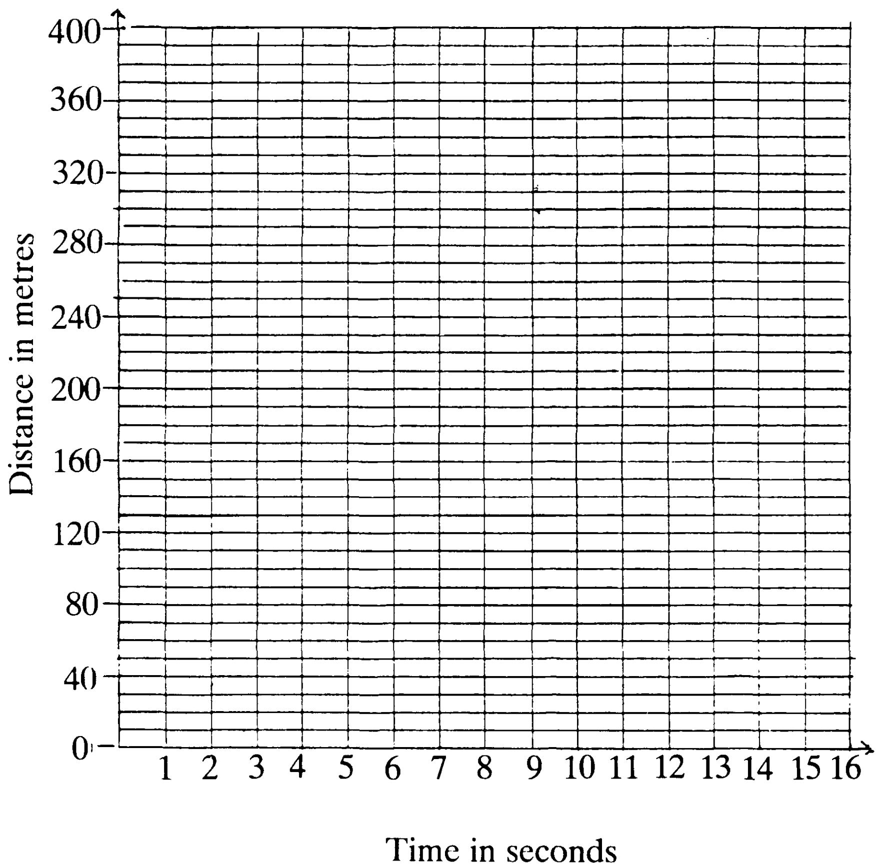
Each question is a description of a situation.

You must try to draw a distance-time graph to illustrate each situation.

(Use a copy of the graph paper shown overleaf. If you get stuck, try using a sheet of 'snapshot blanks' to help you sort your ideas out.)

1. A heavy lorry is driving along at 30 metres/second. After 8 seconds, it reaches a steep hill and so reduces its speed to 10 metres/second.

2. A car is travelling along a country road at 30 metres/second. Five seconds after passing a signpost, the driver suddenly completes a swift "U" turn in the road and retraces her journey (still at 30 metres/second).



3. While a learner driver is motoring along (at 30 metres/second), his instructor asks him to perform a simple reversing exercise. The driver continues on his way for 6 seconds, then stops the car for 4 seconds (while changing gear), and then reverses at a steady 10 metres/second.

4. A motorbike is speeding along a town street at 20 metres/second. After 8 seconds, a small child suddenly steps into the road. Immediately the rider slams on his brakes and screams to a halt. When the child has crossed the road safely, 5 seconds later, the rider continues on his journey again at 20 metres/second.

5. Another learner driver is crawling along a road at 10 metres/second. After 5 seconds of frustration, the instructor tells him to stop at the side of the road, where she shouts at him for a very noisy 5 seconds and then tells him to continue. The learner then nervously continues his journey at 20 metres/second.

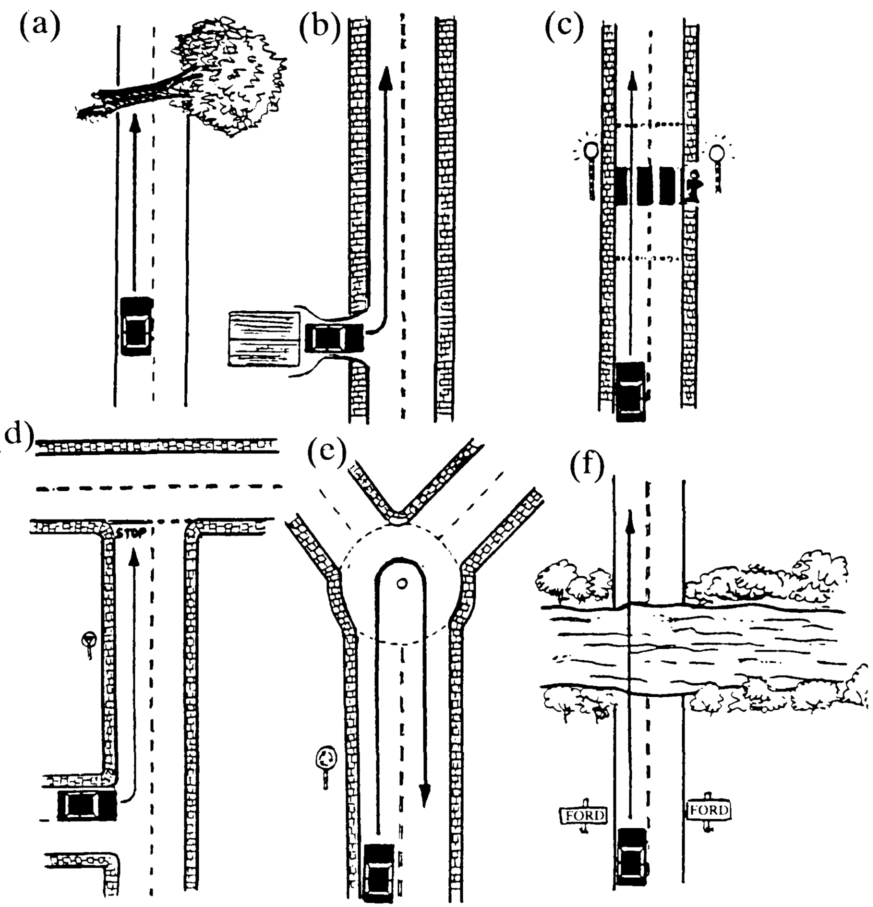
Now try making up a similar situation of your own. Draw a graph to illustrate it, on a separate sheet of paper.

Give just your written description to your neighbour, and ask him or her to draw a graph as well.

Compare your two graphs. Do they agree?

If not, why not?

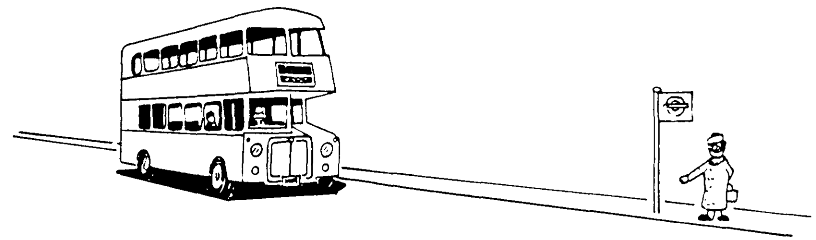
For each of the situations drawn below, *sketch* a realistic distance-time graph to describe the events of the next few seconds.



Now make up some of your own examples.

T5. ACCELERATION AND DECELERATION

A bus is travelling at 10 m/s towards a bus stop 50 metres away.

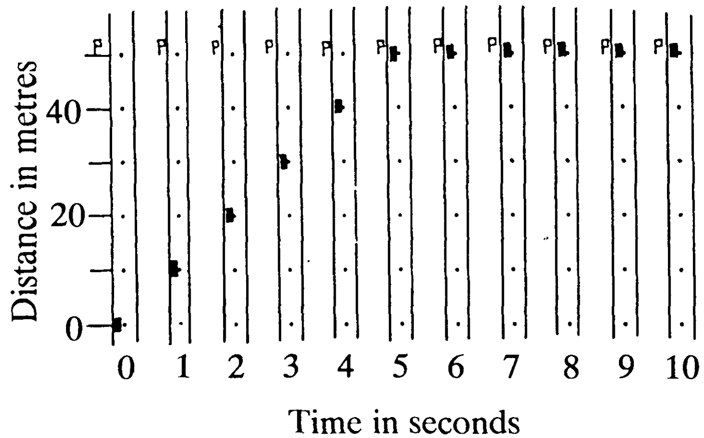


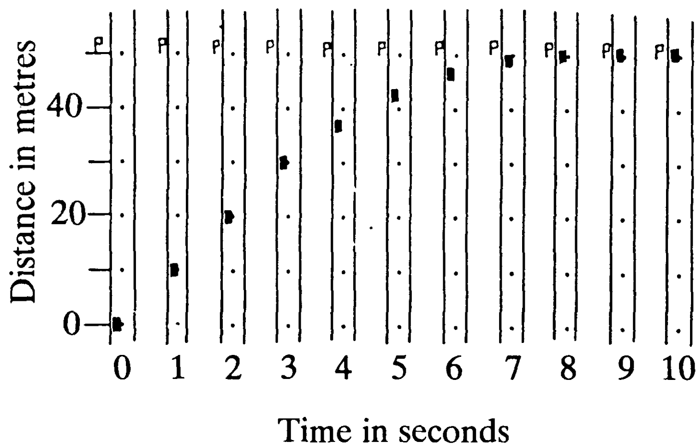
Using a sheet of 'snapshot blanks', draw a series of photographs taken at one second intervals to show how you think the bus will approach the bus stop.

In all the examples we have considered so far, vehicles have either been travelling at constant speeds, or they have suddenly changed from one speed to another. Is this realistic?

How do vehicles *really* behave?

Your answer to the question on page 1 may have looked like one of the following.





Which is the most realistic? Why?

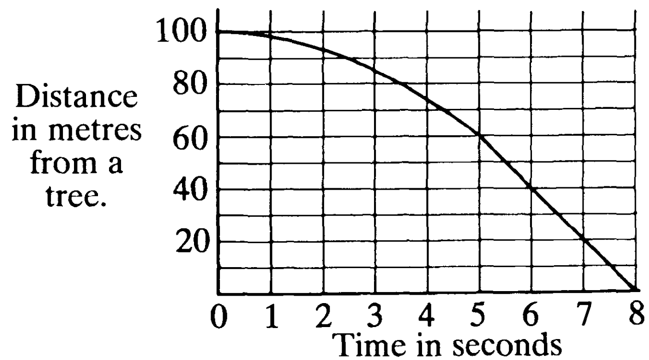
2

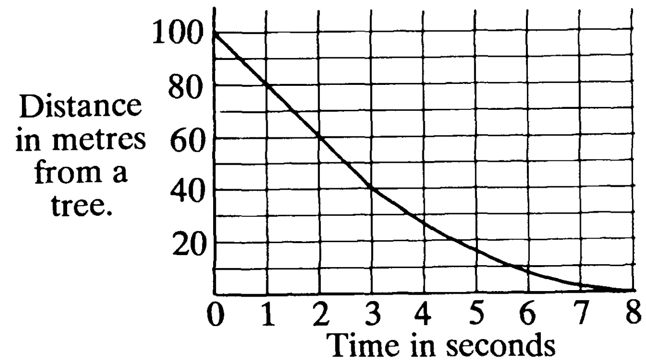
Study the following two graphs.

One shows a car slowing down (decelerating)

One shows a car speeding up (accelerating)

- Which is which? Explain your answer.
- Can you tell *when* the drivers begin and stop braking or accelerating?





3

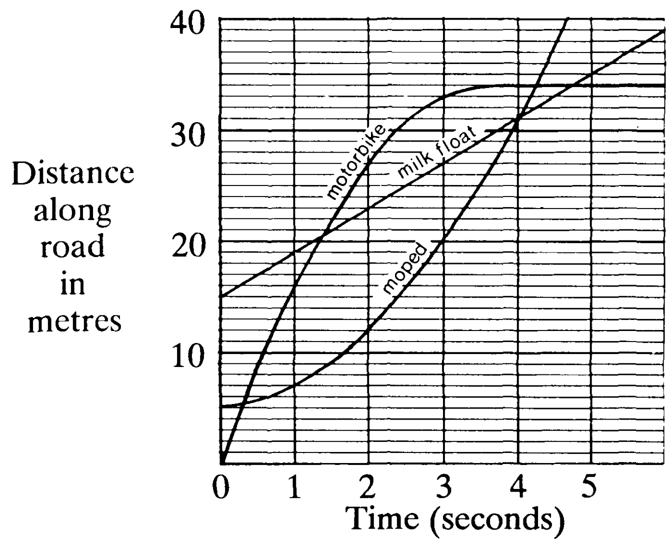
The questions on page 3 can be answered more easily if we draw up a "difference table"

Time in secs	0	1	2	3	4	5
Distance in metres	0	5	15	30	50	75

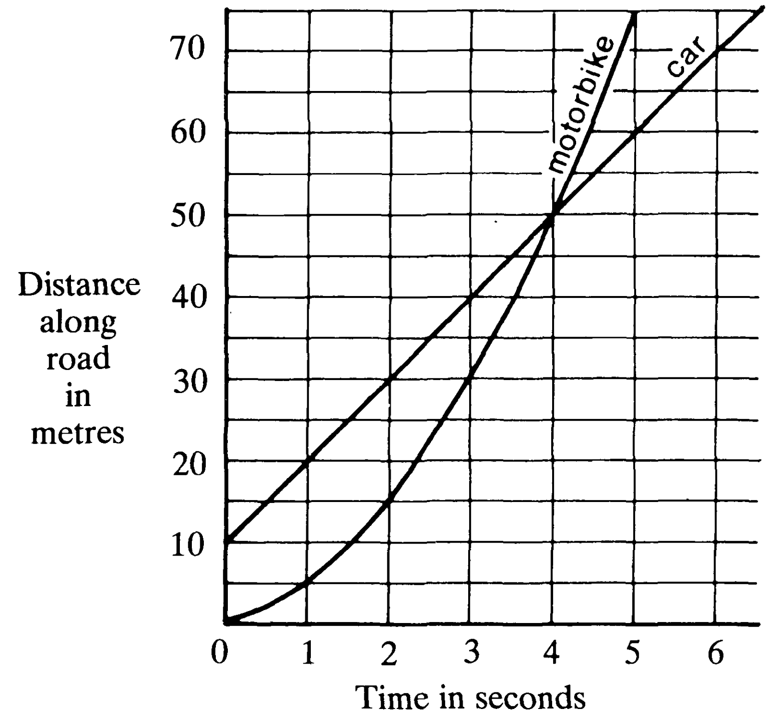
Average speed in m/s	+5	+10	+15	+20	+25
Acceleration in m/s^2		+5	+5	+5	+5

Notice that the first row of differences measures the average speed of a motorbike in successive seconds and the second row of differences measures the acceleration, which has a constant value of 5 m/s^2.

Now describe what is happening in the following situation. Find speeds and accelerations or decelerations wherever you can.



T6. MEASURING SPEED AND ACCELERATION



This graph shows a car and a motorbike, travelling along a country road.

Describe what is happening, as fully as possible.

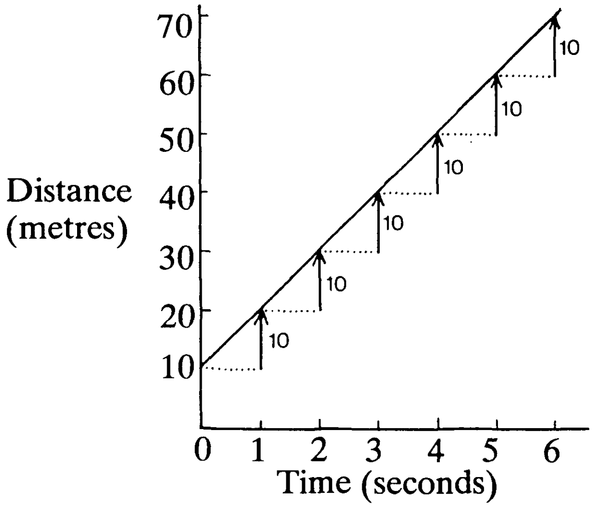
Compare their distances apart and their speeds at different times.

— When are they furthest apart during the first 4 seconds?

— When are they travelling at the same speed?

Measuring speeds and accelerations using difference tables

Let's look at the graph of the *car* in more detail:



We have already seen that the speed of the car can be found from the gradient (or steepness) of the straight line.

Another way of looking at this is to draw up a 'difference table':

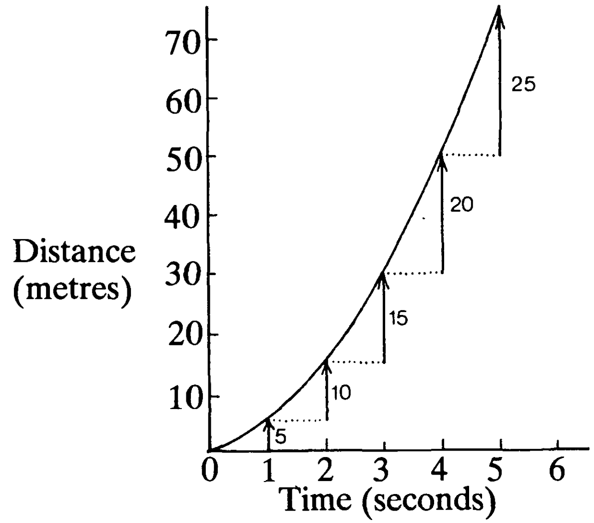
Time in seconds	0	1	2	3	4	5	6
Distance in metres	10	20	30	40	50	60	70

+10 +10 +10 +10 +10 +10

Either method shows us that after each second, the position of the car has changed by 10 metres.

The speed of the car is therefore 10 m/s.

Now let's look at the motorbike:



Time in seconds	0	1	2	3	4	5
Distance in metres	0	5	15	30	50	75

Try to answer the following questions, before turning to the next page:

How far does the motorbike travel during the 1st second, 2nd second, ...?
(So, what is its *average speed* during these intervals of time?)

By how much does the average speed increase in each second?
(So, what is the acceleration of the motorbike?)

Speed-Time Graphs

What do the speed-time graphs look like for the situations described on pages 1 and 3?

How do they differ from the corresponding distance-time graphs?

In practice, are accelerations or decelerations always constant?

If not, what do the speed-time graphs look like?

Write down all your thoughts on these questions, and illustrate your answers with sketch graphs.

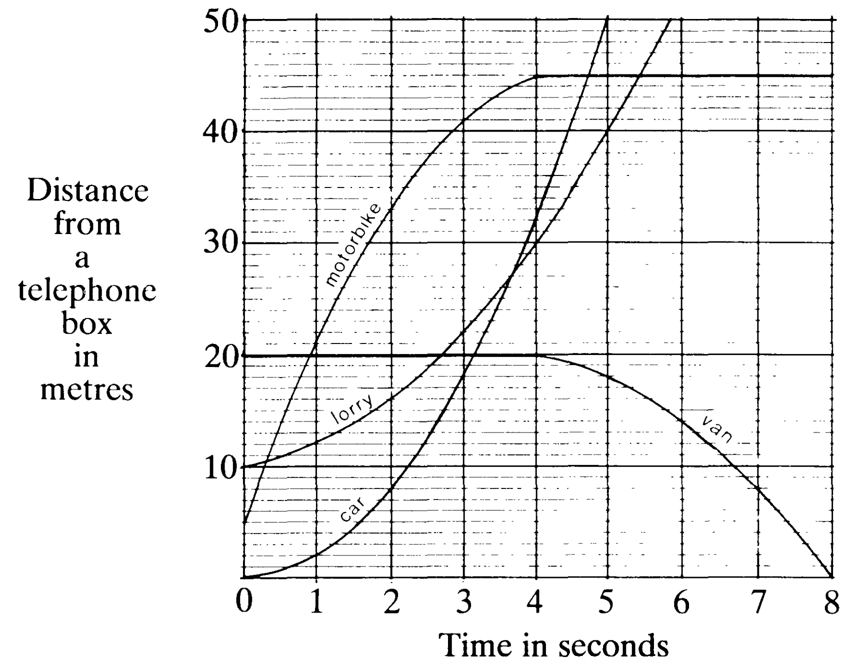
T7. SOME FURTHER SITUATIONS TO EXPLORE

Look carefully at the graph shown below.

Describe what is happening in detail.

Ask yourself questions such as:

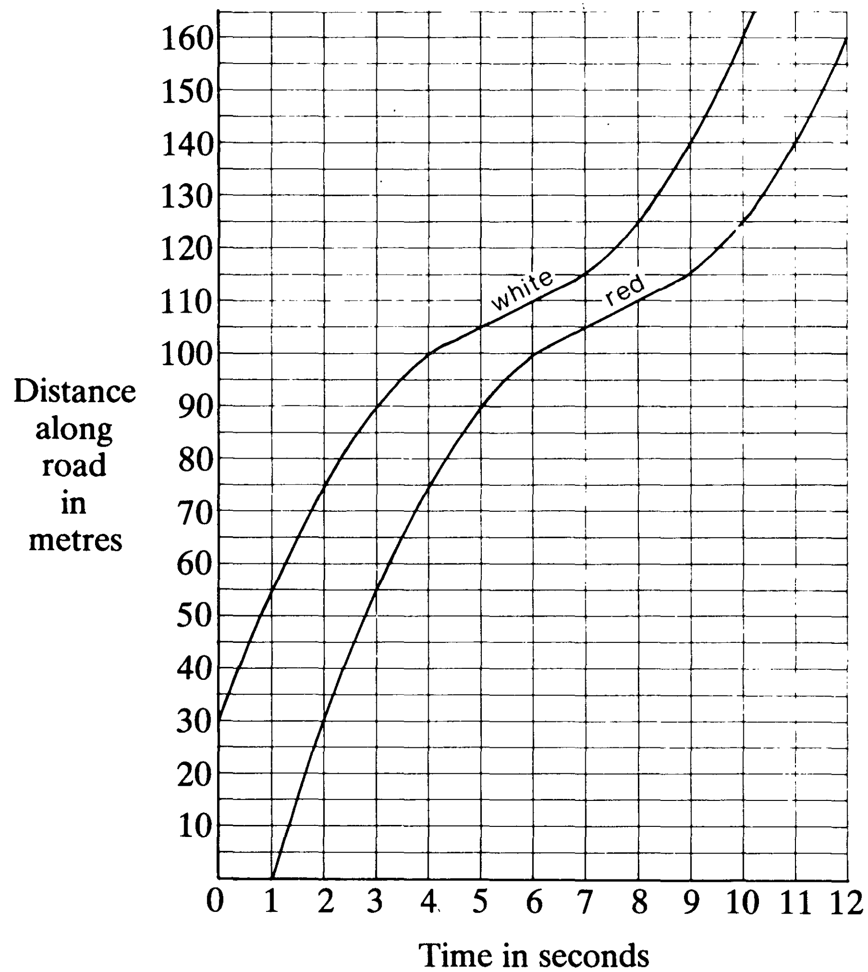
- who overtakes who
- are vehicles accelerating or decelerating?
- can I measure speeds and accelerations?

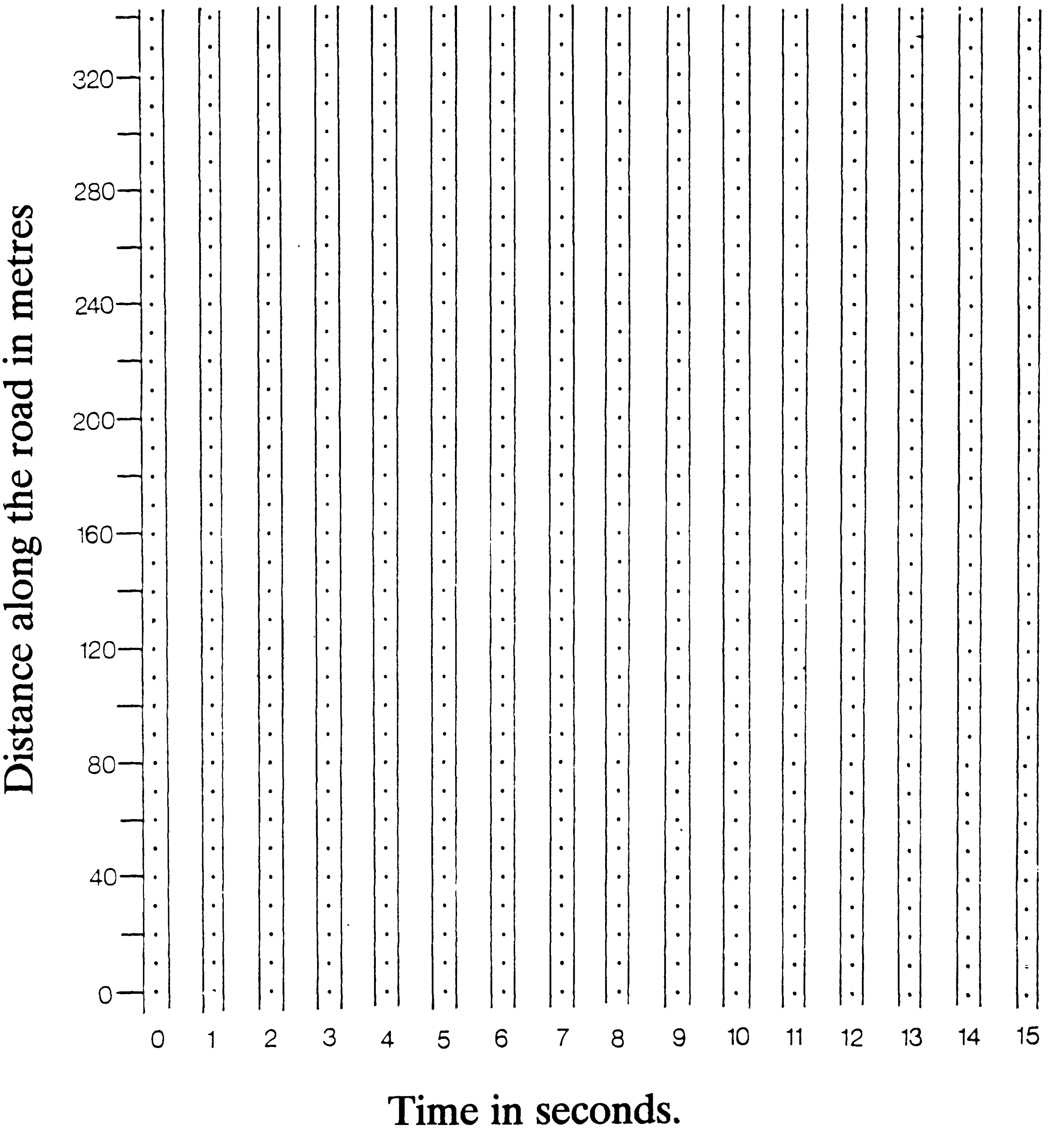


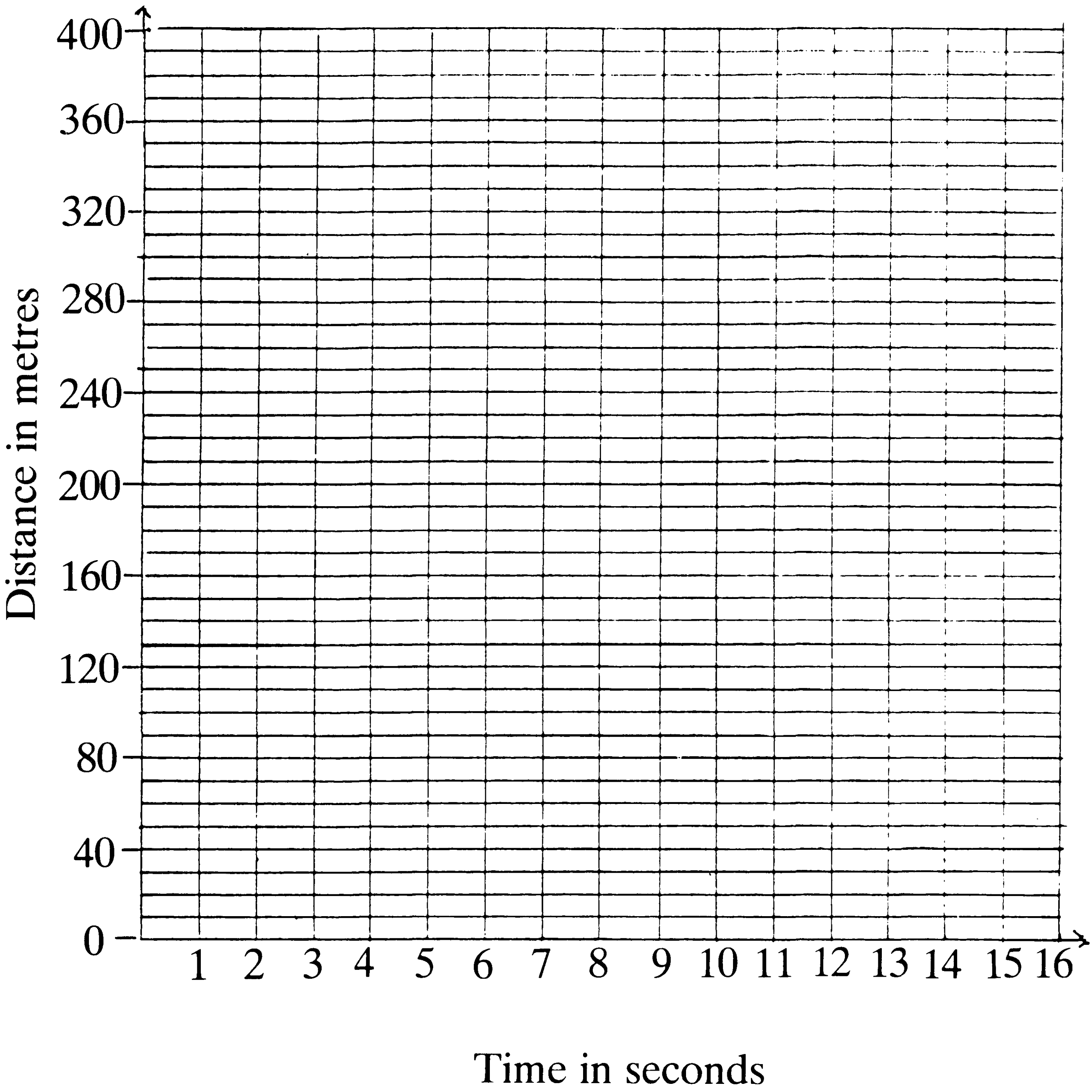
The graph shown opposite describes the motion of two cars as they approach and negotiate a bend in the road.

- Where is the bend?
- What is the deceleration of each car?
- What is the acceleration of each car?
- How does the distance between the two cars vary? (Draw up a table, sketch a graph, or do both!)
- How does the time interval between the two cars vary?

During motor racing events, it often seems that the car in front loses some of its lead when it approaches a corner, and then opens up a gap again afterwards. Why is this? Are the chasing cars really 'catching up' with the leading car?




Distance along the road in metres
320
280
240
200
160
120
80
40
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
Time in seconds.


400
360
320
280
240
200
160
120
80
40
0
Distance in metres
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Time in seconds

Teaching Strategic Skills – Publications List

Problems with Patterns and Numbers – the "blue box" materials

- School Pack – *Problems with Patterns and Numbers* 165 page teachers' book and a pack of 60 photocopying masters.
- Software Pack – Teaching software and accompanying teaching notes. The disc includes SNOOK, PIRATES, the SMILE programs CIRCLE, ROSE and TADPOLES, and four new programs* KAYLES, SWAP, LASER and FIRST. Available for BBC B & 128, Nimbus, Archimedes and Apple II (* the Apple disc only includes the five original programs).
- Video Pack – A VHS videotape with notes.

The Language of Functions and Graphs – the "red box" materials

- School Pack – *The Language of Functions and Graphs* 240 page teachers' book, a pack of 100 photocopying masters and an additional booklet *Traffic: An Approach to Distance-Time Graphs.*
- Software Pack – Teaching software and accompanying teaching notes. The disc includes TRAFFIC, BOTTLES, SUNFLOWER and BRIDGES. Available for BBC B & 128, Nimbus, Archimedes and PC.
- Video Pack – A VHS videotape with notes.

For current prices and further information please write to: Publications Department, Shell Centre for Mathematical Education, University of Nottingham, Nottingham NG7 2RD, England.